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EXPLORING PLANE-SYMMETRIC SOLUTIONS IN $f(R)$ GRAVITY

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The modified theories of gravity, especially the $f(R)$ gravity, have attracted much attention in the last decade. This paper is devoted to exploring plane-symmetric solutions in the context of metric $f(R)$ gravity. We extend the work on static plane-symmetric vacuum solutions in $f(R)$ gravity already available in the literature [1,2]. The modified field equations are solved using the assumptions of both constant and nonconstant scalar curvature. Some well-known solutions are recovered with power-law and logarithmic forms of $f(R)$ models.

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1. INTRODUCTION

Astrophysical data coming from different sources, such as Cosmic Microwave Background fluctuations [3], supernovae type-Ia experiments [4], X-ray experiments [5], and large-scale structure [6] have revealed a completely different picture of our universe. All these observations suggest that the universe is expanding with an accelerating rate. The phenomenon of dark energy and dark matter is another topic of discussion [7]. It was Einstein who first gave the concept of dark energy and introduced the small positive cosmological constant in the field equations. But after sometime he referred to it as the biggest mistake in his life. However, it is now believed that our universe is filled with an exotic cosmic fluid known as dark energy, with strong negative pressure, and the cosmological constant may be a suitable candidate for dark energy. There exist two basic models for dark energy. In the first model, it is associated with empty space and remains constant throughout the spacetime, suggesting the need of the cosmological constant in the field equations. The second model proposes that it varies over the spacetime and cosmic expansion is achieved by a scalar field. Different models have been proposed involving a scalar field, i. e., quintessence [8], k-essence [9], Chaplygin gas [10], and phantom models [11]. It has been predicted that 96 % of energy of the universe is either dark energy or dark matter (76 % dark energy and 20 % dark matter) [7].

Matter and energy domination seems to be a justified reason for this accelerating phase. We can describe dark energy with an equation of state (EOS) parameter $\omega = p/\rho$, where ρ and p represent the energy density and pressure of dark energy. It has been established that the expansion of the universe is accelerating when $\omega \approx -1$ [12]. The universe is found to have a quintessence dark era when $\omega > -1$, while the phantom-like dark energy exists in the region where $\omega < -1$. The universe with phantom-like dark energy ends up with a finite time future singularity known as the Big Rip or cosmic doomsday [13].

Some other observations like rotational velocities of galaxies, the temperature distribution of hot gas in galaxies, gravitational lensing of background objects by galaxy clusters, and the observed fluctuations in the cosmic microwave background radiation have indicated the presence of additional gravity, which may be justified by the existence of dark matter in the universe. According to some astrophysicists, modified theories of gravity may explain this phenomenon of dark matter and dark energy, which seem to be responsible for the current cosmic expansion.

Nowadays, an extended theory known as the $f(R)$ theory of gravity has attracted much attention of the researchers. It is believed that modification of Einstein's theory with some inverse curvature terms may cause an increase in gravity that justifies the accelerated expansion [14]. However, modified gravity is known to be unstable with inverse curvature terms and does not pass some solar-system tests

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[15]. This discrepancy can be addressed by using higher-derivative terms. Moreover, squared curvature terms can be used to achieve viability [16]. It is now expected that the cosmic expansion can be explained if some suitable powers of curvature are included in the Einstein–Hilbert action. The dark matter problems can also be addressed using viable $f(R)$ gravity models [17]. Thus, it would be interesting to investigate modified or alternative theories of gravity. The $f(R)$ theory of gravity, which involves a generic function of the Ricci scalar in the standard Einstein–Hilbert lagrangian, is an attractive choice.

Recent literature [18–28] shows keen interest in exploring different issues in modified $f(R)$ theories of gravity. Spherically symmetric solutions are the most widely explored exact solutions in $f(R)$ gravity. Spherically symmetric vacuum solutions were investigated in [29] and it was found that the set of the field equations in $f(R)$ gravity provided the Schwarzschild–de Sitter metric. The same authors [30] found the perfect fluid solutions and concluded that a unique form of $f(R)$ was not obtained. Spherically symmetric solutions in $f(R)$ gravity using the Noether symmetry have been explored in [31]. The exact static spherically symmetric spacetime solutions in $f(R)$ gravity coupled to nonlinear electrodynamics have been analyzed in [32]. It seems interesting, at least from the theoretical standpoint, to consider other exact solutions of the field equations in $f(R)$ gravity. Cylindrically symmetric vacuum solutions in this theory were found in [33]. In [34], more general results were given using cylindrical symmetry in the $f(R)$ theory.

Here, we investigate the exact solutions of static plane-symmetric solutions in metric $f(R)$ gravity. In particular, some solutions are found using the assumptions of both constant and nonconstant scalar curvature. Some well-known solutions already available in general relativity (GR) are recovered in the presence of important $f(R)$ gravity models. The paper is organized as follows. In Sec. 2, we introduce the field equations in the context of $f(R)$ gravity. Section 3 is used to find plane-symmetric solutions. In Sec. 4, we briefly discuss the physical importance of the solutions. The results are summarized and concluded in the last section.

2. SOME BASICS OF $f(R)$ GRAVITY

Two main approaches exist in $f(R)$ theories of gravity. The first is known as the "metric approach", in which the connection is the Levi-Civita connection and the action is varied with respect to the metric. The second approach is called the "Platini formalism", in which the connection and the metric are considered independent of each other and the variation is done for the two parameters independently. Here we use the metric approach to explore the exact solutions.

The $f(R)$ theory of gravity is actually a generalization or modification of GR. The action for $f(R)$ gravity is given by [29]

$$S = \int \sqrt{-g}\left[\frac{1}{16\pi G} f(R) + L_m\right] d^4x. \qquad (1)$$

Here, $f(R)$ is a generic function of the Ricci scalar and L_m is known as the matter Lagrangian. It may be observed that this action is obtained by just replacing R with $f(R)$ in the standard Einstein–Hilbert action. The corresponding field equations are found by varying the action with respect to the metric $g_{\mu\nu}$,

$$F(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + + g_{\mu\nu} \Box F(R) = \kappa T_{\mu\nu}, \qquad (2)$$

where $T_{\mu\nu}$ is the standard matter energy–momentum tensor and

$$F(R) \equiv df(R)/dR, \quad \Box \equiv \nabla^\mu \nabla_\mu \qquad (3)$$

with ∇_μ being the covariant derivative. Equations (2) are fourth-order partial differential equations in the metric tensor. These equations reduce to the field equations of GR if we take $f(R) = R$.

Contracting the field equations yields

$$F(R)R - 2f(R) + 3\Box F(R) = \kappa T. \qquad (4)$$

In the vacuum case, this reduces to

$$F(R)R - 2f(R) + 3\Box F(R) = 0. \qquad (5)$$

Equation (5) gives an important relation between $F(R)$ and $f(R)$, which can be used to simplify the field equations and to evaluate $f(R)$. It can be seen from Eq. (5) that any metric with a constant Ricci scalar $R = R_0$ is a solution of contracted equation (5) if the following equation holds:

$$F(R_0)R_0 - 2f(R_0) = 0. \qquad (6)$$

This condition is known as the "constant-curvature condition". Further, differenting Eq. (5) with respect to x gives

$$F'(R)R - R'F(R) + 3(\Box F(R))' = 0. \qquad (7)$$

The conditions in Eqs. (6) and (7) were first derived in [35].

3. PLANE-SYMMETRIC SOLUTIONS

In this section, we find plane-symmetric static solutions of the field equations in metric $f(R)$ gravity. We first use the constant scalar curvature ($R = \text{const}$) to find the solutions. We also take a nonconstant-curvature condition to obtain solutions of static plane-symmetric spacetimes in $f(R)$ gravity.

3.1. Plane-symmetric spacetimes

The general static plane-symmetric spacetime is

$$ds^2 = A(x)dt^2 - C(x)dx^2 - B(x)(dy^2 + dz^2). \quad (8)$$

For simplicity, we set $C(x) = 1$, and hence the above spacetime becomes

$$ds^2 = A(x)dt^2 - dx^2 - B(x)(dy^2 + dz^2). \quad (9)$$

The Ricci scalar turns out to be

$$R = \frac{1}{2}\left[\frac{2A''}{A} - \left(\frac{A'}{A}\right)^2 + \frac{2A'B'}{AB} + \frac{4B''}{B} - \left(\frac{B'}{B}\right)^2\right], \quad (10)$$

where the prime denotes the derivative with respect to x. Equation (4) can be rearranged as

$$f(R) = \frac{3\Box F(R) + F(R)R - \kappa T}{2}. \quad (11)$$

With this value of $f(R)$ used in the field equations, it follows that

$$\frac{F(R)R_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) - \kappa T_{\mu\nu}}{g_{\mu\nu}} = \frac{F(R)R - \Box F(R) - \kappa T}{4}. \quad (12)$$

The dependence of metric (9) on x suggests that we can consider Eq. (12) as a set of differential equations for A, B, F, ρ, and p. From Eq. (12), we can see that the combination

$$A_\mu \equiv \frac{F(R)R_{\mu\mu} - \nabla_\mu \nabla_\mu F(R) - \kappa T_{\mu\nu}}{g_{\mu\mu}} \quad (13)$$

is independent of the index μ and hence $A_\mu - A_\nu = 0$ for all μ and ν. Then $A_0 - A_1 = 0$ yields

$$\left[\frac{A'B'}{AB} + \left(\frac{B'}{B}\right)^2 - \frac{2B''}{B}\right]F - 2F'' + \frac{A'}{A}F' - 2\kappa(\rho + p) = 0. \quad (14)$$

Also, $A_0 - A_2 = 0$ gives

$$\left[\frac{A''}{A} - \frac{1}{2}\left(\frac{A'}{A}\right)^2 + \frac{A'B'}{2AB} - \frac{B''}{B}\right]F + \left(\frac{A'}{A} - \frac{B'}{B}\right)F' - 2\kappa(\rho + p) = 0. \quad (15)$$

Thus we obtain two differential equations with five unknowns, namely, A, B, F, ρ, and p. These equations seem difficult to solve due to their highly nonlinear nature. However, we investigate some solutions using the assumptions of both constant and nonconstant curvature.

3.2. Solutions with the constant-curvature assumption

Here, we consider the constant-curvature case $R = R_0$. We then have

$$F'(R_0) = 0 = F''(R_0). \quad (16)$$

It is clear that any solution found for GR must be found for specific version of $f(R)$ theory. In particular, the constant-curvature solutions found in $f(R)$ are already available solutions in GR.

Case I. With condition (16), Eqs. (14) and (15) reduce to

$$\left[\frac{A'B'}{AB} + \left(\frac{B'}{B}\right)^2 - \frac{2B''}{B}\right]F_0 - 2\kappa(\rho + p) = 0. \quad (17)$$

$$\left[\frac{A''}{A} - \frac{1}{2}\left(\frac{A'}{A}\right)^2 + \frac{A'B'}{2AB} - \frac{B''}{B}\right]F_0 - 2\kappa(\rho + p) = 0. \quad (18)$$

We can describe the dark energy with the EOS parameter $\omega = p/\rho$, where ρ and p represent the energy density and pressure of dark energy. It has been established that the expansion of the universe is accelerating when $w \approx -1$ [12]. In this case, Eqs. (17) and (18) reduce to

$$\frac{A'B'}{AB} + \left(\frac{B'}{B}\right)^2 - \frac{2B''}{B} = 0, \quad (19)$$

$$\frac{A''}{A} - \frac{1}{2}\left(\frac{A'}{A}\right)^2 + \frac{A'B'}{2AB} - \frac{B''}{B} = 0. \quad (20)$$

These equations can be solved using the power-law assumption $A \propto x^r$ and $B \propto x^l$, where r and l are any real numbers. Thus we use $A = k_1 x^r$ and $B = k_2 x^l$, where k_1 and k_2 are constants of proportionality. It follows that

$$r = -\frac{2}{3}, \quad l = \frac{4}{3}, \tag{21}$$

and hence the solution becomes

$$ds^2 = k_1 x^{-2/3} dt^2 - dx^2 - k_2 x^{4/3}(dy^2 + dz^2). \tag{22}$$

These values of r and l lead to $R = 0$. This is the most basic possible solution and somehow trivial in the constant-curvature case. We can redefine the parameters as $\sqrt{k_1}\, t \to \tilde{t}$, $\sqrt{k_2}\, y \to \tilde{y}$, and $\sqrt{k_2}\, z \to \tilde{z}$, such that the above metric takes the form

$$ds^2 = x^{-2/3} d\tilde{t}^2 - dx^2 - x^{4/3}(d\tilde{y}^2 + d\tilde{z}^2) \tag{23}$$

which is the same as Taub's metric [36].

Case II. We now assume that $B = A^n$. Then the subtraction of Eq. (17) and Eq. (18) gives

$$(3n+1)A'^2 - 2(n+1)AA'' = 0. \tag{24}$$

This equation yields the solution

$$A = k_3[(n-1)x + 2k_4(n+1)]^{2(n+1)/(1-n)}, \tag{25}$$

where k_3 and k_4 are integration constants. Without loss of generality, we can choose $k_3 = 1$ and $k_4 = 0$. Then Eq. (25) takes the form

$$A = [(n-1)x]^{2(n+1)/(1-n)}. \tag{26}$$

Hence

$$B = [(n-1)x]^{2n(n+1)/(1-n)}, \tag{27}$$

and the solution metric takes the form

$$ds^2 = [(n-1)x]^{2(n+1)/(1-n)} dt^2 - dx^2 - [(n-1)x]^{2n(n+1)/(1-n)}(dy^2 + dz^2). \tag{28}$$

We note that we can recover Taub's solution when $n = -2$.

3.3. Solutions without the constant-curvature assumption

We now explore the solutions of modified field equations without using the constant-curvature assumption. Subtracting Eqs. (14) and (15), we obtain

$$\left[\frac{A'B'}{AB} + \left(\frac{A'}{A}\right)^2 + 2\left(\frac{B'}{B}\right)^2 - 2\left(\frac{A''}{A} + \frac{B''}{B}\right)\right] F + 2\frac{B'}{B}F' - 4F'' = 0. \tag{29}$$

Due to the highly nonlinear nature of Eq. (29), here we also use the assumption $B = A^n$. Then Eq. (29) reduces to

$$(3n+1)\left(\frac{A'}{A}\right)^2 - 2(n+1)\left(\frac{A''}{A}\right) + 2n\frac{A'F'}{AF} - 4\frac{F''}{F} = 0, \tag{30}$$

and the Ricci scalar turns out to be

$$R = \frac{1}{2}\left[(3n^2 - 2n - 1)\left(\frac{A'}{A}\right)^2 + (4n+2)\frac{A''}{A}\right]. \tag{31}$$

We follow the approach of Nojiri and Odintsov [37] and take the assumption $F(R) \propto f_0 R^m$, where f_0 is an arbitrary constant. Then using Eqs. (30) and (31), after some tedious calculations, we obtain

$$\begin{aligned}
&[8m(1+4n-2n^2-12n^3+9n^4) + 16m^2(1+4n-2n^2-12n^3+9n^4) - \\
&- 1 - 3n + 6n^2 + 10n^3 - 21n^4 + 9n^5]A'^6 + \\
&+ [-8m(5+20n+2n^2-36n^3+9n^4) - 32m^2(2+8n-n^2-18n^3+9n^4) + \\
&+ 2(3+11n-4n^2-22n^3+21n^4-9n^5)] \times A'^4A''A + [-8m(1+4n+4n^2-9n^4) + \\
&+ 32m^2(1+4n+n^2-6n^3) + 4n(1+4n+n^2-6n^3)]A'^3A'''A^2 + \\
&+ [8m(8+32n+17n^2-30n^3-9n^4) + 16m^2(4+16n+4n^2-24n^3+9n^4) - \\
&- 4(3+13n+10n^2-8n^3)]A'^2A''^2A^2 + [16m(1+4n+n^2-6n^3) - \\
&- 32m^2(2+8n+5n^2-6n^3) - 8n(1+4n+4n^2)] \times A'A''A'''A^3 + 16m(1+4n+4n^2)A''''A''A^4 - \\
&- 8m(1+4n+n^2-6n^3)A''''A'^2A^3 + \\
&+ [16m^2(1+4n+4n^2) - 16m(1+4n+4n^2)]A'''^2A^4 + [-16m(2+8n+5n^2-6n^3) + \\
&+ 8(1+5n+8n^2+4n^3)]A''^3A^3 = 0.
\end{aligned} \tag{32}$$

Many solutions can be reconstructed using this equation. However, we discuss only three cases here.

Case III. In this case, we try to recover Taub's solution. For this, we substitute $A = x^{-2/3}$ in Eq. (32). After some lengthy calculations, we obtain a constraint equation

$$18m^2 + 9m - 1 + n = 0. \tag{33}$$

We can obtain $B = x^{4/3}$ for $n = -1/2$. Then Eq. (33) reduces to

$$12m^2 + 6m - 1 = 0. \tag{34}$$

The roots of Eq. (34) are $m = (-3 \pm \sqrt{21})/12$. Thus, we have

$$F(R) = f_0 R^{(-3\pm\sqrt{21})/12}. \quad (35)$$

After integration, we obtain

$$f(R) = \hat{f}_0 (R)^{(9+\sqrt{21})/12} + k_5,$$
$$f(R) = \check{f}_0 (R)^{(9-\sqrt{21})/12} + k_6, \quad (36)$$

where

$$\hat{f}_0 = \frac{12 f_0}{9+\sqrt{21}}, \quad \check{f}_0 = \frac{12 f_0}{9-\sqrt{21}},$$

and k_5 and k_6 are integration constants. It has been proved that the terms with positive powers of the curvature support the inflationary epoch [16]. The corresponding Ricci scalar becomes

$$R = \frac{2}{3x^2}. \quad (37)$$

For the first root $m = (-3+\sqrt{21})/12$, EOS parameter ω, and Eqs. (14) and (15), the energy density of the universe turns out to be

$$\rho = \frac{-f_0}{18\kappa(1+\omega)} \left[\frac{21\left(\frac{2}{3}\right)^{(-3+\sqrt{21})/12}}{x^{(1+\sqrt{21})/6}} + \right.$$
$$\left. + \frac{5\left(\frac{-3+\sqrt{21}}{3}\right)\left(\frac{2}{3}\right)^{(-15+\sqrt{21})/12}}{x^{(9+\sqrt{21})/6}} + \frac{4}{3x^6} \right]. \quad (38)$$

We can choose the sign of f_0 depending on the values of ω to obtain the positive-energy density. Similarly, we can find an expression for the energy density for the other root $m = (-3-\sqrt{21})/12$.

Case IV. Here, we take $A = 1/x$ in Eq. (32) to obtain the constraint equation

$$16m^2 + 8m - 3n + 3 = 0. \quad (39)$$

With this equation, it follows that

$$B = x^{-(16m^2+8m+3)/3}. \quad (40)$$

The solution metric takes the form

$$ds^2 = \frac{1}{x} dt^2 - dx^2 - x^{-(16m^2+8m+3)/3}(dy^2+dz^2). \quad (41)$$

The corresponding Ricci scalar becomes

$$R = \frac{3(n^2+2n+1)}{2x^2}. \quad (42)$$

We can construct different $f(R)$ models for different values of m satisfying Eq. (39). An interesting logarithmic form of $f(R)$ models is obtained for $m = -1$:

$$f(R) = f_0 \ln(R) + k_7, \quad (43)$$

where k_7 is an integration constant. Such a logarithmic form was first introduced by Nojiri and Odintsov [38]. In this case, the Ricci scalar becomes $R = 98/3x^2$ and the solution metric takes the form

$$ds^2 = \frac{1}{x} dt^2 - dx^2 - \frac{1}{x^{11/3}}(dy^2+dz^2), \quad (44)$$

and the matter density turns out to be

$$\rho = \frac{-f_0}{441\kappa(1+\omega)} \left[93 + \frac{470596}{x^6} \right]. \quad (45)$$

Similarly, for $m = -2$, we obtain

$$f(R) = -f_0 R^{-1} + k_8, \quad (46)$$

where k_8 is an integration constant. This model is also cosmologically important because it has been proved that a negative power of the curvature serves as an effective dark energy supporting the cosmic acceleration [16]. Obviously, one can work out the Ricci scalar, the energy density, and the solution metric in this case.

Case V. Here, we consider $A = e^x$ in Eq. (32). In this case, we obtain the constraint equation

$$(n-1)(3n^2+2n+1)^2 = 0, \quad (47)$$

which does not involve the parameter m. Hence, this choice yields a solution for any $f(R)$ model in a power-law or logarithmic form. The roots of Eq. (47) are

$$n = 1, \ \frac{-3 \pm i\sqrt{3}}{6}. \quad (48)$$

We discard the imaginary roots and consider the real value of n to obtain a physical solution. In this case, the Ricci scalar turns out to be nonzero constant, $R = 3$. The energy density is also constant here and the solution metric becomes

$$ds^2 = e^x(dt^2 - dy^2 - dz^2) - dx^2. \quad (49)$$

This corresponds to the well-known anti-de Sitter spacetime in GR [39].

3.4. Physical importance of the solutions

The spacetime admitting the three-parameter group of motions of the Euclidean plane is said to have a plane symmetry and is known as a plane-symmetric spacetime. Such a spacetime has many properties equivalent to those of spherical symmetry. The plane-symmetric spacetime has been extensively investigated by many researchers from various standpoints. Taub [40], Bondi [41], Bondi and Pirani–Robinson [42] defined and studied plane-wave solutions. They considered the concept of the spacetime group of motions, which played a fundamental role in plane gravitational waves. It has been established that the spacetime in Eq. (9) admits plane-wave solutions of the GR field equations [43].

In this study, we have explored plane-symmetric solutions in the context of $f(R)$ gravity. This is actually an extension of the already done work [1, 2] where the solutions were given in the vacuum and constant-curvature case only. Here, we do not relax the conditions and generalize the already obtained solutions. The nonvacuum plane-symmetric solutions provide Taub's universe with a singularity at $x = 0$, which suggests the presence of black hole. Another solution, Eq. (41), suggests that an object falling into a black hole approaches the singularity at $x = 0$. However, a nonsingular solution is obtained in the form of the anti-de Sitter spacetime. The anti-de Sitter space is a GR-like spacetime, where in the absence of matter or energy, the spacetime curvature is naturally hyperbolic. From the geometrical standpoint, an anti-de Sitter space has a curvature analogous to a flat cloth sitting on a saddle, with a very slight curvature because it is so large. Thus, it would correspond to a negative cosmological constant. The anti-de Sitter space can also be thought of as an empty space having negative energy, which causes this spacetime to collapse at a greater rate. The existence of a quantum-corrected de Sitter space has been predicted as an outcome of a nontrivial solution for a constant curvature R_0 in $f(R)$ gravity [35]. One may play with the parameters of the theory under consideration in such a way that the de Sitter space provide a solution of the cosmological constant problem. Thus, the physical relevance of the solutions is obvious.

4. SUMMARY AND CONCLUSION

In this paper, we explore the plane-symmetric solutions in $f(R)$ gravity. We have considered the metric version of the theory to find exact solutions of the field equations. We note that most of the work in $f(R)$ gravity has been done for the vacuum static cases with the constant-curvature condition. It can be interesting to find solutions for nonstatic and nonvacuum cases without using the constant curvature condition. As a first step, we investigate plane-symmetric solutions in the nonvacuum case. To our knowledge, this is the first attempt to investigate nonvacuum plane-symmetric solutions in $f(R)$ gravity without using the constant-curvature assumption. We can assume a function of the Ricci scalar to solve the field equations, but this gives fourth-order highly nonlinear differential equations. The assumption of a constant curvature (which may be zero or nonzero) seems to be most suitable and we can obtain some solutions for a constant scalar curvature. We have found two solutions under this assumption and recovered the well-known Taub's solution.

The solutions without the assumption of a constant scalar curvature provide some important $f(R)$ gravity models. We have mainly explored three solutions in this context. The first solution gives Taub's spacetime with power-law forms of $f(R)$ models having positive curvature. We note that the terms with positive powers of the curvature support the inflationary epoch. The Ricci scalar is nonconstant in this case. The second solution also yields a nonconstant curvature, and two important $f(R)$ models have been constructed in this case. The first model is in a logarithmic form while the second corresponds to a negative power of the curvature. We note that a negative power of the curvature serves as an effective dark energy supporting the current cosmic acceleration. The third case yields a well-known solution that corresponds to the anti-de Sitter spacetime. It provides an arbitrary $f(R)$ model in a power-law or logarithmic form. The Ricci scalar in this case is a nonzero constant. We have discussed five cases in this paper. However, many other cases can also be explored and different cosmologically important $f(R)$ models can be reconstructed.

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