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FEASIBILITY OF A FEEDBACK CONTROL OF ATOMIC SELF-ORGANIZATION IN AN OPTICAL CAVITY

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Many interesting nonlinear effects are based on the strong interaction of motional degrees of freedom of atoms with an optical cavity field. Among them is the spatial self-organization of atoms in a pattern where the atoms group in either odd or even sites of the cavity-induced optical potential. An experimental observation of this effect can be simplified by using, along with the original cavity-induced feedback, an additional electronic feedback based on the detection of light leaking the cavity and the control of the optical potential for the atoms. Following our previous study, we show that this approach is more efficient from the laser power perspective than the original scheme without the electronic feedback.

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1. INTRODUCTION

Although the laser cooling employing resonant transition has been successful especially for alkali atoms, the absence of cyclic transitions in other species makes the applications of this approach difficult for a wide range of atoms and especially molecules [1]. Therefore, searching for other more universal cooling methods is still an important activity in atomic and molecular physics.

A universal method for cooling microparticles that involves optical fields should be based on the off-resonant dipole interaction, which is conservative. Thus, an additional decay channel for leaking the energy and cooling the particles should be provided. These requirements are fulfilled in two approaches that we call cavity cooling and feedback cooling. Both these names refer to a wide variety of methods that still have some essential similarities.

In cavity cooling [2, 3], the key ingredient is the coupled dynamics of the motional degrees of freedom of particles and the optical cavity mode. The motional excitation of particles is transferred to the cavity photons. The photons leak the cavity, which results in energy dissipation and the net cooling of the particles. A very interesting and promising variant of cavity cooling employs pumping of particles with light directed transversely to the cavity mode [4]. The scattering of pump photons to the cavity mode becomes cooperative in this case and the cooling of the ensemble acquires features of a self-organization process. An efficient realization of this strategy can be obtained if the coupling between the particles and the cavity is strong enough, which is possible but technically challenging to realize [5–9].

The feedback cooling [10–13] is another approach to cool a wide range of species. In this method, a classical measurement device and the classical signal processing are used to organize a feedback loop designed to extract energy from the ensemble of the particles. The variety of feedback cooling methods originates from the method of stochastic cooling [14] successfully applied in high-energy physics and proposed for the cooling of atoms [15, 16]. There is a variety of particular realizations of this method different in the quantity to be controlled [17–19]. The drawback of this approach is the fact that the quantity that can be accessed in a feedback loop is generally a collective observable of an ensemble. The modes that correspond to the relative motion of particles remain unchanged by the feedback loop. Hence, a remixing mechanism that couples different motional modes of the particles is required for efficient cooling.

Recently [20], we proposed to combine the cavity and feedback cooling to enhance the cavity self-organization. The idea behind this is to measure the photon

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flux leaking from the cavity to modify the external optical potential so as to stimulate further scattering from the pump into the cavity.

This procedure makes the bunching of particles in the potential minimums favorable and simultaneously cools the particles. It is not clear, however, whether the use of the electronic feedback loop as proposed is more efficient than the original configuration [4], where the internal cavity-induced feedback works. The easiest way to analyze this is to restrict the consideration to the case where only a few degrees of freedom can be taken into account. This holds for the quantum degenerate regime where all the particles can be described by a single wavefunction. In this case, not the cooling of particles but self-organization of the degenerate gas should be analyzed to prove the feasibility of the electronic feedback. Below, we show that the feedback enhancement of the original self-organization scheme [4] reduces the laser power needed to observe the particle self-organization.

To demonstrate the feasibility of the feedback control in the cavity-induced self-organization setup, we numerically solve a coupled system of evolution equations for the atoms and the field. These equations are derived on the basis of the positive P-representation [21]. This is done in Sec. 2. The linear stability analysis of the classical approximation is discussed in Sec. 3, and numerical simulation results and the discussions are given in Sec. 4.

2. MODEL

The model that we analyze is similar to the model considered in Ref. [22]. We assume that the 1D Bose–Einstein condensate (BEC) of atoms is placed in an optical cavity along the cavity axis. The transverse field at the frequency resonant to the cavity pumps the atoms. According to the standard approach [4], the atoms scatter the pump photons into the cavity mode and the cavity field forms an optical-lattice potential for the atoms. The effective scattering and hence a strong potential appear when the atoms are localized in groups separated by the field wavelength (in each second well of the optical potential) because such configuration satisfies the Bragg condition.

In our consideration, we supply the setup with a feedback loop additional to the cavity, which is organized as follows (see Fig. 1). First, the cavity photon number is measured and then this electric signal is used to control the intensity of the laser creating an additional optical potential for the atoms. The feedback

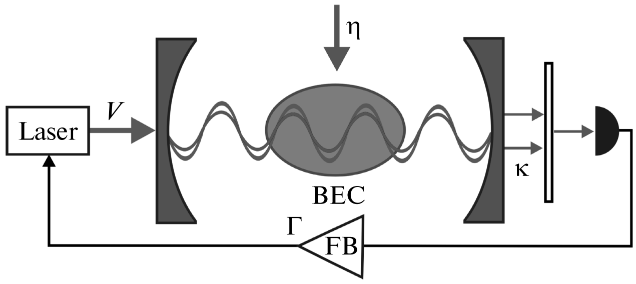

Fig. 1. Setup of the self-organization experiment with an additional electronic feedback. Measurement of the cavity photon number is used to control the strength of the optical potential for the atoms

loop is designed to increase the optical potential as the number of detected photons increases. Furthermore, we assume that the main part of the optical potential acting on the atoms is due to the feedback-controlled laser, while the cavity merely serves as a collector of the scattered photons. This allows relaxing the requirements for the interaction between the cavity and the atoms and therefore simplifying the experiment.

To describe the operation of the setup, we start with the quantum consideration. The Hamiltonian describing the evolution of the BEC and the cavity filed without the feedback is given by

$$H_0 = \hbar\omega_0 a^\dagger a + \int dx\, \Psi^\dagger(x)\Big[-\frac{\hbar^2}{2m}\partial_x^2 + \frac{\hbar g_0^2}{\Delta}U_0^2(x)a^\dagger a + \\ + \frac{\hbar g_0^2}{\Delta}U_0(x)(\eta^* a + \eta a^\dagger)\Big]\Psi(x), \quad (1)$$

where ω_0 is the frequency of the cavity mode, a is the photon annihilation operator of the cavity; $\Psi(x)$ is the annihilation operator of the BEC that obeys the commutation relation $\big[\Psi(x'), \Psi^\dagger(x)\big] = \delta(x - x')$, m is the atomic mass, g_0 is the atom–field coupling constant, Δ is the atom–cavity detuning, and $U_0(x) = \cos(\omega_0 x/c)$ is the cavity mode function. We here assume that Δ is much larger than the spontaneous decay rate γ. To be specific, we consider the D2 transition in Rubidium-85. The decay rate γ is then about $2\pi \cdot 6$ MHz [23]. Thus detuning of about several hundred MHz suffices for the required condition to be satisfied. We also ignore possible optical pumping effects on Zeeman sublevels. The transverse pumping field is considered classically and described by the field amplitude η.

The quantum description of the feedback based on photodetection has been developed in Refs. [24, 25]. According to this theory, the unconditioned evolution of the quantum state of the system is described by the master equation

$$\dot{\varrho} = -\frac{i}{\hbar}\left[H_0, \varrho\right] - \frac{\kappa}{2}\left(a^\dagger a\varrho + \varrho a^\dagger a\right) + \kappa e^{\mathcal{L}\tau} a\varrho a^\dagger, \qquad (2)$$

where κ is the measurement strength, $\mathcal{L}$ is a feedback super-operator acting on the system, and τ is the feedback interaction time. To provide the feedback that changes the atomic potential proportionally to the cavity photon number, we assume that the super-operator $\mathcal{L}$ is implicitly given by

$$e^{\mathcal{L}\tau}\varrho = i\frac{g_0^2}{\Delta\kappa}\Gamma\left[V, \varrho\right] - \varrho. \qquad (3)$$

The operator V in this equation describes the feedback action on the system, which is the optical potential for the atoms formed by the controlled laser. We assume that it is given by

$$V = \hbar \int dx\, \Psi^\dagger(x) U_0^2(x) \Psi(x). \qquad (4)$$

The parameter Γ in (3) is the gain coefficient of the feedback loop. It describes the depth of the feedback-induced potential for the atoms. Experimentally, the feedback-induced potential is realized with an additional laser field that cannot have an arbitrarily large intensity. Due to this restriction, the gain Γ cannot be made very large at all stages of the system evolution. As the atoms become self-organized, the feedback signal, which linearly depends on the cavity photon number, can become too large to be practical. To take this natural limitation into account, we assume that Γ is not a constant parameter but can change as the controlled system evolves. More on the choice of Γ is explained in Sec. 4.

Instead of dealing with the master equation, we can use the positive P-representation [21], which allows transforming the master equation into the Fokker–Planck equation. The use of the positive P-representation implies a doubled phase space of the system, such that the atoms are described by the pair of fields $\phi(x,t)$, $\psi(x,t)$ and the cavity mode is described by the pair of amplitudes α, β. This ensures the semi-positivity of the diffusion matrix in the Fokker–Planck equation and as a consequence the possibility to transform the Fokker–Planck equation into a set of stochastic differential equations. The final Ito-type stochastic differential equations for atomic $\phi(x,t)$, $\psi(x,t)$ and optical α, β fields are

$$\begin{aligned}
&i\hbar\partial_t\phi(x,t) = -\frac{\hbar^2}{2m}\partial_x^2\phi(x,t) + \\
&+ \frac{\hbar g_0^2}{\Delta}\left(\eta^*\alpha + \eta\beta\right)\cos(k_0x)\phi(x,t) + \\
&+ \frac{\hbar g_0^2}{\Delta}\left(1+\Gamma\right)\sin(k_0x)^2\phi(x,t) + \sqrt{\frac{\hbar^2 g_0^2}{\Delta}} \times \\
&\times \left[\alpha\sin(k_0x)^2 + \eta\cos(k_0x)\right]\xi_1(t)\phi(x,t), \\
&-i\hbar\partial_t\psi(x,t) = -\frac{\hbar^2}{2m}\partial_x^2\psi(x,t) + \\
&+ \frac{\hbar g_0^2}{\Delta}\left(\eta^*\alpha + \eta\beta\right)\cos(k_0x)\psi(x,t) + \\
&+ \frac{\hbar g_0^2}{\Delta}\left(1+\Gamma\right)\sin(k_0x)^2\psi(x,t) + \sqrt{\frac{\hbar^2 g_0^2}{\Delta}} \times \\
&\times \left[\beta\sin(k_0x)^2 + \eta^*\cos(k_0x)\right]\xi_2(t)\psi(x,t), \\
&\dot{\alpha}(t) = -\frac{\kappa}{2}\alpha(t) - i\frac{g_0^2}{\Delta}\int \psi(x,t)\left[\alpha(t)\sin(k_0x)^2 + \right. \\
&\left. + \eta\cos(k_0x)\right]\phi(x,t)\,dx + \sqrt{\frac{g_0^2}{4\Delta}}\xi_1^*(t), \\
&\dot{\beta}(t) = -\frac{\kappa}{2}\beta(t) + i\frac{g_0^2}{\Delta}\int \phi(x,t)\left[\beta(t)\sin(k_0x)^2 + \right. \\
&\left. + \eta^*\cos(k_0x)\right]\psi(x,t)\,dx + \sqrt{\frac{g_0^2}{4\Delta}}\xi_2^*(t).
\end{aligned} \qquad (5)$$

Here, $\xi_i(t)$ is the white noise with the correlation function $\langle\xi_i(t)\xi_j(t')\rangle = \delta_{i,j}\delta(t-t')$. In principle, Eqs. (5) take all quantum effects into account via the stochastic terms. However, these equations can be solved numerically for a rather small time interval due to the well-known problem of vanishing boundary terms [26]. Averaging these equations yields the system of semiclassical equations. The equation for the atomic field is analogous to the Gross–Pitaevskii equation with the potential dependent on the cavity field. In the semiclassical approximation, there is no need in a doubled phase space, and therefore only one equation for the atoms and one for the field remain. These equations are given by

$$\begin{aligned}
&i\hbar\partial_t\phi(x,t) = -\frac{\hbar^2}{2m}\partial_x^2\phi(x,t) + \frac{\hbar g_0^2}{\Delta}\left(\eta^*\alpha + \eta\beta\right) \times \\
&\qquad \times \cos(k_0x)\phi(x,t) + \frac{\hbar g_0^2}{\Delta}\left(1+\Gamma\right)|\alpha|^2 \times \\
&\qquad\qquad \times \sin(k_0x)^2\phi(x,t), \\
&\dot{\alpha}(t) = -\frac{\kappa}{2}\alpha(t) - i\frac{g_0^2}{\Delta}\int \psi(x,t)\left[\alpha(t)\sin(k_0x)^2 + \right. \\
&\qquad\qquad \left. + \eta\cos(k_0x)\right]\phi(x,t)\,dx.
\end{aligned} \qquad (6)$$

As can be seen from these equations, the application of feedback is not completely equivalent to a scaling of the atom–field coupling constant g_0. Hence, the evolution of the system is not completely analogous to the dynamics of cavity self-organization without the feedback. It is therefore important to specifically address the feasibility of the feedback to understand what advantages it may have.

3. STABILITY ANALYSIS IN THE THREE-SITE APPROXIMATION

Although the system of equations (6) can be directly simulated, we can have an additional insight into the dynamics of feedback-enhanced self-organization by approximating the spatial dependence of the condensate wavefunction. First of all, as described in [20], we neglect the long-wavelenghth modes of the condensate excitations, considering only those that have the same periodicity as the cavity mode function. Then we divide the distance corresponding to the cavity mode spatial period into four equal parts and represent each part with the amplitude

$$\phi_i = \frac{1}{\sqrt{\lambda/4}} \int\limits_{\lambda(i-1)/4}^{\lambda i/4} dx\, \phi(x), \tag{7}$$

where $i = 1, \ldots, 4$ is the mode number and λ is the wavelength of the cavity field. Two of these modes correspond to odd and even wells of the cavity potential (proportional to the square of the mode function), and two others correspond to the potential maxima between them. It can be shown that the equations of motion for these last modes coincide, and we therefore keep only one of them. Finally, we arrive at a system of four dynamical equations, of which the first three describe the evolution of the atomic modes:

$$\begin{aligned}
&\dot{\varphi}_1 = i(\varphi_2 - \varphi_1) - i\frac{\varepsilon\eta}{\sqrt{\Gamma+1}}(\alpha + \alpha^*)\varphi_1, \\
&\dot{\varphi}_2 = i(\varphi_1 + \varphi_3 - 2\varphi_2) - i|\alpha|^2\varphi_2, \\
&\dot{\varphi}_3 = i(\varphi_2 - \varphi_3) + i\frac{\varepsilon\eta}{\sqrt{\Gamma+1}}(\alpha + \alpha^*)\varphi_3, \\
&\dot{\alpha} = -\frac{\kappa}{2}\alpha - i\varepsilon|\varphi_2|^2\alpha - i\varepsilon\eta\sqrt{\Gamma+1}(|\varphi_1|^2 - |\varphi_3|^2).
\end{aligned} \tag{8}$$

Here, we introduce a dimensionless coupling strength $\varepsilon = g_0^2/(\omega_r\Delta)$, where $\omega_r = \hbar k_0^2/2m$ is the recoil frequency. The cavity decay rate κ in Eqs. (8) is also measured in units of ω_r, and the time is scaled by the recoil period $1/\omega_r$. For convenience, the cavity field amplitude has been renormalized as $\alpha \to \alpha/\sqrt{\Gamma+1}$. This transformation simplifies the analysis of the interesting limit of a small coupling ε. The aim of the feedback is to compensate for $\varepsilon \to 0$, assuming simultaneously that $\sqrt{1+\Gamma} \to \infty$ so as to partially preserve Eqs. (8). However, the last terms in the first and third equations of system (8) vanish in the limit $\varepsilon \to 0$ even if the feedback strength Γ is large. These terms represent the collective recoil of the atoms during the scattering of pump photons to the cavity mode. They drive the first and the third atomic modes in a different way, leading to a spatial redistribution of atoms.

To understand the role of collective recoil in the process of self-organization, we perform the stability analysis of the uniform atomic distribution and determine the conditions required for the self-organization to emerge. For the 3-mode approximation, a quite complete analytic investigation of this issue can be performed. To deal with real numbers, we transform the equations for the complex field amplitudes to the equations for the quadratures defined as

$$x = \frac{\alpha + \alpha^*}{2}, \quad y = \frac{\alpha - \alpha^*}{2i},$$

$$X_i = \frac{\varphi_i + \varphi_i^*}{2}, \quad Y_i = \frac{\varphi_i - \varphi_i^*}{2i}.$$

It is easy to verify that the uniform initial distribution of the atoms is a stationary solution of Eqs. (8) with $\alpha^{(0)} = 0$. To be specific, we assume that the quadrature components of this uniform distribution are $X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = X_0 = \sqrt{N}$, where N is the number of atoms per site, while $Y_1^{(0)} = Y_2^{(0)} = Y_3^{(0)} = 0$. The linearized equations for the small deviations $\delta X_i = X_i - X_i^{(0)}$, $\delta Y_i = Y_i$, $\delta x = x$, and $\delta y = y$ from the stationary solution are

$$\frac{d}{dt}(\delta\mathbf{Z}) = \mathbf{A}\delta\mathbf{Z}, \tag{9}$$

where the vector $\delta\mathbf{Z}$ comprises all the quadratures of the atomic and cavity modes. The evolution matrix is given by

$$\mathbf{A} = \begin{pmatrix} -\kappa/2 & \varepsilon X_0^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\varepsilon X_0^2 & -\kappa/2 & -2\varepsilon\eta\sqrt{\Gamma+1}X_0 & 0 & 0 & 0 & 2\varepsilon\eta\sqrt{\Gamma+1}X_0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ -2\dfrac{\varepsilon\eta}{\sqrt{\Gamma+1}}X_0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 2\dfrac{\varepsilon\eta}{\sqrt{\Gamma+1}}X_0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}. \tag{10}$$

For the linear stability analysis, we should solve the characteristic equation for eigenvalues of the evolution matrix

$$\lambda^2\left(9+\lambda^2\right)\Big[4\lambda^4+4\kappa\lambda^3+\left(\kappa^2+4X_0^4\varepsilon^2+4\right)\lambda^2+ \\ +4\kappa\lambda+4X_0^4\varepsilon^2\left(1-8\varepsilon\eta^2\right)+\kappa^2\Big]=0. \tag{11}$$

Characteristic equation (11) does not contain the feedback gain constant Γ. Thus, the presence of the feedback does not affect the stability of the system in the linear approximation. This, in particular, means that the physical process responsible for the instability and self-organization in the linear approximation is the collective atomic recoil. It is possible to find an approximate condition for the ultimate instability of the system. The system becomes unstable when a combination of parameters X_0, ε, κ, and η makes the real part of at least one of other four eigenvalues positive.

All the roots of Eq. (11) can be given in radicals. However, some of the results are quite lengthy and difficult to analyze. We therefore use an approximate treatment of the part of Eq. (11) in square brackets. After some rearrangement in this part, we obtain the following equation for the remaining four eigenvalues:

$$4X_0^4\varepsilon^2\left(\lambda^2+1-8\varepsilon\eta\right)+\left(1+\lambda^2\right)(2\lambda+\kappa)^2=0. \tag{12}$$

We are interested in the critical values of the parameters, that is, the values corresponding to bifurcation, where nonzero real parts of some eigenvalues appear. To proceed with the approximate treatment of this part of the characteristic equation, we assume that the eigenvalues are small compared to the cavity decay rate κ at least near the self-organization threshold. Then the analytic approximations for two eigenvalues can be found and are given by

$$\lambda_\pm = \pm\sqrt{\frac{32\varepsilon^3\eta^2X_0^4}{\kappa^2+4\varepsilon^2X_0^4}-1}. \tag{13}$$

This approximate result can be verified by comparison with the exact solution of the 4th order equation found, for example, using some computer algebra system. For small absolute values of these eigenvalues, the agreement is very good.

If the expression in the radicand in Eq. (13) is positive, then at least one of the eigenvalues is real and positive. This ensures the instability of the uniform distribution. The critical value of the transverse pumping η is then given by

$$\eta_c = \frac{\kappa^2+4\varepsilon^2N^2}{32\varepsilon^3N^2}. \tag{14}$$

We note that in the limit of a large number of atoms $N\to\infty$, the critical pumping reaches the finite value $\eta_c = 1/8\varepsilon$. This result clearly demonstrates the importance of the atom–field coupling constant and explains possible challenges in the experimental observation of self-organization. Taking the difference in the notation into account, the result in Eq. (14) agrees with the condition derived in Ref. [22], Eq. (10). This indicates that in spite of its simplicity, the three-site approximation seems to capture essential features of the system.

To conclude this part, the feedback based on the photodetection cannot stimulate the instability of the uniform distribution and initiate the growth of the scattered signal. However, we can show that in case of simultaneous action of the collective recoil and the feedback, a considerable reduction in the total power required to reach self-organization can be obtained.

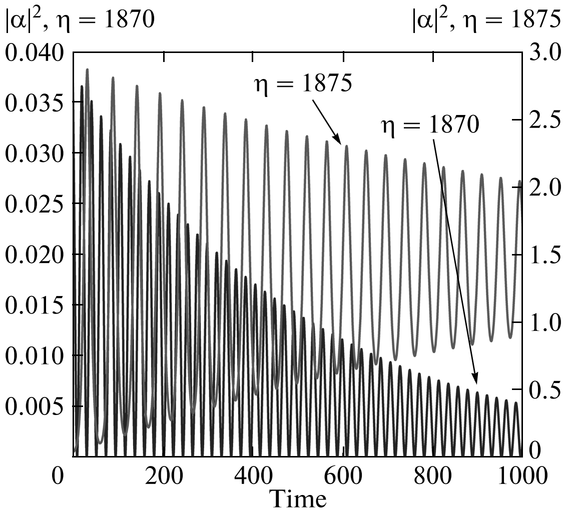

Fig. 2. The field inside the cavity for $\eta = 1870$ (black curve) and $\eta = 1875$ (grey curve). No feedback enhancement. The parameters of the system are $\varepsilon = 1.3 \cdot 10^{-3}$, $\kappa = 5 \cdot 10^3$, and $N = 10^4$

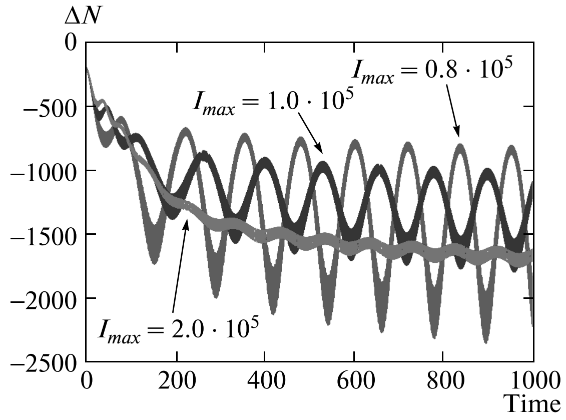

Fig. 3. The difference between the numbers of the atoms in odd and even sites, ΔN, as a function of time for different values of the maximal intensity of the feedback laser. The parameters for the simulations are $\varepsilon = 1.3 \cdot 10^{-3}$, $\eta = 10^3$, and $\Gamma = 2 \cdot 10^6$

4. NUMERICAL SIMULATIONS

Having the three-site model with only four bosonic modes, we can perform straightforward numerical analysis of the nonlinear problem. Since the number of the degrees of freedom is small, the simulations can be done on a conventional workstation computer. In particular, we numerically solve system (8) with the help of XMDS2 [27].

It is natural first to test the threshold condition, Eq. (14), without the feedback loop. In Fig. 2, the evolution of the intracavity intensity $|\alpha|^2$ is shown for two values of the transverse pump η, both of which are close to the critical value η_c. It is seen that for the tested conditions, self-organization occurs for the pump value in the range $\eta_c \in (1870, 1875)$. This result is in a reasonable agreement with the result predicted by Eq. (14), which is $\eta_c = 1885$.

Numerical values are obtained for $\varepsilon = 1.3 \cdot 10^{-3}$, $\kappa = 5 \cdot 10^3$, and $N = 10^4$, which should be easily realized in experiment. In particular, these values correspond to $\kappa = 38$ MHz, $g_0 = 32$ kHz, and $\Delta = 100$ MHz. They are taken as an example of the weak atom–field coupling regime. Hence, the cavity decay rate κ is taken to be larger, while the interaction constant g_0 is taken to be smaller than in a typical atom–cavity experiment. For example, in Ref. [28], $\kappa = 2\pi \cdot 1.4$ MHz and $g_0 = 2\pi \cdot 16$ MHz. The parameters values given above are used for all the numerical results presented below.

The evolution of the system and the effect of feedback with different constant strengths Γ have been investigated in [20]. There, the enhancement of self-organization has been clearly demonstrated. However, one important aspect of the application of feedback remains unclear. The application of feedback requires an additional optical power. Therefore, it is not obvious that the feedback is energetically more favorable than the straightforward increase in the transverse pump η.

To resolve this question, we restrict the intensity that is produced inside the cavity by the feedback laser by some value I_{max}. To implement this restriction in the simulation program, we assume that the gain Γ is a variable quantity and depends on the intracavity intensity $|\alpha|^2$ as

$$\Gamma = \frac{I_{max}}{|\alpha|^2}\left[1 - \exp\left(-\frac{\Gamma_0}{I_{max}}|\alpha|^2\right)\right]. \tag{15}$$

This expression represents a model dependence and is not based on any real feedback mechanism. The idea behind Eq. (15) is to make the feedback action to be approximately linear for small signals ($\Gamma \approx \Gamma_0$) and provide saturation if the cavity intensity $|\alpha|^2$ is large.

The results of numerical simulations for the atom number difference in odd and even sites ΔN are shown in Fig. 3. The transverse pump is set to be well below the critical value, $\eta = 1000$. The feedback gain is chosen to ensure self-organization for the unrestricted feedback laser intensity, $\Gamma = 2 \cdot 10^6$. It is seen that the self-organization occurs already for $I_{max} = 8 \cdot 10^4$, which is more than one order of magnitude smaller than the intensity of the transverse pump $|\eta|^2 = 10^6$. Hence, the total laser intensity required to observe the self-organization is approximately 3.5 times smaller than that without the feedback, because in the latter case, the required total intensity is $|\eta_c|^2 \approx 3.5 \cdot 10^6$. Thus, the application of feedback allows reducing the optical power requirements for the observation of atomic self-organization.

The curves in Fig. 3 corresponding to $I_{max} = 8 \cdot 10^4$ and $I_{max} = 1 \cdot 10^5$ demonstrate oscillations, which are due to the nonlinearity of the feedback response. These oscillations practically disappear for larger maximum intensities of the feedback laser I_{max} (see the curve corresponding to $I_{max} = 2 \cdot 10^5$). Interestingly, the oscillatory behavior is also typical for self-organization without a feedback, as is demonstrated in Fig. 2. But it does not appear if the feedback is on and the feedback laser intensity is not limited (see Fig. 3 in Ref. [20]). To give an estimate of the absolute values of the laser power required to obtain self-organization, we assume that the cavity length is $L \approx 10$ cm. Then the photon flux Φ that provides the maximal optical potential can be estimated as $\Phi = I_{max}L/c \approx 10^{15}$ photons/s. This is equivalent to about 0.1 mW of laser power, which is easliy obtained with modern tunable diode lasers.

5. SUMMARY

We have analyzed the use of feedback in the atom–cavity self-organization setup. The feedback is based on the measurement of the field leaking from the cavity and the appropriate change in the cavity-induced potential. The stability of the uniform atomic distribution has been studied in the three-site approximation, where long-wavelength excitations of atoms have been ignored. It has been shown that the feedback adds essentially nonlinear terms to the evolution equations and does not affect the linear stability of the uniform atomic distribution. The instability of the uniform atomic distribution is therefore solely due to the collective atomic recoil effect.

Thus, in the limit of an extremely small atom–field coupling, the feedback alone can hardly produce self-organization. However, the application of feedback allows considerably reducing the total optical power needed for self-organization. To demonstrate this, we have analyzed the performance of the setup in the case of a limited intensity of the feedback laser. This restriction has been implemented in simulations via a nonlinear feedback response with saturation. It has been shown that in spite of this intensity restriction, the feedback results in a fast onset of self-organization. The total laser intensity required for self-organization with the feedback, at least for some tested parameters, is a few times smaller than the minimal total intensity required for observing self-organization without a feedback.

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