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LOCALIZED MODES IN OPTICS OF PHOTONIC LIQUID CRYSTALS WITH LOCAL ANISOTROPY OF ABSORPTION

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The localized opical modes in spiral photonic liquid crystals are theoretically studied for the certainty at the example of chiral liquid crystals (CLCs) for the case of CLC with an anisotropic local absorption. The chosen here model (absence of dielectric interfaces in the studied structures) allows one to get rid off the polarization mixing at the surfaces of the CLC layer and the defect structure (DMS) and to reduce the corresponding equations to the equations for the light of diffracting in the CLC polarization only. The dispersion equations determining connection of the EM and DM frequencies with the CLC layer parameters (anisotropy of local absorption, CLC order parameter) and other parameters of the DMS are obtained. Analytic expressions for the transmission and reflection coefficients of CLC layer and DMS for the case of CLC with an anisotropic local absorption are presented and analyzed. It is shown that the CLC layers with locally anisotropic absorption reduce the EM and DM lifetimes (and increase the lasing threshold) by the way different from the case of CLC with an isotropic local absorption. Due to the Borrmann effect revealing of which is different at the opposite stop-band edges in the case of CLC layers with an anisotropic local absorption the EM life-times for the EM frequencies at the opposite stop-bands edges may be significanately different. The options of experimental observations of the theoretically revealed phenomena are brierfly discussed.

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1. INTRODUCTION

Recently there have been much activities in the field of localized optical modes, in particular, edge modes (EM) and defect modes (DM) in chiral liquid crystals (CLCs) mainly due to the possibilities to reach a low lasing threshold for the mirrorless distributed feedback (DFB) lasing [1–4] in CLCs. The EM and DM existing as a localized electromagnetic eigenstate with its frequency close to the forbidden band gap and in the forbidden band gap, respectively, were investigated initially in the periodic dielectric structures [5]. The corresponding EM and DM in CLCs, and more general in spiral media, are very similar to the EM and DM in one-dimensional scalar periodic structures. They reveal abnormal reflection and transmission [1, 2], and allow distributed feedback (DFB) lasing at a low lasing threshold [3].

Almost all theoretical studies of the EM and DM in chiral and scalar periodic media were performed by means of a numerical analysis with the exceptions [6, 7], where the known exact analytical expression for the eigenwaves propagating along the helix axis [8–10] were used for a general study of the DM. The approach used in [6, 7] looks as fruitful one because it allows to reach an easy understanding of the DM and EM physics, and this is why it deserves further implementation in the study of the EM and DM.

In the cited studies the optical absorption in CLC was regarded as an isotropic. However the case of isotropic absorption in CLC does not cover all options happening in CLC. For example, quite common is alignment of dye molecules with clearely presented absorption lines in liquid crystals. If the director distribution in a liquid crystal sample is not homogenious (what is the case of CLC) a local anisotropy of absorption in the sample may exist and manifest itself in some circumstances [11, 12]. The corresponding effects depend on the value of liquid crystal order parameter and disappear if the order parameter value is zero,

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i. e., at the point of liquid crystal phase transition to liquid. The corresponding effects in the CLC transmission and reflection spectra, in particular Borrmann effect, were studied both experimentally [13, 14] and theoretically [11–13]. In the present paper the influence of local anisotropy of absorption on characteristics of localized modes is theoretically studied. Below analytical solutions for the EM and DM (associated with a local anisotropy of absorption in CLCs) are presented and some limiting cases simplifying the problem are considered.

2. THE BOUNDARY-VALUE PROBLEM

To investigate EM in a CLC we have to consider a boundary problem, i. e., transmission and reflection of light incident on a CLC layer along the spiral axis [10–12]. We assume that the CLC is represented by a planar layer with a spiral axis perpendicular to the layer surfaces (Fig. 1). We also assume that the average CLC dielectric constant coincides with the dielectric constant of the ambient medium. This assumption practically prevents conversion of one circular polarization into another at the layer surfaces [11, 12], and allows to have only two eigenwaves with diffracting circular polarization taken into account.

The procedure of obtaining the reflection R and transmission T coefficients for a CLC layer with a local anistropy of absorption is similar to the one for a nonabsorbing CLC however some complications arise due to the fact that the elements of CLC dielectric tensor are complex quantities now.

Let us begin from discussing of the dielectric tensor of a substance with locally anisotropic absorption. The principal values of corresponding dielectric tensor are complex and have different imaginary parts depending in the case of CLC on the liquid crystal order parameter S [10]. In general case all three imaginary parts are different. For a CLC the imaginary parts are different for $S = 1$ and are equal for $S = 0$. Returning to the CLC we assume for simplification of the problem that only one principal value of dielectric tensor is complex at $S = 1$. This corresponds to the assumption that the absorption in CLC is due only to dye's molecules and at $S = 1$ a complete ordering of the dye's molecules occurs. We assume also that the axis corresponding to the real principal value of the dielectric tensor is directed along the spiral axis and two other axes are rotating around the spiral axis. These rotating axes determine the local, depending on the coordinate along spiral axis, direction of absorption anisotropy. Now we have to insert complex principal values of dielectric tensor in the expressions for dielectric anisotropy [11, 12, 15]. As the result the dielectric anysotropy $\Delta\varepsilon$ (see below) becomes a complex quantity. Luckelly, the expressions for reflection and transmission coefficients for light propagating in a CLC layer along the cholesteric axis [11, 12, 15] are exact and are applicable to the case of anisotropic absorption which is under consideration here.

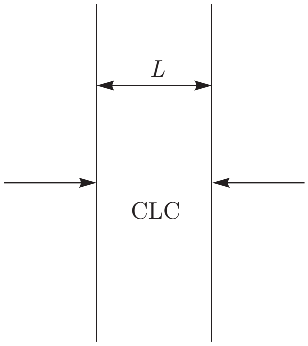

Fig. 1. Schematic of the boundary problem for edge mode

Below in the calculations and the formulas for limiting values of the problem parameters is assumed that the most favorable for the Borrmann effect situation occurs. Namely, it is assumed that the CLC absorption is due to the dye molecules solved in the CLC only and the corresponding molecular dye absorption oscillators are directed along the long molecular exes and, more over, their orientational ordering may be complete, i. e., the order parameter S can reach 1. Under the accepted assumption the imaginary parts of principal values of the dielectric tensor for S differing from one are by the following way expressed via the imaginary part of the single complex principal value e_1, or e_2 at $S = 1$ for the cases of absorption oscilatore axis parallel and perpendicular to the long molecular axis, respectively:

$$\begin{gathered}\mathrm{Im}[\varepsilon_{\|}] = \mathrm{Im}[\epsilon_1](1+2S)/3,\\ \mathrm{Im}[\varepsilon_{\perp}] = \mathrm{Im}[\epsilon_1](1-S)/3\end{gathered} \tag{1a}$$

and the local dielectric anisotropy is

$$\Delta\varepsilon = \varepsilon_{\|} - \varepsilon_{\perp} = \mathrm{Re}[\varepsilon_{\|} - \varepsilon_{\perp}] + i\epsilon_1 S, \tag{2a}$$

for the oscilatore axis parallel to the long molecular axis;

$$\begin{gathered}\mathrm{Im}[\varepsilon_{\perp}] = \mathrm{Im}[\epsilon_2](1+2S)/3,\\ \mathrm{Im}[\varepsilon_{\|}] = \mathrm{Im}[\epsilon_2](1-S)/3\end{gathered} \tag{1b}$$

and the local dielectric anisotropy is

$$\Delta\varepsilon = \varepsilon_{\|} - \varepsilon_{\perp} = \mathrm{Re}[\varepsilon_{\|} - \varepsilon_{\perp}] - i\,\mathrm{Im}[\epsilon_2]S, \tag{2b}$$

for the oscillator axis perpendicular to the long molecular axis.

In what will be followed below we shall present the results related to the first option, for the case of absorption oscillator axis parallel to the long molecular axis. If for the first option the Borrmann effect reveals itself close to the high frequency stop-band edge, for the second option a similar revealing of the Borrmann effect happens close to the low frequency stop-band edge [11, 12].

In the view of Refs. [10–12, 15, 16], we state here only the final expressions for the amplitude transmission T and reflection R coefficients for light incident on a CLC layer of thickness L. These are given as

$$R(L) = i\delta \sin qL / \{(q\tau/\kappa^2)\cos qL + \\ + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\}, \quad (3a)$$

$$T(L) = \exp[i\kappa L](q\tau/\kappa^2) / \left\{(q\tau/\kappa^2)\cos qL + \\ + i[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1]\sin qL\right\}, \quad (3b)$$

where

$$q = \kappa\left\{1 + (\tau/2\kappa)^2 - [(\tau/\kappa)^2 + \delta^2]^{1/2}\right\}^{1/2} \quad (4)$$

and,

$$\epsilon_0 = (1 + i\gamma)\,\mathrm{Re}[\varepsilon_\parallel + \varepsilon_\perp]/2,$$

$$\delta = \mathrm{Re}[\varepsilon_\parallel - \varepsilon_\perp]/\mathrm{Re}[\epsilon_0] + i\epsilon_1 S/\mathrm{Re}[\epsilon_0]$$

and

$$\gamma = \epsilon_1(1 - S)/3\,\mathrm{Re}[\epsilon_0]$$

is the parameter describing the locally isotropic part of absorption. Here δ is the dielectric anisotropy with $\varepsilon_\parallel$ and $\varepsilon_\perp$ as the local principal values of the CLC dielectric tensor [10–12], $\kappa = \omega\epsilon_0^{1/2}/c$ with c as the speed of light, and $\tau = 4\pi/p$ with p as the cholesteric pitch.

Note, that in the case of absorption due solely to the dye moleciles solved in CLC only one principal value of dielectric tensor is complex at $S = 1$ if the absorption oscillators of dye molecules are directed along a sole axis in the molecule and the degree of orientational oreder of the dye molecules is the same as for CLC. In principle, the orientational order parameter for dye molecules may be different from the CLC order parameter S.

As one expects the CLC local absorption anisotropy reveals itself in the reflection and transmission spectra due to the Borrmann effect (well known in the X-ray diffraction) [11–13] revealing itself in the absorption suppression at one of the stop band edges. Contrary to the case of isotropic absorption with the reflection and transmission spectra being symmetric relative the stop band centre in the case of locally anisotropic absorption in CLC the spectra are nonsymmetric relative the stop band centre. In the case of the CLC with a locally anisotropic absorption [11–13] the absorption suppression at one of the stop band edges, as in the X-ray case, also takes place and is due to orthogonality of the whole optical electric field in the sample to the dye molecular absorbing oscillators at the frequency of one stop band edge. The Figs. 2 and 3 demonstrate the Borrman effect in reflection and transmission, respectively, in a CLC layer with locally anisotropic absorption (the calculations performed according Eqs. (3) for the order

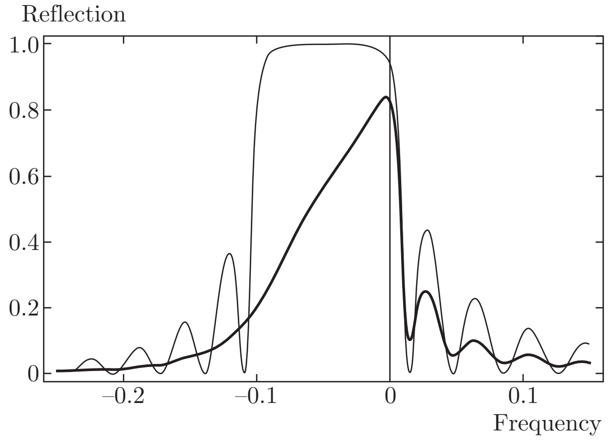

Fig. 2. Reflection (thin curve corresponds to the complete absence of absorption) versus the dimensionless frequency $v = \delta[2(\omega - \omega_B)/(\delta\omega_B) - 1]$ used also in all figures below for locally anisotropic absorption in CLC layer (see Fig. 1) at $\delta = 0.05 + 0.03i$, $l = 300$ ($l = L\tau = 2\pi N$, where N is the director half-turn number at the CLC layer thickness L)

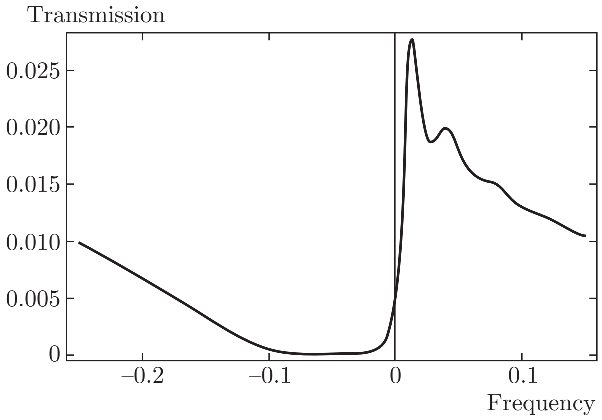

Fig. 3. Transmission versus frequency for locally anisotropic absorption in CLC layer (see Fig. 1) at $\delta = 0.05 + 0.015i$, $l = 300$

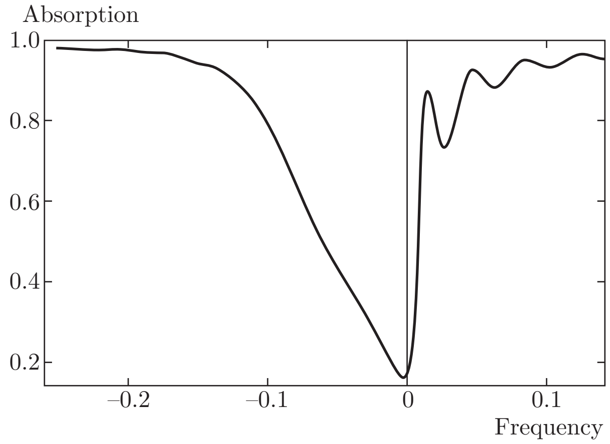

Fig. 4. Total absorption $1–R–T$ versus frequency for locally anisotropic absorption in CLC layer (see Fig. 1) at $\delta = 0.05 + 0.015i$, $l = 300$

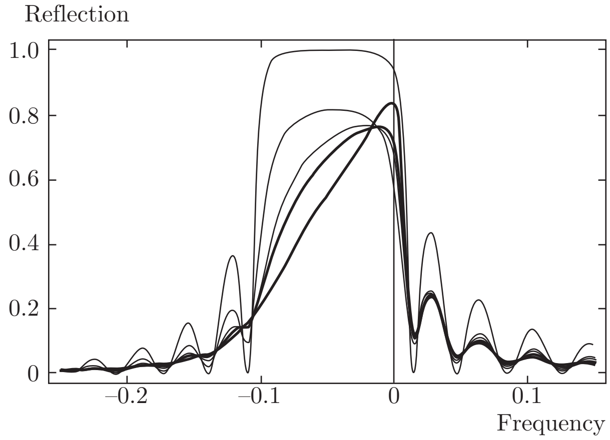

Fig. 5. Reflection for a CLC layer with local anisotropy of absorption versus the frequency for $S = 0,\ 0.3,\ 0.5,\ 1$ (the curve thickness is growing with increase of S) at $\mathrm{Im}[\epsilon_1]/\mathrm{Re}[\epsilon_0] = 0.03$, $\delta = 0.05$, $l = 300$ (the thinest curve corresponds to complete absence of absorption)

parameter $S = 1$, however, here and below the notations R and T are different from the ones in the Eqs. (3) and relate to the intensity reflection and transmission coefficients).

The Borrmann effect demonstrates itself most clearly in the spectra of total absorption in a sample given by $1–R–T$ (see Fig. 4).

The Figs. 2–4 demonstrate increasing of reflection and transmission at one stop band edge compared to the other stop band edge and a strong suppression of the absorption at this stop band edge. To illustrate the dependence of the Borrmann effect on the value of the order parameter S Figs. 5, 6 present the calculation results for the reflection and total absorption $1–R–T$ for the values of $S = 1$, 0.5, 0.3, 0, respectively. These figures show that the suppression of absorption effect is decreasing with decrease of the CLC order parameter S. Here should be mentioned that in the above calculations was assumed that the pitch value and δ are the same for all values of S which means that the accepted assumption allows the dye molecules order parameter variations to be different from the corresponding variations of the CLC molecule's order parameter. In real situations the variations of the order parameter is connected usually with the temperature variations (and as it is known the pitch is a temperature dependent CLC parameter). Figure 7 demonstrates that the total absorption at the EM frequency decreases with the CLC layer thickness increase (L at the Fig. 7 is 10 times larger than at the Fig. 6). Figure 8 demonstrates that the transmission T at the EM frequency decreases with the CLC order parameter S increase and a more pronounced decrease happens at the stop-band edge opposite to the edge where the reflection is increased due to the Borrmann effect. It is worthing to note that the calculation results presented at the Figs. 5–8 take into account that the all three principal values of the dielectric tensor are complex values and only for the order parameter $S = 1$ a single principal value of the local dielectric tensor corresponding to the long molecular axes direction is complex.

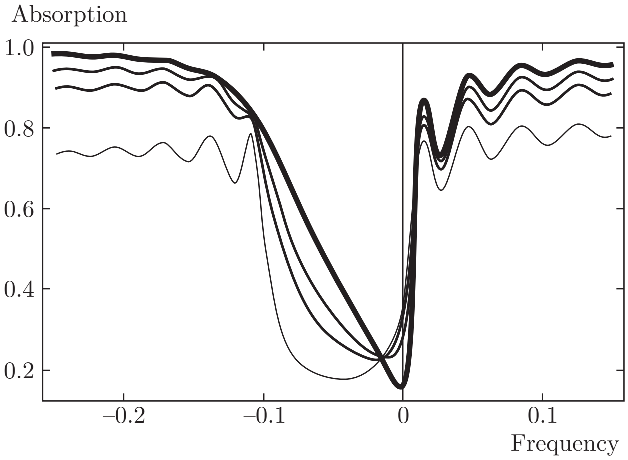

Fig. 6. Absorption for a CLC layer with local anisotropy of absorption versus the frequency for $S = 0,\ 0.3,\ 0.5,\ 1$ (the curve thickness is growing with increase of S) at $\mathrm{Im}[\epsilon_1]/\mathrm{Re}[\epsilon_0] = 0.03$, $\delta = 0.05$, $l = 300$

The calculated at Figs. 6, 7 absorption spectra are directly related to the luminescence spectra if the dye absorption line overlaps with the stop-band. Because

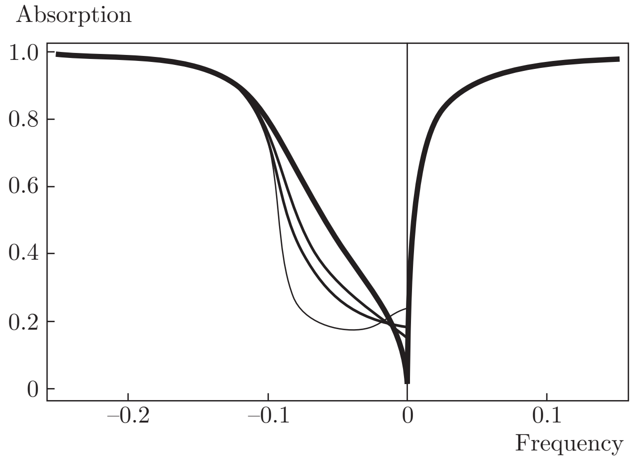

Fig. 7. Absorption for a CLC layer with local anisotropy of absorption versus the frequency for $S = 0$, 0.3, 0.5, 1 (the curve thickness is growing with increase of S) at $\mathrm{Im}[\epsilon_1]/\mathrm{Re}[\epsilon_0] = 0.03$, $\delta = 0.05$, $l = 3000$

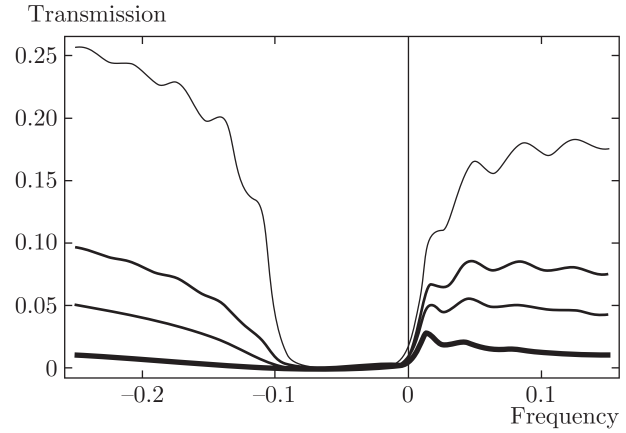

Fig. 8. CLC layer transmission versus frequency for locally anisotropic absorption in CLC layer for $S = 0$, 0.3, 0.5, 1 (the curve thickness is growing with increase of S) at $\mathrm{Im}[\epsilon_1]/\mathrm{Re}[\epsilon_0] = 0.03$, $\delta = 0.05$, $l = 300$

the absorbed optical photons of the incident beam are reemitted as luminescence photons the absorption maximum results in a luminescence intensity maximum, naturally, at the frequency being shifted in the direction of lower frequencies. By this way the luminescence spectra occure to be dependent on the local absorption anisotropy revealing different luminescence intensity for the pumping wave frequency coinciding with the opposite stop-band edges frequencies.

3. EDGE MODE

In a non-absorbing CLC $\gamma = 0$ in the general expression for the dielectric constant $\varepsilon = \epsilon_0(1 + i\gamma)$. The calculations of the reflection and transmission coefficients as functions of the frequency give the well-known results [8–12] different from the curves presented at the Figs. 3–6, in particular, at $\gamma = 0$, $T + R = 1$ for all frequencies. Let us study how a locally anisotropic absorption in CLC influences on the properties of EM existing at discrete frequencies outside the stop band [15, 16], in particular, on the EM life-time. To do this we have to solve the corresponding dispersion equation. Following the reference [15] one gets the dispersion equation in the form similar to the case of zero absorption

$$\mathrm{tg}\, qL = i(q\tau/\kappa^2)/\left[(\tau/2\kappa)^2 + (q/\kappa)^2 - 1\right], \qquad (5)$$

however the entering in this equation parameters occur to be given by the Eqs. (1), (2), (4), i. e., are dependent on some additional quantities compared to the case of zero absorption

Solutions to Eq. (5) for the EM frequencies ω_{EM} situated outside the stop band edges are discrete and in the general case can be found only numerically. The EM frequencies ω_{EM} turn out to be complex and may be be presented as $\omega_{EM} = \omega(1 + i\Delta)$, where in real situations Δ is a small parameter determining the EM life-time. From a general analysis of Eq. (5) one concludes that the EM life-times at the frequencies outside the opposite stop band edges are different in the case of locally anisotropic absorption (there is no symmetry in theier values relative to the stop band centre).

Fortunately, an analytical solution can be found for thick CLC layers and a sufficiently small Δ ensuring the condition $L\,\mathrm{Im}(q) \ll 1$. In this case, ω and Δ are determined by the conditions $qL = n\pi$ and the EM life-times at the case of isotropic absorption may be presented as:

$$\tau_{EM} = 1/\Delta\omega_{EM} = \\ = 2\omega_{EM}\left[\delta(n\pi)^2(\pi\delta L/p)^{-3} + \gamma\right], \qquad (6)$$

where the integer number n is the edge mode number [15] ($n = 1$ corresponds EM frequency (reflection coefficient minimum) closest to the stop-band edge) and γ is the parameter determining the isotropic fraction of absorption in CLC. So, in the case of isotropic absorption the localized mode life-time is limited by $2/\omega_{EM}\gamma$.

In the case of a locally anisotropic absorption in CLC the EM life-time depends on the EM frequency position ralative to the stop-band center and may exeed the value given by Eq. (6) due to the Borrmann

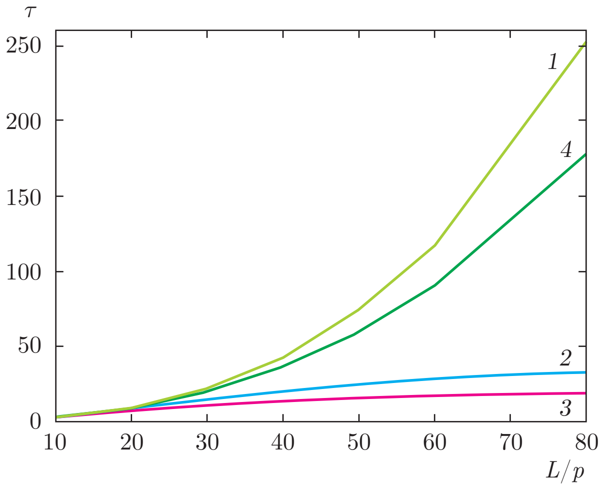

Fig. 9. Calculated life-time of the first EM (in the units $\epsilon_0^{1/2}$ p/c) versus the CLC layer thickness: *1* — zero absorption; *2* — isotropic absorption; *3* — locally anisotropic absorption $\mathrm{Im}[\epsilon_1] = 0.01$, $\mathrm{Im}[\epsilon_2] = 0$; *4* — locally anisotropic absorption $\mathrm{Im}[\epsilon_1] = 0$, $\mathrm{Im}[\epsilon_2] = 0.01$ (see Eqs. (1) at $S = 1$), $\delta = 0.05$

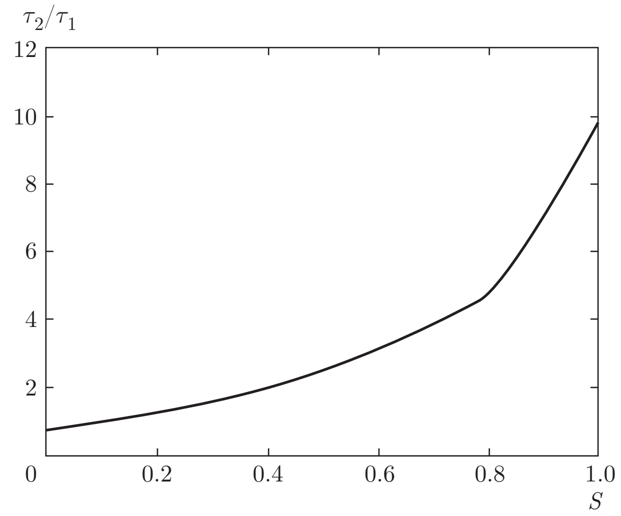

Fig. 10. Calculated life-times ratio of the first EM at the high and low frequency stop band edge versus the value of the order paprameter S for locally anisotropic absorption $\mathrm{Im}[\epsilon_1] = 0$, $\mathrm{Im}[\epsilon_2] = 0.02$ (see Eqs. (1)), $\delta = 0.05$, $L/p = 80$

effect. For example, if the CLC layer thickness L is sufficiently large and the condition $L\,\mathrm{Im}(q) \ll 1$ holds the ratio of life-times at the opposite stop band edge frequencies may be estimated by the following expression:

$$\tau_B/\tau_{AB} = [\omega_{EM}(\epsilon_0\gamma + 2\epsilon_0\,\mathrm{Im}\,\delta) + \\ + 1/\tau_m]\,/(\omega_{EM}\epsilon_0\gamma + 1/\tau_m), \quad (7)$$

where τ_B, τ_{AB}, τ_m are the life-time at the stop-band edge where the Borrmann effect happens, at the opposite edge, at the edges in the case of non-absorbing CLC (see (6) at $\gamma = 0$), respectively, and γ is determining the isotropic component of absorption. If γ is approaching zero what is happening when the order parameter is approaching 1 (if we neglect all sources of absorption except the dye) τ_B is coinciding with τ_m given by (6) at $\gamma = 0$ what corresponds suppression of absorption for EM at the stop-band edge frequency in the case of CLC with local anisotropy of absorption. Note, to be accurate, that a complete suppression of absorption for EM is achivable in the limit of infinity thick CLC layer only. At the opposite stop-band edge frequency the absorption is enhanced and the EM life-time being proportional to $1/2\epsilon_0\,\mathrm{Im}\,\delta$ is shorter than τ_m.

The results of numerical solutions to Eq. (5) for the EM frequencies ω_{EM} determining the EM life-time at the low frequency stop band edge for nonabsorbing, isotropically and locally anisotropic absorbing CLC are presented at the Figs. 9, 10. The Fig. 9 demonstrates suppression of absorption at $\epsilon_1 > \epsilon_2$ (curve *4* for the absorption oscillators directed along axis *2*) and absorption enhancement (curve *3* for the absorption oscillators directed along axis *1*) resulting in a shorter life-time for this case than for isotropic absorption (curve *2*). The Fig. 10 presents dependence of the life-times ratio of the first EM at the high and low frequency stop band edge on the value of the order parameter S. It shows that the growth of the the EM life-time at $S = 1$ due to the Borrmann effect at one stop band edge compared to the oppsite one disappears at $S = 0$.

What is concerned of the luminescence spectra if the dye absorption line overlaps with the stop-band the intensity of the luminescence is enhanced for the pump wave frequency at the stop band edge corresponding to the shorter EM life-time and suppressed for the pump wave frequency at the stop band edge corresponding longer EM life-time. That gives the estimate of the pumping wave intensity ratio ensuring equal luminescence intensity for the pump wave frequency coinciding with the opposite stop-band edges according to the Eq. (7) i. e., $I_B/I_{AB} = \tau_B/\tau_{AB}$, where I_B/I_{AB} are the puping wave intensities ensuring equal luminescence intensity at the opposite stop band edges.

4. LASING THRESHOLD AT LOCAL ANISOTROPY OF ABSORPTION

The reflection, transmssion and absorption spectra studied above give hints that the DFB lasing in the case of a locally anisotropic absorption in CLC is also significantly influenced by the anisotropy. To study the lasing threshold we have to solve dispersion equation at the EM frequency relative to the negative imaginary part of dielectric tensor.

The imaginary addition to dielectric tensor in the case of assumed absorption isotropy may be taken into account by introducing into the dielectric tensor a factor of the form $(1 - ig)$ where g is a small positive quantity. In the case of nonabsorbing CLC the threshold problem was studied in [15] and in the limit of a thick CLC layer the threshold is given by the following expression:

$$g = \delta(n\pi)^2(\delta L\tau/4)^{-3}. \tag{8}$$

In the case of isotropically absorbing CLC in the same limit the threshold is given by the next expression:

$$g = \delta(n\pi)^2(\delta L\tau/4)^{-3} + \gamma, \tag{9}$$

where γ is a small positive parameter determiming the isotropic absorption in CLC (see Eq. (4)).

So, naturally, the threshold gain becomes higher if an absorption in the CLC exists. In the case of a locally anisotropic absorption the values of the threshold gain are dependent on the order parameter S and are different for the EM frequencies at the opposite stop-band edges. At one edge (where the Borrmann effects reveals) it may approach to the value given by the Eq. (8) and at the opposite stop-band edge the threshold gain may be essentially higher. If the absorption is due only to the dye molecules the threshold gain becomes dependent on the order parameter S and its minimal value is given by the following expression

$$g = \delta(n\pi)^2(\delta L\tau/4)^{-3} + \epsilon_1(1 - S)/3\,\mathrm{Re}[\epsilon_0], \tag{10}$$

where ϵ_1 is the imaginary part of the dielectric tensor principal value at $S = 1$ determined by the dye molecules absorption, i. e., under the asumption that the absorption is due only to the dye molecules. It is useful to realize that the lazing thresholds corresponding to Eqs. (9), (10) relate to a vanishingly small population inversion of the lasing transition levels (vanishingly low amplification at the lasing frequency).

Note, that anomalously strong absorption effect [11, 17] at the frequency of pumping wave may influence the lowering of lasing threshold gain in the case of a locally anisotropic absorption in CLC even stronger than in the case of an isotropic absorption [18, 19]. Really, if the absorption oscillators (at the pumping wave frequency) in the dye molecules and the oscilators corresponding to the lasing frequency have the same orientations the lasing wave absorption will be suppressed and the pumping wave absorption will be enhanced if the lasing frequency coincides with the low frequency stop-band edge where the Borrman effect takes place and the pumping wave frequency coincides with the high frequency stop-band edge where the absorption is enhanced.

5. DEFECT MODE

The defect mode studied recently by many authors [20–29] is a localized solution of the boudary problem for the structure (DMS) shown at the Fig. 11. The solution to the boundary problem is carried out in the similar way as for a CLC layer above. As such, we give below the final results for the case of a locally anisotropic absorption in CLC layers (all the simplifications assumed above for the CLC layer are implemented for the DMS too).

There is an option to obtain formulas determining the optical properties of the structure depicted in Fig. 11 via the solutions found for a single CLC layer [15]. If one uses the expressions for the amplitude transmission $T(L)$ and reflection $R(L)$ coefficients for a single cholesteric layer (3a) and (3b), the transmission and reflection intensity coefficients for the whole structure may be presented in the following forms:

$$T(d,L) = \\ = \left|[T_e T_d \exp(ikd)] \,/\, [1 - \exp(2ikd) R_d R_u]\right|^2, \tag{11}$$

$$R(d,L) = |\{R_e + R_u T_e T_u \times \\ \times \exp(2ikd)/\,[1 - \exp(2ikd) R_d R_u]\}|^2, \tag{12}$$

where d is the defect layer thickness, $R_e(T_e)$, $R_u(T_u)$, and $R_d(T_d)$ (Fig. 11) are the amplitude reflection (transmission) coefficients of the CLC layers (Fig. 1) for the light incidences on the outer (top) layer surface, the inner top CLC layer surface from the inserted defect layer, and the inner bottom CLC layer surface

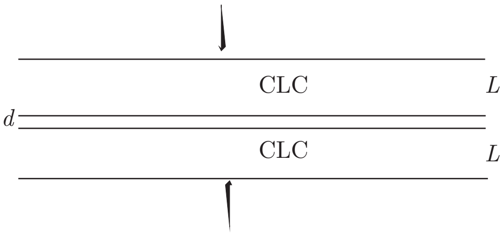

Fig. 11. Schematic of the structure with a defect layer (DMS)

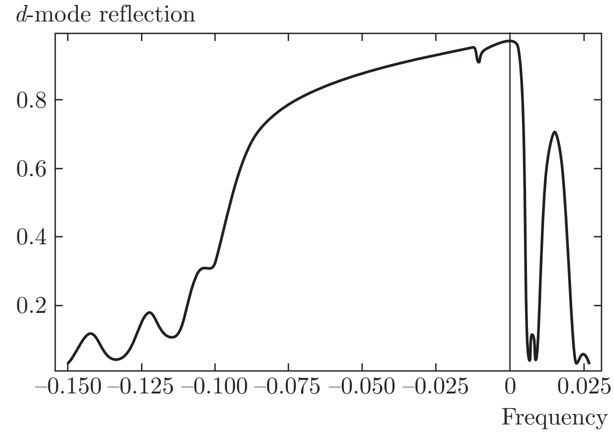

Fig. 12. Borrmann effect in reflection for DMS (see Fig. 11) for locally anisotropic absorption in CLC at $\delta = 0.05 + 0.003i$, $N = 75$, $d/p = 0.1$

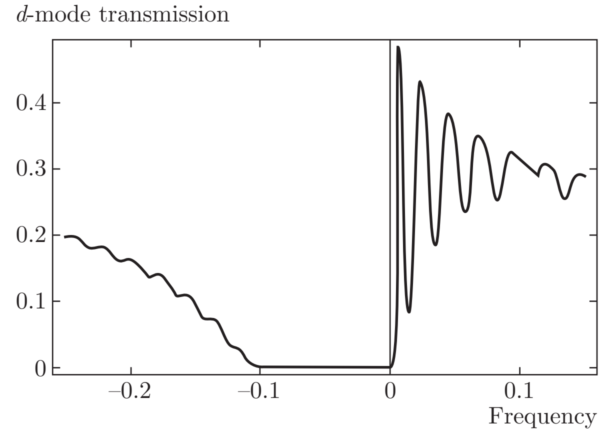

Fig. 13. Borrmann effect in transmission for DMS for locally anisotropic absorption in CLC at $\delta = 0.05 + 0.003i$, $N = 75$, $d/p = 0.1$

from the inserted defect layer, respectively. It is assumed in deriving Eqs. (11) and (12) that the external beam is incident on the structure (Fig. 11) from the above only.

Because we are concentrated on CLCs with a locally anisotropic absorption the parameters applied in the expressions for the DMS reflection (12) and transmission (11) coefficients have to be taken from Eqs. (1), (2). The cases of DM with nonabsorbing and isotropically absorbing CLC were studied in [20–22].

Similarly to the case of EM, the DM frequency ω_D is determined by zeros of the determinant of the system (disperesion equation) corresponding to the boundary-value solution for the structure depicted in the Fig. 11 [20–22]. The corresponding disperesion equation is given as

$$\Big\{\exp(2ikd)\sin^2 qL - \exp(-i\tau L)\Big[(\tau q/\kappa^2)\cos qL + {} + i\left((\tau/2\kappa)^2 + (q/\kappa)^2 - 1\right)\sin qL\Big]^2/\delta^2\Big]\Big\} = 0. \qquad (13)$$

It is to be noted that, for a non-absorbing CLC, at a finite length L the Eq. (13) does not reach zero for a real value of ω. However, it can reach zero when ω is complex.

As we expect the local absorption anisotropy reveal itself in DMS reflection and transmission spectra due to the Borrmann effect [11–13]. The Fig. 12 demonstrate Borrmann effect in reflection for a DMS with a local absorption anisotropy. The Fig. 12 presenting the calculation results shows that the stop-band edges become nonequivalent in scattering spectra (for no-nabsorbing CLC or CLC with isotropic absorption the reflection is symmetric relative to the stop-band center).

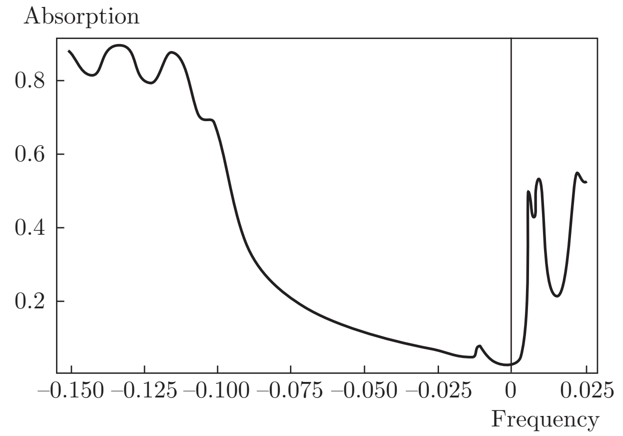

Fig. 14. Total absorption in DMS for locally anisotropic absorption in CLC (see Fig. 11) at $\delta = 0.05 + 0.003i$, $N = 75$, $d/p = 0.1$

The reflection coefficient value close to the frequencies of one stop-band edge is much more large than the one close to the frequencies of the other stop-band edge. The Fig. 13 demonstrates Borrmann effect in the transmission for a DMS with a local absorption anisotropy. Figure 13 shows that the stop-band edges are also nonequivalent in transmission spectra. The transmission coefficient value close to the same frequency as for the reflection stop-band edge showing an enhancement of reflection is much larger than close to the frequency of the opposite stop-band edge.

Naturally, that the total absorption (1–R–T) has to be different at the stop-band edges.

The Fig. 14 demonstrates that due to the Borrmann effect existing in transmission and reflection spectra for

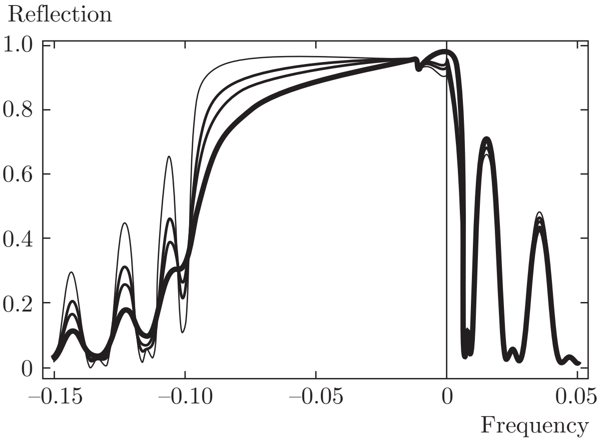

Fig. 15. Reflection for DMS (see Fig. 11) at locally anisotropic absorption in CLC for $S = 0,\ 0.3,\ 0.5,\ 1$ (the curve thickness is growing with increase of S) at $\mathrm{Im}[\epsilon_1]/\mathrm{Re}[\epsilon_0] = 0.003$, $\delta = 0.05$, $N = 75$, $d/p = 0.1$

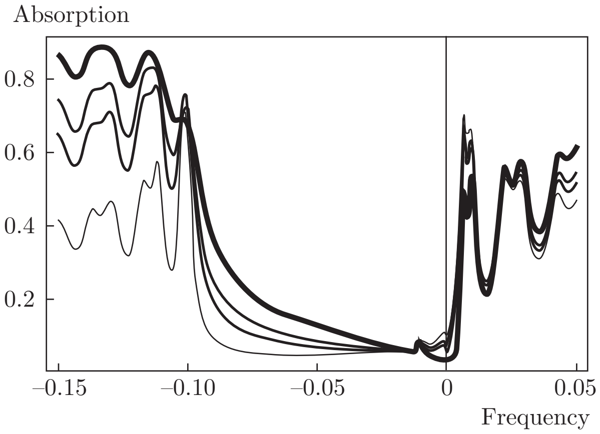

Fig. 17. Absorption for DMS at locally anisotropic absorption in CLC for $S = 0,\ 0.3,\ 0.5,\ 1$ (the curve thickness is growing with increase of S) at $\mathrm{Im}[\epsilon_1]/\mathrm{Re}[\epsilon_0] = 0.003$, $\delta = 0.05$, $N = 75$, $d/p = 0.1$

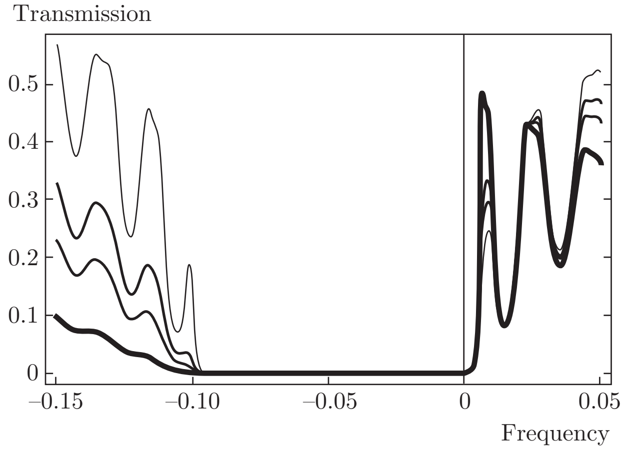

Fig. 16. Transmission for DMS at locally anisotropic absorption in CLC for $S = 0,\ 0.3,\ 0.5,\ 1$ (the curve thickness is growing with increase of S) at $\mathrm{Im}[\epsilon_1]/\mathrm{Re}[\epsilon_0] = 0.003$, $\delta = 0.05$, $N = 75$, $d/p = 0.1$

a CLC with local absorption anisotropy a suppression of the total absorption occurs at the frequencies close to one of the stop-band edges. The strength of the discussed absorption suppression effect depends on the order parameter value S and the CLC layers thickness. The effect disappears for zero order parameter value being at the maximum at its value equal to 1 (complete absorption suppression happens for infinitely thick CLC layers).

Similarly to the case of a CLC layer the manifestation of the Borrmann effect is strongly dependent on the CLC order paprameter S. The calculation results of DMS reflection, transmission and absorption spectra for the varying order paprameter ($S = 0,\ 0.3,\ 0.5,\ 1$) in assumption that the absorption is due to the dye molecules only and the ordering degree of dye molecules is relevant to S are presented at Figs. 15–17, respectively.

The presented reflection and transmission spectra for DMS show that the DM life time in the case of a local absorption anisotropy in CLC depends on the localized mode frequency position relative the stop-band edge frequency growing with approach of the localized mode frequency to the stop-band edge frequency where manifestation of the Borrmann effect takes place. To find the DM life time in the general case one has to solve numerically the dispersion equations (13). However, for sufficiently thick CLC layers an analytical solution may be found.

In the case of isotropic absorption characterized by γ (see Eq. (4)) the DM life time τ_{DM} decreases compared to the case of zero absorption ($\gamma = 0$) and for sufficiently thick CLC layers in DMS an analytic expression for the DM life-time may be found (see [21]). Because for a general position of the DM frequency ω_{DM} inside the stop-band this expression is rather complicated we give here τ_{DM} for the case of an isotropic CLC absorption for the ω_{DM} position coinciding with the stop-band center:

$$\tau_{DM} = 2/\omega_{DM}\times \times \left[(4/3\pi)(p/\delta L)\exp[-2\pi\delta L/p] + \gamma\right]. \quad (14)$$

As the calculations show in the case of anisotropic local absorption the DM life-time is growing at aproching the DM frequency ω_{DM} to the stop-band edge demonstrating the Borrmann effect and the DM life-time is decreasing at approching ω_{DM} to the opposite stop-band edge. For the case of locally anisotropic (depending on the order parameter S) absorption one finds the following dependent on the order parameter S limitation on the DM life time for the ω_{DM} position coinciding with the stop-band center:

$$\tau_{DM} < 2/\left[\omega_{DM}\epsilon_1(1-S)/3\,\mathrm{Re}[\epsilon_0]\right], \qquad (15)$$

where ϵ_1 is the imaginary part of the CLC dielectric tensor principal value at $S = 1$ determined by the dye molecules absorption (at the assumption that the whole absorption in CLC is due to the dye molecules only).

What is concerned of the DFB lasing threshold at locally anisotropic absorption in CLC it may be found similarly to the case of EM from a numerical solution of the dispersion equation (13). Nevertheless, some qualitative conclusions about the threshold increasing compared to the case of nonabsorbing CLC may be formulated from the analytical solution found in the case of thick CLC layers. Namely, the DFB lasing threshold at DM frequency ω_{DM} increases differently for different position of ω_{DM} in the stop-band and is strongly dependent on the order parameter S value. For $S = 1$ the DFB lasing threshold may approach to its value for the case of nonabsorbing CLC [21] if the ω_{DM} position is close to the stop-band edge where the Borrmann effect occurs.

6. CONCLUSION

In the preceeding sections the influence of the Borrmann effect (studied previously for CLC transmission and reflection spectra [11–14]) on the properties of the localized modes in the case of local absorption anisotropy in CLC was studied in the first time. The performed analytical description of the EM and DM (neglecting the polarization mixing) allows one to reveal typical EM and DM features related to existence of a local absorption anisotropy in CLC. For example, dependence of the localized mode live-time and lasing threshold on the position of localized mode frequency relative to the stop-band edges.

The results obtained here for the EM and DM (see also [30]) clarify the physics of the absorption suppression due to the Borrmann effect and the dependence of the localized modes life-times on their frequency positions relative to the stop-band edges. The predictions formulated above (for example, on different lasing threshold at the lasing frequncies at opposite stop-band edges) are open for an experimental verification and may be used for the optimisation of experiments related to the DFB lasing in CLC. As a positive specific of the corresponding experiments in CLCs should be mentioned the option to study lasing at the opposite stop-band edges without actual changing the lasing frequency changing instead of this the CLC pitch by temperature variations or by applcation of magnetic or electric field at LCL.

The studied above Borrmann effect influence on the CLC layer absorption spectra predicts new options for experimental investigation of this effect by the means of the luminescence technique (or vice versa to study the luminescence under the conditions of the Borrmann effect realization).

Note, that the obtained results are qualitatively applicable to the corresponding localized electromagnetic modes in any periodic media, and may be regarded as a useful guide in the studies of the localized modes under the conditions of the Borrmann effect existence. For example, the results related to CLCs with local anisotropy of absorption may be useful for optimizing of DFB lasing in general. Really, the corresponding theoretical predictions show which one of the two stop-band frequencies is preferable for obtaining the most lower lasing threshold.

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