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ORDER PARAMETER IN ELECTRON SYSTEMS: ITS FLUCTUATIONS AND OSCILLATIONS

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The concept of the order parameter introduced by Landau in his theory of phase transitions [1] plays the central role in condensed matter and statistical physics. It has become clear later with development of the scaling theory that, in order to describe a phase transition, one should integrate over the order parameter with a weight determined by a Ginzburg–Landau–Wilson free energy functional [2]. This field of research has attracted a lot of interest at the Landau Institute in particular because scaling ideas had been proposed previously by Patashinskii and Pokrovskii [3] and by Kadanoff [4], and first renormalization group study of the phase transitions had been performed by Larkin and Khmel'nitski [5].

Simultaneously, it was getting clear that fluctuations of the order parameter could be very important not only near a phase transition but also in low-dimensional systems. The dimension of the system was determined by geometry of the sample, while the dimension of the electron bands was not always important. It was demonstrated in 1975 in my works with Anatoly Larkin [6, 7] that the behavior of two-point correlation functions of one-dimensional electron systems at large distances or times was completely determined by sound-like gapless quantum fluctuations. Although many thermodynamical physical quantities could be calculated microscopically using rather sophisticated Bethe-Ansatz methods, one could not determine the correlation functions using that technique. Instead, it was demonstrated that it was sufficient to compute correlations of superconducting or insulating order parameters taking into account their fluctuations. The publications [6, 7] have allowed us to demonstrate that exact results obtained in previous works for rather special models [8–10] are general and one could obtain results for realistic models using the general scheme. Actually, this was one of the first steps in the subsequent development of powerful bosonization techniques [11] (for review, see, e. g. [12]).

The idea that many interesting effects could be efficiently described by considering low energy fluctuations of the order parameter motivated me later to study physics of granular superconductors [13]. In these materials, the superconductivity in a single grain can be well described by a phase of the order parameter fluctuating in time. The modulus of the order parameter was assumed to be a constant and there was no need to consider space variations of the phase inside the grain. In order to describe the macroscopic superconductivity in the array of the grains one had to account for the Josephson coupling between the grains. Coulomb interaction turns out to be very important enhancing the phase fluctuations in time and eventually leading to a superconductor–insulator transition

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dubbed later "Coulomb blockade". Since then, I have made several other important works on granular superconductors and participated in writing a review on this subject. Physics of granular superconductors is the same as physics of artificially designed Josephson networks and the description developed by me in 1980 [13] has been used and further developed later in a huge number of publications. Actually, the approach developed in this works allows one to describe not only superconductors but also Coulomb blockade effects in normal granular metals.

The idea of fluctuations of the order parameter turned out to be fruitful in problems of Anderson localization and mesoscopics. Originally, it was conjectured in a "bosonic" replica reformulation of models with disorder by Wegner [14] but soon it has been followed by "fermionic" replica representation in my work with Larkin and Khmelnitskii [15]. Within this approach one reduces summation of certain classes of diagrams (so called "diffusons" and "cooperons") to study of fluctuations of a matrix. The matrix looked formally as an order parameter and a "free energy functional" looked very similar to the one describing fluctuations of the phase in superconductor. The free energy functional had a form of a non-linear σ-model. The "fermionic" σ-model [15] has been later modified for studying disordered electron systems with electron–electron interaction [16] and quantum Hall effect in [17].

Although the σ-model allowed one to perform very efficiently perturbation theory and renormalization group calculations, it was not possible to do non-perturbative calculations. In order to circumvent this difficulty, I have derived in 1982 [18] a supermatrix σ-model that was free of these problems and allowed one to perform essentially non-perturbative calculations. The first application of the zero-dimensional σ-model to studying level–level correlations in small metal particles was successful, and I demonstrated [19] that level–level correlation functions agreed with those obtained from the Wigner–Dyson random matrix theory [20, 21]. These works were followed by a review where the supersymmetry method was applied to problems of compound nuclei [22]. Somewhat later the random matrix theory has been applied in study of quantum chaos [23] and the supersymmetry turned out to be useful there. Since then, a huge amount of works has been published where the zero-dimensional σ-model was used for mesoscopic and quantum chaos problems [24–26]. Somewhat later the supersymmetric σ-model has been successfully applied to study of localization in disordered quantum wires and on the Bethe lattice [27–32]. I continued to work in this direction for quite a long time because the method really worked. This subject has been presented in numerous publications and reviews [33–37] and in a book [38].

The supermatrix "order parameter" Q that appears in the σ-model approach must be averaged with the free energy functional, and it has no physical sense without carrying out this procedure. Actually, the average $\langle Q\rangle$ with the action of the σ-model is not an interesting quantity because it is the average density of state and the latter quantity is a smooth function of energy. Conductivity, level correlations, density–density correlations, etc. can be written in terms of a product of several Q like, e. g., $\langle QQ\rangle$. Therefore, the matrix Q is not an order parameter in its usual sense. In order to obtain an interesting physical quantity, one should integrate a product of several Q over all configurations.

Recently, I have encountered a rather similar situation in my investigation of a possibility of existence of a thermodynamically stable "Time-crystal". The time-crystal is expected to demonstrate an oscillating behavior of physical quantities in time. The concept of a time-crystal has been proposed several years ago by Wilczek [39] in a simple model but later it was realized that the time crystal state proposed there was not the ground state and therefore could not be stable. Moreover, it was even argued that the thermodynamically stable quantum crystal could not exist at all [40].

However, in a recent preprint [41] I have suggested and investigated a model that can undergo a transition into a state with an order parameter b depending on both real and imaginary times. As a result, effective wave functions also depend on the real and imaginary time. The position of this non-trivial order parameter in time is arbitrary and the averaging over the positions gives zero. At the same time, the average of a product of the order parameters can be finite and can be measured experimentally. For example, the average $\langle bb\rangle$ can be measured in quantum scattering experiments. This situation resembles the one with fluctuation of order parameters considered previously but now the order parameter oscillates in time rather than fluctuates.

The present paper is not a review of my publications in several different fields of physics. I simply wanted to use an opportunity to emphasize considering several examples that many new results in several fields of research can be obtained with the help of the generalized concept of the order parameter. Using this approach, one can considerably simplify calculations because it is sufficient to consider large distances without going into details of band structures and interactions. Very often this route gives a possibility to solve problems that have not been solved before and predict new physical

phenomena. I have realized the efficiency of this approach during my years at the Landau Institute and used it later in many works.

In contrast to the standard notion of a static long range order in an ordered phase, one may encounter situations when there is no static long-range order. One can see from the results of the investigation of several models of electrons with interaction or moving in a random potential that there can be interesting nontrivial physics. The properties of the models have been understood considering either fluctuations or oscillations in space and time of a generalized order parameter. Coulomb blockade, Anderson localization, space-time quantum crystals, etc., are clearly quite different phenomena but their theoretical description has many common features. Of course, it is always pleasant to obtain new physical results and have a possibility to compare them with experiments. However, unification of the description of many rather different phenomena using a single concept gives also a strong esthetic satisfaction.

An essential part of my results presented in this paper has either been done at the Landau Institute or followed from ideas developed there. I have started my scientific carrier and worked for many years at the Landau Institute in its best time, and I am personally very grateful to Isaac Markovich for the creation of the Institute, for the support of my research, and for giving me the possibility to work at the Institute.

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