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POSSIBLE ACCELERATION OF COSMIC RAYS IN A ROTATING SYSTEM: UEHLING–UHLENBECK MODEL

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We illustrate the possible acceleration of cosmic rays passing through a kind of amplification channel (via diffusion modes of propagating plane-wave fronts) induced by a rotating system. Our analysis is mainly based on the quantum discrete kinetic model (considering a discrete Uehling–Uhlenbeck collision term), which has been used to study the propagation of plane (e. g., acoustic) waves in a system of rotating gases.

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1. INTRODUCTION

Radiation that enhances with altitude in the atmosphere were discovered by V. F. Hess in 1912, and it was clear by the early 1930s that this radiation comes from outer space. The all-particle energy spectrum of primary cosmic rays extends from 1 GeV (10^{12} eV) to above 10^{20} eV (100 EeV), the highest energies of known individual particles in the Universe [1–3]. The highest-energy particles are so rare that they are detectable only by means of the giant cascades or extensive air showers they create in the atmosphere. Details of how these extensive air showers are observed and the parameters of importance are measured can be found in [1]. In spite of many efforts, we still have only a limited understanding of where these particles are coming from, how they are accelerated to such extremely high energies, and how they propagate through interstellar space [2].

The astrophysical environments that are able to accelerate particles to such high energies, including active galactic nuclei, large-scale galactic wind termination shocks, relativistic jets and hot-spots of Fanaroff–Riley radio galaxies, pulsars, magnetars, quasar remnants, starbursts, colliding galaxies, and gamma-ray burst fireballs were discussed in [3]. Many researchers believe that cosmic rays are accelerated in a process called diffusive shock acceleration. Suitable astrophysical shocks occur in supernova explosions, and ionized nuclei gain energy as they are repeatedly overtaken by the expanding shock wave. Such mechanisms lead in fact to a power-law spectrum with a maximum energy of about $Z \cdot 10^{15}$ eV (Z is the atomic number), which roughly agrees with the observed steepening (although the theoretically predicted spectrum proves to be steeper than actually observed) [2].

In this short paper, the highly nonlinear discrete Uehling–Uhlenbeck equations [4] together with the model of free orientations (θ is the relative direction of particle scattering with respect to the normal of the propagating plane-wave front) are solved to study the diverse dispersion relations of plane waves in a system of rotating gases (or disk-like gases). Anomalous amplification channels can occur for diffusion modes of propagating plane-wave fronts. As a result of intensive computations, we propose that the acceleration of cosmic rays might happen via this kind of channel.

2. THEORETICAL FORMULATIONS

We make the following assumptions before we investigate the general equations of our model:

(1) We consider a system of rotating disks or a gas of identical particles of unit mass and a shape of a disk of diameter d; each particle i, $i = 1, \ldots, N$, is characterized by the position q_i of its center and its velocity $\bar{u}_i$. We also have the geometric limitations $|q_i - q_j| \geq d$, $i \neq j$.

(2) Each particle moves in the plane with a velocity belonging to a discrete set $\mathcal{V}$ of 4-velocities with only one speed in the plane. The velocity modulus c is a reference speed depending on the reference frame and

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specific distribution of particles; c is normally linked to the internal energy of molecules in thermodynamic equilibrium.

(3) The collisional mechanism is that of hard spheres, that is, the particles scatter elastically and change their phase states instantaneously, preserving momentum. Only binary collisions are considered, since a multiple collision is a negligible event.

A collision between two particles (say, i and j) takes place when they are located at q_i and $q_j = q_i - d\mathbf{n}$, where $\mathbf{n}$ is the unit vector joining their centers. After collisions, the particles scatter, preserving momentum, in the directions allowed by the discrete set $\mathcal{V}$. In other words, particles change according to

$$(q_i, \bar{u}_i) \to (q_i, \bar{u}_i^*), \quad (q_j, \bar{u}_j) \to (q_j, \bar{u}_j^*).$$

The collision is uniquely determined if the incoming velocity and the impact angle $\psi \in [-\pi/2, \pi/2]$ are known; the angle is defined as the angle between $\bar{u}_i$ and $\mathbf{n}$ or

$$\mathbf{n}(\psi) = (\cos[\psi + (k-1)\pi/2], \sin[\psi + (k-1)\pi/2]),$$

$$k = 1, \ldots, 4$$

($k = 4$ means during a binary encounter, there are in general two incoming velocities and two departing velocities). From the selected velocities, we have two classes of encounters: (a) $\langle \bar{u}_i, \bar{u}_j \rangle = 0$ and (b) $\langle \bar{u}_i, \bar{u}_j \rangle = -c^2$.

(a) In the first class, the momentum conservation implies an encounter at $\pi/2$ with the exchange of velocities

$$\bar{u}_i = \bar{u}^k \to \bar{u}_i^* = \bar{u}^{k+1}, \quad \bar{u}_j = \bar{u}^{k+1} \to \bar{u}_j^* = \bar{u}^k$$

in the case $\psi \in [-\pi/2, 0]$, and

$$\bar{u}_i = \bar{u}^k \to \bar{u}_i^* = \bar{u}^{k+3}, \quad \bar{u}_j = \bar{u}^{k+3} \to \bar{u}_j^* = \bar{u}^k$$

in the case $\psi \in [0, \pi/2]$.

(b) Similarly, $\langle \bar{u}_i, \bar{u}_j \rangle = -c^2$;

(i) Head-on encounters with the impact angle $\psi = 0$ such that

$$\bar{u}_i = \bar{u}^k \to \bar{u}_i^* = \bar{u}^{k+2}, \quad \bar{u}_j = \bar{u}^{k+2} \to \bar{u}_j^* = \bar{u}^k,$$

(ii) Head-on encounters with an impact angle $\psi \neq 0$ such that if $\psi \in [-\pi/2, 0]$, then

$$\bar{u}_i = \bar{u}^k \to \bar{u}_i^* = \bar{u}^{k+1}, \quad \bar{u}_j = \bar{u}^{k+2} \to \bar{u}_j^* = \bar{u}^{k+3},$$

and if $\psi \in [0, \pi/2]$, then

$$\bar{u}_i = \bar{u}^k \to \bar{u}_i^* = \bar{u}^{k+3}, \quad \bar{u}_j = \bar{u}^{k+2} \to \bar{u}_j^* = \bar{u}^{k+1}.$$

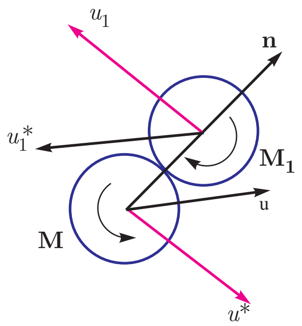

Fig. 1. Schematic plot of a collision. $\mathbf{M}$ and $\mathbf{M}_1$ are the associated angular momenta with opposite sign

For grazing collisions, that is $\langle \mathbf{n}, \bar{u}_i \rangle = \langle \mathbf{n}, \bar{u}_j \rangle = 0$, we put $u_i^* = \bar{u}_i$, $\bar{u}_j^* = \bar{u}_j$. A schematic presentation is illustrated in Fig. 1.

With the general collision rules, we assume that the hard-disk gas is composed of identical particles of the same mass. The velocities of these particles are restricted, e. g., to $\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_p$, where p is a finite positive integer. The discrete number density of particles with the velocity $\mathbf{U}_i$ at a point $\mathbf{x}$ and time t is denoted by $\bar{N}_i(\mathbf{x}, t)$. If only nonlinear binary collisions are considered, then the evolution of $\bar{N}_i$ ($i = 1, \ldots, p$) is described by

$$\frac{\partial \bar{N}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \bar{N}_i = \sum_{j,k,l} \hat{A}_{kl}^{ij} [\bar{N}_k \bar{N}_l (1 + \sigma \bar{N}_i)(1 + \sigma \bar{N}_j) - - \bar{N}_i \bar{N}_j (1 + \sigma \bar{N}_k)(1 + \sigma \bar{N}_l)] \bar{N}_i \bar{N}_j), \quad (1)$$

where (k, l) are admissible sets of collisions, $i \in \Lambda = = \{1, \ldots, p\}$, and the summation is taken over all $j, k, l \in \Lambda$, where $\hat{A}_{kl}^{ij}$ are nonnegative constants satisfying $\hat{A}_{kl}^{ji} = \hat{A}_{kl}^{ij} = \hat{A}_{lk}^{ij}$ (indistinguishability of the particles in collision), $\hat{A}_{kl}^{ij}(\mathbf{u}_i + \mathbf{u}_j - \mathbf{u}_k - \mathbf{u}_l) = 0$ (conservation of momentum in the collision), and $\hat{A}_{kl}^{ij} = \hat{A}_{ij}^{kl}$ (microreversibility condition). Here, σ is a Pauli blocking parameter [4]: for $\sigma < 0$ (normally, $\sigma = -1$), we can obtain a gas of Fermi–Dirac particles; for $\sigma > 0$ (normally, $\sigma = 1$), we obtain a gas of Bose–Einstein particles; and for $\sigma = 0$, we obtain a gas of Boltzmann particles. The conditions stated for the discrete velocity above require elastic, binary collisions, such that the momentum and energy are preserved.

For binary collision only (two-disk encounter each time) with $\sigma = 0$, the equation of discrete kinetic models proposed in [4] is a system of $2n(= p)$ semilinear partial differential equations of the hyperbolic type ($i = 1, \ldots, 2n$,)

$$\frac{\partial}{\partial t} \bar{N}_i + \mathbf{U}_i \cdot \frac{\partial}{\partial \mathbf{x}} \bar{N}_i = \frac{2cS}{n} \sum_{j=1}^{n} \bar{N}_j \bar{N}_{j+n} - \bar{N}_i \bar{N}_{i+n}, \quad (2)$$

where

$$\mathbf{U}_i = c(\cos[\theta + (i-1)\pi/n], \sin[\theta + (i-1)\pi/n]),$$

$\bar{N}_i = \bar{N}_{i+2n}$ are unknown functions, c is the reference velocity modulus, S is an effective collision cross section for the system of two (rotating) disks, and θ is the free orientation parameter (the orientation starting from the positive x axis to the $\bar{u}_1$ direction and is relevant to the (net) induced scattering measured relative to the sound-propagating direction), which might be linked to the external field or the angular momentum or the rotation effects.

Because passing the plane (e.g., acoustic) wave causes a small departure from equilibrium (of Maxwellian type), resulting in an energy loss owing to internal friction and heat conduction, we linearize the above equations around a uniform Maxwellian state ($\bar{N}_0$) by setting

$$\bar{N}_i(t,x) = \bar{N}_0(1 + P_i(t,x)),$$

where P_i is a small perturbation. (Maxwellian is referred to an equilibrium state here.)

First, we have

$$\frac{\partial P_m}{\partial t} + \mathbf{U}_m \cdot \frac{\partial P_m}{\partial \mathbf{x}} + 2cS\bar{N}_0(P_m + P_{m+n}) = \frac{cS\bar{N}_0}{n}\sum_{k=1}^{2n}[P_k + P_{k+n}], \quad (3)$$

where, $m = 1, \ldots, 2n$. The linearized version of the above equation is (with $f_p = 2cS\bar{N}_0$)

$$\frac{\partial P_m}{\partial t} + \mathbf{U}_m \cdot \frac{\partial P_m}{\partial \mathbf{x}} + f_p(P_m + P_{m+n}) = \frac{f_p}{n}\sum_{k=1}^{2n} P_k. \quad (4)$$

In these equations, after replacing the index m with $m+n$ and using the identities $P_{m+2n} = P_m$, we have

$$\frac{\partial P_{m+n}}{\partial t} - \mathbf{U}_m \cdot \frac{\partial P_{m+n}}{\partial \mathbf{x}} + f_p(P_m + P_{m+n}) = \frac{f_p}{n}\sum_{k=1}^{2n} P_k. \quad (5)$$

Combining the above two equations by first adding and then subtracting, with

$$A_m = (P_m + P_{m+n})/2, \quad B_m = (P_m - P_{m+n})/2,$$

we have ($m = 1, \ldots, 2n$)

$$\frac{\partial A_m}{\partial t} - c\cos\left[\theta + \frac{(m-1)\pi}{n}\right]\frac{\partial B_m}{\partial x} + 2 f_p A_m = \frac{2 f_p}{n}\sum_{k=1}^{2n} A_k, \quad (6)$$

$$\frac{\partial B_m}{\partial t} + c\cos\left[\theta + \frac{(m-1)\pi}{n}\right]\frac{\partial A_m}{\partial x} = 0. \quad (7)$$

We note that $\partial P_m/\partial y = 0$ because P_m only varies along the wave propagating direction, the x-axis direction. From $P_{m+2n} = P_m$, with A_m and B_m mentioned above, we have $A_{m+n} = A_m$ and $B_{m+n} = -B_m$.

After some manipulations, we then have

$$\left(\frac{\partial^2}{\partial t^2} + c^2\cos^2\left[\theta + \frac{(m-1)\pi}{n}\right]\frac{\partial^2}{\partial x^2} + 4cS\bar{N}_0\frac{\partial}{\partial t}\right)R_m = \frac{4cS\bar{N}_0}{n}\sum_{k=1}^{n}\frac{\partial}{\partial t}R_k, \quad (8)$$

where $R_m = (P_m + P_{m+n})/2$, $m = 1, \ldots, n$, because $R_1 = R_m$ for $1 = m \pmod{2n}$.

Now we are ready to look for solutions in the plane-wave form

$$R_m = r_m \exp[i(kx - \omega t)], \quad m = 1, \ldots, n,$$

with $\omega = \omega(k)$. This could be related to the dispersion relations of one-dimensional forced plane-wave (e.g., ultrasound) propagation problem (of dilute monatomic hard-sphere gases). We then have

$$\left(1 + ih - 2\lambda^2\cos^2\left[\theta + \frac{(m-1)\pi}{n}\right]\right)r_m - \frac{ih}{n}\sum_{k=1}^{n} r_k = 0, \quad (9)$$

where $\lambda = kc/\sqrt{2}\omega$, $h = 4cS\bar{N}_0/\omega \propto 1/\mathrm{K}_n$ is the rarefaction parameter of the gas, K_n is the Knudsen number defined as the ratio of the mean free path in a hard-disk gase to the wave length of the plane (e.g., acoustic) wave (here $m = 1, \ldots, n$).

Let

$$r_m = \frac{\mathcal{C}_\prime}{1 + ih - 2\lambda^2\cos^2[\theta + (m-1)\pi/n]},$$

where $\mathcal{C}_\prime$ is an arbitrary unknown constant, since we are only interested in eigenvalues of above relation. The eigenvalue problem for the $2\times n$ velocity model reduces to $F_n(\lambda) = 0$, or

$$1 - \frac{ih}{n}\sum_{m=1}^{n}\frac{1}{1 + ih - 2\lambda^2\cos^2[\theta + (m-1)\pi/n]} = 0. \quad (10)$$

We solve the $n = 2$ case, i.e., the 4-velocity case. The admissible collision $(1,3) \leftrightarrow (2,4)$ for the system of rotating disks during a binary encounter is shown schematically in Fig. 2. The corresponding eigenvalue equations become algebraic equations with complex roots.

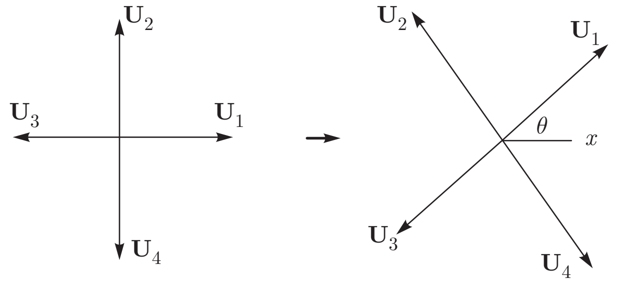

Fig. 2. Schematic plot for the regular scattering and the orientational scattering. Plane waves propagate along the x-direction. Binary encounters of $\mathbf{U}_1$ and $\mathbf{U}_3$ and their departures after head-on collisions ($\mathbf{U}_2$ and $\mathbf{U}_4$). The number densities N_i are associated with $\mathbf{U}_i$. θ is the free orientation parameter (the orientation starting from the positive x-axis to the $\mathbf{U}_1$ direction and is relevant to the (net) induced scattering measured relative to the plane wave-propagating direction), which might be linked to the external field or the angular momentum or the rotation effects

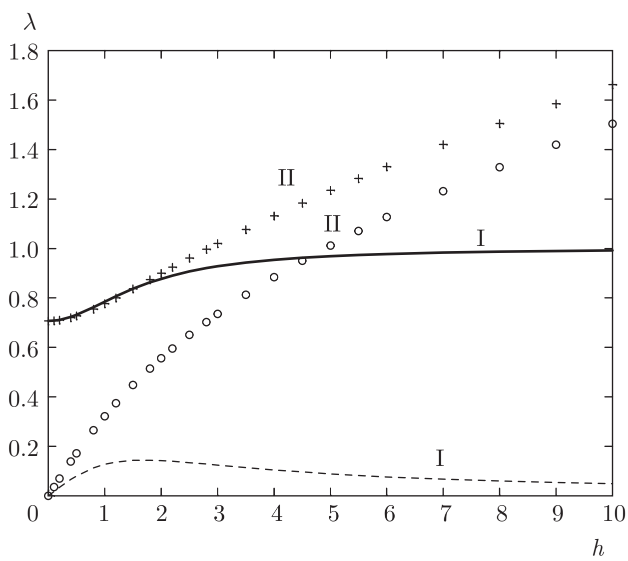

Fig. 3. Dispersion relations of branches I (solid curve is for dispersion, dashed curve is for attenuation) and II (crosses are for dispersion, circles are for amplification) over a range of h (rarefaction measure): $\lambda = \lambda_r + i\lambda_i$, λ_r is the phase speed dispersion, and λ_i is attenuation or amplification; $\theta = 0$ here. Branch II (diffusion mode) shows anomalous amplification

3. NUMERICAL RESULTS AND DISCUSSION

Using the numerical approach and with intensive validations, we can obtain the complex roots $\lambda = \lambda_r + i\lambda_i$ for the polynomial equations above. The roots are the values of the dimensionless dispersion (positive real part) and the attenuation or absorption (positive imaginary part). We plot those of $\theta = 0$ in Fig. 3. Curves of branch I follow the conventional dispersion relation of plane-wave (e. g., ultrasound) propagation in dilute monatomic hard-sphere gases [5]. We remind the readers that θ is relevant to the external field or the angular momentum or the rotation effects and possible effects of $\theta \neq 0$ can be traced in [4].

Curves of branch II, however, show an entirely different trend. The dispersion part seems to follow the diffusion mode reported in [5]. It increases, but never reaches a limit. The anomalous attenuation or amplification might be due to the intrinsic resonance (an eigen-oscillation) or the implicit behavior of the angular momentum relation during an encounter of two rotating disks (with opposite-sign rotating directions or the angular momenta such that the total (net) angular momentum is zero); the latter is absent or of no need in the formulation of 2-body collisions. We note that the directions of the rotation axes of two disks might be opposite in sign rather than being of the same sign. We do not know at present whether the former or the latter can favor the anomalous attenuation or amplification.

We noticed that some researchers argue that there must be some source of free energy to drive the growth. Meanwhile, as argued in [6], the rotational energy of a young pulsar with period P that remains after a supernova explosion is estimated to be $2 \times 10^{50}(10\text{ ms}/P)^2$ erg. It is an additional energy reservoir for particle acceleration (we note that for the energy in a source capable of accelerating particles to 10^{20} eV, the energy in the magnetic field < 0.1 G is $\gg 10^{57}$ erg [1]), and in particular it could be the source of very high energy electron–positron pairs. However, the cosmic rays passing through this plane-wave-amplified channel might be accelerated (including neutrinos)! In fact, as remarked in [7], "Cosmic rays must be involved in the general Galaxy rotation ...". Finally, the results presented in Fig. 3 show that the possible acceleration (due to amplification) is proportional to $h \propto c\,S\,\bar{N}_0$, which is also proportional to Z (a nucleus charge number, considering the effective scattering cross section S). This matches qualitatively with [8]: the PAO collaboration data strongly favor the nuclei composition becoming progressively heavier at energies 4–40 EeV.

We discuss an example of a rotating system. The existing observations of a few events at energies greater than 10^{20} eV require the birth of some magnetars with $\eta_1\Omega_4^2 > 0.1/Z$; then each particle gains the energy [9]

$$E(\Omega) = Q\eta\Phi_{mag} = 3 \cdot 10^{21} Z\eta_1\Omega_4^2\mu_{33} \ \text{ eV},$$

where Q is the charge, μ_{33} is the dipole moment, and Ω is the angular velocity of the underlying rotating system or star. We note that $\Phi_{mag} = \Psi_{mag}/R_L$, where

$\Psi_{mag} = R_L^2 B(R_L)$ is the magnetic flux in the open field that extends beyond the light cylinder, located at $r = R_L = c/\Omega = 30/\Omega_4$ km, that connects the magnetosphere to the outside world, with $\Omega = 10^4 \Omega_4$ s^{-1} (here c is the light speed in the vacuum). Here, η is the fraction of the open field line voltage experienced by each particle on its way from the star to the outside world and $\eta_1 \equiv \eta/0.1$. Nevertheless, the Greisen–Zatsepin–Kuzmin losses limit the observable volume to a size $V_E \geq 50$ Mpc, for maximum energies appropriate to the example mentioned here [9]. The number of source events per unit time that we can detect, in general, is

$$\dot{n}_0 = \frac{4\pi}{3} V_E^3 n_g \nu_m^{fast},$$

where n_g is a galaxy density and ν_m^{fast} is the birthrate of fast magnetars.

4. CONCLUSION

We adopt the quantum kinetic approach that includes a highly nonlinear discrete Uehling–Uhlenbeck operator to investigate the possible origin of the acceleration of cosmic rays. Based on our numerical results, we propose that the acceleration of cosmic rays occurs once cosmic rays pass through a kind of amplification channel induced by a system of rotating gases (or disk-like gases). We shall investigate other interesting issues [10–17] in the near future.

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