

DRAG FORCE AND SUPERFLUIDITY IN THE SUPERSOLID STRIPE PHASE OF A SPIN–ORBIT-COUPLED BOSE–EINSTEIN CONDENSATE

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Received April 22, 2018

Contribution for the JETP special issue in honor of L. P. Pitaevskii's 85th birthday

DOI: 10.1134/S004445101811007X

The phenomenon of supersolidity is characterized by a simultaneous presence of superfluidity and crystalline order (see [1] for a review). Originally, it was investigated theoretically in a general context and in the context of helium [2–8], and then in systems of bosons with soft-core two-body potentials [9–12] and in two-dimensional dipolar Bose–Einstein condensates (BECs) [13–16]. However, only recently supersolid configurations have been produced in experiments with ultracold atomic gases [17, 18]. In particular, in Ref. [18] the authors reported the observation of the supersolid stripe phase in a two-component BEC with spin–orbit (SO) coupling, originally predicted in Refs. [19–22] (see also reviews [23–27] and references therein).

The excitation spectrum of a SO-coupled BEC in the stripe phase was calculated in Ref. [28]. It is characterized by a band structure with two gapless modes, associated with the two symmetries (gauge and translational) that are spontaneously broken in the stripe phase. For these two modes, the frequency of the excitations propagating along the direction of the SO coupling vanishes at the edge of the Brillouin zone. Because of this structure, the Landau criterion for anisotropic systems [29] predicts a zero critical velocity if the motion is not in the direction parallel to the stripes. This raises the problem of understanding how to characterize the superfluid behavior of the system. In the present work we study the superfluidity of the stripe phase by calculating the drag force acting on a moving defect. We follow the procedure originally employed in Ref. [30] for a standard single-component BEC. The method consists in calculating the response of the system to an external δ-like potential describing the coupling of the BEC to a pointlike moving impurity.

The single-particle Hamiltonian of a two-component SO-coupled BEC was first realized in the experiment of Ref. [31], and then employed in Ref. [18] to detect the stripe phase. It reads

$$h_{SO} = \frac{(p_x - \hbar k_R \sigma_z)^2}{2m} + \frac{p_\perp^2}{2m} + \frac{\hbar\Omega_R}{2}\sigma_x + \frac{\hbar\delta_R}{2}\sigma_z\,. \quad (1)$$

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This Hamiltonian results from the coupling of the BEC with a pair of Raman beams, which yields transitions between the two pseudospin states. The strength of the SO coupling is fixed by the momentum transfer $\hbar k_R \hat{\mathbf{e}}_x$ due to the lasers ($\hat{\mathbf{e}}_x$ is the unit vector along x), while the Raman coupling $\hbar\Omega_R$ is related to their intensity. The detuning $\hbar\delta_R$ of the atomic transition from Raman resonance plays the role of a Zeeman shift (which we take equal to zero). The quantity m is the atom mass, $\sigma_{x,y,x}$ are the usual Pauli matrices, and $p_\perp^2 = p_y^2 + p_z^2$.

Hamiltonian (1) is static and translationally invariant, and one can look for a complete set of eigenstates in the form of plane waves with momentum $\mathbf{p}$. The energy dispersion as a function of $\mathbf{p}$ is made of two branches. The minima of the lower branch, yielding the single-particle ground state, are located at finite momenta $\mathbf{p} = \pm\hbar\mathbf{k}_1^0 = \pm\hbar k_1^0 \hat{\mathbf{e}}_x$ if $\hbar\Omega_R < 4E_R$ (here $E_R = (\hbar k_R)^2/2m$ is the Raman recoil energy), or at $\mathbf{p} = 0$ if $\hbar\Omega_R \geq 4E_R$.

By introducing the two-component bosonic quantum field

$$\hat{\Psi}(\mathbf{r}) = (\hat{\Psi}_\uparrow(\mathbf{r})\hat{\Psi}_\downarrow(\mathbf{r}))^T$$

(we denote the pseudospin components as $\uparrow$ and $\downarrow$), the many-body Hamiltonian for a system of N particles in a volume V, with contact interparticle interaction, can be written as

$$\hat{H} = \int_V d^3r \left[\hat{\Psi}^\dagger h_{SO} \hat{\Psi} + \frac{g_{dd}}{2}(\hat{\Psi}^\dagger\hat{\Psi})^2 + \right. \\ \left. + \frac{g_{ss}}{2}(\hat{\Psi}^\dagger \sigma_z \hat{\Psi})^2\right]. \quad (2)$$

Here $g_{dd} = 4\pi\hbar^2 a_{dd}/m$ and $g_{ss} = 4\pi\hbar^2 a_{ss}/m$ are the density-density and spin–spin interaction strengths, respectively, with $a_{dd} = (a + a_{\uparrow\downarrow})/2$ and $a_{ss} = (a - a_{\uparrow\downarrow})/2$ the corresponding s-wave scattering length (we assume $a_{\uparrow\uparrow} = a_{\downarrow\downarrow} = a$).

We apply the Gross–Pitaevskii (GP) mean-field approach and the Bogoliubov theory, and write the field operator in the Heisenberg representation as

$$\hat{\Psi}(\mathbf{r}, t) = e^{-i\mu t/\hbar}\left[\Psi_0(\mathbf{r}) + \delta\hat{\Psi}(\mathbf{r}, t)\right],$$

where Ψ_0 is a classical field (condensate wavefunction) and $\delta\hat{\Psi}(\mathbf{r}, t)$ parameterizes quantum fluctuations. The chemical potential μ is fixed by the condition $\int_V d^3r\, n_0(\mathbf{r}) = N$ with $n_0(\mathbf{r}) = \Psi_0^\dagger(\mathbf{r})\Psi_0(\mathbf{r})$ being the local particle density. In the stripe phase, the classical field Ψ_0 that minimizes the total energy has the form of a Bloch wave [28],

$$\Psi_0(\mathbf{r}) = \exp(ik_c x) \sum_{\bar{m}\in\mathbb{Z}} \tilde{\Psi}_{\bar{m}} \exp(2i\bar{m}k_1 x). \quad (3)$$

Here $\hbar\mathbf{k}_c = \hbar k_c \hat{\mathbf{e}}_x$ is the quasimomentum, and the role of the reciprocal lattice vectors is played by the quantities $\{2\bar{m}\mathbf{k}_1\}_{\bar{m}\in\mathbb{Z}}$, where $\mathbf{k}_1 = k_1\hat{\mathbf{e}}_x$. The two-component spinor coefficients of the Bloch expansion are denoted as $\tilde{\Psi}_{\bar{m}}$. One can easily show that for $\delta_R = 0$ and $a_{\uparrow\uparrow} = a_{\downarrow\downarrow}$, the equalities $k_c = k_1$ and $\tilde{\Psi}_{-\bar{m}} = (\sigma_x \tilde{\Psi}_{\bar{m}-1})^*$ hold, which yields a vanishing magnetic polarization [20, 22, 28]

$$\langle\sigma_z\rangle = \int_V d^3r\, \Psi_0^\dagger(\mathbf{r})\sigma_z\Psi_0(\mathbf{r}).$$

Qualitatively, the stripe phase can be regarded as a macroscopic occupation of an equal-weighted superposition of the two states lying at the minima of the single-particle dispersion. However, because of the interparticle interaction, higher-order harmonic terms with wave vectors $\pm 3k_1$, $\pm 5k_1$, ... have to be included in the wavefunction (3). Notice also that the interaction shifts the momentum k_1 from the single-particle value k_1^0 [22].

The fluctuation term of the field operator in the stripe phase can be written as [28]

$$\delta\hat{\Psi}(\mathbf{r}, t) = \sum_{\ell,\mathbf{k}\in\mathrm{BZ}} \Big[U_{\ell,\mathbf{k}}(\mathbf{r})\hat{b}_{\ell,\mathbf{k}} \exp(-i\omega_{\ell,\mathbf{k}}t) + \\ + V_{\ell,\mathbf{k}}^*(\mathbf{r})\hat{b}_{\ell,\mathbf{k}}^\dagger \exp(i\omega_{\ell,\mathbf{k}}t)\Big], \quad (4)$$

where $\hat{b}_{\ell,\mathbf{k}}$ ($\hat{b}_{\ell,\mathbf{k}}^\dagger$) are the annihilation (creation) operators of a quasiparticle with quasimomentum $\hbar\mathbf{k}$ and energy $\hbar\omega_{\ell,\mathbf{k}}$, the index ℓ labels different bands of the excitation spectrum, and BZ is the Brillouin zone. The two-component Bogoliubov amplitudes, which obey the ortho-normalization condition

$$\int_V d^3r \left[U_{\ell,\mathbf{k}}^\dagger(\mathbf{r})U_{\ell',\mathbf{k}'}(\mathbf{r}) - V_{\ell,\mathbf{k}}^\dagger(\mathbf{r})V_{\ell',\mathbf{k}'}(\mathbf{r})\right] = \delta_{\ell\ell'}\delta_{\mathbf{k}\mathbf{k}'},$$

can be expressed as Bloch waves of the form

$$\begin{aligned} U_{\ell,\mathbf{k}}(\mathbf{r}) &= \exp(i\mathbf{k}\cdot\mathbf{r})\exp(ik_c x) \times \\ &\times \sum_{\bar{m}\in\mathbb{Z}} \tilde{U}_{\ell,\mathbf{k}+2\bar{m}\mathbf{k}_1} \exp(2i\bar{m}k_1 x), \\ V_{\ell,\mathbf{k}}(\mathbf{r}) &= \exp(i\mathbf{k}\cdot\mathbf{r})\exp(-ik_c x) \times \\ &\times \sum_{\bar{m}\in\mathbb{Z}} \tilde{V}_{\ell,\mathbf{k}+2\bar{m}\mathbf{k}_1} \exp(2i\bar{m}k_1 x), \end{aligned} \quad (5)$$

where $\tilde{U}_{\ell,\mathbf{k}+2\bar{m}\mathbf{k}_1}$ and $\tilde{V}_{\ell,\mathbf{k}+2\bar{m}\mathbf{k}_1}$ are expansion coefficients. They can be deduced, together with the frequencies $\omega_{\ell,\mathbf{k}}$, by solving the linearized Bogoliubov–de Gennes equations in the stripe phase [28]. Notice that the summation in Eq. (4) is restricted to the quasimomenta with the x component in the first Brillouin zone, i. e., $0 \le k_x \le 2k_1$.

The Bogoliubov spectrum of the stripe phase was originally calculated in Ref. [28]. It has a band structure with two gapless branches. At low k these two branches exhibit a linear dispersion. The corresponding sound velocity is anisotropic, with minimum and maximum values for excitations propagating along x and in the yz plane, respectively. Furthermore, in the case of excitations with $\mathbf{k}$ parallel to the x axis, the frequency of the two gapless bands vanishes when k_x approaches $2k_1$, i. e., at the edge of the first Brillouin zone.

Let us now add an external perturbation of the form $U_{imp}(\mathbf{r},t) = g_{imp}\delta(\mathbf{r}-\mathbf{v}t)$ to the single-particle Hamiltonian (1). This perturbation accounts for the presence of a heavy impurity moving with constant velocity $\mathbf{v}$. The impurity is coupled to the BEC with strength $g_{imp} = 2\pi\hbar^2 b/m$ (b is the atom–impurity scattering length). Within the accuracy of the Bogoliubov approach, the many-body Hamiltonian describing the perturbation is

$$\hat{H}_{imp}(t) = g_{imp} n_0(\mathbf{v}t) + \frac{g_{imp}}{V} \times \\ \times \sum_{\ell,\mathbf{k}\in \mathrm{BZ}} \sum_{\bar{m}\in\mathbb{Z}} \exp\left[i(\mathbf{k}+2\bar{m}\mathbf{k}_1)\cdot\mathbf{v}t\right] \times \\ \times f_{\ell,\mathbf{k}+2\bar{m}\mathbf{k}_1} \hat{b}_{\ell,\mathbf{k}} + \mathrm{H.c.}, \quad (6)$$

where

$$f_{\ell,\mathbf{k}+2\bar{m}\mathbf{k}_1} = \int_V d^3r \exp\left[-i(\mathbf{k}+2\bar{m}\mathbf{k}_1)\cdot\mathbf{r}\right] \times \\ \times \sum_{\mathbf{k}'\in \mathrm{BZ}} \left[\Psi_0^\dagger(\mathbf{r}) U_{\ell,\mathbf{k}'}(\mathbf{r}) + V^T_{\ell,\mathbf{k}'}(\mathbf{r})\Psi_0(\mathbf{r})\right]$$

is the matrix element of the perturbation. From the linear response theory, we know that the time-averaged dissipation rate for the impurity is $W = \mathbf{F}\cdot\mathbf{v}$, where the time-averaged drag force takes the following expression [30, 32]:

$$\mathbf{F} = -\frac{2\pi g_{imp}^2}{\hbar V^2} \sum_{\ell,\mathbf{q}} \mathbf{q}\,|f_{\ell,\mathbf{q}}|^2\,\delta\left(\omega_{\ell,\mathbf{q}} - \mathbf{q}\cdot\mathbf{v}\right). \quad (7)$$

Computing the drag force we find that it never vanishes at finite v, unless the impurity moves parallel to the stripes (Fig. 1). Moreover, unlike in standard Bose gases, here the force does not generally act in the direction opposite to $\mathbf{v}$, but lies in the plane spanned by $\mathbf{v}$ and the direction of the SO coupling [29, 33–35] ($\mathbf{F}$ is antiparallel to $\mathbf{v}$ only if the latter is parallel or perpendicular to the x axis). Without loss of generality, we can take $\mathbf{v} = v(\cos\theta_v, \sin\theta_v, 0)$ with $0 \le \theta_v \le \pi/2$. Then, the force is $\mathbf{F} = -F(\cos\theta_F, \sin\theta_F, 0)$, where $\theta_v - \pi/2 \le \theta_F \le \theta_v + \pi/2$. For low values of v, the main contributions to the force are from the modes of the gapless branches with momentum close to the edge of the Brillouin zone. In this regime, F is larger for small values of θ_v, and it increases with Ω_R. Furthermore, θ_F is close to zero, i. e., the force predominantly acts along the direction of the SO coupling. If the anisotropy of the sound velocity is negligible, as happens at small Ω_R, one finds the following expressions for the two non-vanishing components of the force at small v:

$$F_x \approx -\frac{16\pi\hbar^2 k_1^2 b^2 \bar{n}|\tilde{f}_1|^2}{m^2 c_1^2} v_x \left(1 + \frac{v_y^2}{c_1^2}\right), \\ F_y \approx -\frac{16\pi\hbar^2 k_1^2 b^2 \bar{n}|\tilde{f}_1|^2}{m^2 c_1^4} v_x^2 v_y. \quad (8)$$

Here $\bar{n} = N/V$ is the average density, c_1 is the typical value of the sound velocity of the lowest gapless branch, and the coefficient $|\tilde{f}_1|^2$ is related to the behavior of matrix element $|f_{1,\mathbf{q}}|^2$ close to the Brillouin point:

$$|f_{1,\mathbf{q}}|^2 \approx \frac{|\tilde{f}_1|^2}{\hbar\sqrt{(q_x-2k_1)^2+q_\perp^2}}.$$

With increasing v, the contribution of the low-q modes of the gapless bands of the Bogoliubov spectrum, as well as the contribution of high gapped bands, becomes dominant. This tends to orient the force in the direction antiparallel to the velocity. At the same time, the magnitude of $\mathbf{F}$ tends to the value found in [30] in the absence of SO coupling. Finally, we point out that if the impurity moves in the y–z plane, i. e., parallel to the stripes, the drag force vanishes if v is smaller than the sound velocity for the lower gapless branch along the same direction.

If the number of impurities is N_{imp}, the time-averaged mean-field energy per particle is $\varepsilon = \varepsilon_0 + \chi g_{imp}\bar{n}$. Here ε_0 is the mean-field energy per particle in the absence of impurities, and $\chi = N_{imp}/N$ is the impurity concentration. We make ε_0 always positive by subtracting the energy of the single-particle ground state, $\varepsilon_-(\pm\hbar\mathbf{k}_1^0) = -(\hbar\Omega_R)^2/16E_R$. The term proportional to g_{imp} in ε comes from the time average of the mean-field contribution to the perturbation Hamiltonian (6), multiplied by N_{imp}.

We can now estimate the time scale over which the energy dissipation occurs, $\tau = \varepsilon_0/(\chi|W|)$. For a typical situation with a low concentration of impurities χ and small scattering lengths b, a_{dd}, we find that $\tau \gtrsim 0.1$ s for a wide range of velocities (Fig. 2). This is of the

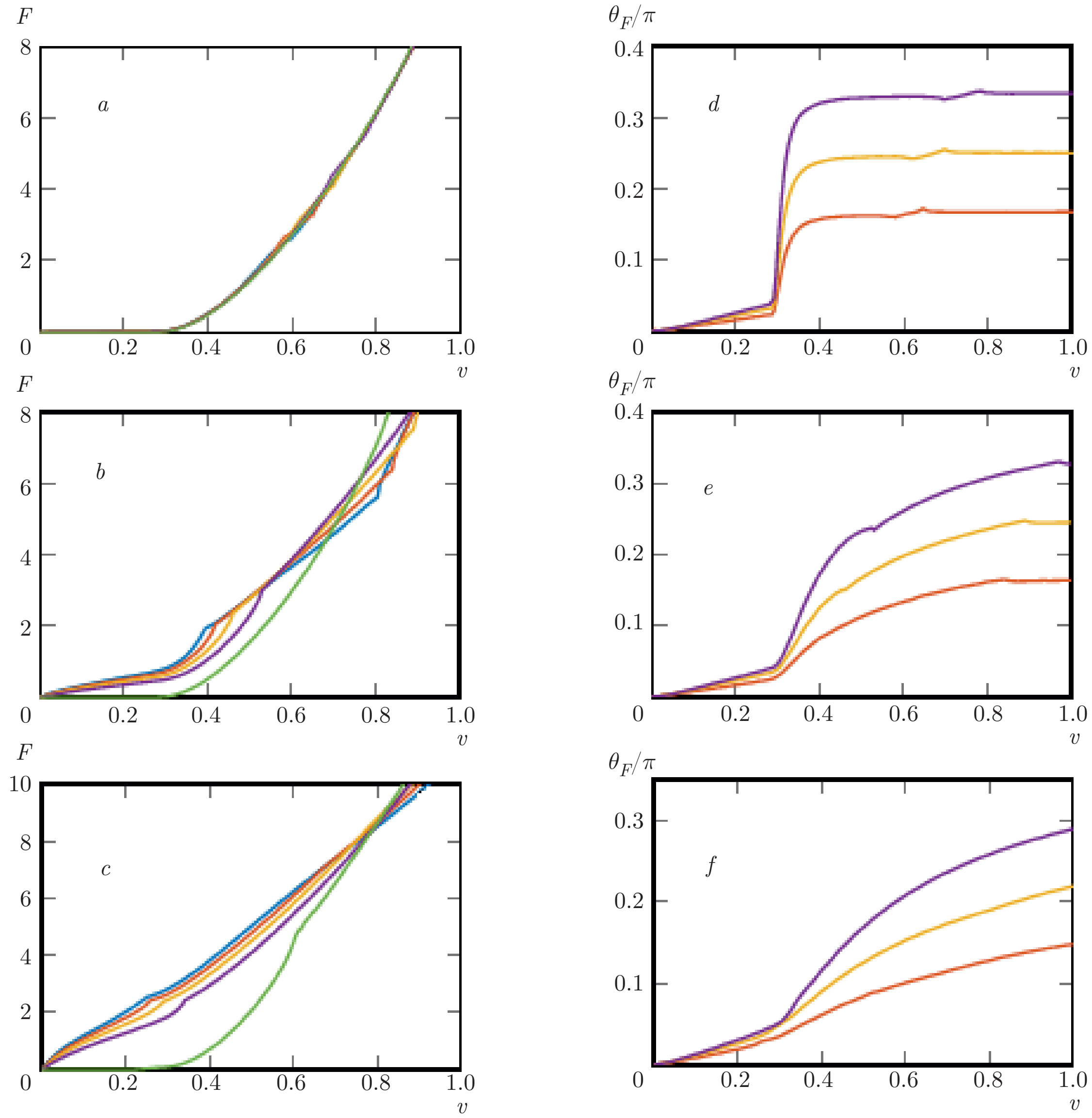

Fig. 1. (Color online) Magnitude a–c and orientation d–f of the drag force versus the velocity of the impurity. Each couple of panels corresponds to a different value of the Raman coupling: $\hbar\Omega_R/E_R = 0.2$ (a,d), 1.0 (b,e), and 2.0 (c,f). The different curves show the results for $\theta_v = 0$ (blue), $\pi/6$ (red), $\pi/4$ (yellow), $\pi/3$ (violet), and $\pi/2$ (green). In the panels in the right column, we only display the curves for the nontrivial $\theta_v = \pi/6, \pi/4, \pi/3$ cases. The density $\bar{n}/k_R^3 = 0.75$ and the interaction parameters $g_{dd}\bar{n}/E_R = 0.08$, $g_{ss}\bar{n}/E_R = 0.075$ correspond to those of the experiment [18] with k_R increased by a factor of 2. The velocity v is in units of $\hbar k_R/m$, and the force F is in units of $(\hbar k_R)^2\bar{n}b^2/m$

order of or even larger than the typical duration of experiments with ultracold atomic gases.

In conclusion, we have calculated the drag force acting on an impurity moving in the stripe phase of a SO-coupled BEC. The force never vanishes unless the motion occurs in the direction parallel to the stripes. Moreover, it is not generally antiparallel to the direction of the motion. At large impurity speeds, the results approach those in the absence of the SO coupling. By computing the time scale over which the energy dissipation occurs at small velocities, we find that it takes fairly large values, comparable to the duration of current experiments. We can thus say that the motion of a slow impurity through the stripe phase can be considered, to a large extent, as dissipationless, similarly to what happens in ordinary uniform superfluids.

In future, we plan to study the effects of the friction force on the moving striped BEC. Unlike in uniform

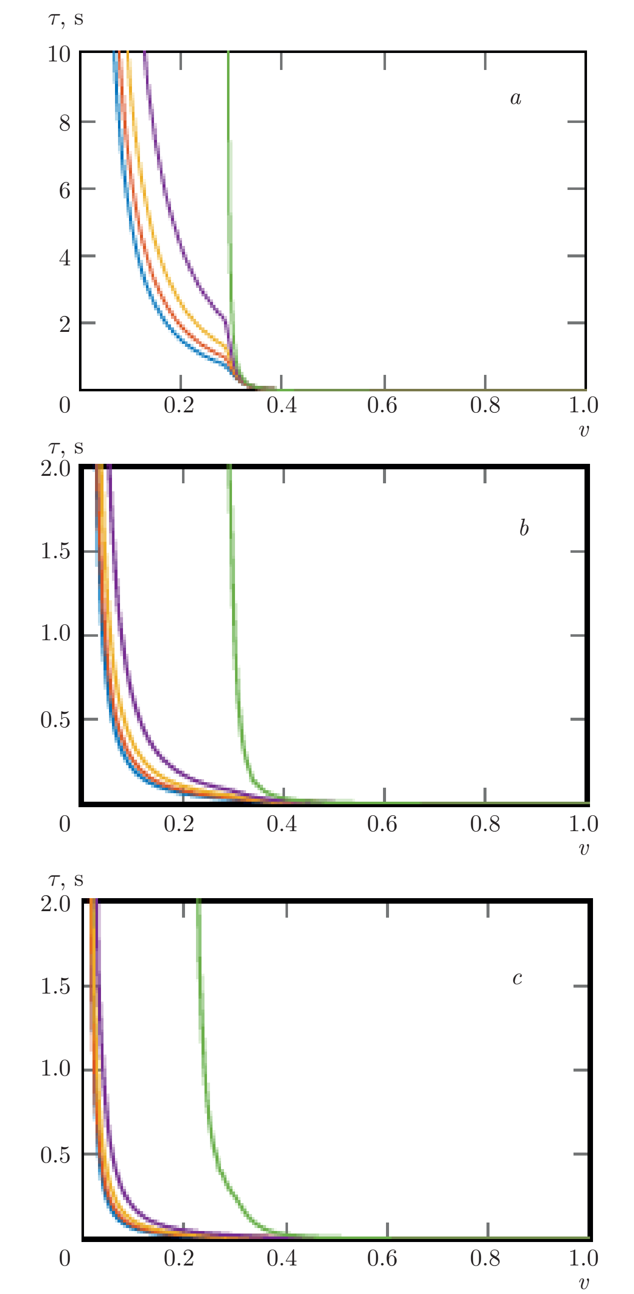

Fig. 2. (Color online) Time scale for the energy dissipation as a function of the velocity of the impurities for a fixed impurity concentration $\chi = 0.5$, and for $b/a_{dd} = 1.0$. For the other parameters, we use the same values as in Fig. 1. Each panel corresponds to a different value of the Raman coupling: $\hbar\Omega_R/E_R = 0.2$ (a), 1.0 (b), and 2.0 (c). The different curves show the results for $\theta_v = 0$ (blue), $\pi/6$ (red), $\pi/4$ (yellow), $\pi/3$ (violet), and $\pi/2$ (green). The density $\bar{n}/k_R^3 = 0.75$ and the interaction parameters $g_{dd}\bar{n}/E_R = 0.08$, $g_{ss}\bar{n}/E_R = 0.075$ correspond to those of the experiment [18] with k_R incseased by a factor 2. The velocity v is in units of $\hbar k_R/m$

superfluids, where it can only reduce the velocity of the flow, we expect that in the stripe phase the friction may act in the direction of weakening or eliminating the density modulations.

We acknowledge useful discussions with N. Pavloff, L. P. Pitaevskii, and S. Stringari. The research leading to these results has received funding from the European Research Council under European Community's Seventh Framework Programme (FP7/2007-2013 Grant Agreement No. 341197).

The full text of this paper is published in the English version of JETP.

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