

DIMENSIONLESS PHYSICS

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Received June 30, 2020,
revised version August 11, 2020
Accepted for publication August 12, 2020

Contribution for the JETP special issue in honor of I. E. Dzyaloshinskii's 90th birthday

DOI: 10.31857/S0044451021040246

It is not excluded that the Standard Model of particle physics together with general relativity are effective theories which emerge at low energy for the collective modes of the extreme ultraviolet [1]. Sakharov gravity [2], which emerges in the fermionic vacuum, provides the characteristic example. The scenario, where all the known symmetries in our Universe emerge on the macroscopic scale, but disappear in the highly trans-Planckian microscopic regime, takes place in different many-body condensed matter systems. For example, the analogs of Lorentz invariance and the curved spacetime are developed for some low energy fermionic and bosonic modes [3], but these phenomena disappear at large energy, where the microscopic degrees of freedom intervene (the analog of the trans-Planckian degrees of freedom). The condensed matter systems with topologically stable Weyl points in the fermionic spectrum, such as Weyl semimetals and the chiral superfluid phase of liquid ^{3}He, demonstrate simultaneous emergence of chiral fermions, gauge bosons, and tetrad gravity [4–8], which do not survive on the high-energy atomic level.

We do not know the structure of the trans-Planckian world, but we can try different possible scenarios of emergent physics and search for the common properties in the low energy corner. Here we consider two scenarios of emergent gravity, which are very different, but have the important common property.

The first one is the tetrad gravity, where the tetrad fields emerge as bilinear combinations of the fermionic fields by symmetry breaking. This scenario has been investigated by Diakonov [9], Vladimirov and Diakonov [10, 11], and Obukhov and Hehl [12]. The analog of Diakonov–Vladimirov (DV) scenario takes place in topological superfluid ^{3}He-B [13].

The other one is the analog of gravity in the elasticity theory of crystals [14–20], where the elastic deformations are described in terms of the tetrads of elasticity [16]. In principle, this analogy can be extended to the real gravity, if the quantum vacuum is considered as a plastic (malleable) fermionic crystalline medium and the elasticity tetrads become the gravitational tetrads [21, 22]. The condensed matter analog of such vacuum is the quantum crystal with fermionic quasiparticles, such as vacancies [23–25].

The common property of these two approaches to quantum gravity is that the tetrad fields in both theories have dimension of inverse length. As a result, most of the physical quantities which obey diffeomorphism invariance become dimensionless [11, 26–28]. Since the two very different scenarios lead to the same phenomenon, it is natural to suggest that the gravity in our universe also follows this common rule. Here we consider some consequences which come from this rule.

Let us consider the theory of the crystal elasticity using approach of Ref. [16]. The deformed crystal structure can be described as a system of three crystallographic surfaces of a constant phase, $X^a(x) = 2\pi n^a$, $n^a \in Z$ with $a = 1, 2, 3$. The intersection of the surfaces

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$$X^1(\mathbf{r},t) = 2\pi n^1, \quad X^2(\mathbf{r},t) = 2\pi n^2, \\ X^3(\mathbf{r},t) = 2\pi n^3 \tag{1}$$

are the nodes of the deformed crystal lattice. For the undeformed crystal, $X^a(\mathbf{r},t) = \mathbf{K}^a \cdot \mathbf{r}$, where $\mathbf{K}^a$ are the (primitive) reciprocal lattice vectors. The deformations of the crystal can be described in terms of the elasticity tetrads, the gradients of the phase functions:

$$E_i{}^a(x) = \partial_i X^a(x)\,. \tag{2}$$

In the absence of dislocations, $E_i{}^a(x)$ is an exact differential:

$$\partial_k E_l{}^a(x) - \partial_l E_k{}^a(x) = 0\,. \tag{3}$$

In the presence of the topological defects – dislocations, the density of dislocations plays the role of torsion:

$$T^a_{kl} = (\partial_k E_l{}^a - \partial_l E_k{}^a)\,. \tag{4}$$

Such construction can be extended to the 3+1 quantum vacuum with $a = 0, 1, 2, 3$, assuming that the vacuum looks like the plastic crystalline medium. In this model the elasticity tetrads $E_\mu{}^a$ become the gravitational tetrads [21, 22]. The deformed vacuum crystal with dislocations describes the curved geometry of the teleparallel Weitzenböck gravity with vanishing curvature and nonzero torsion. On the macroscopic coarse grained level, where the separate dislocations are not resolved, the torsion field $T^a_{\mu\nu}$ can be considered as a continuous function of coordinates. The metric $g_{\mu\nu}$ originates from the elasticity tetrads:

$$g_{\mu\nu} = \eta_{ab} E_\mu{}^a E_\nu{}^b\,, \tag{5}$$

where $\eta_{ab} = (-,+,+,+)$.

The important property of the elasticity tetrads is that being the derivatives, they have the canonical dimensions of inverse length, $[E_\mu{}^a] = [l]^{-1}$. Correspondingly, the metric has dimension $[g_{\mu\nu}] = 1/[l]^2$, while the interval is dimensionless: $ds^2 = -g_{\mu\nu}dx^\mu dx^\nu$, $[\Delta s] = 1$. The distance between the two nodes of the deformed crystal is determined by the integer number of crystal surfaces between the points of the grid and thus does not depend on the length scale. In such quantum vacua, the size of the unit cell is not fixed and can be arbitrary.

Let us introduce notation d for conventional dimension of the physical quantities and the notation d_{DV} for the shifted dimension of the same quantities. The shift of dimensions means that $[l]^{-d} \to [l]^{-d_{DV}}$ in the DV approach. For the interval ds, the conventional dimension $d = -1$ and the shifted dimension $d_{DV} = 0$, see Table 1. For the torsion field $d = 1$, and $d_{DV} = 2$.

The shift of the dimensions of the physical quantities leads to the new properties of the quantum vacua and also topological insulators, which allows extending the application of the topological anomalies. For example, the Chern–Simons term describing the 3+1 intrinsic quantum Hall effect becomes dimensionless. As a result, the prefactor of this term is given by the integer momentum-space topological invariants as in the case of $2+1$ dimension [26–28]. The shift is also important for the Nieh–Yan anomaly [28].

In the DV theory [9–12], the tetrads are composite fields emerging as bilinear combinations of fermionic fields. Tetrads appear as the order parameter of the symmetry breaking transition (see also Ref. [29]):

$$e^a_\mu = i\left\langle \psi^\dagger \Gamma^a \nabla_\mu \psi - \nabla_\mu \psi^\dagger \Gamma^a \psi \right\rangle\,. \tag{6}$$

The corresponding symmetry breaking scheme is $L_L \times L_S \to L_{L+S}$, where L_L and L_S are two separate symmetries under Lorentz rotations of the coordinate and spin space, respectively. These two symmetries are broken to the diagonal subgroup — the Lorentz group of the combined rotations in two spaces, L_{L+S}. In addition, this order parameter breaks the PT symmetry, see also Ref. [30]. The similar scheme of symmetry breaking of three-dimensional rotations in the orbital and spin spaces takes place in the superfluid ^{3}He-B [13, 31]:

$$SO(3)_L \times SO(3)_S \to SO(3)_{S+L}.$$

According to Eq. (6), the frame field e^a_μ transforms as a derivative in the same manner as the elasticity tetrads. That is why it has the dimension of inverse length, $[e^a_\mu] = 1/[l]$, i. e., its dimension is shifted from $d = 0$ to $d_{DV} = 1$, see Table 2. In such vacua, it is natural to assume that the fermionic field ψ as well as the bosonic fields Φ are scalars under diffeomorphisms [9, 10], i. e., their dimensions are shifted from $d = 3/2$ and $d = 1$ to $d_{DV} = 0$, see Tables 2 and 3, respectively. For Weyl or massless Dirac fermions one has the conventional action:

$$S = \int d^4x |e| e^{a\mu} \left(\psi^\dagger \Gamma^a \nabla_\mu \psi + \text{H.c.}\right).$$

This action expressed in terms of the DV tetrads remains dimensionless, since $[e] = [l]^{-4}$, $[e^{a\mu}] = [l]$, and $[\psi] = 1$, see Table 2. This suggests that the DV dimension of tetrads is natural, which is also supported by the elasticity tetrads.

The nontrivial dimension of the metric suggests that metric is not the quantity, which describes the spacetime, but the quantity, which determines the dynamics of effective low energy fields in the background of microscopic quantum vacuum.

The shifts of dimensions are shown in Tables 1, 2, and 3 correspondingly for gravity, fermions, and scalar fields. Many quantities, which obey diffeomorphism invariance, become dimensionless. The action is dimensionless and remains dimensionless in the DV dimensions, since the action is a diffeomorphism invariant. Another example of the diffeomorphism invariant quantity is the rest mass M of particles. In the case of mass, the dimension is shifted from $d = 1$ to $d_{DV} = 0$. That the DV dimension of mass is $[M] = 1$ can be seen from the classical equation for the particle spectrum: $g^{\mu\nu} p_\mu p_\nu = M^2$. According to Table 1, M^2 has dimension $d_{DV} = -2 + 1 + 1 = 0$, i. e., $[M] = 1$, and the action and the mass terms in fermionic and bosonic actions are dimensionless:

$$S = M \int ds \,, \tag{7}$$

$$S = \int d^4x |e| M \psi^\dagger \psi \,, \tag{8}$$

$$S = \int d^4x \sqrt{-g} \left(g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi + M^2 \Phi^2 \right) \,, \tag{9}$$

$$S = \frac{1}{4} \int d^4x \sqrt{-g} \left(F_{\mu\nu} F^{\mu\nu} + M^2 g^{\mu\nu} A_\mu A_\nu \right) \,. \tag{10}$$

This follows from the DV dimensions

$$[e] = [\sqrt{-g}] = [l]^{-4}, \quad [\Phi] = [\psi] = 1, \quad [g^{\mu\nu}] = [l]^{-2},$$
$$[ds] = [M] = 1, \quad [A_\mu] = [l]^{-1}.$$

Since the scalar curvature in general relativity is diffeomorphism invariant, it is dimensionless in the DV approach, $[\mathcal{R}] = 1$. Its dimension is shifted from $d = 2$ to $d_{DV} = 0$. Other examples of the diffeomorphism invariant quantities are the Newton constant G and the cosmological constant Λ in the Einstein–Hilbert action:

$$S_{GR} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, \mathcal{R} + \int d^4x \sqrt{-g}\, \Lambda \,. \tag{11}$$

The dimensions of G and Λ are shifted from correspondingly $d = -2$ and $d = 4$ to $d_{DV} = 0$, i. e., $[\Lambda] = [G] = 1$. These dimensionless quantitites are determined by the ratio of the mass scales [32] or by the functions of scalar fields [33]. In principle, only the ratio between the mass parameters makes sense [10]. In a given case, only the combination ΛG^2 matters. According to Zeldovich [32], this combination is expressed in terms of QCD mass scale: $\Lambda G^2 \sim \Lambda_{QCD}^6 G^3$ (see also Refs. [34–37]). In the other approaches, the electroweak energy scale [38,39] and the neutrino mass scale [40] enter, $\Lambda G^2 \sim M_W^8 G^4$ and $\Lambda G^2 \sim M_n^4 G^2$, respectively.

The spacetime volume $V = \int d^4x \sqrt{-g}$ is dimensionless, $[V] = 1$, and may have quantized values. Then Λ as the corresponding Lagrange multiplier may have universal quantized values with $\Lambda = 0$ in the equilibrium Minkowski vacuum.

The dimensionless Lagrange multiplier appears also in the q-theory of the quantum vacuum [41], if the q-theory is based on the 4-form gauge field introduced by Hawking for phenomenological description of the quantum vacuum [42],

$$q^2 = F^{\mu\nu\alpha\beta} F_{\mu\nu\alpha\beta}.$$

In the DV units the vacuum variable q and the Lagrange multiplier μ_q (the corresponding chemical potential of the conserved quantity) are dimensionless, see Table 3. If μ_q is fundamental, it becomes the general characteristics of the quantum vacuum. While the variable q determines the variable vacuum energy $\Lambda(q) = \epsilon(q) - \mu_q q$, the universal chemical potential provides the nullification of Λ in the Minkowski vacuum. At this value of μ_q, all the initial states (even those with the Planck scale Λ) finally relax to Minkowski vacuum with $\Lambda = 0$ [41], thus providing the solution of the cosmological constant problems.

In the DV approach, mass and energy have different dimensions. While mass is dimensionless, $[M] = 1$, the energy has dimension of frequency, $[E] = [\omega] = [\sqrt{g_{00}}] = 1/[l]$. Correspondingly, the temperature is dimensionless, $[T] = 1$, while the constant temperature, which enters the Tolman law [43], $T(\mathbf{r})\sqrt{-g_{00}(\mathbf{r})} = T_{Tolman}$, has dimension of frequency, $[T_{Tolman}] = [\omega] = [\sqrt{g_{00}}] = 1/[l]$, see Table 1. Tolman temperature is the integration constant in equilibrium in a stationary spacetime [44].

The Unruh temperature of the accelerated body is $T_U = a/2\pi$ [45], where a is covariant acceleration,

$$a^2 = g_{\mu\nu} \frac{d^2x^\mu}{ds^2} \frac{d^2x^\nu}{ds^2}.$$

Since a is diffeomorphism-invariant, it is dimensionless together with the Unruh temperature, $[a] = [T_U] = 1$. The same is with the Hawking temperature of a black hole. For the Schwartzschild black hole with rest energy M_{BH}, Bekenstein entropy S_{BH}, Hawking temperature T_{BH}, and horizon area A_{BH}:

$$T_{BH} = \frac{1}{8\pi G M_{BH}}, \qquad S_{BH} = 4\pi G M_{BH}^2 = \frac{A_{BH}}{4G} \,. \tag{12}$$

Table 1. Dimension shifts for gravity

General relativity	Dimension d	d_{DV}
$g^{\mu\nu}$	0	−2
$g_{\mu\nu}$	0	2
$\sqrt{-g}$	0	4
$d^4x\sqrt{-g}$	−4	0
$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$	−2	0
$dA = \sqrt{dS^{\mu\nu}dS_{\mu\nu}}$	−2	0
M	1	0
$S = M\int ds$	0	0
∂_μ	1	1
p_μ	1	1
$R = g^{\mu\nu}R_{\mu\nu}$	2	0
G_{Newton}	−2	0
R/G_{Newton}	4	0
$\Lambda_{cosmological}$	4	0
$T_{Hawking}$	1	0
T_{Tolman}	1	1
$T_{Tolman}/\sqrt{g_{00}}$	1	0

Table 2. Dimension shifts for fermions

Fermions	Dimension d	d_{DV}
e^a_μ	0	1
e^μ_a	0	−1
$e = \sqrt{-g}$	0	4
ψ	3/2	0
$M\bar{\psi}\psi$	4	0
$i\bar{\psi}\Gamma^a e^\mu_a D_\mu\psi$	4	0
$\mathcal{T}_a$	1	2
$\mathcal{T}_a\mathcal{T}^a$	2	4
$\lambda^2_{Nieh-Yan}$	2	0
$e^a_\mu A_\nu \tilde{F}^{\mu\nu}$	3	4
$ie^\mu_a e^\nu_b \bar{\psi}(\Gamma^a\Gamma^b - \Gamma^b\Gamma^a)\psi F_{\mu\nu}$	5	0
$QQQL$	6	0

Table 3. Dimension shifts for scalar fields

Scalar/Vector	Dimension d	d_{DV}
Φ	1	0
$g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$	4	0
$M^2\phi^2$	4	0
A_μ	1	1
$F_{\mu\nu}$	2	2
$F_{\mu\nu}F^{\mu\nu}$	4	0
$(F_{\mu\nu}F^{\mu\nu})^k$	$4k$	0
$F_{\mu\nu}\tilde{F}^{\mu\nu}$	4	4
$F_{\mu\nu\alpha\beta}$	4	4
$F^{\mu\nu\alpha\beta}$	4	−4
$q^2 = F^{\mu\nu\alpha\beta}F_{\mu\nu\alpha\beta}$	8	0
μ_q	0	0

All the quantities, that enter Eq. (12), are dimensionless in the DV approach, $[T_{BH}] = [S_{BH}] = [M_{BH}] = [A_{BH}] = [G] = 1$. The area of the black hole is dimensionless, because the covariant form of the scalar area element is

$$dA = \sqrt{dS^{\mu\nu}dS_{\mu\nu}}.$$

Since

$$[S^{\mu\nu}] = [l]^2 \quad \text{and} \quad [S_{\mu\nu}] = [S^{\mu\nu}][g_{\mu\nu}]^2 = 1/[l]^2,$$

one obtains $[A] = 1$, which supports the idea that the area of the black hole horizon is quantized [46–48].

Similar quantization may occur for the de Sitter spacetime, which is the submanifold of Minkowski spacetime in the $4+1$ dimension:

$$g^{4+1\,Mink}_{\mu\nu}x^\mu x^\nu = \alpha^2.$$

Since $[g_{\mu\nu}] = [l]^{-2}$, the parameter α is dimensionless as well as the scalar curvature $R = 12/\alpha^2$, i.e., in the DV dimensions $[R] = [\alpha] = 1$. The dimensionless parameter α of the de Sitter spacetime emphasizes the unique symmetry of this spacetime and supports quantization of this parameter (see, e.g., [49]), which could be similar to the Bekenstein quantization of the black hole area [46]. However, in case of the superplastic vacuum, the quantization of area can be very different from the quantization in terms of the Planck area, because the elementary cell of the underlying lattice may have nothing to do with the Planck scale.

Table 2 contains the operators with the mass dimensions 3, 5, and 6. The non-renormalisable dimension 5 operator gives a contribution to the electron magnetic moment [1]:

$$G_5 = ie^\mu_a e^\nu_b \bar{\psi}(\Gamma^a\Gamma^b - \Gamma^b\Gamma^a)\psi F_{\mu\nu},$$

and the non-renormalisable dimension 6 four-fermion operator describes the baryon number violation:

$$G_6 = QQQL,$$

where L and Q are the lepton and quark doublets. Since in the DV approach the mass is dimensionless, these operators become dimensionless: their $d_{DV} = 0$. The prefactors in these terms are determined either by the ratio of the mass scales ("ultraviolet" and "infrared") or by the functions of scalar fields. The same is with the $4k$ mass operator for $k > 1$ in Table 3: $G_{4k} = (F_{\mu\nu} F^{\mu\nu})^k$.

In terms of the DV dimensionalities, the operators with $d_{DV} = 4$ are topological. The operators of the type $G_4 = F_{\mu\nu}\tilde{F}^{\mu\nu}$ are topological in both classes of dimensions, since for them $d = d_{DV} = 4$. They are accompanied by the fundamental integer or fractional prefactors. There are also operators, which have original dimension $d \neq 4$ but acquire dimension $d_{DV} = 4$ in the DV approach. This means that they are not topological in conventional approach, but may become topological in the DV dimensions.

The former dimension $d = 2$ operator $\mathcal{T}_a \mathcal{T}^a$ and the dimension $d = 3$ operator $e^a_\mu A_\nu \tilde{F}^{\mu\nu}$ acquire dimension $d_{DV} = 4$ in the DV dimensionalities. As a result, they become topological and their prefactors become the topological quantum numbers. The operator $e^a_\mu A_\nu \tilde{F}^{\mu\nu}$ determines the quantum Hall response in $3+1$ topological insulators [26, 27, 50] described by the following Chern–Simons topological term [27]:

$$S_{4D}[A_\mu] = \\ = \frac{1}{4\pi^2} \sum_{a=1}^{3} N_a \int d^4x \; E_\mu{}^a \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta \,. \quad (13)$$

It explicitly contains the elasticity tetrads $E_\mu{}^a$. The integer coefficients N_a are three topological invariants in terms of the Green's functions:

$$N_a = \frac{1}{8\pi^2} \epsilon_{ijk} \int_{-\infty}^{\infty} d\omega \int_{BZ} dS^i_a \times \\ \times \mathrm{Tr}[(G_\omega G^{-1})(G_{k_i} G^{-1})(G_{k_j} G^{-1})] \,. \quad (14)$$

Invariants N_a describe the quantized response of Hall conductivity to crystal deformations:

$$\frac{d\sigma_{ij}}{dE_k{}^a} = \frac{e^2}{2\pi h} \epsilon_{ijk} N_a \,. \quad (15)$$

In terms of the conventional tetrads, the gravitational Nieh–Yan anomaly related to torsion [51–61],

$$\partial_\mu j^\mu_5 = \lambda^2 \left(\mathcal{T}^a \wedge \mathcal{T}_a - e^a \wedge e^b \wedge R_{ab} \right) \,, \quad (16)$$

contains the nonuniversal prefactor — the ultraviolet cut-off parameter λ with dimension of inverse length, $[\lambda] = 1/[l]$. Since λ may depend on coordinates, which explicitly violates the topology, the Nie–Yan contribution to the anomaly is still rather subtle (see recent literature [27, 61–65]). In the DV tetrads, the torsion in Eq. (4) has dimension $[T^a_{kl}] = 1/[l]^2$, and the prefactor λ^2 becomes dimensionless, $[\lambda] = 1$, which suggests that the prefactor is universal and is quantized.

The Chern–Simons term describing the 3+1 quantum Hall effect can be extended to the 3+1+1 Wess–Zumino actions:

$$S^{\mathrm{aab}}_{WZ} = \frac{1}{8\pi^2} \int_{X^5} d^4x \, d\tau \, \epsilon^{\mu\nu\alpha\beta\gamma} e_\gamma{}^a F^{\mathrm{a}}_{\mu\nu} F^{\mathrm{b}}_{\alpha\beta} \,. \quad (17)$$

$$S^{abc}_{NY} \propto \int_{X^5} d^4x \, d\tau \, \epsilon^{\mu\nu\alpha\beta\gamma} e_\gamma{}^a T^b_{\mu\nu} T^c_{\alpha\beta} \,. \quad (18)$$

$$S^{ab}_{WZ} \propto \int_{X^5} d^4x \, d\tau \, \epsilon^{\mu\nu\alpha\beta\gamma} e_\gamma{}^a T^b_{\mu\nu} F_{\alpha\beta} \,. \quad (19)$$

In terms of DV tetrads, these dimensionless terms are universal and do not depend on the cut-off parameters.

In two scenarios of emergent gravity, the superplastic vacuum and the DV theory with bilinear tetrad field, the invariance under diffeomorphisms leads to the dimensionless physics. In the DV theory, this invariance is assumed as fundamental. In the superplastic vacuum, the diffeomorphism invariance corresponds to the proposed invariance under arbitrary deformations of the 4D vacuum crystal. In words of ′t Hooft (applied originally to the local conformal symmetry) "this could be a way to make distance and time scales relative, so that what was dubbed as "small distances" ceases to have an absolute meaning" [66]. This suggests that the dimensionless physics can be the natural consequence of the diffeomorphism invariance, and can be the general property of any gravity emerging in quantum vacuum. The dimensionless physics leads to new topological terms in action with the universal integer valued topological quantum numbers of the quantum vacuum.

The universality takes place only for the topological numbers and the symmetry parameters. The other dimensionless quantitites are not universal, being described by the functions of the ratios of different mass scales. In this respect, the answer to the question of how many fundamental constants are there in physics [67–70] can be trivial: there are no fundamental constants, and the ratios of parameters and the ratio of the length scales are the only meaningful quantities [10].

Funding. This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement No. 694248).

The full text of this paper is published in the English version of JETP.

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