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QUANTUM ENTANGLEMENT IN THE MULTIVERSE

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We show that the quantum state of a multiverse made up of classically disconnected regions of the space–time, whose dynamical evolution is dominated by a homogeneous and isotropic fluid, is given by a squeezed state. These are typical quantum states that have no classical counterpart and therefore allow analyzing the violation of classical inequalities as well as the EPR argument in the context of the quantum multiverse. The thermodynamical properties of entanglement are calculated for a composite quantum state of two universes whose states are quantum mechanically correlated. The energy of entanglement between the positive and negative modes of a scalar field, which correspond to the expanding and contracting branches of a phantom universe, are also computed.

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1. INTRODUCTION

In quantum optics, there are quantum states that violate some inequalities that should be satisfied in the classical theory of light. The effect of photon antibunching, the violation of the Cauchy–Schwartz inequality and, first and foremost the violation of Bell's inequalities, clearly reveal the corpuscular nature of the photon and the existence of nonlocal correlations in the quantum state of the electromagnetic field [1]. Thus, quantum states with no classical analogue allow better understanding the distinguishing concept of complementarity and the nonlocal character of quantum theory.

The interpretation of such nonclassical states in the context of the quantum multiverse would be significantly different from that given in quantum optics. First, there is no need of a common space–time among the universes of the multiverse, and hence the concepts of complementarity and nonlocality have to be revised or extended. One of the aims of this paper is the analysis of such an extension. Second, we do not observe other universes rather than ours, and therefore the nonclassicality of the multiversal states can only be inferred from the properties of our single universe.

The other aim of the paper is to study the thermodynamics of entanglement of a pair of universes whose quantum mechanical states are entangled. As it is well known, entanglement is a quantum feature without a classical analogue. Actually, gravitational and cosmic entanglement are clearly related to quantum effects that have no classical counterpart because they can be related to the origin of the black hole thermodynamics [2, 3], and, on cosmological grounds, to the current accelerated expansion of the universe [4, 5]. Quantum entanglement can also be considered between the modes of matter fields that belong to different universes in a multiverse that shares a common space–time [6, 7]. Therefore, the entanglement between the states of two universes in a more general multiverse scenario also provides us with quantum effects having no classical analogue, one of which could be the small value of the cosmological constant nowadays [8].

The origin of the inter-universal entanglement is a question that deserves a better understanding of the physical processes that occur in the multiverse. It might well be that the universes of the multiverse could

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be created in pairs [9], some of which could stay in an entangled state, or it could also be that inter-universal entanglement is a relic effect of a dimensional reduction from any multidimensional theory. We can generally consider the existence of entangled states between two or more universes in any multiverse scenario. The key question is whether inter-universal entanglement can have dynamical or thermodynamical consequences in the properties of each single universe. That would make the whole multiverse proposal testable.

On the other hand, the quantum multiverse is a new paradigm that requires introducing statistical boundary conditions in cosmology. These can be given, for instance, by imposing a constant number of universes in the multiverse, a constant energy or a constant entropy, conditions that can be partially determined by the choice of the representation taken to describe the state of single universes. The representation plays a significant role in the degree of entanglement between the universes (described by that representation) and, thus, the boundary condition imposed on the state of the whole multiverse also determine, at least partially, the degree of entanglement between different universes.

The outline of the paper is as follows. In Sec. 2, we specify the multiverse scenario that we discuss in the context of a third-quantization formalism. A generalized quantum formulation of thermodynamics is analyzed in Sec. 3. In Sec. 4, some examples of entangled and squeezed states in the context of the quantum multiverse are computed and their thermodynamical magnitudes are considered of entanglement for different cases. The possible violation of classical inequalities and the analog of the EPR argument in the multiverse are analyzed in Sec. 5. Finally, in Sec. 6, we draw some tentative conclusions and make further comments.

2. THE MULTIVERSE SCENARIO IN THE THIRD-QUANTIZATION FORMALISM

Different multiverse scenarios can be found in the literature [7, 10–12] and, therefore, it is first needed to specify the model of the multiverse we are dealing with. Precisely, we shall consider the multiverse formed by different regions of the space–time which are causally disconnected from each other: (i) because the existence of cosmic singularities, like it happens in a phantom-dominated universe [13, 14] where the big-rip singularity splits the whole space–time into two disconnected pieces, (ii) because the very definition of the whole space–time manifold entails a nonsimply topology, like it would happen in a multiverse formed by a disconnected set of n simply-connected regions of space- time; or (iii) because the existence of a quantum barrier that makes meaningless any causal relationship between different regions of the space–time. Each simply-connected region of the space–time will then be considered a single universe throughout this paper[1)].

From a classical standpoint, we should just consider the causal piece of the space–time that we inhabit and regard the rest of the manifold as not being physically admissible. However, and this is one of the main claims of the present work, there might be quantum correlations among the otherwise disconnected regions of the space–time (similarly to how quantum correlations may appear in the composite state of two distant particles), which would have observable consequences on the properties of our single universe. In that case, other universes different from ours should also be considered to physically describe the universe.

In that context, the natural formulation of the quantum multiverse is a third-quantization scheme [15–17], where creation and annihilation operators of universes can be defined and a many-universe description of the wave function of the multiverse can be given similarly to the many-particle description naturally arising in quantum field theory. The basic idea of the third-quantization formalism is to consider the wave function of the universe as a field that propagates in the superspace of geometries and matter fields and, then, to study the state of the multiverse as quantum field theory in the superspace. Such a quantum field theory is not well-defined in the general superspace. But in the case of a homogeneous and isotropic space–time minimally coupled to n scalar fields, $\boldsymbol{\varphi} \equiv (\varphi_1, \ldots, \varphi_n)$, the Wheeler–De Witt equation can be written as [18]

$$\left(-\frac{\hbar^2}{\sqrt{-G}}\partial_A\left(\sqrt{-G}G^{AB}\partial_B\right)+\mathcal{V}(q^A)\right)\phi(q^A)=0, \quad (1)$$

where

$$\phi(q^A)\equiv\phi(a,\boldsymbol{\varphi})$$

is the wave function of the universe, which is defined on the configuration variables

$$\{q^A\}\equiv\{a,\boldsymbol{\varphi}\},$$

the potential $\mathcal{V}(q^A)$ is given by

$$\mathcal{V}(q^A)=a^3\Lambda-a+a^3(V_1(\varphi_1)+\ldots+V_n(\varphi_n)),$$

[1)] Of course, other multiverse scenarios can also be posed within each single universe of the quantum multiverse considered in this paper.

with $V_i(\varphi_i)$ being the potential of the scalar field φ_i, and G^{AB} is the inverse of the minisupermetric

$$G_{AB} = \mathrm{diag}(-a, a^3, \ldots, a^3), \tag{2}$$

whose determinant is

$$G = -a^{3n+1}.$$

Wheeler–De Witt equation (1) can be seen as a Klein–Gordon equation in the minisuperspace [15–17] and the Lorentzian signature of the minisupermetric (2) allows us to consider a formal analogy with quantum field theory in a curved space–time with the scale factor playing the role of a time-like variable of the minisuperspace and the scalar fields $\boldsymbol{\varphi} \equiv (\varphi_1, \ldots, \varphi_n)$, the spatial coordinates. The role of the scale factor as the time variable within a single universe can generally be a tricky task (see Refs. [18–24] for the customary discussions on the subject). However, we mainly restrict our attention to large parent universes with semiclassical space–times that undergo a monotonic expansion and for which, therefore, there is a one-to-one correspondence between the scale factor and the cosmic time, given by the Friedmann equation. In the context of the multiverse being considered, however, the cosmic time t becomes meaningless and the scale factor turns out to be the intrinsic time-like variable of the minisuperspace.

Following a description parallel to quantum field theory (see, e. g., Sec. 4.6 in Ref. [25]), the wave function of the whole multiverse is given by the generalized third-quantized Schrödinger equation

$$i\hbar\frac{\partial\Psi(a,\phi)}{\partial a} = \hat{H}\Psi(a,\phi) \equiv$$
$$\equiv -\frac{1}{2}\int d^n\boldsymbol{\varphi}\,\frac{\delta^2\Psi(a,\phi)}{\delta\phi(\boldsymbol{\varphi},a)\delta\phi(\boldsymbol{\varphi},a)} +$$
$$+\frac{1}{2}\int d^n\boldsymbol{\varphi}d^n\boldsymbol{\chi}\phi(\boldsymbol{\varphi},a)M(\boldsymbol{\varphi},\boldsymbol{\chi},a)\phi(\boldsymbol{\chi},a)\Psi(a,\phi), \tag{3}$$

where $M(\boldsymbol{\varphi},\boldsymbol{\chi},a)$ is the kernel of Eq. (1). The quantum state of the multiverse, in which different species of universes can coexist, is then given by a linear combination of product states of the form [26]

$$\Psi_{\mathbf{N}}(a,\boldsymbol{\phi}) \equiv$$
$$\equiv \Psi_{N_1}^{\boldsymbol{\alpha}_1}(a,\phi_1)\Psi_{N_2}^{\boldsymbol{\alpha}_2}(a,\phi_2)\ldots\Psi_{N_m}^{\boldsymbol{\alpha}_m}(a,\phi_m), \tag{4}$$

where

$$\boldsymbol{\phi} \equiv (\phi_1, \phi_2, \ldots, \phi_m), \quad \mathbf{N} \equiv (N_1, N_2, \ldots, N_m),$$

with N_i being the number of universes of type i, represented by the wave function

$$\phi_i \equiv \phi(a, \boldsymbol{\varphi}_i)$$

that corresponds to a universe that is described in terms of $\boldsymbol{\varphi}_i$ matter fields and

$$\boldsymbol{\alpha}_i \equiv (\alpha_{i,1}, \ldots, \alpha_{i,k})$$

parameters. For instance, it may represent a landscape of de Sitter universes with different values Λ_i of their vacuum energies. The functions $\Psi_{N_i}^{\Lambda_i}(a,\phi_i)$ in Eq. (4) are then the wave functions of the number eigenstates of the third-quantized Schrödinger equation

$$i\hbar\frac{\partial}{\partial a}\Psi_{N_i}^{\Lambda_i}(a,\phi_i) = \hat{H}_i(a,\phi,p_\phi)\Psi_{N_i}^{\Lambda_i}(a,\phi_i), \tag{5}$$

where $\hat{H}_i(a,\phi,p_\phi)$ is the third-quantized Hamiltonian [17, 26] that corresponds to each kind of universe, with

$$p_\phi \equiv \sqrt{-G}G^{0B}\nabla_B\phi$$

and ∇_B being respectively the third-quantized momentum and the covariant derivative in the minisuperspace. We note that we could also consider Hamiltonians of interaction between different species of universes, adding a more exhaustive phenomenology to the model of the multiverse [8].

Generally, the customary interpretation of the wave function of the multiverse, Eq. (4), is as follows [17]: we consider the expansion of the wave function in the orthonormal basis of number states, i. e.,

$$\Psi = \sum_{\mathbf{N}}\Psi_{\mathbf{N}}(a,\boldsymbol{\phi})|\mathbf{N}\rangle, \tag{6}$$

and $\Psi_{\mathbf{N}}(a_0,\boldsymbol{\phi})$ here is the probability amplitude to find $\mathbf{N}$ universes in the state of the multiverse with the value of the scale factor $a = a_0$. We note that the state given by Eq. (6) only represents the quantum state of a multiverse made up of homogeneous and isotropic universes. But the homogeneity and isotropy of the universes of the multiverse are conditions that can be assumed in the first approximation if we are dealing with large parent universes where macroscopic observers can inhabit. We can then assume that quantum state (6) is rather general and, indeed, it represents the most general quantum state of the multiverse in this paper.

We then consider a Friedmann–Robertson–Walker (FRW) metric whose evolution is dominated by a perfect fluid with the equation of state $p = w\rho$, where p and ρ are the pressure and the energy density of the fluid. We also consider an auxiliary scalar field φ that can represent the homogeneous and isotropic modes of a matter field whose potential is subdominant, at least as a first approximation. This is helpful in analyzing the influence of the inter-universal entanglement in

the matter fields of single universes. Wheeler–De Witt equation (1) can then be conveniently written as

$$\hbar^2\ddot{\phi} + \frac{\hbar^2}{a}\dot{\phi} - \frac{\hbar^2}{a^2}\phi'' + \omega^2(a)\phi = 0, \tag{7}$$

where $\phi \equiv \phi(a,\varphi)$ is the wave function of the universe, with

$$\dot{\phi} \equiv \frac{\partial\phi}{\partial a}$$

and

$$\phi' \equiv \frac{\partial\phi}{\partial\varphi}.$$

The potential term $\omega^2(a)$ is given by

$$\omega^2(a) = \omega_0^2 a^{2(q-1)} - \kappa a^2, \tag{8}$$

where ω_0^2 is a constant that is proportional to the energy density of the fluid on a given hypersurface

$$\Sigma_0 \equiv \Sigma(a_0), \quad q \equiv \frac{3}{2}(1-w)$$

parameterizes different kinds of fluids that permeate the universe, for instance, with the values $w = -1$, $w = 0$, and $w = 1/3$, which respectively mimic a vacuum-like fluid, a dust-like fluid, and a radiation-like fluid, and $\kappa = 0, \pm 1$ for a space–time with flat, closed, and open spatial sections, respectively. More realistic degrees of freedom are desirable in Eq. (7). But some interesting models of the universe can already be described by Eqs. (7) and (8), e. g., a flat or a closed de Sitter universe with

$$w = -1, \quad \omega_0^2 \equiv \Lambda,$$

$$\kappa = 0 \quad \text{or} \quad \kappa = +1,$$

and a universe with a slow-roll field χ for which

$$\frac{\partial\chi}{\partial t} \approx 0$$

with $w = -1$ and $\omega_0^2 \equiv V(\chi_0)$.

The quantum state of the multiverse can then be expressed in terms of an orthonormal basis of number states $|N\rangle$ that represent the number of universes (see, Eq. (6)). However, different representations can be taken for the number states $|N\rangle$ and it is not clear at all which one can properly represent the number of universes of the multiverse. A boundary condition has to be imposed on the state of the multiverse in order to (partially) fix the appropriate representation to be considered.

In the multiverse we are dealing with, there is no common space–time among the universes of the multiverse, and therefore no real observer can exist *outside* the universes. However, it is expected that the measurements performed by an idealized "super-observer", i. e., someone who lives in the multiverse, would not depend on the spatial and temporal properties of a particular single universe if these are internal properties of the universes with a meaning supplied by a particular reference system. It is then expected that the global properties of the multiverse, like the number of universes, would be invariant under changes, for instance, in the scale factor of a particular single universe. The boundary condition that the properties of the multiverse be independent of the value of the scale factor of a particular single universe then restricts the possible representations of universes to the set of invariant representations. These are given by annihilation and creation operators $\hat{b}$ and $\hat{b}^\dagger$ for which [27]

$$\frac{d\hat{b}^{(\dagger)}}{da} = \frac{i}{\hbar}[\hat{H}, \hat{b}^{(\dagger)}] + \frac{\partial\hat{b}^{(\dagger)}}{\partial a} = 0. \tag{9}$$

The solution of Eq. (9) is not unique [28], but for each solution, the eigenvalues of an operator constructed from a combination of invariant operators are independent of the value of the scale factor of a single universe. The boundary condition imposed is therefore an appropriate boundary condition to represent a multiverse with a constant number of universes, although it is worth noting that it is not the only boundary condition that can be imposed on the state of the multiverse.

Furthermore, the invariant representation does not have to be the most appropriate representation to describe the state of a singled-out universe from the standpoint of an internal observer who lives in the asymptotic regime of a large classical universe. We therefore consider two representations: the invariant representation induced by the boundary condition, which is imposed on the state of the whole multiverse, and the asymptotic representation that describes the state of one single universe from the standpoint of an internal observer. We then show that both representations are related by a Bogoliubov transformation, which entails entanglement effects between the states of two universes.

Like in quantum optics [1] and quantum gravity [9], entanglement among the states of two or more universes can be seen as a quantum feature that has no classical counterpart because it is a feature that does not appear in a classical multiverse of causally disconnected universes. But it is worth noting that the universes we are dealing with are large regions of space–time where it behaves classically, and therefore the quantum effects we are describing are not quantum properties of the space–time of a single universe, i. e., we are dealing not

with quantum gravity as such but with novel features that exclusively appear in the quantum multiverse.

3. QUANTUM THERMODYNAMICS IN THE MULTIVERSE

In quantum information theory, the generation of quantum entanglement follows some formal analogies with respect to the classical formulation of thermodynamics [29–33]. In particular, the impossibility of increasing the amount of entanglement in a bipartite system by means of local operations and classical communications alone has been claimed to be an analogue of the second principle of thermodynamics in quantum information theory [29]. This and other parallelisms have motivated the search for a quantum formulation of the thermodynamics of entanglement [29, 31, 32, 34] that would generalize the classical formulation of thermodynamics much as quantum theory is a more general framework from which the classical one can be recovered as a particular limit case.

The multiverse scenario allows considering entangled states among the states of two or more universes and computing the thermodynamical properties of entanglement between them. Inter-universal entanglement might then have observable consequences in the thermodynamical properties of a single universe if the relation between the quantum formulation of the thermodynamics of entanglement and the classical formulation of thermodynamics is eventually found. That would represent a major achievement for testing the multiverse scenario.

We first discuss the basics of the thermodynamics of entanglement in the multiverse. Following Refs. [30, 33], we define thermodynamical quantities for a closed system that is quantum mechanically represented by a density matrix $\hat{\rho}$, with the dynamics determined by a Hamiltonian operator $\hat{H}$:

$$E(a) = \mathrm{Tr}\left(\hat{\rho}(a)\hat{H}(a)\right), \tag{10}$$

$$Q(a) = \int^{a} \mathrm{Tr}\left(\frac{d\hat{\rho}(a')}{da'}\hat{H}(a')\right) da', \tag{11}$$

$$W(a) = \int^{a} \mathrm{Tr}\left(\hat{\rho}(a')\frac{d\hat{H}(a')}{da'}\right) da', \tag{12}$$

where $\mathrm{Tr}(\hat{O})$ means the trace of an operator $\hat{O}$, and, in the case of the multiverse, the time variable is replaced by the scale factor, which is a time-like variable of the minisuperspace. In these definitions, E is the quantum informational analog of energy, Q is the analogue of heat, and W is the analogue of work. Then the first principle of thermodynamics

$$dE = \delta W + \delta Q, \tag{13}$$

is directly satisfied. The quantum entropy is customarily defined by the von Neumann formula

$$S(\hat{\rho}) = -\,\mathrm{Tr}\left(\hat{\rho}(a)\ln\hat{\rho}(a)\right), \tag{14}$$

where the logarithmic function of an operator must be understood as its series expansion, i. e.,

$$S(\hat{\rho}) = \sum_{k=1}^{\infty}\frac{1}{k}\sum_{l=0}^{k}(-1)^l\begin{pmatrix} k \\ l \end{pmatrix}\mathrm{Tr}\left(\hat{\rho}^{l+1}\right) = \\ = -\sum_i \lambda_i \ln\lambda_i, \tag{15}$$

with λ_i being the eigenstates of the density matrix, and

$$0\cdot\ln 0 \equiv 0.$$

For a pure state, $\hat{\rho}^n = \hat{\rho}$ and $\lambda_i = \delta_{ij}$ for some value j, and therefore the entropy vanishes.

It is worthy of note that the quantum thermodynamical analogues to energy and entropy are invariant under a unitary evolution of the state of the multiverse. Using the cyclic property of the trace, we can write

$$S(\hat{\rho}) = \sum_{k=1}^{\infty}\frac{1}{k}\sum_{l=0}^{k}(-1)^l\begin{pmatrix} k \\ l \end{pmatrix}\times \\ \times\,\mathrm{Tr}\left(\hat{\mathcal{U}}_S^{\dagger}(a)\hat{\rho}_0^{l+1}\hat{\mathcal{U}}_S(a)\right) = S(\hat{\rho}_0), \tag{16}$$

and analogously for the energy E if no dissipative processes are considered in the dynamics of the multiverse. Such processes can make the state of a single universe effectively undergo an nonunitary evolution, increasing the entropy of an expanding universe [35, 36].

The invariance expressed by Eq. (16) is not necessarily applicable to the heat Q and work W. For instance, we consider two representations A and B that were related by a unitary transformation $\hat{\mathcal{U}}$ such that

$$\hat{\rho}_B = \hat{\mathcal{U}}\hat{\rho}_A\hat{\mathcal{U}}^{\dagger}, \quad \hat{H}_B = \hat{\mathcal{U}}\hat{H}_A\hat{\mathcal{U}}^{\dagger}.$$

It then follows that $E_A = E_B$ and $S_A = S_B$. In particular,

$$\delta Q_A(a)+\delta W_A(a) = \mathrm{Tr}\left(\frac{\partial\hat{\rho}_A}{\partial a}\hat{H}_A\right) + \\ + \mathrm{Tr}\left(\hat{\rho}_A\frac{\partial\hat{H}_A}{\partial a}\right) = \mathrm{Tr}\left(\frac{\partial\hat{\rho}_B}{\partial a}\hat{H}_B\right) + \\ + \mathrm{Tr}\left(\hat{\rho}_B\frac{\partial\hat{H}_B}{\partial a}\right) = \delta Q_B(a) + \delta W_B(a), \tag{17}$$

where we use that

$$\dot{\hat{\mathcal{U}}}\hat{\mathcal{U}}^{\dagger} = -\hat{\mathcal{U}}\dot{\hat{\mathcal{U}}}^{\dagger}.$$

However, if $\hat{\mathcal{U}} \equiv \hat{\mathcal{U}}(a)$, then it is not necessarily true that $\delta Q_A = \delta Q_B$ and $\delta W_A = \delta W_B$. In classical thermodynamics, heat and work, unlike the energy and the entropy, are not functions of state because their values depend on the path of integration of δQ and δW. The analogy in quantum thermodynamics is that Q and W depend on the representation that is taken to compute them.

Two terms can be distinguished in the change of entropy: one due to the variation of heat and the other caused by an adiabatic process, i. e., [18, 30]

$$\frac{dS}{da} = \frac{1}{T}\frac{\delta Q}{da} + \sigma(a), \tag{18}$$

where the second term $\sigma(a)$ is called production of entropy [30]. The second principle of thermodynamics states that the entropy of a system cannot decrease under any adiabatic process, which is equivalent to saying that the production of entropy must be non-negative, i. e.,

$$\sigma(a) \geq 0. \tag{19}$$

We note that in quantum thermodynamics of open systems [18, 30], the change of entropy is also expressed as

$$\frac{dS}{dt} = \left(\frac{dS}{dt}\right)_{ext} + \left(\frac{dS}{dt}\right)_{int},$$

where

$$\left(\frac{dS}{dt}\right)_{ext} = \frac{\delta Q}{T}$$

is interpreted as the change in the entropy because of the interaction with an external bath (or reservoir) at a temperature T; and

$$\left(\frac{dS}{dt}\right)_{int} \geq 0$$

is interpreted as the change of entropy because of the change in the internal degrees of freedom. But in the multiverse, the terms external and internal have no meaning because in a closed system all the thermodynamical quantities are by definition internal to the system. We can still formally define the thermodynamical quantities in Eqs. (10)–(12) and (14) similarly to how this is done in open systems, although their interpretation is rather different for a closed system like the multiverse. Here, the heat Q and work W cannot be interpreted as ways of exchanging energy with a

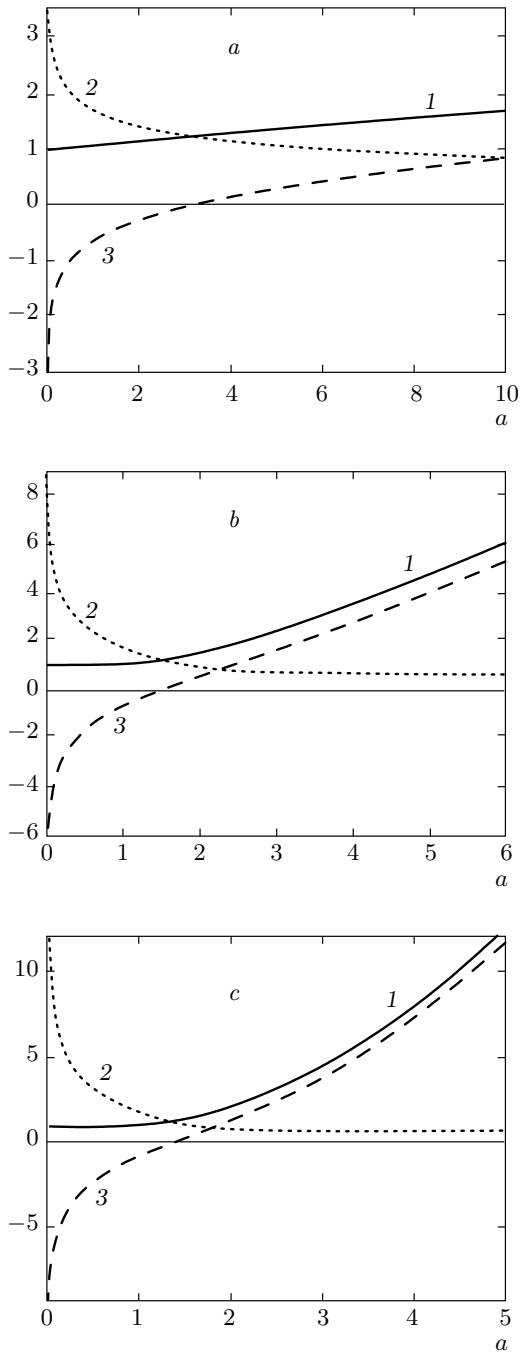

Fig. 1. Energy E (*1*), heat Q (*2*), and work W (*3*), Eqs. (21)–(24), for different values of the parameter $w = 0$ (*a*), -0.6 (*b*), and -1 (*c*) in the equation of state $p = w\rho$. The first principle of thermodynamics $E = Q + W$ is always satisfied

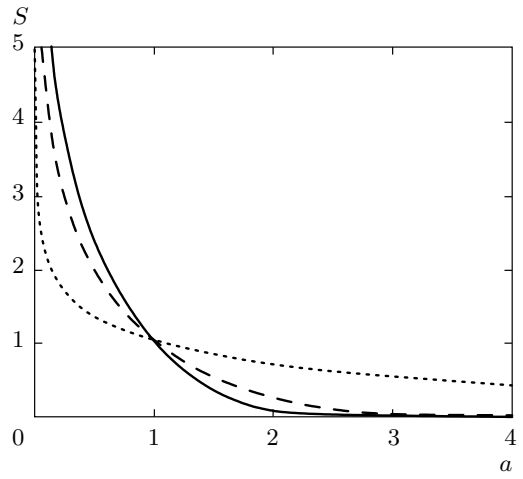

Fig. 2. Quantum entropy of the universe, Eq. (25), for different values of the parameter w: $w = -1$ (solid line), -0.6 (dashed line), 0 (dotted line). The entropy decreases as the scale factor increases. However, the second principle of thermodynamics is still satisfied because the process is not adiabatic and the production of entropy is zero

reservoir because there is no such reservoir. Similarly, the analogue of the temperature T does not represent the temperature of an external bath. All the thermodynamical quantities of a closed system are internal properties of the system.

In some appropriate limit, we should recover the classical formulation of thermodynamics, whose paradigmatic state is a thermal state at a constant temperature with the density matrix given by

$$\hat{\rho} = \frac{1}{Z}\sum_N \exp\left[-\frac{\hbar\omega(a)}{T}\left(N+\frac{1}{2}\right)\right]|N,a\rangle\langle N,a|, \quad (20)$$

where

$$Z^{-1} \equiv 2\,\mathrm{sh}\,\frac{\hbar\omega(a)}{2T}$$

and

$$\omega(a) = \frac{\omega_0}{\hbar}a^{q-1}$$

is the frequency of the Hamiltonian that determines the evolution of one single universe (see Eq. (8)). In that case, the thermodynamical quantities involved in the first principle of thermodynamics (13) turn out to be

$$E(a) = \frac{\hbar\omega(a)}{2}\,\mathrm{cth}\,\frac{\hbar\omega(a)}{2T}, \quad (21)$$

with

$$Q(a) = T\left(\frac{\hbar\omega(a)}{2T}\,\mathrm{cth}\,\frac{\hbar\omega(a)}{2T} - \ln\mathrm{sh}\,\frac{\hbar\omega(a)}{2T}\right), \quad (22)$$

$$W(a) = T\ln\mathrm{sh}\,\frac{\hbar\omega(a)}{2T}. \quad (23)$$

It can be verified that

$$dE = \delta Q + \delta W. \quad (24)$$

For a constant value of the frequency, the total energy is also a constant, and then

$$\delta Q = \delta W = 0.$$

In the multiverse, however, the heat production term δQ appears because of the dependence of the frequency on the scale factor. Hence, the entropy

$$S = \frac{\hbar\omega(a)}{2T}\,\mathrm{cth}\,\frac{\hbar\omega(a)}{2T} - \ln\mathrm{sh}\,\frac{\hbar\omega(a)}{2T} - \ln 2 \quad (25)$$

is no longer constant and the change of entropy

$$dS = -\frac{\hbar^2\omega\dot{\omega}}{4T^2}\frac{1}{\mathrm{sh}^2(\hbar\omega(a)/2T)}\,da \quad (26)$$

turns out to be negative as the scale factor increases. However, the second principle of thermodynamics is still satisfied because the change of entropy corresponds precisely to the change of heat (divided by the temperature T), and the production of entropy is therefore zero,

$$\sigma = \frac{dS}{da} - \frac{1}{T}\frac{\delta Q}{da} \equiv 0, \quad (27)$$

as is expected in a closed system with no dissipative process (which we do not consider here). Therefore, the second principle of thermodynamics does not impose any arrow of time in the case being considered because it is satisfied for an expanding universe as well as for a contracting one, simply because Eq. (27) is identically satisfied in both cases. We note that the customary arrow of time appears in cosmology as a consequence of taking some coarse graining over the matter fields that we do not consider here. The relation between the arrow of time of entanglement thermodynamics and the usual arrow of time in cosmology is a subject that deserves further investigation.

4. ENTANGLED AND SQUEEZED STATES IN THE MULTIVERSE

4.1. FRW universe filled with a fluid and a massless scalar field

We consider a massless scalar field in a flat FRW space–time whose dynamics is dominated by a perfect fluid with the equation of state

$$p = w\rho,$$

where p and ρ are the pressure and the energy density of the fluid, and w is a constant parameter. A massless scalar field φ can represent the homogeneous and isotropic modes of a local matter field whose potential energy is subdominant and negligible, in the first approximation. Then, with an appropriate factor ordering [18] and rescaling the scalar field to absorb unimportant constants, the Wheeler–De Witt equation can be written as Eq. (7), with Eq. (8) for $\kappa = 0$. We recall that

$$q \equiv \frac{3}{2}(1-w)$$

in Eq. (8) just parameterizes the kind of fluid that permeates the universe. We mostly consider the values $w = -1$, which mimics a flat de Sitter space–time with $\Lambda = \omega_0^2$, $w \gtrsim -1$, which corresponds to a quintessence-like fluid, and $w \lesssim -1$, which corresponds to a phantom-like fluid. However, we note that the formalism equally applies to any other constant value of w.

In the third-quantization formalism, the wave function of the universe is promoted to an operator that can be decomposed in normal modes as

$$\hat{\phi}(a,\varphi) = \int dk \left[e^{ik\varphi} A_k(a) \hat{c}_k^\dagger + e^{-ik\varphi} A_k^*(a) \hat{c}_k \right], \quad (28)$$

where the amplitudes $A_k(a)$ satisfy the Bessel equation

$$a^2 \ddot{A}_k + a\dot{A}_k + (\tilde{\omega}_0^2 a^{2q} + k^2) A_k = 0, \quad (29)$$

with

$$\dot{A}_k \equiv \frac{\partial A_k}{\partial a}, \quad \tilde{\omega}_0 \equiv \frac{\omega_0}{\hbar}.$$

The constant operators

$$\hat{c}_k \equiv \sqrt{\frac{\tilde{\omega}_{0k}}{2\hbar}} \left(\hat{\phi} + \frac{i}{\tilde{\omega}_{0k}} \hat{p}_\phi \right), \quad \hat{c}_k^\dagger \equiv \sqrt{\frac{\tilde{\omega}_{0k}}{2\hbar}} \left(\hat{\phi} - \frac{i}{\tilde{\omega}_{0k}} \hat{p}_\phi \right)$$

can respectively be interpreted in Eq. (28) as the annihilation and creation operators of a universe whose energy density is proportional to

$$\tilde{\omega}_{0k}^2 \equiv \tilde{\omega}_0^2 a_0^{2q} + k^2,$$

at the boundary hypersurface

$$\Sigma_0 \equiv \Sigma(a_0).$$

The kind of universes created or annihilated by $\hat{c}_k^\dagger$ and $\hat{c}_k$ depends on the boundary condition that is imposed on the probability amplitude $A_k(a)$. If the operators $\hat{c}_k^\dagger$ and $\hat{c}_k$ in (28) respectively create and annihilate expanding branches of the universe, then the probability amplitude A_k is given by

$$A_k(a) = \sqrt{\frac{\pi}{4q}} e^{\pi k/2q} \mathcal{H}^{(2)}_{ik/q} \left(\frac{\omega_0}{q\hbar} a^q \right), \quad (30)$$

where $\mathcal{H}^{(2)}_\nu(x)$ is the Hankel function of second kind and order ν. The normalization constant in Eq. (30) is chosen such that the usual orthonormality conditions

$$(\phi_k, \phi_l) = \delta_{kl}, \quad (\phi_k^*, \phi_l^*) = -\delta_{kl}, \quad (\phi_k, \phi_l^*) = 0, \quad (31)$$

hold for the modes

$$\phi_k(a,\varphi) \equiv e^{ik\varphi} A_k(a),$$

with the scalar product,

$$(\phi, \psi) \equiv -i \int_{-\infty}^{\infty} d\varphi W^{-1} (\phi \partial_a \psi^* - \psi^* \partial \phi), \quad (32)$$

where $W = 1/a$ is the Wronskian of Bessel equation (29). The modes in Eq. (30) correspond to the expanding branches of the universe because in the semiclassical regime [37],

$$\mathcal{H}^{(2)}_\nu \left(\frac{\omega_0}{q\hbar} a^q \right) \sim a^{-q/2} e^{-i/\hbar S_c(a)}, \quad (33)$$

where

$$S_c(a) = \frac{\omega_0}{q} a^q$$

is the classical action. Then the momentum operator, which is defined by the equation

$$\hat{p}_a \phi(a) \equiv -i\hbar \frac{\partial \phi(a)}{\partial a},$$

is highly peaked around the value of the classical momentum [38],

$$p_a^c \equiv -a \frac{\partial a}{\partial t},$$

and it then follows that

$$\frac{\partial a}{\partial t} \approx \frac{1}{a} \frac{\partial S_c}{\partial a},$$

which corresponds to the expanding branch of the Friedmann equation.

We could have imposed a different boundary condition on the probability amplitudes $A_k(a)$ such that the creation and annihilation operators, $\hat{c}_k^\dagger$ and $\hat{c}_k$ in Eq. (28), would create and annihilate entangled pairs of expanding and contracting branches of the universe. The modes A_k would then be given by

$$\bar{A}_k = \left(\frac{2q}{\pi}\,\mathrm{sh}\,\frac{k\pi}{q}\right)^{-1/2} \mathcal{J}_{-ik/q}\left(\frac{\omega_0}{q\hbar}a^q\right), \tag{34}$$

where $\mathcal{J}_\nu(x)$ is the Bessel function of first kind and order ν. The two sets of modes are related by the Bogoliubov transformation

$$\bar{A}_k = \alpha_k A_k + \beta_k A_k^*, \tag{35}$$

where α_k and β_k are given by

$$\alpha_k = e^{\pi k/q}\beta_k, \quad \beta_k = \left(\frac{e^{-\pi k/q}}{2\,\mathrm{sh}(\pi k/q)}\right)^{1/2}, \tag{36}$$

with

$$|\alpha_k|^2 - |\beta_k|^2 = 1.$$

The vacuum state of the bar modes, $|\bar{0}_k\rangle$, turns out to be a squeezed state in the representation of the modes without bar. The mean value of the number operator

$$\hat{\bar{N}}_k \equiv \hat{\bar{c}}_k^\dagger \hat{\bar{c}}_k,$$

computed in the vacuum state $|0_k\rangle$,

$$\langle 0_k|\hat{\bar{N}}_k|0_k\rangle = |\beta_k|^2 = \frac{1}{e^{2\pi k/q}-1}, \tag{37}$$

turns out to represent a thermal distribution with the temperature given by

$$T \equiv \frac{q}{2\pi}\xi_0\left(\frac{\hbar}{k_B}\right), \tag{38}$$

where

$$q \equiv \frac{3}{2}(1-w)$$

and ξ_0 is a constant of dimension s^{-1}. The above thermal distribution is formally similar to thermal radiation that appears in quantizing a scalar field in a Milne universe in the context of quantum field theory in a curved space–time (see Ref. [39]). However, unlike for the Milne universe, it is not clear in the case of a multiverse made up of parent universes which vacuum state corresponds to a "preferred observer" (i. e., to an adiabatic vacuum), because the modes of the wave function of the universe are defined on the minisuperspace rather than on the space–time variables. For the same reason, it seems difficult to estimate T in Eq. (38). However, the remarkable result is that the universe might stay in the thermal state as a consequence of quantum entanglement between different branches.

Indeed, the interpretation in the multiverse is rather different from that of the quantum field theory in the Milne universe. The "no-boundary" condition proposed by Hartle and Hawking [40] implies that the quantum state of the universe is described by a real wave function given by the superposition of an expanding and a contracting branch [41]. Then the universes of the multiverse would be quantum mechanically represented by the modes $\bar{A}_k(a)$ in Eq. (34), and the state of each single branch by the modes $A_k(a)$ and $A_k^*(a)$ in Eq. (30), for the expanding and the contracting branches, respectively.

The expanding and the contracting branches of the universe would subsequently undergo a very effective decoherence process [42, 43] by means of which their states rapidly become causally disconnected. However, squeezing relation (35) between the two sets of modes $\bar{A}_k$ and A_k does not depend on the value of the scalar factor, and is therefore still valid even when the two semiclassical branches of the universe are rather independent from each other. In that case, an observer inhabiting one of the semiclassical branches would describe the state of her universe by a reduced density matrix that is the result of tracing out the degrees of freedom of the partner branch from the composite state of the two branches.

A particularly interesting case where the causal disconnection between the branches of the universe is even more explicit is where the evolution of the universe is dominated by a phantom-like fluid (with $w < -1$). Then the big rip singularity [13, 44] splits the whole space–time manifold into two regions, before and after the singularity. These two regions are causally disconnected because of the breaking down of the classical laws of physics in the singularity, which prevents each region from any physical signaling to the partner region. The universe expands before the big rips occurs and contracts after it. Therefore, the composite quantum state of the universe is given by a superposition of the expanding and contracting branches in Eq. (34). However, for an observer inhabiting one of the branches, the quantum state of the corresponding branch is given by a reduced density matrix that is obtained by tracing out the degrees of freedom of the partner region. Within the formal analogy with quantum field theory in a curved space–time [25], if a composite state corresponds to the vacuum state $|\bar{0}_{k,-k}\rangle$, then the total density matrix can be written as

$$\hat{\rho} = |\bar{0}_{k,-k}\rangle\langle\bar{0}_{k,-k}| = \\ = \frac{1}{|\alpha_k|^2}\sum_{n,m=0}^{\infty}\left(\frac{\beta_k}{\alpha_k}\right)^{n+m}|n_k, n_{-k}\rangle\langle m_k, m_{-k}|, \quad (39)$$

where the modes k and $-k$ respectively correspond to the expanding and contracting branches of the universe. The reduced density matrix for the expanding region before the singularity turns out to be

$$\hat{\rho}_r = \mathrm{Tr}_{-k}\,\rho = \\ = \frac{1}{Z}\sum_{n=0}^{\infty}\exp\left[-\frac{2\pi k}{q}\left(n_k+\frac{1}{2}\right)\right]|n_k\rangle\langle n_k|, \quad (40)$$

with

$$Z \equiv \sum_{n=0}^{\infty}\exp\left[-\frac{2\pi k}{q}\left(n_k+\frac{1}{2}\right)\right] = 2\,\mathrm{sh}\,\frac{\pi k}{q}.$$

It represents a thermal state with the temperature in (38).

Using the reduced density matrix and the equations developed in this section, we can obtain the thermodynamical quantities that correspond to thermal state (40). Entanglement entropy (94) turns out to be

$$S_{ent} = |\alpha_k|^2\ln|\alpha_k|^2 - |\beta_k|^2\ln|\beta_k|^2 = \\ = \frac{\pi k}{q}\,\mathrm{cth}\,\frac{\pi k}{q} - \ln\left(2\,\mathrm{sh}\,\frac{\pi k}{q}\right), \quad (41)$$

which coincides with Eq. (25) if $\omega_k = k$ and $T = q/2\pi$. From Eqs. (??) and (23), we can verify that $Q = TS_{ent}$, and the energy and work are given by

$$E = \frac{k}{2}\,\mathrm{cth}\,\frac{\pi k}{q}, \quad (42)$$

$$W = \frac{q}{2\pi}\ln\mathrm{sh}\,\frac{\pi k}{q}. \quad (43)$$

The change of the entropy with respect to the value of the mode k for the constant temperature $T = q/2\pi$ is (see Eq. (26))

$$\frac{dS}{dk} = -\frac{\pi^2 k}{q^2}\frac{1}{\mathrm{sh}^2\,\dfrac{\pi k}{q}} = \frac{1}{T}\frac{\delta Q}{dk}. \quad (44)$$

Therefore, the production of entropy σ is zero. In that case, the energy of entanglement can be identified with the heat Q, i. e.,

$$E_{ent} = Q = \frac{k}{2}\,\mathrm{cth}\,\frac{\pi k}{q} - \frac{q}{2\pi}\ln\left(2\,\mathrm{sh}\,\frac{\pi k}{q}\right), \quad (45)$$

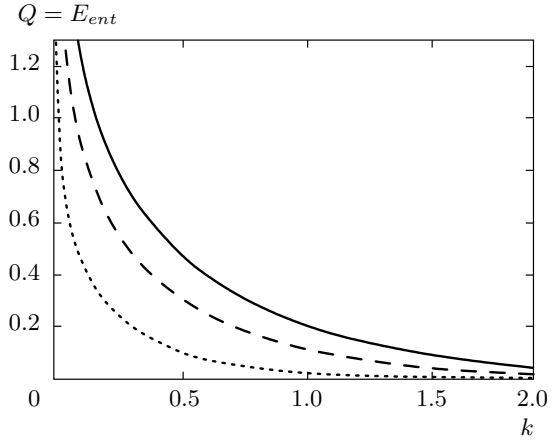

Fig. 3. Energy of entanglement between the positive and negative modes k of the scalar field, Eq. (45), for different values of the parameter w of the equation of state of the fluid that dominates the expansion of the universe: $w = -1$ (solid line), -0.6 (dashed line), 0 (dotted line)

where

$$q = \frac{3}{2}(1-w),$$

with w being the proportionality constant of the equation of state of the fluid that dominates the expansion of the universe, $p = w\hat{\rho}$ (we recall that $q = 3$ for vacuum-dominated universes). The energy of entanglement is depicted in Fig. 3 for different values of the parameter w.

Similar results should be expected for a closed FRW space–time because the geometric term in Eq. (8) becomes negligible for large values of the scale factor. Furthermore, the squeezing relation given by Eq. (35) does not depend on the value of the scale factor, and it can therefore be expected that the entanglement between the branches of the universe also survives at small values of the scale factor.

We consider the particular case of a massless scalar field in a closed de Sitter space–time endowed with a cosmological constant Λ. Then the Wheeler–De Witt equation can be written as Eq. (7) with

$$\kappa = 1, \quad q = 3, \quad \omega_0^2 \equiv \Lambda$$

in Eq. (8):

$$\hbar^2\ddot{\phi} + \frac{\hbar^2}{a}\dot{\phi} - \frac{\hbar^2}{a^2}\phi'' + (\Lambda a^4 - a^2)\phi = 0, \quad (46)$$

where we recall that

$$\phi \equiv \phi(a,\varphi), \quad \dot{\phi} \equiv \frac{\partial\phi}{\partial a}, \quad \phi' \equiv \frac{\partial\phi}{\partial\varphi}.$$

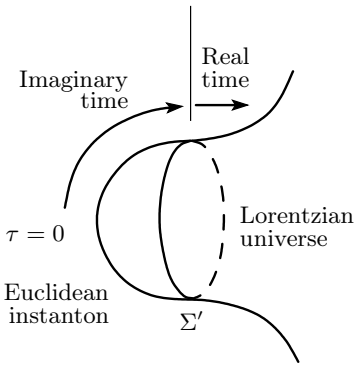

Fig. 4. The creation of a de Sitter universe from a de Sitter instanton

The probability amplitude of wave function (28), $A_k(a)$, now satisfies the equation of a damped harmonic oscillator,

$$\hbar^2 \ddot{A}_k(a) + \frac{\hbar^2}{a}\dot{A}_k(a) + \omega_k^2(a)A_k(a) = 0, \tag{47}$$

with the mode-dependent frequency $\omega_k(a)$ given by

$$\omega_k(a) = \sqrt{\Lambda a^4 - a^2 + \frac{\hbar^2 k^2}{a^2}}. \tag{48}$$

The corresponding Friedmann equation for each single mode turns out to be

$$\frac{\partial a}{\partial t} = \frac{\omega_k(a)}{a},$$

and hence real values of frequency (48) essentially define a Lorentzian domain of the wave function of a single universe, and complex values define the Euclidean region of the universe. We first consider the zero-mode wave function, i. e., $k = 0$. Then, for

$$a > a_+ \equiv \frac{1}{\sqrt{\Lambda}},$$

the solution of the Friedmann equation describes the evolution of a closed de Sitter space–time with an eventual exponential expansion of the scale factor with the Friedmann time,

$$a(t) \sim e^{\sqrt{\Lambda}t}.$$

For $a < a_+$, the solution of the Euclidean Friedmann equation corresponds to a de Sitter instanton that eventually collapses at the Euclidean time $\tau = 0$ (see Fig. 4). This is the customary picture of a de Sitter universe created from a de Sitter instanton [18, 19, 45, 46].

Other modes different from zero should also be considered [47]; the quantum correction given by the last term in Eq. (48) then introduces an important difference. For

$$k_m > k > 0,$$

where

$$k_m^2 \equiv \frac{4}{27\hbar^2\Lambda^2},$$

there are two transition hypersurfaces from the Euclidean to the Lorentzian region,

$$\Sigma' \equiv \Sigma(a_+), \quad \Sigma'' \equiv \Sigma(a_-),$$

with

$$a_+ \equiv \frac{1}{\sqrt{3\Lambda}}\sqrt{1 + 2\cos\left(\frac{\theta_k}{3}\right)}, \tag{49}$$

$$a_- \equiv \frac{1}{\sqrt{3\Lambda}}\sqrt{1 - 2\cos\left(\frac{\theta_k + \pi}{3}\right)}, \tag{50}$$

where, in units for which $\hbar = 1$,

$$\theta_k \equiv \operatorname{arctg}\frac{2k\sqrt{k_m^2 - k^2}}{k_m^2 - 2k^2}. \tag{51}$$

The picture is then rather different from the one depicted in Fig. 4. First, on the transition hypersurface

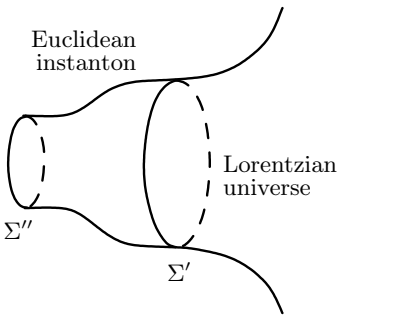

Fig. 5. Before reaching the collapse, the instanton finds the transition hypersurface Σ''

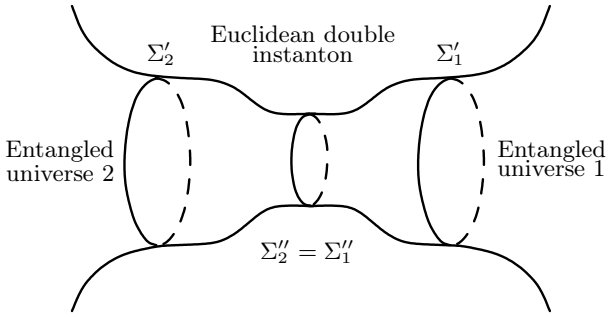

Fig. 6. The creation of a pair of entangled universes from a pair of instantons

Σ', the universe finds the Euclidean region (we note that $a_+ \to 1/\sqrt{\Lambda}$, and $a_- \to 0$ as $k \to 0$). However, before reaching the collapse, the Euclidean instanton finds the transition hypersurface Σ'' (see Fig. 5). Then, following a mechanism that parallels that proposed in Refs. [48–50], two instantons can be matched by identifying their hypersurfaces Σ'' (see Fig. 6). A double instanton like the one depicted in Fig. 6 would eventually give rise to an entangled pair of universes because the matching hypersurface

$$\Sigma'' \equiv \Sigma''(a_-),$$

where

$$a_- \equiv a_-(\theta_k)$$

is given by Eq. (50) with Eq. (51), depends on the value of the mode k. Hence, the matched instantons can only be joined for an equal value of the mode of their respective scalar fields, i. e., for an equal value of the momentum of the scalar field. The pair of universes created from such a double instanton is then entangled, with the composite quantum state given by

$$\phi_{I,II} = \int dk \left[\exp\left(ik(\varphi_I + \varphi_{II})\right) \times \right.$$
$$\times A_{I,k}(a) A_{II,k}(a) \hat{c}^\dagger_{I,k} \hat{c}^\dagger_{II,k} + \exp\left(-ik(\varphi_I + \varphi_{II})\right) \times$$
$$\left. \times A^*_{I,k}(a) A^*_{II,k}(a)\, \hat{c}_{I,k}\, \hat{c}_{II,k}\right], \quad (52)$$

where φ_I and φ_{II} are the values of the scalar fields of each single universe, labeled I and II. The cross terms like $A_{I,k} A^*_{II,k}$ cannot be present in the state of the pair of universes because of orthonormality relations (31). Then the composite quantum state must necessarily be the entangled state represented by Eq. (52).

We note that this is a quantum effect having no classical analogue because the quantum correction term in Eq. (48) does not appear in the classical theory. Furthermore, we also note that there is no Euclidean regime for $k \geq k_m$, and it can therefore be assumed that no universes were created from the space–time foam with such values of the mode. The value k_m would then become the natural cut-off of the theory.

For each single universe of the entangled pair, we should expect a behavior similar to that in the case of a flat space–time, at least for large values of the scale factor. However, Eq. (47) is not exactly solvable. For a scale factor $a \gg a_-$, the quantum correction term in Eq. (48) can be disregarded and the WKB approximation can be considered. Then the solutions of Eq. (47) are given, up to the order $\hbar$, by

$$A_k(a) \approx \frac{1}{\sqrt{2a\omega(a)}} \exp\left(\pm \frac{i}{\hbar} S_c(a)\right), \quad (53)$$

where

$$\omega(a) \equiv \omega_{k=0}(a)$$

and

$$S_c(a) = \int^a da'\, \omega(a') = \frac{(a^2\Lambda - 1)^{3/2}}{3\Lambda} \quad (54)$$

is the solution of the corresponding Hamilton–Jacobi equation. However, the dependence on the mode k has disappeared in WKB approximation (53) and no explicit computation can be made to relate the different modes of the scalar field for different boundary conditions.

4.2. Slowly varying field in a closed FRW space–time

We now consider the case of a slowly varying field in a closed FRW space–time. In that regime,

$$\frac{\partial \varphi}{\partial t} \approx 0, \quad V(\varphi) \approx V(\varphi_0),$$

and Wheeler–De Witt equation (7) can be written as

$$\ddot{\phi}(a, \varphi_0) + \frac{\dot{\mathcal{M}}}{\mathcal{M}} \dot{\phi}(a, \varphi_0) + \tilde{\omega}^2 \phi(a, \varphi_0) = 0, \quad (55)$$

where

$$\dot{\mathcal{M}} \equiv \frac{\partial \mathcal{M}}{\partial a}, \quad \mathcal{M} \equiv \mathcal{M}(a) = a$$

and

$$\tilde{\omega} \equiv \tilde{\omega}(a, \varphi_0) = \frac{a}{\hbar} \sqrt{a^2 V(\varphi_0) - 1}.$$

The Wheeler–De Witt equation is explicitly written in form (55) to stress the formal similarity with the equation of a damped harmonic oscillator with a time-dependent frequency, where the scale factor formally plays the role of the time variable of the minisuperspace spanned by the variables (a, φ). The term with ϕ'' in Eq. (7) does not appear in Eq. (55) because it comes from the quantization of the classical momentum

$$p_\varphi \propto \frac{\partial \varphi}{\partial t},$$

which is zero in the slow-roll approximation.

Following the analogy between Wheeler–De Witt equation (55) and the standard equation for the harmonic oscillator, we can use different representations to describe the quantum state of the multiverse. However, as is well known, the Hamiltonian of a harmonic oscillator with a time-dependent frequency is not an invariant operator [27], and its eigenstates evolve as squeezed states [27, 51–56]. The representation given

by the eigenstates of the Hamiltonian of harmonic oscillator (55) is not an appropriate representation for describing a given number of universes in the multiverse because the number of universes of the multiverse would then depend on the value of the scale factor of a particular single universe. Similarly, the representation chosen in Sec. 2 in terms of the constant operators $\hat{c}_k$ and $\hat{c}_k^\dagger$, defined after Eq. (28), is not an appropriate number representation because the eigenvalues of the constant number operator

$$\hat{N}_k \equiv \hat{c}_k^\dagger \hat{c}_k$$

are not scale-factor invariant either, a property which is expected in the multiverse.

The boundary condition of the multiverse that the number of universes does not depend on the value of the scale factor of a particular single universe determines the representation that has to be chosen. This has to be an invariant representation [27, 28]. For instance, we consider the invariant representation defined by the annihilation and creation operators [28]

$$\hat{b}(a) = \frac{i}{\sqrt{\hbar}}\left(u^*\hat{p}_\phi - \mathcal{M}\dot{u}^*\hat{\phi}\right), \tag{56}$$

$$\hat{b}^\dagger(a) = \frac{-i}{\sqrt{\hbar}}\left(u\hat{p}_\phi - \mathcal{M}\dot{u}\hat{\phi}\right), \tag{57}$$

where

$$u(a) = \frac{1}{\sqrt{2}}R(a)e^{-i\theta_R(a)},$$

with $R(a)$ satisfying the auxiliary equation

$$\ddot{R} + \frac{\dot{\mathcal{M}}}{\mathcal{M}}\dot{R} + \tilde{\omega}^2 R = \frac{1}{\mathcal{M}^2 R^3}, \tag{58}$$

and

$$\dot{\theta}_R = \frac{1}{\mathcal{M}R^2}.$$

It can be verified that a solution of Eq. (58) is given by

$$R = \sqrt{\phi_1^2 + \phi_2^2},$$

where ϕ_1 and ϕ_2 are two linearly independent solutions of Eq. (55) satisfying the normalization condition

$$\phi_1\dot{\phi}_2 - \phi_2\dot{\phi}_1 = \frac{1}{\mathcal{M}}.$$

In the WKB approximation, they can be chosen as

$$\begin{aligned}\phi_1(a,\varphi_0) &\approx \frac{1}{\sqrt{\mathcal{M}\tilde{\omega}}}\cos\frac{S_c}{\hbar},\\ \phi_2(a,\varphi_0) &\approx \frac{1}{\sqrt{\mathcal{M}\tilde{\omega}}}\sin\frac{S_c}{\hbar},\end{aligned} \tag{59}$$

where

$$S_c = \frac{(a^2V(\varphi_0)-1)^{3/2}}{3V(\varphi_0)},$$

whence

$$R \approx \frac{1}{\sqrt{\mathcal{M}\tilde{\omega}}} \propto V^{-1/4}a^{-3/2}, \tag{60}$$

for large values of the scale factor. The operators defined in Eqs. (56) and (57) satisfy the usual relations

$$\hat{b}(a)|N,a\rangle = \sqrt{N}|N-1,a\rangle, \tag{61}$$

$$\hat{b}^\dagger(a)|N,a\rangle = \sqrt{N+1}|N+1,a\rangle, \tag{62}$$

$$\hat{b}^\dagger(a)\hat{b}(a)|N,a\rangle = N|N,a\rangle, \tag{63}$$

where $|N,a\rangle$ are the eigenstates of the invariant operator,

$$\hat{I} \equiv \hat{b}^\dagger(a)\hat{b}(a) + \frac{1}{2},$$

and therefore $N \neq N(a)$. Thus, N can be interpreted as the number of universes in the multiverse, and $\hat{b}^\dagger(a)$ and $\hat{b}(a)$ as the creation and annihilation operators of universes.

The creation and annihilation operators defined by Eqs. (56) and (57) can be related to the creation and annihilation operators $\hat{c}^\dagger$ and $\hat{c}$ of the harmonic oscillator with the constant mass $\mathcal{M}_0 \equiv a_0$ and frequency

$$\omega_0 \equiv \omega(a,\varphi_0)_{|a=a_0}$$

by the squeezing transformation

$$\hat{b}(a) = \mu_0\hat{c} + \nu_0\hat{c}^\dagger, \tag{64}$$

$$\hat{b}^\dagger(a) = \mu_0^*\hat{c}^\dagger + \nu_0^*\hat{c}, \tag{65}$$

where

$$\mu_0 = \frac{e^{i\theta_R}}{2\sqrt{\mathcal{M}_0\omega_0}}\left(\frac{1}{R} + \mathcal{M}_0\omega_0 R - i\mathcal{M}\dot{R}\right), \tag{66}$$

$$\nu_0 = \frac{e^{i\theta_R}}{2\sqrt{\mathcal{M}_0\omega_0}}\left(\frac{1}{R} - \mathcal{M}_0\omega_0 R - i\mathcal{M}\dot{R}\right), \tag{67}$$

with

$$|\mu_0|^2 - |\nu_0|^2 = 1.$$

In quantum optics, the squeezed states of light are also called two-photon coherent states [57, 58] because they can be interpreted as coherent states of an entangled pair of photons. This allows interpreting the squeezed states of the multiverse as the state of a correlated pair of universes. We note that in invariant representation (56), (57), the Hamiltonian of the multiverse that leads to Wheeler–De Witt equation (55), i. e.,

$$\hat{H} = \frac{1}{2\mathcal{M}}\hat{p}_\phi^2 + \frac{\mathcal{M}\tilde{\omega}^2}{2}\hat{\phi}^2,$$

becomes

$$\hat{H} = \hbar \left[\beta_- \hat{b}^2 + \beta_+ (\hat{b}^\dagger)^2 + \beta_0 \left(\hat{b}^\dagger \hat{b} + \frac{1}{2} \right) \right], \qquad (68)$$

where

$$\beta_+^* = \beta_- \equiv \frac{\mathcal{M}}{2} \left(\dot{u}^2 + \tilde{\omega}^2 u^2 \right), \qquad (69)$$

$$\beta_0 \equiv \mathcal{M} \left(|\dot{u}|^2 + \tilde{\omega}^2 |u|^2 \right). \qquad (70)$$

It is worth noting that the Hamiltonian given by Eq. (68) is formally equivalent to the Hamiltonian of a degenerate parametric amplifier in quantum optics, which is associated with the creation and annihilation of pairs of photons. Similarly, the quadratic terms in $\hat{b}^\dagger$ and $\hat{b}$ in Hamiltonian (68) can be associated with the creation and annihilation of correlated pairs of universes in the quantum state of the multiverse.

We can define the creation and annihilation operators $\hat{B}$ and $\hat{B}^\dagger$ of pairs of degenerate universes, i. e., those with the same properties and $\hat{b}_1 \equiv \hat{b}_2$, as

$$\hat{B}(a) = \operatorname{ch} r \, \hat{b} + \exp(-i\theta/2) \operatorname{sh} r \hat{b}^\dagger, \qquad (71)$$

$$\hat{B}^\dagger(a) = \operatorname{ch} r \hat{b}^\dagger + \exp(i\theta/2) \operatorname{sh} r \, \hat{b}, \qquad (72)$$

where

$$\operatorname{ch} 2r = \frac{\beta_0}{\omega}, \qquad (73)$$

$$\operatorname{sh} 2r = \frac{2|\beta_\pm|}{\omega}, \qquad (74)$$

$$\theta = i \ln \frac{\beta_+}{\beta_-}, \qquad (75)$$

and $\beta_\pm$ and β_0 are defined in Eqs. (69), (70). In terms of the creation and annihilation operators of correlated pairs of universes, the Hamiltonian is diagonal,

$$\hat{H} = \hbar\omega \left(\hat{B}^\dagger \hat{B} + \frac{1}{2} \right).$$

This can be interpreted such that the quantum correlations between the states of the multiverse, which are given by the nondiagonal terms in the Hamiltonian, disappear when the universes are considered in pairs. However, a pair of universes forms an entangled state for which the thermodynamical properties of entanglement of each individual universe can be computed.

We now define two other representations with a clear physical interpretation of the state of the multiverse. We can consider large parent universes [17] with a characteristic length of the order of the Hubble length of our universe. For large values of the scale factor, the nondiagonal terms in Hamiltonian (68) vanish and the coefficient β_0 asymptotically coincides with the proper frequency of the Hamiltonian [26]. Equivalently, it can be verified that $r \to 0$ in Eqs. (71) and (72), and therefore the operators $\hat{B}^\dagger$ and $\hat{B}$ are the creation and annihilation operators of single universes. Then the quantum correlations between the number states disappear and, thus, the quantum transitions among number states are asymptotically suppressed for parent universes. In terms of the creation and annihilation operators of parent universes, asymptotically defined as

$$\hat{b}_p^\dagger \equiv \sqrt{\frac{\mathcal{M}\omega}{2\hbar}} \left(\hat{\phi} - \frac{i}{\mathcal{M}\omega} \hat{p}_\phi \right),$$

$$\hat{b}_p \equiv \sqrt{\frac{\mathcal{M}\omega}{2\hbar}} \left(\hat{\phi} + \frac{i}{\mathcal{M}\omega} \hat{p}_\phi \right),$$

with $\mathcal{M} \equiv \mathcal{M}(a)$ and $\omega \equiv \omega(a, \varphi_0)$, the invariant creation and annihilation operators given by Eqs. (56) and (57) are given by

$$\hat{b}(a) = \mu_p \, \hat{b}_p + \nu_p \, \hat{b}_p^\dagger, \qquad (76)$$

$$\hat{b}^\dagger(a) = \mu_p^* \, \hat{b}_p^\dagger + \nu_p^* \, \hat{b}_p, \qquad (77)$$

where

$$\mu_p = \frac{e^{i\theta_R}}{2\sqrt{\mathcal{M}\omega}} \left(\frac{1}{R} + \mathcal{M}\omega R - i\mathcal{M}\dot{R} \right), \qquad (78)$$

$$\nu_p = \frac{e^{i\theta_R}}{2\sqrt{\mathcal{M}\omega}} \left(\frac{1}{R} - \mathcal{M}\omega R - i\mathcal{M}\dot{R} \right), \qquad (79)$$

with

$$|\mu_p|^2 - |\nu_p|^2 = 1,$$

and are therefore also related by a squeezing transformation.

For completeness, we also describe the quantum fluctuations of the space–time of a parent universe, whose contribution to the wave function of the universe is important at the Planck scale [60]. Some of these fluctuations can be viewed as tiny regions of the space–time that branch off from the parent universe and rejoin the large regions thereafter; thus, they can be interpreted as virtual baby universes [17]. In that case, $\mathcal{M} \approx l_b$ and $\omega \approx \omega_b$ are two constants that are given by the characteristic length and energy of the baby universe. Quantum correlations then play an important role in the state of the gravitational vacuum. This is represented by a squeezed state, an effect that can be related to that previously pointed out by Grishchuck and Sidorov [61], who also showed that the squeezed state of the gravitational vacuum can be interpreted as the creation of gravitational waves in an

expanding universe. In terms of the creation and annihilation operators of baby universes,

$$\hat{b}_b^\dagger \equiv \sqrt{\frac{l_b\omega_b}{2\hbar}}\left(\hat{\phi} - \frac{i}{l_b\omega_b}\hat{p}_\phi\right),$$
$$\hat{b}_p \equiv \sqrt{\frac{l_b\omega_b}{2\hbar}}\left(\hat{\phi} + \frac{i}{l_b\omega_b}\hat{p}_\phi\right),$$

invariant representation (56), (57) becomes

$$\hat{b}(a) = \mu_b\,\hat{b}_b + \nu_b\,\hat{b}_b^\dagger, \tag{80}$$
$$\hat{b}^\dagger(a) = \mu_b^*\,\hat{b}_b^\dagger + \nu_b^*\,\hat{b}_b, \tag{81}$$

with

$$\mu_b = \frac{e^{i\theta_R}}{2\sqrt{l_b\omega_b}}\left(\frac{1}{R} + l_b\omega_b R - i\mathcal{M}\dot{R}\right), \tag{82}$$
$$\nu_b = \frac{e^{i\theta_R}}{2\sqrt{l_b\omega_b}}\left(\frac{1}{R} - l_b\omega_b R - i\mathcal{M}\dot{R}\right), \tag{83}$$

and

$$|\mu_b|^2 - |\nu_b|^2 = 1.$$

We finally pose the general quantum state of a multiverse made up of pairs of entangled universes. As noted in Sec. 2, the universes of the multiverse can generally have different values of their parameters. However, these parameters have the same value for an entangled pair of universes by the very definition of the boundary condition imposed on the state of the whole multiverse. Then the general quantum state of the multiverse would evolve in accordance with Schrödinger equation (5), with the Hamiltonians $\hat{H}_i$ given by

$$\hat{H}_i(a,\phi,p_\phi) = \hbar\left(\beta_-^{(i)}\hat{b}_1^{(i)}\hat{b}_2^{(i)} + \beta_+^{(i)}(\hat{b}_1^{(i)})^\dagger(\hat{b}_2^{(i)})^\dagger + \right.$$
$$\left. + \frac{1}{2}\beta_0^{(i)}\left((\hat{b}_1^{(i)})^\dagger\hat{b}_1^{(i)} + (\hat{b}_2^{(i)})^\dagger\hat{b}_2^{(i)} + 1\right)\right), \tag{84}$$

where $\beta_\pm^{(i)}$ and $\beta_0^{(i)}$ are given by Eqs. (69) and (70), and the index i labels the different species of pairs of universes that can be present in the multiverse. In the case considered in this section, it can label the different values of the effective vacuum energy determined by $V(\varphi_0^{(i)})$.

4.2.1. Energy and entropy of entanglement

The plausible existence of entangled and squeezed states in the context of a quantum multiverse allow considering general correlated states between two universes. It has to be noted that entanglement is highly dependent on the choice of modes, [which] is mainly dictated by the physics of the given situation (cf. Ref. [62, p. 88]). We therefore mainly consider two sets of modes in the multiverse: one is given by the invariant representation in Eqs. (56) and (57), which is consistent with the boundary condition imposed on the state of a multiverse with a fixed number of universes, and the other corresponds to the asymptotic representation of a large parent universe like ours, where observers can exist. As we have seen, these two representations are related by the squeezing transformation given by Eqs. (76) and (77). For completeness, we also consider the representation of baby universes given by Eqs. (80) and (81).

In both cases, the squeezing relations given by Eqs. (76) and (80), (81) allow writing the composite state of two entangled universes as

$$\hat{\rho}(a) = \hat{\mathcal{U}}_S^\dagger(a)|0_1 0_2\rangle\langle 0_1 0_2|\hat{\mathcal{U}}_S(a), \tag{85}$$

where the evolution operator is the squeezing operator given by

$$\hat{\mathcal{U}}_S(a) = \exp\left(r(a)e^{i\theta}\hat{b}_1\hat{b}_2 - r(a)e^{-i\theta}\hat{b}_1^\dagger\hat{b}_2^\dagger\right), \tag{86}$$

with $r(a)$ and $\theta(a)$ being the squeezing parameters that depend on the value of the scale factor. In Eq. (85),

$$|0_1 0_2\rangle \equiv |0_1\rangle|0_2\rangle,$$

with $|0_1\rangle$ and $|0_2\rangle$ being the ground states of each single universe in their asymptotic representations. We first obtain the thermodynamical properties of entanglement in terms of the squeezing parameters r and θ, and then compute the value of these parameters for baby and parent universes and their thermodynamical properties of entanglement.

The reduced density matrix for each single universe is given by

$$\hat{\rho}_{(1,2)} \equiv \mathrm{Tr}_{(2,1)}\,\hat{\rho} = \sum_{N_{(2,1)}=0}^{\infty}\langle N_{(2,1)}|\hat{\rho}|N_{(2,1)}\rangle. \tag{87}$$

We note that $N_{(2,1)}$ in Eq. (87) does not label the universes because it is not an eigenvalue of the number operator in the invariant representation. Instead, it represents the excitation level of one single universe as seen by an internal observer [63]. We focus, for instance, on universe 1 (they both are identical anyway). Its state is then given by

$$\hat{\rho}_1 = \sum_{N_2=0}^{\infty}\langle N_2|\hat{\mathcal{U}}_S^\dagger|0_2\rangle|0_1\rangle\langle 0_1|\langle 0_2|\hat{\mathcal{U}}_S|N_2\rangle. \tag{88}$$

Using the disentangling theorem [64, 65]

$$\hat{\mathcal{U}}_S^\dagger(a) = \exp\left(\Gamma(a) e^{i\theta} \hat{b}_1^\dagger \hat{b}_2^\dagger\right) \times \\ \times \exp\left(-g(a)(\hat{b}_1^\dagger \hat{b}_1 + \hat{b}_2^\dagger \hat{b}_2 + 1)\right) \times \\ \times \exp\left(-e^{-i\theta}\Gamma(a)\hat{b}_1 \hat{b}_2\right), \quad (89)$$

where

$$\Gamma(a) = \operatorname{th} r(a), \quad g(a) = \ln \operatorname{ch} r(a), \quad (90)$$

we obtain that each single universe is quantum mechanically represented by the thermal state given by

$$\hat{\rho}_1(a) = \exp(-2g(a)) \sum_{N=0}^{\infty} \exp(2N \ln \Gamma(a)) |N\rangle\langle N| = \\ = \frac{1}{\operatorname{ch}^2 r} \sum_{N=0}^{\infty} \left(\operatorname{th}^2 r\right)^N |N\rangle\langle N| = \\ = \frac{1}{Z} \sum_{N=0}^{\infty} \exp\left(-\frac{\omega(a)}{T(a)}\left(N + \frac{1}{2}\right)\right) |N\rangle\langle N|, \quad (91)$$

where $|N\rangle \equiv |N\rangle_1$ (and similarly for $\hat{\rho}_2$ with $|N\rangle \equiv \equiv |N\rangle_2$), and

$$Z^{-1} = 2 \operatorname{sh} \frac{\omega}{2T}.$$

The two universes of the entangled pair evolve in thermal equilibrium with respect to each other, with a temperature that depends on the scale factor:

$$T \equiv T(a) = \frac{\omega(a)}{2 \ln(1/\Gamma(a))}. \quad (92)$$

The entanglement entropy, which is defined as

$$S_{ent} = -\operatorname{Tr}(\hat{\rho}_1 \ln \hat{\rho}_1), \quad (93)$$

turns out to be

$$S_{ent}(a) = \operatorname{ch}^2 r \ln \operatorname{ch}^2 r - \operatorname{sh}^2 r \ln \operatorname{sh}^2 r. \quad (94)$$

It is an increasing function of the squeezing parameter r (see Fig. 7). The second principle of quantum thermodynamics, given by Eq. (19), is satisfied because the change in the entropy of entanglement corresponds precisely to the change of heat divided by the temperature, and the production of entropy σ vanishes. This can be verified by computing the thermodynamical quantities in Eqs. (10)–(12). From Eq. (10), the energy of the state represented by $\hat{\rho}_1$ $(= E(\hat{\rho}_2))$ is given by

$$E_1(a) = \operatorname{Tr} \hat{\rho}_1 \hat{H}_1 = \omega\left(\operatorname{sh}^2 r + \frac{1}{2}\right) = \\ = \omega\left(\langle \hat{N}(a)\rangle + \frac{1}{2}\right), \quad (95)$$

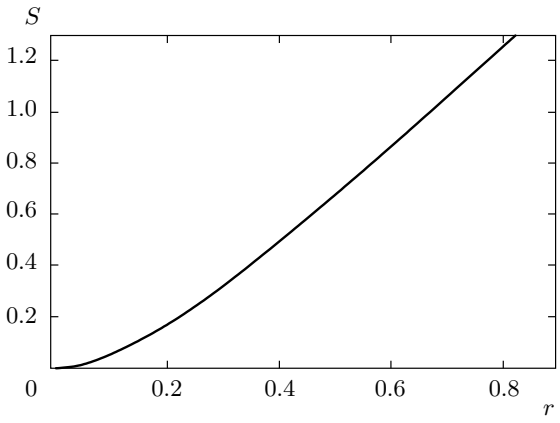

Fig. 7. Entanglement entropy, Eq. (94), as a function of the squeezing parameter r

where

$$\hat{H}_1 \equiv \omega\left(\hat{b}_1^\dagger \hat{b}_1 + \frac{1}{2}\right).$$

The change in the heat and work, given by Eqs. (11) and (12), are

$$\delta W_1 = \operatorname{Tr}\left(\hat{\rho}_1 \frac{d\hat{H}_1}{da}\right) = \frac{\partial \omega}{\partial a}\left(\langle \hat{N}(a)\rangle + \frac{1}{2}\right), \quad (96)$$

$$\delta Q_1 = \operatorname{Tr}\left(\frac{d\hat{\rho}_1}{da}\hat{H}_1\right) = \omega \frac{\partial \langle \hat{N}(a)\rangle}{\partial a}, \quad (97)$$

whence it follows that

$$dE_1 = \delta W_1 + \delta Q_1.$$

From Eqs. (97) and (94), it also follows that the production of entropy is zero,

$$\sigma = \frac{dS_{ent}}{da} - \frac{1}{T}\frac{\delta Q}{da} = 0, \quad (98)$$

where

$$T = \frac{\omega}{2} \ln^{-1} \frac{1}{\Gamma}$$

is defined in Eq. (92). Moreover, Eq. (98) can be compared with the expression that is standardly used to compute the energy of entanglement (see Refs. [2–4]),

$$dE_{ent} = T dS_{ent}. \quad (99)$$

It allows establishing that the energy of entanglement is given by

$$dE_{ent} = \delta Q = \omega \operatorname{sh} 2r \, dr. \quad (100)$$

The results can be interpreted as follows. For an entangled pair of large parent universes, the squeezing parameter, r, given by

$$r = \operatorname{arcsh} |\nu_p|$$

with ν_p in Eq. (79), turns out to be a decreasing function of the scale factor. We note that in the case of the parent universes,

$$\langle\hat{N}(a)\rangle \approx \frac{\mathcal{M}\dot{R}^2}{4\omega} = \frac{1}{16\omega^2}\left(\frac{\dot{\mathcal{M}}}{\mathcal{M}} + \frac{\dot{\omega}}{\omega}\right)^2 \approx \approx \frac{9}{16V}a^{-6}. \quad (101)$$

Then the energy of entanglement given by the integration of Eq. (100) becomes

$$E_{ent} = Q \propto V^{-1/2}a^{-4}, \quad (102)$$

and the entropy of entanglement, Eq. (94), is

$$S_{ent} \approx -\langle\hat{N}(a)\rangle \log\langle\hat{N}(a)\rangle \propto V^{-1}a^{-6}\log a. \quad (103)$$

These are the expected results because the universes of an entangled pair become more and more disentangled from each other as the universes expand, becoming asymptotically independent for an infinite value of the scale factor. Thus, the entropy and the energy of entanglement are also decreasing functions of the scale factor. The entropy of entanglement turns out to be a monotonic function, thus providing us with an arrow of time for each single universe [66]. The energy of entanglement between the pair of universes would contribute to the energy density of each single universe if it can effectively be considered a kind of energy that fills the universe. It would yield a large contribution at the early stage of the universe, and it becomes extremely small at large values of the scale factor, i. e., for more evolved universes.

In the case of baby universes that describe vacuum fluctuations of the space–time of a parent universe, the results can be related to those previously obtained in [61]. Actually, the effective number of vacuum fluctuations,

$$\langle\hat{N}_b(a)\rangle \propto \frac{1}{R^2} \approx Va^3, \quad (104)$$

scales with the volume of the space of the parent universe. The energy of the vacuum fluctuations therefore increases as the universe expands, as does the entropy of entanglement, which hence provides us with the customary behavior of the arrow of time in cosmology [18, 61].

5. VIOLATION OF CLASSICAL INEQUALITIES AND THE EPR ARGUMENT IN THE MULTIVERSE

Entangled and squeezed states have no classical analogue and provide us with an example in which the EPR argument could be applied in a cosmological context. However, there is no need of a common space–time to be shared by the universes in the quantum multiverse, and therefore the concepts of locality and nonlocality become meaningless. The entangled states in the quantum multiverse are rather related to the quantum interdependence of the states that represent disconnected regions or branches of the universe. Nevertheless, it has been shown that quantum correlations between two disconnected universes might have observable consequences for the properties of each single universe, one of which might well be the existence of a contribution to the vacuum energy of each single universe. That would make the whole multiverse proposal testable, at least in principle.

In the preceding sections, it has been shown that entangled and squeezed states can generally be considered in the quantum multiverse. In quantum optics, these quantum states are called nonclassical states [1] because they can violate some inequalities that should be satisfied in the classical description of light. For instance, the second-order coherence function $g^{(2)}(0)$, which classically should satisfy

$$g^{(2)}(0) \geq 1$$

(see [1, 67]), quantum mechanically is given, for a single mode, by [1]

$$g^{(2)}(0) = \frac{\langle(\hat{b}^\dagger)^2\hat{b}^2\rangle}{\langle\hat{b}^\dagger\hat{b}\rangle^2},$$

where $\hat{b}$ and $\hat{b}^\dagger$ are boson operators satisfying the commutation relation

$$[\hat{b}, \hat{b}^\dagger] = 1.$$

In the quantum state of the multiverse, taking relations (64) and (65) for the operators $\hat{b}$ and $\hat{b}^\dagger$ into account, the second-order coherence function can be written as

$$g^{(2)}(0) = 1 + \frac{14x^4 + 9x^2 - 2}{25x^4 + 20x^2 + 4}, \quad (105)$$

where $x \equiv |\nu_0|$. Function (105) is plotted in Fig. 8 for different values of the parameter w and for

$$N_0 \equiv \langle N_0|\hat{c}^\dagger\hat{c}|N_0\rangle = 2.$$

For values of the scale factor that are close to the value $a_0 = 10$,

$$N_{eff} \equiv \langle N_0|\hat{b}^\dagger\hat{b}|N_0\rangle = 5x^2 + 2 \approx 2,$$

and the second-order coherence function is less than unity (see Fig. 8), which is consistent because for values $a \approx a_0$, with $a_0 \gg 1$,

$$\hat{b} \approx \hat{c}, \quad \hat{b}^\dagger \approx \hat{c}^\dagger.$$

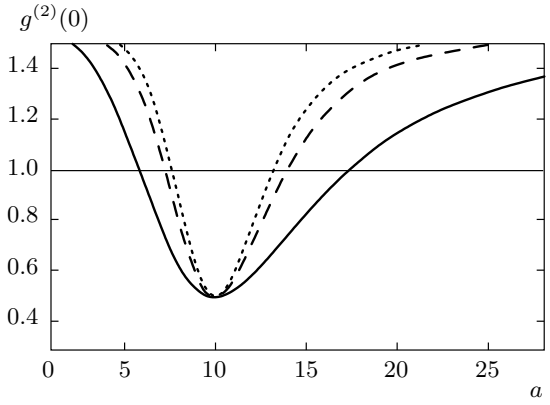

Fig. 8. Antibunching effect in the multiverse, Eq. (105), with Eq. (65) and the values $\mathcal{M}_0 = a_0$, $\omega_0 = a_0^{q-1}$, $a_0 = 10$ ($\gg 1$), and $R \approx 1/\sqrt{\mathcal{M}\omega}$, for different values of the parameter w: $w = -1$ ($q = 3$, dotted line), $-2/3$ ($q = 2.5$, dashed line), 0 ($q = 1.5$, solid line)

For smaller and larger values of the scale factor, $a \gg a_0$ or $a \ll a_0$, the effect disappears because the effective number N_{eff} is large and the quantum correlations disappear. This clearly reveals a strong dependence of the violation of the classical inequalities on the representation that is chosen to describe the quantum state of the multiverse.

Squeezed states violate the Cauchy–Schwartz inequality for any value of the squeezing parameters [1], and they can also violate Bell's inequalities. The latter violation is even more important because it is directly related to nonlocal characteristic of the quantum theory. Bell's inequalities are violated, for a two-mode state, when [1]

$$C = \frac{\langle \hat{b}_1^\dagger \hat{b}_1 \hat{b}_2^\dagger \hat{b}_2 \rangle}{\langle \hat{b}_1^\dagger \hat{b}_1 \hat{b}_2^\dagger \hat{b}_2 \rangle + \langle (\hat{b}_1^\dagger)^2 \hat{b}_1^2 \rangle} \geq \frac{\sqrt{2}}{2}. \tag{106}$$

In the multiverse, taking Eqs. (64) and (65) into account, we obtain

$$\langle (\hat{b}_1^\dagger)^2 \hat{b}_1^2 \rangle = N^2(6x^4 + 6x^2 + 1) + \\ + N(6x^4 + 2x^2 - 1) + 2x^4, \tag{107}$$

$$\langle \hat{b}_1^\dagger \hat{b}_1 \hat{b}_2^\dagger \hat{b}_2 \rangle = N^2(6x^4 + 6x^2 + 1) + \\ + N(6x^4 + 4x^2) + x^2(2x^2 + 1), \tag{108}$$

where

$$x \equiv |\nu_0| = \operatorname{sh} r, \quad N_1 = N_2 \equiv N.$$

In Eqs. (107) and (108), it is assumed that the universes are identical except for the existence of conscious observers that make each single universe distinguishable, and hence $[\hat{b}_i, \hat{b}_j^\dagger] = 0$ for $i \neq j$. For the initial vacuum state, $r = 0$ and $N = 0$, $C = 1 > 0.7$, which implies a maximum violation of Bell's inequalities [1]. For $N = 1$, i. e., for a pair of entangled universes, we obtain

$$C = \frac{14x^4 + 11x^2 + 1}{28x^4 + 19x^2 + 1}, \tag{109}$$

and Bell's inequalities are violated ($C \geq 0.7$) for $0 < \operatorname{sh} r < 0.31$, i. e., for small values of the squeezing parameter.

However, it is not clear at all how the violation of Bell's inequalities could be checked in the quantum multiverse because the customary procedure would involve measuring properties of the two universes of an entangled pair. That could only be done by a hypothetical observer who would live in the multiverse. For a real observer living in a single universe, the entanglement of the universe could only be inferred by comparing the thermal properties derived from the theory of interuniversal entanglement with the thermal properties of her universe, at least in principle.

Furthermore, it is worth noticing that what is violated in an experiment with photons involving squeezed and entangled states are some classical assumptions like the wave description of light or the local character of classical particles. Those experiments clearly show the fundamental character of the concept of complementarity in quantum theory: quantum systems have to be complementarily described in terms of particles and waves.

Despite the profound differences between quantum optics and quantum cosmology, mainly due to the role of the observer in both theories, the existence of entangled and squeezed states in the quantum multiverse would also violate some classical assumptions like the independence of disconnected regions of space–time. The extension of the principle of complementarity, which is a fundamental and general feature of quantum theory and should therefore be also assumed in quantum cosmology, would mean that a complementary quantum description of the universe has to exist in terms of "particles" and "waves", the former naturally leading to the multiverse scenario and the latter impelling us to also consider interactions and quantum correlations among the universes of the multiverse.

The existence of squeezed and entangled states in the multiverse also allows proposing an argument analogous to the EPR argument in quantum mechanics. The original EPR argument [68] was an attempt to show the incompleteness of the quantum theory from a realistic standpoint. It was Bell [69] who pointed out that the EPR experiment actually showed nonlocal characteristics of quantum mechanics. Roughly speaking, in an entangled state between two particles, we

can know the properties of a distant particle by means of making a measurement on the other particle of the pair, irrespectively of how far are they separated. In the quantum multiverse, however, there is no need of a common space–time to be shared by the universes and therefore the concepts of locality and nonlocality become meaningless, having to be extended to the concepts of independence or interdependence of the quantum states of the universes. The entangled states of the multiverse are rather related to the concept of nonseparability of the states that correspond to different regions of space–time, which are classically disconnected, however.

Generally speaking, the separability or nonseparability of the modes of a given representation is clearly dependent on that representation. As is pointed out in Ref. [62], the crux is that what is an interacting Hamiltonian for one of the modes may not be so for a different set of modes (see also Ref. [8]). Thus, the objection could be raised that the existence of entangled states in the multiverse can be the result of an incorrect choice of subspaces $\mathcal{H}_1$ and $\mathcal{H}_2$ of the whole Hilbert space $\mathcal{H}$ that corresponds to the complete quantum description of the universe. That is, $\mathcal{H}$ cannot be given by a direct product,

$$\mathcal{H} \neq \mathcal{H}_1 \otimes \mathcal{H}_2,$$

or splitting the whole Hilbert space into two subspaces because it is just a useful mathematical tool to obtain the quantum state in $\mathcal{H}$ that corresponds to a unique single universe. This can be accepted. But the analogous argument in the quantum description of the electromagnetic field would be that entangled states of a pair of photons are just a useful way to represent the state of the field. The violation of classical inequalities in quantum optics reveals the corpuscular nature of the photon, and its existence as an autonomous entity, although not necessarily independent. In the second-quantization formalism, this allows interpreting different modes of the wave function of the universe as different universes in an appropriate representation. The complementarity characteristic of quantum theory impels us to also consider their wave properties and thus quantum interference and correlations between the states of different universes, which can be considered identical, as in the model considered in this paper, except for the plausible existence of conscious observers that might communicate with each other through quantum channels[2)].

[2)] Classical channels to construct the communication protocol could be provided by the existence of wormholes joining different regions.

Of course, the measurement process is even more difficult to be formulated in the context of the multiverse, and this is crucial in determining the appropriate representation of universes as being seen by an observer who lives in the universe. It is therefore not clear at all what representation should be chosen. However, the inter-universal entanglement in the multiverse scenario may provide us with a wide variety of novel features that could account for unexplained and new unexpected cosmic phenomena, and it therefore seems to be worthy of further investigation.

6. CONCLUSIONS AND FURTHER COMMENTS

It has been shown that squeezed and entangled states can generally be posed in the context of the multiverse. Specifically, it has been shown that the quantum state of a multiverse made up of homogeneous and isotropic space–times with a massless scalar field is given by a squeezed state, and that the quantum state of the phantom multiverse turns out to be an entangled state between the modes that correspond to the expanding and contracting branches of each universe, before and after the big rip singularity. A pair of entangled universes can also originate from a double instanton, whose creation is allowed by the presence of quantum corrections in the Wheeler–De Witt equation. Therefore, quantum states with no classical analogue have generally to be considered in the context of the quantum multiverse.

Statistical boundary conditions have to be imposed to determine the quantum state of the multiverse. The boundary condition of the multiverse that the number of universes of the multiverse does not depend on the value of the scale factor of a particular single universe partially fixes the representation to be chosen. This is given by the Lewis states that can be interpreted, in the context of the multiverse, as the states that represent entangled pairs of universes.

If the existence of squeezed states in the multiverse would imply a violation of Bell's inequalities, then, because there is no common space–time to the universes in the quantum multiverse, the nonlocality features of squeezed states would rather be related to the interdependence of the entangled quantum states that represent different universes or regions of the universe, which are classically and thus causally disconnected.

The thermodynamical properties of a closed system like the multiverse have been studied. All the thermodynamical quantities of a closed system are internal

properties of the system and, with the given definitions, the first and second principles of thermodynamics are satisfied for any value of the scale factor. The entropy of the multiverse can decrease, at the same time satisfying the second principle of thermodynamics because the process is not adiabatic, the change of entropy precisely corresponds to the change of heat (divided by the temperature), and hence the entropy production is zero.

Unlike the values of the quantum informational analogies of work and heat, the values of the quantum thermodynamical energy and entropy do not depend on the representation chosen to describe the state of the multiverse if different representations are related to each other by unitary transformations. Therefore, if the universe starts in a pure state, it remains a pure state in the course of the unitary evolution of the universes in the multiverse.

We have also considered a pair of universes whose quantum mechanical states are entangled. The composite state of the pair is given by a pure state. However, the state of each single universe turns out to be given by a thermal state with a temperature that depends on the scale factor. Both universes of the entangled pair therefore stay in thermal equilibrium in the course of the correlated evolutions of their scale factors. Cosmic entanglement thus provides us with a mechanism by which the thermodynamical arrow of time in the multiverse, given by the change of the total quantum entropy, would be zero for a multiverse described in terms of pure states of entangled pairs of universes, a conclusion which could be related to that already pointed out in Ref. [6] (see also Ref. [70]). Each single universe of the multiverse, however, would still have an arrow of time given by the change of the entropy of entanglement with its partner universe. This arrow of time corresponds to a decrease of the entanglement entropy rather than an increase, however. Nevertheless, the second principle of thermodynamics is satisfied because the change of the entropy of entanglement precisely corresponds to the change of the energy of entanglement, which can be identified with the heat of entanglement of each single universe.

The evolution of the temperature and the energy of entanglement depends on the kind of universes that are considered. For baby universes, the energy of entanglement grows with the expansion of the parent universe. It can be interpreted as an effective creation of a large number of vacuum fluctuations in the space–time of the parent universe. For parent universes, the temperature and the energy of entanglement decrease in the course of the expansion of the universe. Thus, the energy density of each single universe can be high in the initial stage, which is expected for an inflationary period, but it can have a smaller value in a more evolved epoch, like the current one.

The energy of entanglement for the positive and negative modes of a massless scalar field, which respectively correspond to the expanding and contracting branches of the universe, behaves similarly to the vacuum energy when the scalar field starts with a small value of the mode and evolves to higher values with the expansion of the universe.

In this paper, it has also been pointed out that the quantum mechanical fundamental concepts of complementarity and nonlocality have to be revised in the context of the quantum multiverse. Thus, multiversal nonlocality has to be extended so as to express the interdependence of different regions of the whole manifold that represents the multiverse. These regions can classically and causally be disconnected from each other, although their composite state can still have quantum correlations. Therefore, the classical concept of causality ought to be revised. The concept of complementarity in the multiverse implies the consideration of interference processes among different universes or branches of the universe. These processes might have observable effects in each single universe, underlying the question of whether the multiverse studied in this paper can be tested, i. e., whether it is in fact a falseable scientific proposal[3)].

In general, regarding the testability of a multiverse proposal, we first note that a multiverse can actually be considered if it allows searching for the effects that other universes might imprint on the properties of our own universe. Furthermore, different ways of potentially observing the effects of the multiverse in our universe have been proposed. In Refs. [71, 72], it has been proposed that giant voids in the sky could be the result of inter-universal interactions; according to Ref. [73], the light pattern of gravitational lensing produced by wormholes, ringholes, and Klein-bottle holes that connect our universe with others would be distinguishable from that made by similar tunnels connecting different regions of our universe, thus providing us with a mechanism for testing the multiverse.

In the model presented in this paper, it has been shown that the inter-universal entanglement can modify the dynamical and thermodynamical properties of single universes. Thus, some additional forms of testability of the quantum multiverse can be envis-

[3)] In words of Ellis [10], the issue of testability underlies the question of whether multiverse proposals are really scientific.

aged. First, the temperature of entanglement might be matched with the temperature of our universe if there exists a relation between the thermodynamics of entanglement and the thermodynamics of the universe. Second, assuming that the energy of inter-universal entanglement is the major contribution to the vacuum energy of a single universe, the evolution rate of the scale factor would have a correlation with the amount of inter-universal entanglement and, particularly, with the rate of change of its energy of entanglement. By using the observational data, the entanglement rate could be fixed. Furthermore, different boundary conditions for the state of the multiverse imply different entanglement rates between the states of single universes. Therefore, it might well be that the evolution rate of the scale factor of our universe would provide us with a criterion for selecting the appropriate boundary condition of the whole multiverse, making not only the multiverse proposal but also the choice of cosmic boundary conditions testable.

Thus, the question of testability of the multiverse, far from being the problem of the multiverse, seems to be the keystone for considering new approaches to traditional questions in quantum cosmology, like the boundary conditions, the arrow of time, or the anthropic principles, among others. It challenges us to adopt new and open-minded points of view about major physical and philosophical preconceptions.

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