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STABILITY OF THE LEPTON BAG MODEL BASED ON THE KERR–NEWMAN SOLUTION

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We show that the lepton bag model considered in our previous paper [10], generating the external gravitational and electromagnetic fields of the Kerr–Newman (KN) solution, is supersymmetric and represents a BPS-saturated soliton interpolating between the internal vacuum state and the external KN solution. We obtain Bogomolnyi equations for this phase transition and show that the Bogomolnyi bound determines all important features of this bag model, including its stable shape. In particular, for the stationary KN solution, the BPS bound provides stability of the ellipsoidal form of the bag and the formation of the ring–string structure at its border, while for the periodic electromagnetic excitations of the KN solution, the BPS bound controls the deformation of the surface of the bag, reproducing the known flexibility of bag models.

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1. INTRODUCTION AND OVERVIEW

It has been discussed since long ago that black holes may be connected with elementary particles. However, the spin/mass ratio of elementary particles is extremely large, and the corresponding black hole loses the horizons, turning into an ultra-extreme (over-charged and over-rotating) Kerr–Newman (KN) solution with a naked singular ring, which forms a topological defect of space–time. As usual, emergence of a singularity is a hint for a generalization of the theory, and the Kerr singular ring created the problem of the source of the KN solution. This problem proved to be very complicated, and this year we can mark the 50th anniversary of its discussions. Earlier attempts to build a source of the KN solution where discussed by Israel in [1], and Israel referred to the paper by Newman and Janis [2], wherein the nontriviality of this problem was first indicated. Carter obtained in [3] that the KN solution has the gyromagnetic ratio $g = 2$, corresponding to that of the Dirac electron, and starting from this fact, Israel [1] suggested a classical model of the electron based on a rotating disk-like source of the KN solution, enclosed by the Kerr singular ring.

The consistent regular model of the KN source was suggested by López, who built the KN source as a rotating vacuum bubble, covering the Kerr singular ring. At the same time, many properties of the KN source indicated its close relationships to string models [4–7], and a resolution of this duality was coming from the disk-like soliton model [8], in which the vacuum internal state of the López bubble source was replaced by a superconducting false vacuum formed by the Higgs mechanism of symmetry breaking. The ring–string emerged in this model as a narrow tube of the electromagnetic (EM) potential concentrated at the sharp boundary of the disk-like source, similar to the well-known Nielsen–Olesen vortex string model in the Landau–Ginzburg theory [9].

Recently, this model was generalized to a gravitating bag model [10], for which one of the known features is the flexibility and ability to create string-like structures [1)].

A principal peculiarity of the model considered in [10] was the requirement to retain the external gravitational EM field of the KN solution, which is known [3, 13] to have the gyromagnetic ratio $g = 2$, corresponding to that of the Dirac electron. Such a bag can be considered as a semiclassical model for some particles of the electroweak sector of the Stan-

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[1)] Extended particle-like soliton models based on the Higgs mechanism of symmetry breaking, such as Q-balls, skirmions, bags, and vortex strings, are widely discussed now. Flexibility of the bag models is used, in particular, for the flux-tube string models [11, 12].

dard Model, such as the electron or the muon, since the external gravitational and EM field of these particles corresponds to the KN solution with very good precision.

In this paper, we show that this bag model is supersymmetric and represents a BPS-saturated soliton interpolating between a supersymmetric pseudo-vacuum state inside the bag and the external field of the exact KN solution. We obtain that all the important features of this soliton considered in [10] follow unambiguously from the Bogomolnyi equations corresponding to the BPS-saturated solution.

1.1. Source of the KN solution as a spinning soliton

The Kerr–Schild form of the KN metric is [13]

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is the metric of an auxiliary Minkowski space[2)] M^4,

$$H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta} \tag{2}$$

is a scalar function, r and θ are ellipsoidal coordinates, and k_μ is the null vector field, $k_\mu k^\mu = 0$, forming the principal null congruence (PNC) $\mathcal{K}$, a vortex polarization of the Kerr space–time. The surface $r = 0$ represents a disk-like "door" from the negative sheet $r < 0$ to the positive one $r > 0$. A smooth extension of the solution from the retarded to advanced sheet (together with a smooth extension of the Kerr PNC) occurs via the disk $r = 0$ spanned by the Kerr singular ring $r = 0$, $\cos\theta = 0$ (see Fig. 1) and creates another PNC on the negative sheet. The null vector fields $k^{\mu\pm}(x)$ turns out to be different on these sheets, and two different null congruences $\mathcal{K}^\pm$ create two different metrics

$$g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2H k^\pm_\mu k^\pm_\nu$$

on the same Minkowski background.

The mysterious two-sheeted structure of the Kerr geometry motivated the search for various models for the source of the KN solution avoiding the negative sheet. A relevant "regularization" of this space was suggested by López [14], who excised a singular region together with the negative sheet and replaced it by a regular core with a flat internal metric $\eta_{\mu\nu}$. The resulting vacuum bubble should be matched with the exter-

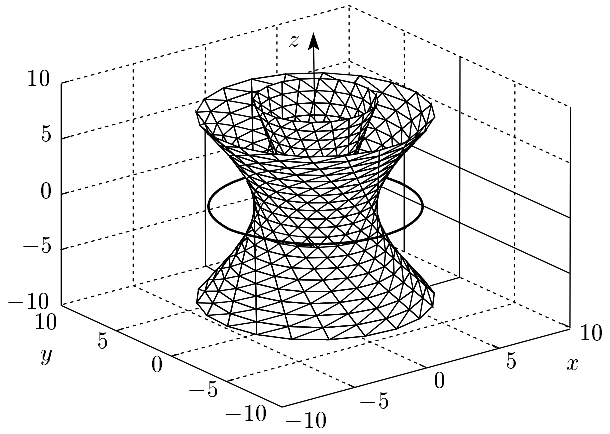

Fig. 1. Null directions of the Kerr congruence k^μ are focused on the Kerr singular ring, forming a two-sheeted space of the advanced and retarded fields

nal KN solution along the boundary $r = R$, determined by the condition

$$H|_{r=R}(r) = 0, \tag{3}$$

which in accordance with (1) and (2) leads to

$$R = r_e = \frac{e^2}{2m}. \tag{4}$$

Since r is Kerr's oblate radial coordinate (see Fig. 2), the bubble source takes an ellipsoidal form and covers the Kerr singular region, forming a flat space inside the disk of the radius $r_c \approx a = \hbar/mc$ and thickness r_e, with the degree of flatness $r_e/r_c \sim e^2 = \alpha \approx 137^{-1}$ corresponding to the fine structure constant.

Developing this model led in [8] to a soliton model with a domain-wall phase transition, in which gravity controls the external classical space–time, while quantum theory forms a supersymmetric pseudo-vacuum state inside the bubble. The conflict between quantum theory and gravity is resolved by the principle of the separation of their zones of influence:

PI: space–time should be flat inside the core,

PII: the exterior should be the exact KN solution,

PIII: the boundary between regions PI and PII is determined by López condition (4).

In [8, 15], a mysterious effectiveness of this principles was mentioned, which uniquely defines the form of this soliton and two its peculiarities:

(A) the Higgs field is oscillating with the frequency $\omega = 2m$, and therefore belongs a type of oscillons,

(B) angular momentum is quantized, $J = n/2$, $n = 1, 2, 3, \ldots$

[2)] We use the signature $(-+++)$.

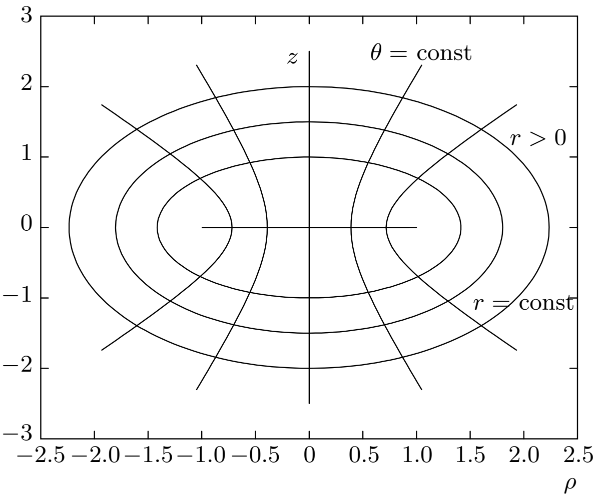

Fig. 2. Kerr's oblate spheroidal coordinates cover the space–time twice, for $r > 0$ and $r < 0$

In this paper, we show that the KN bubble source forms a BPS-saturated soliton, and both peculiarities (A) and (B) are uniquely determined by the Bogomolnyi equations, which also determine the shape of the soliton and therefore its dynamics and stability.

Starting in Sec. 2 from the description of our approach used in previous paper [10], we derive the Bogomolnyi equations adapted to specific Kerr's coordinates in Sec. 3, and integrate them by reducing the problem to two dimensions (t, r), time and the Kerr radial coordinate.

In Sec. 4, we generalize the stationary KN bag to the bag model flexible to deformations and obtain that these deformations are also controlled by the Bogomolnyi bound. Considering stringy deformations of the bag caused by EM excitations of the KN solution, we show that traveling waves may create deformations that break smoothness of the solution and create a traveling singular pole connected with a traveling circular wave. We conclude in Sec. 5.

2. GRAVITATING BAG MODEL AND THE SUPERSYMMETRIC SCHEME OF PHASE TRANSITION

The bubble source formed by the López boundary was generalized to a soliton [8], and then to a gravitating bag model [10, 16]. The concept of a bag model assumes incorporating the fermionic sector, in which the Dirac equation acquires mass through a Yukawa coupling to the Higgs field [11, 12]. As a consequence, the mass turns out to be a variable function of the space–time distribution of the Higgs condensate. The boundary of the bag is modeled by a domain wall interpolating between the external KN solution and the flat internal pseudo-vacuum state, and the phase transition between these states is controlled by the Higgs mechanism of symmetry breaking, which is used in many soliton models as well as in the well-known Nielsen–Olesen model [9], which is in fact the Landau–Ginzburg (LG) field model for the vortex string in a superconducting media.

As it was shown in [10], the typical quartic potential Φ,

$$V(|\Phi|) = g(\bar{\sigma}\sigma - \eta^2)^2, \quad \sigma = \langle|\Phi|\rangle, \tag{5}$$

used for the Higgs field in all these models, is not suitable for the source of the KN solution because the external Higgs field distorts the external KN solution, turning the EM field into a short-range one.

Contrary to the standard bag model forming a cavity in the Higgs condensate [11], condition PII requires the Higgs condensate to be enclosed inside the bag. This cannot be done with potential (5), and a more complex scheme of a phase transition was used in [10], which contained three chiral fields $\Phi^{(i)}$, $i = 1, 2, 3$. In fact, it is a supersymmetric generalization of the LG model [17].

One of the fields, say $\Phi^{(1)}$, was identified as the Higgs field Φ. Hence the new notation

$$(\Phi, Z, \Sigma) \equiv (\Phi^1, \Phi^2, \Phi^3) \tag{6}$$

was used.

Due to condition PI, the bag is to be placed in the flat region, and the domain wall phase transition may be considered with the flat background metric, $g_{\mu\nu} = \eta_{\mu\nu}$. Therefore, the domain-wall boundary of the bag and the bag as a whole are not dragged by rotation. Because of that, the chiral part of the Hamiltonian is simplified to

$$H^{(ch)} = T_0^{\,0(ch)} = \\ = \frac{1}{2}\sum_{i=1}^{3}\left[\sum_{\mu=0}^{3}|\mathcal{D}_\mu^{(i)}\Phi^i|^2 + |\partial_i W|^2\right], \tag{7}$$

where the covariant derivatives

$$\mathcal{D}_\mu^{(i)} \equiv \partial_\mu + ieA_\mu^i$$

are flat. As in [18], the potential V is determined by the superpotential

$$V(r) = \sum_i |\partial_i W|^2. \tag{8}$$

It was shown in [10] that the superpotential

$$W(\Phi^i, \bar{\Phi}^i) = Z\left(\Sigma\bar{\Sigma} - \eta^2\right) + (Z+\mu)\Phi\bar{\Phi} \tag{9}$$

suggested by Morris [19], where μ and η are real constants, provides the necessary concentration of the Higgs field inside the bag, and from the supersymmetry condition $\partial_i W = 0$, two vacuum states were determined:

(I) internal: $r < R - \delta$,

$$\begin{gathered} V(r) = 0, \quad |\Phi| = \eta = \text{const}, \quad Z = -\mu, \\ \Sigma = 0, \quad W_{in} = \mu\eta^2, \end{gathered} \tag{10}$$

(II) external: $r > R + \delta$,

$$\begin{gathered} V(r) = 0, \quad \Phi = 0, \quad Z = 0, \\ \Sigma = \eta, \quad W_{ext} = 0, \end{gathered} \tag{11}$$

(III) the transition zone $R - \delta < r < R + \delta$, where vacua (I) and (II) are separated by a positive spike of the potential V.

The principal result obtained here is that the position of the domain wall boundary satisfying requirements PI–PIII is uniquely determined by the Bogomolnyi bound, and therefore these requirements determine stability of the bag, leading to a supersymmetric and BPS-saturated source of the KN solution.

As was discussed in [10] (and earlier in [8]), inside the bag and in the transition zone (III), the space is flat, the fields Φ^2 and Φ^3 are constant, and only the complex Higgs field $\Phi(x) = |\Phi(x)|e^{i\chi(x)}$, interacting with the vector potential of the KN solution A_μ penetrating inside has a nontrivial dynamics. As a result, the field model in this zone reduces to the Abelian field model in flat space–time, which has only one chiral field Φ and coincides with the model for the vortex string used by Nielsen–Olesen [9]. The corresponding Lagrangian leads to the equations

$$\partial_\nu \partial^\nu \Phi = \partial_{\bar{\Phi}} V, \tag{12}$$

$$\partial_\nu \partial^\nu A_\mu = I_\mu = e|\Phi|^2(\chi_{,\mu} + eA_\mu), \tag{13}$$

which are consistent with the vacuum states in zones (I) and (II).

Equation (13), which is indeed Eq. (2.4) of the Nielsen–Olesen model [9], indicates that the current must not penetrate inside the bag beyond a thin surface layer. Setting $I_\mu = 0$ inside the bag, we obtain $\partial_\nu \partial^\nu A_\mu = 0$ and

$$\chi_{,\mu} + eA_\mu = 0, \tag{14}$$

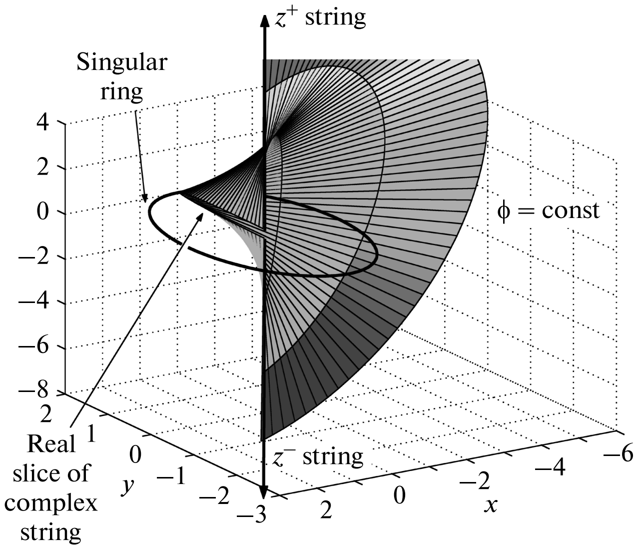

Fig. 3. The Kerr surface $\phi = \text{const}$. The Kerr congruence is dragged by rotation even in the zero-mass limit. In the equatorial plane, the congruence is tangent to the Kerr singular ring, and the vector potential forms a closed Wilson loop wrapped around the boundary of the spheroidal bag

which shows that the gradient of the phase of the Higgs field $\chi_{,\mu}$ must compensate the penetrating vector potential A_μ of the KN field. We emphasize that although the KN gravitational field vanishes near the boundary of the bag, its strong effect on the EM field is maintained. Since the KN vector potential

$$A_\mu = -\operatorname{Re}\frac{e}{r + ia\cos\theta}k_\mu \tag{15}$$

is aligned to directions of the Kerr congruence k_μ, it must be dragged by the Kerr singular ring even in the flat limit (see Fig. 3).

The boundary of the bag at $r = R = e^2/2m$ regularizes vector potential (15), and it takes the maximal value in the equatorial plane $\cos\theta = 0$:

$$A_\mu^{(max)} = -\frac{2m}{e}k_\mu. \tag{16}$$

There are only the longitudinal and the timelike components of the vector potential in the stationary KN solution. Since $k_0 = 1$, the timelike component takes the maximal value $A_0 = -2m/e$, which in accordance with (14) should be compensated by the phase of the Higgs field $\chi_{,0}$, which leads to the important result (A): oscillations of the Higgs field with the frequency $\omega = 2m$.

At the same time, the longitudinal part of the vector potential A_μ forms a closed loop along the boundary of the bag in the equatorial plane, and in accordance with (14) it should also be compensated by the change in the phase of the Higgs field $\chi_{,\phi}$. In [8], using the Kerr relation $J = ma$, we obtained the second remarkable consequence (B): angular momentum is quantized, $J = n/2$, $n = 1, 2, 3, \ldots$

We now consider these result as a consequence of the supersymmetry of the bag model. We use the recipe described in [20, 21] for a similar problem for a planar domain wall with one chiral field and reduce the problem to solvable first-order Bogomolnyi equations, in particular implying (A) and (B).

3. SOURCE OF THE KN SOLUTION AS A BPS-SATURATED SOLITON

The full Lagrangian corresponding to the bosonic part of the N=1 supersymmetric model with three chiral fields $\Phi^{(i)} = \{\Phi, Z, \Sigma\}$, $i = 1, 2, 3$, has the form [18]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \\ - \frac{1}{2}\sum_i (\mathcal{D}^{(i)}_\mu \Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^* - V. \quad (17)$$

As we mentioned earlier, the part of the Lagrangian related to the field $\Phi^{(i)} = \Phi^{(1)} \equiv \Phi$ is the same as in the Nielsen–Olesen model.

The corresponding stress–energy tensor decomposes into a pure EM part $T^{(em)}_{\mu\nu}$ and contributions from the chiral fields $T^{(ch)}_{\mu\nu}$:

$$T^{(tot)}_{\mu\nu} = T^{(em)}_{\mu\nu} + \sum_i (\mathcal{D}^{(i)}_\mu \Phi^i)\overline{(\mathcal{D}^{(i)}_\nu \Phi^i)} - \\ - \frac{1}{2}g_{\mu\nu}\left[\sum_i (\mathcal{D}^{(i)}_\lambda \Phi^i)\overline{(\mathcal{D}^{(i)\lambda}\Phi^i)} + V\right]. \quad (18)$$

The flatness of the metric inside the bubble and in the vicinity of the domain wall boundary leads to the disappearance of dragging of the chiral fields, and similarly to previous treatment, we can use the chiral part of the Hamiltonian in form (7).

The domain-wall boundary of the bag and the bag as a whole do not rotate. Nevertheless, the influence of gravity is saved in the shape of the bag and also as a drag effect acting of the KN EM field, which retains correlation with a twisted Kerr congruence even in the flat-space limit. We have to take it into account, and it is advisable to use the Kerr coordinate system

$$x + iy = (r + ia)e^{i\phi}\sin\theta, \\ z = r\cos\theta, \quad t = \rho - r, \quad (19)$$

which is adapted to the shape of the bag, and where KN vector potential (15) takes the simple form ([13], Eq. (7.7))

$$A_\mu dx^\mu = -\,\mathrm{Re}\left(\frac{e}{r + ia\cos\theta}\right) \times \\ \times (dr - dt - a\sin^2\theta\, d\phi). \quad (20)$$

As we have seen, the components A_ϕ and A_t have very specific behavior, and are compensated by the phase of the oscillating Higgs field

$$\Phi(x) \equiv \Phi^1(x) = |\Phi^1(r)|e^{i\chi(t,\phi)}, \quad (21)$$

which is equivalent to the equations

$$\mathcal{D}^{(1)}_t \Phi^1 = 0, \quad \mathcal{D}^{(1)}_\phi \Phi^1 = 0, \quad (22)$$

which are analogs of (13), and lead to respective consequences (A) and (B). As a result, these terms drop out from expression (7), and all the remainder chiral fields depend only on the Kerr radial coordinate r:

$$\Phi^2 = \Phi^2(r), \quad \Phi^3 = \Phi^3(r). \quad (23)$$

The sum $\sum_{\mu=0}^3 |\mathcal{D}^{(i)}_\mu \Phi^i|^2$ in (7) reduces to a single term,

$$H^{(ch)} = T_0^{\;0(ch)} = \frac{1}{2}\sum_{i=1}^3 [|\mathcal{D}^{(i)}_r \Phi^i|^2 + |\partial_i W|^2], \quad (24)$$

where the coordinate r parameterizes the oblate surface of the bag and, similarly to parallel surfaces of the planar domain walls, the surfaces r and $r + dr$ can be regarded as “locally parallel” to each other (see Figs. 4 and 5).

Following [20, 21], we now use a “trick”, by introducing the angles α_i, which allow us to rewrite expression (24) in the equivalent form

$$H^{(ch-r)} = \sum_{i=1}^3 \frac{1}{2}\left|\mathcal{D}^{(i)}_r \Phi^i - e^{i\alpha_i}\frac{\partial \bar{W}}{\partial \bar{\Phi}^i}\right|^2 + \\ + \mathrm{Re}\exp(-i\alpha_i)\frac{\partial \bar{W}}{\partial \bar{\Phi}^i}\mathcal{D}^{(i)}_r \Phi^i, \quad (25)$$

where the phases α_i should be independent of r and be chosen so as to ensure the vanishing of the square terms, i. e.,

$$\mathcal{D}^{(i)}_r \Phi^i = \exp(i\alpha_i)\frac{\partial \bar{W}}{\partial \bar{\Phi}^i}. \quad (26)$$

The functions W and Z are real, and without loss of generality we can also set a real Φ^3, which allows us to take $\alpha_2 = \alpha_3 = 0$. For the Higgs field, represented by the function

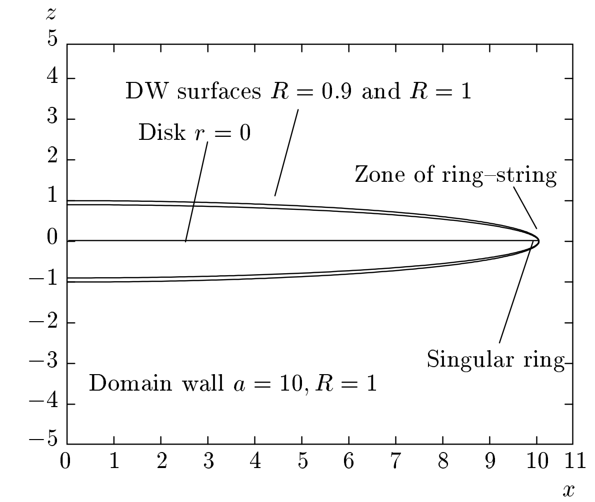

Fig. 4. Axial section of the spheroidal domain-wall (DW) phase transition

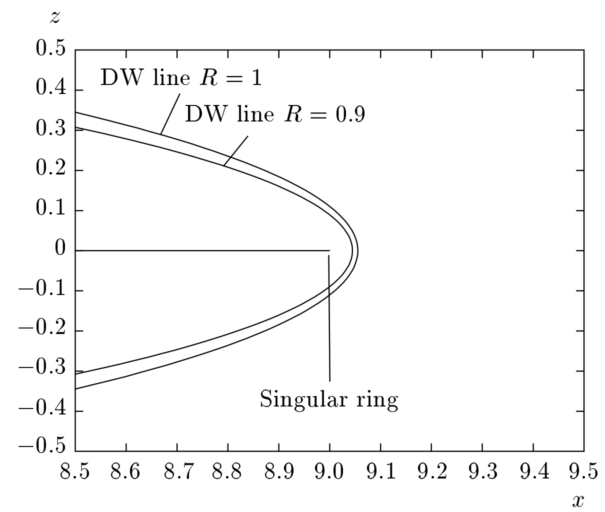

Fig. 5. Enlarged fragment of the disk boundary, the ring–string zone

$$\Phi \equiv \Phi^1 = |\Phi| e^{i\chi(t,\phi)},$$

we have

$$\frac{\Phi^1}{\bar{\Phi}^1} = e^{2i\chi(t,\phi)},$$

and from (26) and (21) we obtain

$$\alpha_1 = 2\chi(t, \phi). \quad (27)$$

In this case,

$$\mathcal{D}_r^{(1)} \Phi = e^{2i\chi(t,\phi)} \mathcal{D}_r^{(i)} \bar{\Phi},$$

and (25) takes the form

$$H^{(ch-r)} = \sum_{i=1}^{3} \frac{1}{2} \left| \partial_r \Phi^i - \frac{\partial W}{\partial \Phi^i} \right|^2 + \\ + \operatorname{Re} \left(\frac{\partial W}{\partial \Phi^i} \right) \partial_r \Phi^i, \quad (28)$$

where the replacement of the covariant derivatives $\mathcal{D}_r^{(1)}$ with the partial ∂_r is valid due to the concrete form of superpotential (9).

A minimum of the energy density $H^{(ch-r)}$ is achieved for

$$\mathcal{D}_r^{(i)} \Phi^i = \frac{\partial W}{\partial \Phi^i}, \quad \mathcal{D}_r^{(i)} \bar{\Phi}^i = \frac{\partial \bar{W}}{\partial \bar{\Phi}^i}, \quad (29)$$

which are the Bogomolnyi equations corresponding to the saturated Bogomolnyi bound. Expression (28) turns into a full differential,

$$H^{(ch-r)} = \operatorname{Re} \left(\frac{\partial W}{\partial \Phi^i} \right) \partial_r \Phi^i = \frac{\partial W}{\partial r}. \quad (30)$$

We can now obtain the mass–energy of the bag together with its domain-wall boundary

$$M_{bag} \equiv M_{ch} = \int dx^3 \sqrt{-g}\, T_0^{\ 0(ch)}. \quad (31)$$

For the Kerr coordinate system,

$$\sqrt{-g} = (r^2 + a^2 \cos^2 \theta) \sin \theta. \quad (32)$$

Axial symmetry allows us to integrate over ϕ, leading to

$$M_{bag} = 2\pi \int dr\, d\theta (r^2 + a^2 \cos^2 \theta) \sin \theta T_0^{\ 0(ch)}. \quad (33)$$

Using (30), we obtain

$$M_{bag} = 2\pi \int d\theta (r^2 + a^2 \cos^2 \theta) \sin \theta \partial_r W\, dr. \quad (34)$$

Taking into account that superpotential $W(r)$ is constant inside and outside the source,

$$W_{int} = \mu\eta^2, \quad W_{ext} = 0, \quad (35)$$

we have $\partial_r W = 0$ inside and outside the bag and, by crossing the bag boundary, we obtain the difference

$$\Delta W = W(R + \delta) - W(R - \delta) = -\mu\eta^2.$$

After integration over $r \in [0, R]$ and then over $X = \cos\theta$, we obtain

$$M_{bag} = 2\pi \Delta W \int_{-1}^{1} dX (R^2 + a^2 X^2) = \\ = 4\pi \left(R^2 + \frac{1}{3} a^2 \right) \Delta W. \quad (36)$$

4. STRINGY DEFORMATIONS OF THE KN BAG

As discussed in [10], taking the bag model concept, we should also accept the dynamical point of view that the bags are to be soft and deformed, acquiring excitations similar to excitations of the dual string models [12, 22, 23]. By deformations, the bags may form stringy structures. Generally considered are the radial and rotational excitations, forming open strings or flux tubes. The old Dirac model of an "extensible" spherical electron [24] may also be considered as a prototype of the bag model with spherically symmetric deformations — radial excitations.

The bag-like source of the KN solution without rotation, $a = 0$, represents the Dirac model of a spherical "extensible" electron, which has the classical electron radius $R = r_e = e^2/2m$ at rest. The KN rotating disk-like bag (see Fig. 1 in [10]) may be considered as the Dirac bag stretched by rotation to a disk of the Compton radius, $a = \hbar/2mc$, which corresponds to the zone of vacuum polarization of a "dressed" electron.

It has been obtained long ago that the Kerr geometry is closely related to strings [7]. In particular, in our old work [4, 5], the Kerr singular ring was associated with a closed ring–string that may carry traveling waves like a waveguide[3)]. In the soliton bag model, the Kerr singularity disappears, but this role is played by the sharp boundary of the disk-like bag. Like the Kerr singular ring [4], it can serve as carrier of traveling waves. It was shown in [6] that the field structure of this string is similar to the structure of the fundamental string, obtained by Sen as a solitonic string-like solution of low energy string theory [26]. As it was shown in [4, 5] and recently in [27], the EM and spinor excitations of the KN solution are concentrated near the Kerr ring, forming string-like traveling waves. For the stationary KN solution, the EM field forms a frozen wave [4], located along the boundary of the disk-like source. Locally, this frozen string is a typical plane-fronted EM wave with null invariants,

$$\mathbf{E} \cdot \mathbf{H} = 0, \quad \mathbf{E}^2 = \mathbf{H}^2, \tag{37}$$

and with the Poynting vector

$$\mathbf{S} = \frac{1}{4\pi} \mathbf{E} \times \mathbf{H}$$

directed along the tangent to the Kerr singular ring, $\mathbf{k} \cdot \mathbf{S} > 0$. In the regularized KN solution, the Kerr singular ring is regularized, acquiring a cut-off parameter R, which for the axially symmetric KN solution is the constant $R = r_e$, Eq. (4) (see Fig. 6*a*).

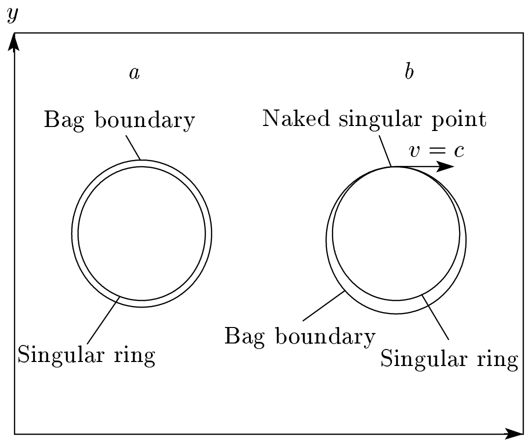

Fig. 6. Regularization of the KN EM field. A section of the disk-like bag in the equatorial plane. The distance from positions of the boundary of the bag from the position of the (former) singular ring acts as a cut-off parameter R. *a*) Axially symmetric KN solution gives a constant cut-off $R = r_e$. *b*) The boundary of the bag is deformed by a traveling wave, creating a circulating singular point of tangency (zitterbewegung)

Since the null vector of the Kerr congruence k_μ is tangent to the Kerr singular ring, and since $R \ll a$, the ring–string at the boundary is almost light-like, and its structure is very close to the known *pp*-wave strings [28–30]. However, for an external observer, the light-like closed string should shrink to a point due to Lorentz contraction, [27]. The extended KN string, positioned along the boundary of the bag, cannot be closed, [31], since the end points of the string worldsheet $x^\mu(\phi, t)$ and $x^\mu(\phi + 2\pi, t)$ must not coincide[4)]. There are two ways to make a consistent extended string structure:

1) to consider this string as an open one and to complete it to a consistent sum comprising the left and right modes,

2) to form an orientifold string, which means that the open string is built from a closed one by folding its worldsheet [31]: the interval $\phi \in [0, 2\pi]$ is represented as a half-interval $\phi^+ \in [0, \pi]$, doubled by the reversed half-interval $\phi^- \in [\pi, 2\pi]$, with $x^\mu(\phi^-, t) = x^\mu(2\pi - \phi^-, t)$.

3) Another, complex string appears in the complex structure of the Kerr geometry [7, 25].

4) Otherwise the worldsheet becomes a worldline. We are faced here with an odd peculiarity of the Kerr spinning particle, where the chiral fields form an extended bag, while the associated EM field forms a light-like string that looks like a point for an external observer.

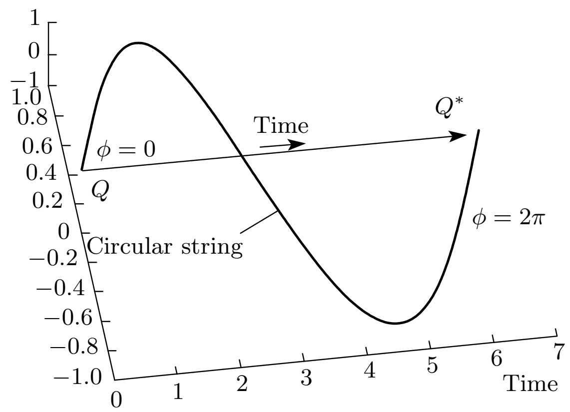

Fig. 7. The circular left mode formed by a traveling wave along the KN string is completed by the time-like right mode formed by the frozen traveling wave of the stationary KH solution q

Here, we follow the first way, and consider the above "frozen" solution as a right mode of an excitation. We complete it by the left counterpart, which we find among other admissible excitations. All exact solutions for the EM field on the Kerr background were obtained in [13], and they are defined by an analytic function $A = \psi(Y,\tau)/P^2$, where $Y = e^{i\phi}\tan(\theta/2)$ is a complex projective angular variable, $\tau = t - r - ia\cos\theta$ is a complex retarded-time parameter, and $P = 2^{-1/2}(1 + Y\bar{Y})$ for the Kerr geometry at rest. The vector potential is determined by the function ψ as follows [13]:

$$A_\mu dx^\mu = -\operatorname{Re}\left(\frac{\psi}{r + ia\cos\theta}\right) e^3 + \chi d\bar{Y}, \qquad \chi = 2\int (1 + Y\bar{Y})^{-2}\psi\, dY. \tag{38}$$

The simplest function $\psi = -e$ yields the stationary KN solution with function (2). It corresponds to the frozen circular EM wave discussed above (see Fig. 7). This circular traveling mode is locally plane wave "propagates" along the Kerr singular ring. By regularization, the EM field acquires the constant cut-off parameter $R = r_e$ (see Fig. 6*a*).

Along with many other possible stringy waves, an interesting effect is manifested by the lowest wave solutions[5)]

$$\psi = e\left(1 + \frac{1}{Y} e^{i\omega\tau}\right). \tag{39}$$

It is easy to find the back reaction of this excitation. The boundary of the disk is very close to the position of the Kerr singular ring, and regularization of the stationary KN source in fact represents a constant cut-off parameter $R = r_e$, Eq. (4), for the Kerr singularity. The EM traveling waves deform the bag surface, and the boundary of the deformed bag can be determined from the condition $H = 0$, Eq. (3).

Like the stationary KN solution, the function ψ acts on the metric through the function H, which in the general case has the form

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2\cos^2\theta}, \tag{40}$$

and the condition $H = 0$ determines the boundary of disk $R = |\psi|^2/2m$, which acts as the cut-off parameter for EM field. The corresponding deformations of the bag boundary are shown in Fig. 6*b*. We see that solution (39) takes the form

$$\psi = e(1 + e^{-i(\phi - \omega t)}),$$

in the equatorial plane $\cos\theta = 0$ and the cut-off parameter

$$R = \frac{|\psi|^2}{2m} = \frac{e^2}{m}(1 + \cos(\phi - \omega t))$$

depends on $\phi - \omega t$. The vanishing of R at $\phi = \omega t$ creates a singular pole, which circulates along the ring–string together with the traveling wave of the excitation, reproducing light-like zitterbewegung of the Dirac electron. This pole may be interpreted as a single end point of the ring–string: either as a point-like bare electron or as a light-like quark, if it is also present in the associated fermionic sector.

5. CONCLUSION

The mysterious problem of the source of two-sheeted Kerr geometry leads to a gravitating soliton-bubble model, which has to retain the external long-range gravitational and EM field of the KN solution. The requirement of consistency with gravity leads to a supersymmetric field model of a phase transition in which the Higgs condensate forms a supersymmetric core of a spinning particle-like solution. The resulting model considered in [10] has much in common with the famous MIT and SLAC bag models, as well as with the basic concept of the Standard Model, where the initially massless leptons (left and right) acquire a mass inside the bag from the Higgs mechanism of symmetry breaking.

In the present extension of [10], we showed that the KN bag model forms a BPS-saturated solution of the Bogomolnyi equations, and therefore the stationary

5) Remarkable features of this combination were discussed in [4].

bag forms a stable configuration determined by the KN parameters: charge, spin, and the rotation parameter $a = J/m$, while the mass is related to the parameters of the domain-wall bubble encoded in the superpotential W.

Similar to the other bag models, the KN bag is pliant to deformations. The spinning bag takes the shape of a thin disk, whose sharp boundary represents a ring–string, which can support traveling waves. The domain-wall boundary of the disk is determined by the BPS bound, which coincides with the López boundary determined by principles PI–PIII. For the stationary KN solution, this corresponds to the bag of an oblate ellipsoidal form taking the Compton zone of a dressed electron. The boundary of the disk is completed by a "frozen" light-like ring–string of the Compton radius. Since the tangent direction to this string is light-like with great precision, it shrinks by the Lorentz contraction, and its space–time extension cannot be "seen" by an external observer [27, 32][6].

On the other hand, we showed that the ring–string traveling waves lead to deformations of the bag surface, and the lowest EM excitation of the KN solution breaks the regularization of the KN solution, creating a singular pole that reproduces the known zitterbewegung, circulating with speed of light along the ring–string together with traveling wave. The bag model acquires an additional point-like element that may be interpreted as an analog of the bare electron, while the model as a whole turns into a single bag–string–quark system, which should be associated with a dressed electron.

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[6] However, it is was supposed in [27, 32] that the real Compton extension of this string would be observable in some experiments with low-energy scattering.

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