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ON THE IMPLICATION OF BELL'S PROBABILITY DISTRIBUTION AND PROPOSED EXPERIMENTS OF QUANTUM MEASUREMENT

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In derivating Bell's inequalities, the probability distribution is supposed to be a function only of a hidden variable. We point out that the true implication of the probability distribution of Bell's correlation function is the distribution of joint measurement outcomes on the two sides. It is therefore a function of both the hidden variable and the settings. In this case, Bell's inequalities fail. Our further analysis shows that Bell's locality holds neither for dependent events nor for independent events. We think that the measurements of EPR pairs are dependent events, and hence violation of Bell's inequalities cannot rule out the existence of the local hidden variable. To explain the results of EPR-type experiments, we suppose that a polarization-entangled photon pair can be composed of two circularly or linearly polarized photons with correlated hidden variables, and a couple of experiments of quantum measurement are proposed. The first uses delayed measurement on one photon of the EPR pair to demonstrate directly whether measurement on the other could have any nonlocal influence on it. Then several experiments are suggested to reveal the components of the polarization-entangled photon pair. The last one uses successive polarization measurements on a pair of EPR photons to show that two photons with the same quantum state behave the same under the same measuring conditions.

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1. INTRODUCTION

Quantum theory gives only probabilistic predictions for individual events based on the probabilistic interpretation of the wave function, which leads to the suspicion of the incompleteness of quantum mechanics and the puzzle of nonlocality of the measurement of EPR pairs [1]. Indeed, if hidden-variable theory is not introduced into quantum measurement, we can hardly understand the distant correlation of EPR pairs, e. g., quantum teleportation and quantum swapping [2, 3]. Bell pointed out that any theory that is based on the joint assumptions of locality and realism conflicts with the quantum mechanical expectation [4]. Since then, various local and nonlocal hidden-variable models against Bell's inequalities have been proposed (see, e. g., [5–10]), among which the most attractive one is the time-related and setting-dependent model suggested by Hess and Philipp [10], but it was criticized in [11] and [12] for being nonlocal. As a matter of fact, there is an assumption regarding the probability distribution in derivating Bell's inequalities. Bell supposed that it is a function of a hidden variable and irrelevant to the measuring condition. But the validity of this assumption is dubious. It has been pointed out by many authors that if this assumption does not hold, then Bell's inequalities fail [13–15]. On the other hand, it has been shown that even if nonlocality is taken into account, Bell's inequalities can also be violated [16, 17]. We therefore focus on Bell's probability distribution and discuss its validity. We point out that its true implication is the probability distribution of the joint measurement outcomes. Because the measurement outcomes are related to settings, the probability distribution is also related to settings. In this case, Bell's inequalities do not hold. We explore the physical meaning of the hidden variable and suggest the uncertainty of the spatial distribution of the particle as a hidden variable.

In terms of quantum entanglement, the spin (polarization) of a pair of EPR particles is indefinite and interdependent for the two particles. By analyzing the existing experiments of polarization entanglement [18–31], we show that polarization-entangled Bell states (maximally entangled states) can be formed by circu-

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larly or linearly polarized photon pairs with correlated hidden variables. If the hidden variable does exist, then the quantum state of one of the EPR particles does not change when a measurement is made on the other, and the outcomes of a pair of particles with the same quantum state are the same under the same circumstances. We propose three types of experiments to test the above hypotheses. The experiments are easy to realize because the experimental setups are very simple.

2. ON BELL'S PROBABILITY DISTRIBUTION AND SUGGESTED HIDDEN VARIABLES

In local hidden-variable theories, Bell's inequalities play an important role. Bell regarded that his correlation function was founded on the crucial assumption of Einstein that the result of B is independent of the setting of the measuring device a, and similarly A is independent of b, whence [4]

$$P(a,b) = \int A(a,\lambda)B(b,\lambda)\rho(\lambda)\,d\lambda, \qquad (1)$$

where $A(a,\lambda) = \pm 1$, $B(b,\lambda) = \pm 1$, and $\rho(\lambda)$ is the probability distribution of the hidden variable according to Bell. It was suggested in [13] that ρ can depend on measuring conditions. In [14], this idea was expressed using a modified definition of locality. But many researchers insist on the locality of Eq. (1) and believe that the probability distribution of the hidden variable cannot be influenced by the measuring process. Thus, the arguments in [13] and [14] are not widely accepted. If ρ actually represents the probability distribution of the hidden variable, then Eq. (1) seems reasonable. We now analyze mathematical implications of ρ. Equation (1) includes four joint probabilities:

$$P_{++}(A=1,B=1), \quad P_{+-}(A=1,B=-1),$$

$$P_{-+}(A=-1,B=1), \quad P_{--}(A=-1,B=-1).$$

Then we have

$$P(a,b) = P_{++} - P_{+-} - P_{-+} + P_{--}.$$

Because $P(a,b)$ actually implies the joint probabilities of the measurement outcomes of A and B, ρ must be the joint probability density function with respect to the results of A and B, i. e., $\rho = \rho(A=\pm 1, B=\pm 1)$. Because the results of A and B depend on the settings of measuring devices and hidden variables of the pair, we have $\rho = \rho(a,b,\lambda)$. If it does not vary with measuring conditions, then we have the case considered by Bell. For a pair of EPR particles, it is easy

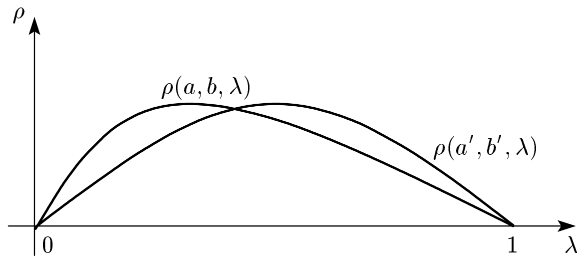

Fig. 1. Possible probability distributions under different measuring conditions

to understand that they share the same hidden variable. But there is no prior reason that the probability distribution of measurement outcomes is irrelevant to the settings. Two curves plotted in Fig. 1 represent the possible probability distributions under different measuring conditions a, b and a', b'.

We emphasize that ρ should not be regarded as the probability distribution of the hidden variable. Instead, it is the probability distribution of the results A and B. Because the joint measurement outcomes are related to a, b and λ, it is natural that the joint probability distribution is a function of a, b, and λ. This is the key to understanding Bell's correlation function. It seems that Bell misunderstood mathematical implications of the probability distribution.

We proceed with the analysis of Bell's correlation function. Bell considered the case of a pair of EPR particles. We extend this to the general case where particles A and B have respective hidden variables λ_A and λ_B. Because the measurement outcome is related to the local condition and the hidden variable, we have $A = A(a,\lambda_A)$ and $B = B(b,\lambda_B)$. In the case where λ_A and λ_B are mutually independent, we obtain

$$\begin{aligned} P(a,b) = \iint A(a,\lambda_A)B(b,\lambda_B)\rho(a,\lambda_A) \times \\ \times \rho(b,\lambda_B)\,d\lambda_A d\lambda_B = \\ = \int A(a,\lambda_A)\rho(a,\lambda_A)\,d\lambda_A \int B(b,\lambda_B)\rho(b,\lambda_B)\,d\lambda_B = \\ = P_a P_b, \qquad (2) \end{aligned}$$

i. e., the joint probability is equal to the product of individual probabilities, which shows that the two events are independent. If there exists a definite relation between λ_A and λ_B, the two events are dependent. In that case, the joint probability density is not equal to the product of individual probability densities. We can only write $\rho(a,b,\lambda_A,\lambda_B)$ for it. Assuming that $\lambda_B = f(a,b,\lambda_A)$, we eliminate the integral variable λ_B to obtain

$$P(a,b) = \int A(a,\lambda_A)B(b,\lambda_B)\rho(a,b,\lambda_A,\lambda_B)\,d\lambda_A = \\ = \int A(a,\lambda)B(a,b,\lambda)\rho(a,b,\lambda)\,d\lambda, \quad (3)$$

where $B(a,b,\lambda) = B(b,\lambda_B)$ denotes the result of B. For a pair of EPR particles, assuming that $\lambda_A = \lambda_B$, we obtain

$$P(a,b) = \int A(a,\lambda)B(b,\lambda)\rho(a,b,\lambda)\,d\lambda. \quad (4)$$

We see that in this case, Eq. (1) should be modified as Eq. (4). Similarly, we have

$$P(a,c) = \int A(a,\lambda)B(c,\lambda)\rho(a,c,\lambda)\,d\lambda, \quad (5)$$

$$P(b,c) = \int A(b,\lambda)B(c,\lambda)\rho(b,c,\lambda)\,d\lambda. \quad (6)$$

With the above expressions, Bell's inequalities cannot be obtained. We do not discuss the detailed derivation process.

We see from above analysis that Bell's correlation function is valid neither for dependent events nor for independent events. For a pair of EPR particles, their hidden variables can be correlated because they are born from the same particle, and hence their measurement outcomes are correlated, i. e., the measurements on the two sides are dependent events. Thus, violation of Bell's inequalities with EPR-type experiments cannot rule out the existence of local hidden variables.

In what follows, we discuss the problem of quantum measurement based on the assumption that the local hidden variable exists. We first explore the physical meaning of the hidden variable. Due to the wave–particle duality and uncertainty principle, a microscopic particle can be regarded as a wave packet, which occupies a certain spatial volume. The hidden variable represents the intrinsic fluctuating state of a particle. Hence, any parameter that can represent the characteristics of the spatial distribution of the particle can be used as a hidden variable. At present, only the uncertainties of position, momentum, and angular momentum can be used this way, and we might as well borrow them to represent hidden variables. We note that intrinsic quantum fluctuations of the particle are not random, they also obey certain laws that are unknown to us.

We take spin (polarization) of a particle as an example. In classical theory, the angular momentum is a vector, whose magnitude and projections on three directions are all well-defined. In quantum mechanics, the angular momentum magnitude is well-defined, and we can determine its projection l_z on one direction. But the angular position ϕ and the other two projections, l_x and l_y, are all indefinite; ϕ and l_z satisfy the uncertainty relation

$$\Delta\phi\Delta l_z \geq \hbar/2.$$

Both $\Delta\phi$ and Δl_z indicate quantum fluctuations of a particle around the projection (measurement) direction, and they can therefore be used as hidden variables. Because spin (polarization) is a relativistic quantum effect, it is likely that the corresponding hidden variables are irrelevant to time. We test this hypothesis in the following experiment.

The hidden variables of spin (polarization) represent quantum fluctuations of the degree of freedom of spin (polarization) in three-dimensional space, which should be independent of external circumstances. But the measurement on the particle always projects the spin (polarization) on a specific direction. The quantum fluctuation of spin (polarization) can be different in different directions, i. e., the hidden variable is multivalued. In this sense, we may also think that the hidden variable varies with the measuring conditions. We now try to explore the measuring process. In classical mechanics and quantum field theory, we have the principle of least action. We can introduce this principle into quantum measurement. We define $\Delta\phi\Delta l_z$ as the action for the spin (polarization) of a particle in the projection (measurement) direction. When a photon is incident on a polarizer, it has two choices. Consequently, there are two possible collapsed polarization directions and two corresponding actions. We suppose that the photon always chooses the direction with the smaller action. For a linearly polarized photon, its polarization direction can be regarded as the direction with the least action, i. e., we have $\Delta\phi\Delta l_z = \hbar/2$ in this direction. Thus, when the polarization direction of a photon is parallel to the orientation of a polarizer, the photon passes through the polarizer with certainty. Similarly, we define the product of the uncertainties of position and momentum as the action for the motion of the center of mass of a photon.

In the general case, when a measurement is made on a particle, its quantum state collapses into another state, and the collapsing process is nonlinear and irreversible. A small change in the external circumstances or the hidden variable may lead to a different result, i. e., the measurement outcome is sensitive to the external circumstances and hidden variables. Hence, the collapse of the quantum state is chaotic. From this

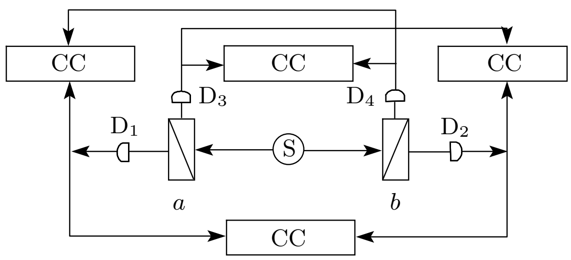

Fig. 2. Experimental test of Bell's inequalities: S is a source; D_i are the counters; CC are the coincidence counters

standpoint, the evolutions of microcosm and macrocosm, and even of the universe are chaotic in essence.

3. INTERPRETATION OF EPR-TYPE EXPERIMENTS

The experiment used to test Bell's inequalities with the polarization state of photon pairs is shown in Fig. 2. A pair of EPR photons is incident on a pair of polarization analyzers a and b. We let "+" and "−" respectively denote the transmitted and reflected channels. The results for the $|\phi^+\rangle$ state in quantum mechanics are [24]

$$P_+(a) = P_-(a) = 1/2, \qquad (7)$$

$$P_+(b) = P_-(b) = 1/2, \qquad (8)$$

$$P_{++}(a,b) = P_{--}(a,b) = \frac{1}{2}\cos^2(a-b), \qquad (9)$$

$$P_{+-}(a,b) = P_{-+}(a,b) = \frac{1}{2}\sin^2(a-b). \qquad (10)$$

In terms of quantum entanglement, the polarization of a pair of EPR photons is indefinite. If the hidden variable exists, the polarization of each photon should be well-defined. We consider the experiment with photon pairs emitted by the $J=0 \to J=1 \to J=0$ cascade of atomic calcium [18, 19]. According to the classical theory, the two photons are circularly polarized. For the experiment of $J=1 \to J=1 \to J=0$ cascade of atomic mercury [20], one photon is linearly polarized and the other is circularly polarized. In the case of down-conversion of a nonlinear crystal [21–31], the wave packets of two orthogonally polarized photons overlap at the crystal or beam splitter. They form two circularly polarized photons under appropriate conditions. The combination of a half-wave plate and a quarter-wave plate can transform the Bell state into other three Bell states [24]. From these facts, we believe that the Bell state can be composed of circularly (or circularly/linearly) polarized photon pairs. For the twin photons generated in cascade radiation or down-conversion, their hidden variables can be regarded as correlated, such that measurements on the two photons are dependent events. To obtain the joint probabilities, we use projective geometry to calculate the conditional probabilities.

We first consider the Bell state composed of circularly polarized photon pairs. For a circularly polarized photon, the probabilities of being transmitted and reflected are both $1/2$, irrespective of the orientatition of the polarizer. For single probabilities, we thus obtain the results in Eqs. (7) and (8). For a pair of correlated photons, we can use conditional probability to obtain

$$P_{++}(a,b) = P_+(a)P_+(b|a) = P_+(b)P_+(a|b), \qquad (11)$$

where $P_+(b|a)$ and $P_+(a|b)$ are conditional probabilities, which can be calculated by the projective method. For the $|\phi^+\rangle$ state, we suppose that

$$P_+(b|a) = P_+(a|b) = \cos^2(a-b).$$

We can understand above method as follows. If the photon on the left-hand side can pass through polarizer a, then the photon on the right-hand side can certainly pass through a polarizer with the same orientation. If the orientation of the polarizer on the right is set at b, the probability that the photon on the right can pass through the polarizer is $\cos^2(a-b)$. Then we have

$$P_{++}(a,b) = \frac{1}{2}\cos^2(a-b),$$

which agrees with Eq. (9). We note that only for a pair of circularly polarized photons with maximally correlated or anticorrelated hidden variables ($\lambda_A = \lambda_B$ or $\lambda_A = -\lambda_B$) can we use this projective method. For circularly polarized photon pairs with independent hidden variables, we have

$$P_{++}(a,b) = P_+(a)P_+(b) = 1/4.$$

As regards the Bell state composed of circularly/linearly polarized photon pairs, we suppose the circularly polarized photons are incident on polarizer a and linearly polarized photons are incident on polarizer b. We first project a onto b. Because $P_+(a) = 1/2$ and the angle between the orientations of the two polarizers is $a-b$, we use projective geometry to obtain $P_{++}(a,b) = (1/2)\cos^2(a-b)$. We then project b onto a. We suppose that the polarization directions of linearly polarized photons are distributed uniformly in space and the angle between the polarization direction of the photon and the orientation of polarizer b is x. The probability that the photon can pass through polarizer b is $\cos^2(b-x)$ according to Malus' law, and then the joint probability is

$$P_{++}(a,b) = \frac{1}{2\pi}\int_0^{2\pi} \cos^2(b-x)\cos^2(a-b)\,dx = \\ = \frac{1}{2}\cos^2(a-b). \quad (12)$$

If the polarization directions of linearly polarized photon are distributed only in two orthogonal directions, we have

$$P_{++}(a,b) = \frac{1}{2}\cos^2 x \cos^2(a-b) + \\ + \frac{1}{2}\sin^2 x\cos^2(a-b) = \frac{1}{2}\cos^2(a-b), \quad (13)$$

which also agrees with the result of quantum mechanics. Additionally, if the linearly polarized direction of photons is set at the angle $\pm 45°$ to the orientation of the polarizer, the probabilities that the linearly and circularly polarized photons can pass through their respective polarizers are both $1/2$. In this case, we also obtain the same result as in quantum mechanics using projective method.

In the general case, linearly polarized photon pairs cannot form a Bell state (which we discuss in detail in the next section). But in a special case, their joint probability can also agree with the result of quantum mechanics. We suppose that two photons have the same polarization direction and the polarization directions of photon pairs are distributed in two orthogonal directions with equal probability, and the orientation of polarizer a is in the x (or y) direction, while the orientation of polarizer b can vary arbitrarily. When the polarization of the pair of photons is in the x (or y) direction, the photon incident on polarizer a can pass through it with certainty, and the probability that the photon incident on polarizer b can pass through it is $\cos^2(a-b)$ according to Malus' law. When the polarization of a pair of photons is in the y (or x) direction, the photon incident on polarizer a cannot pass through. Hence, the joint probability for the photon pair to pass through the polarizers is $P_{++}(a,b) = (1/2)\cos^2(a-b)$. In this case, linearly polarized photon pairs can also form a Bell state.

We summarize as follows: (i) circularly polarized photon pairs with correlated hidden variables form a Bell state; (ii) circularly/linearly polarized photon pairs with correlated hidden variables can form a Bell state under the condition that the polarization directions of linearly polarized photons are distributed uniformly in space or in two orthogonal directions, or the direction of linearly polarized photons is set at the angle $\pm 45°$ to the orientation of the polarizer; and (iii) linearly polarized photon pairs with correlated hidden variables can form a Bell state only when the polarization directions of photon pairs are distributed in two orthogonal directions with equal probability and the orientation of one of the polarizers is parallel to one of the polarization directions of the photon pair.

We have supposed above that the measurement outcome of a photon is determined by the external conditions and hidden variable. In fact, it can also be determined by other properties of the photon. We consider the Bell state composed of circularly polarized photon pairs. Even if the polarization uncertainties of the two photons are the same, their rotation directions can be different. We let λ_A and λ_B respectively denote the hidden variables of the two photons, and d_A and d_B respectively denote the rotation directions of the pair. Then the four Bell states can be denoted by the combination of λ and d. We suppose that for the $|\phi^+\rangle$ state, we have $\lambda_A = \lambda_B$ and $d_A = d_B$, while for the $|\psi^-\rangle$ state, we have $\lambda_A = -\lambda_B$ and $d_A = d_B$. The respective coincidence rates of P_{++} for the four Bell states $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$, and $|\psi^-\rangle$ are

$$\frac{1}{2}\cos^2(a-b), \quad \frac{1}{2}\cos^2(a+b), \\ \frac{1}{2}\sin^2(a+b), \quad \frac{1}{2}\sin^2(a-b)$$

[24]. Then we can infer that the rotation direction determines the plus or minus sign, while the hidden variable determines the expression of sine or cosine. For the $|\psi^+\rangle$ state, we therefore have $\lambda_A = -\lambda_B$ and $d_A = -d_B$, and for the $|\phi^-\rangle$ state, we have $\lambda_A = \lambda_B$ and $d_A = -d_B$. Because the rotation direction of the photon is a measurable quantity, we do not regard it as a hidden variable.

As regards the Bell states composed of circularly/linearly (or linearly) polarized photon pairs, we can use the polarization uncertainty and one of the polarization components (e. g., horizontal or vertical polarization) of the pair to denote the four Bell states. For example, the $|\phi^+\rangle$ state can be denoted by $\lambda_A = \lambda_B$ and $H_A = H_B$ (or $V_A = V_B$). For the $|\phi^-\rangle$ state, we have $\lambda_A = \lambda_B$ and $H_A = -H_B$ (or $V_A = -V_B$).

We now use the above theory to explain the experimental results. The atomic cascade radiation experiments in Refs. [18–20] can be explained by circularly polarized photon pairs or circularly/linearly polarized photon pairs. For the down-conversion of the crystal, the wave packets of a pair of orthogonally polarized photons overlap at the crystal or beam splitter. If their phases are the same in the case when they are separated from each other at the output port, they tend to convert into a pair of circularly polarized photons with different rotation directions. Because the two photons have anti-correlated hidden variables, the experiment generates the $|\psi^+\rangle$ state. If the two photons obtain a phase shift of $\pm\pi/2$ during the propagation process in the crystal due to their different phase velocities, they form a pair of circularly polarized photons with the same rotation direction. Then the experiment generates the $|\psi^-\rangle$ state. This can explain the experimental results in Refs. [21–27]. As regards the Bell state composed of four photons, a pair of orthogonally polarized photons can form a pair of circularly polarized photons with the same rotation direction. The Bell state is then obtained. Even if each pair of photons forms a pair of linearly polarized photons with polarization directions at $\pm 45°$, we can turn them into circularly polarized photons by inserting two quarter-wave plates into the optical paths. This can explain the experimental results in Refs. [28–31]. Because the quantum states of two photons in the same path are the same, even if one photon is lost during the detection process, the coincidence rate remains unaffected. This type of experiments can increase the detection efficiency of correlated photon pairs.

We have supposed that two linearly polarized photons are decomposed into a pair of circularly polarized photons with different rotation directions when their wave packets are separated from each other. Certainly, they can also convert into a pair of linearly polarized photons with the polarization direction of $\pm 45°$. In this case, if one of the orientations of the polarizers is set at $\pm 45°$, the Bell state is also obtained based on our analysis above. Because one of the polarizers is oriented at $\pm 45°$ in most of the experiments, this possibility cannot be ruled out. To test which assumption is correct, we let the orientation of the fixed polarizer deviate from $\pm 45°$, e. g., be $\pm 20°$, while the orientation of the other polarizer can vary arbitrarily. If the Bell state can still be obtained in this case, then the first assumption is correct; otherwise, the second is correct. Because some experiments have already indicated that a Bell state can be obtained when the fixed polarizer is oriented at $0°$ or $90°$ [25, 26], it is likely that the first assumption is correct. This experiment also provides a method for discriminating between quantum theory and our theory. In the experimental setups in Refs. [21–26], we insert a quarter-wave plate into each output path. According to quantum theory, the Bell state then remains unaffected. In our theory, by contrast, linearly polarized photon pairs can be obtained by inserting quarter-wave plates, and the Bell state cannot be obtained in the general case. Then we can decide which theory is correct based on the experimental results.

4. PROPOSED EXPERIMENTS OF QUANTUM MEASUREMENT

4.1. Experimental test of the locality of the measurement of EPR pairs

One of the questions raised by the EPR paradox is: if we have measured one particle of the EPR pair, what is the quantum state of the other? For example, we suppose that the $|\phi^+\rangle$ state is composed of circularly polarized photon pairs. According to quantum entanglement, when we measure one photon and find it linearly polarized, the other instantaneously collapses into linear polarization. In terms of the hidden-variable theory, the other remains circularly polarized until we analyze it with a polarizer. Does this violate the angular momentum conservation? If we only consider the system composed of a pair of photons, the angular momentum of the system is certainly not conserved. In the measuring process, a third component — the measuring device — is involved. If the measuring device is included, the momentum and angular momentum of the system are still conserved.

To discriminate between the two hypotheses, we must seek a material that can exhibit different effects when circularly and linearly polarized photons respectively pass through it. We note that the usual method of inserting a quarter-wave plate into the optical path cannot be used here because the circularly polarized photons in one optical path may have two rotation directions, and we therefore use the roto-optic effect (or the Faraday effect). This is because a linearly polarized photon can be regarded as a combination of left-handed and right-handed circularly polarized components. When it passes through a roto-optic material, the velocities of the two components are different according to Fresnel's roto-optic theory. Then there exists a phase shift between the two components. The polarization plane of the photon rotates and the polarization quantum state changes. As a circularly polarized photon passes through the roto-optic material, its polarization quantum state does not change because it has only one rotation direction.

The experimental setup is shown in Fig. 3, where

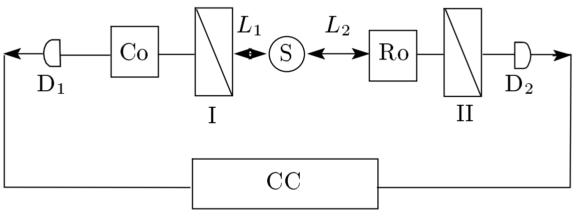

Fig. 3. Experimental test of the locality of the measurement of EPR pairs: S is a source; D_1 and D_2 are the counters; Ro is a roto-optic material; Co is a compensator; CC is a coincidence counter; I and II are polarizers

I and II are two polarizers with the same orientation and Ro is a roto-optic material that rotates the polarization plane of a linearly polarized photon by $\pi/2$. The $|\phi^+\rangle$ state composed of circularly polarized photon pairs can be generated by down-conversion of a nonlinear crystal. When the wave packets of two orthogonally polarized photons overlap at the beam splitter or crystal [21, 22, 24], we can think that the $|\psi^+\rangle$ state generated in the experiments is composed of circularly polarized photon pairs. Then the $|\phi^+\rangle$ state can be obtained by inserting a half-wave plate into one of the optical paths. A circularly polarized photon remains circularly polarized after it passes through a half-wave plate. The $|\phi^+\rangle$ state obtained in this way is therefore composed of circularly polarized photon pairs. If we adopt the method of cascade radiation, then the experiments in [18, 19] just generate the $|\phi^+\rangle$ state. Let the distance between source S and Ro be longer than the distance between S and polarizer I ($L_2 > L_1$). Then the left-traveling photon is analyzed first. An optical path length compensator Co is used to guarantee the simultaneous detection of two photons within the coincidence time window of counters D_1 and D_2. If the roto-optic material is the Faraday rotator, then the compensator can be used with another identical one that is power-off. As a matter of fact, if the optical path length difference between the two sides is appropriately adjusted, the compensator Co can be removed.

We now discuss the expectations of the two theories. According to quantum entanglement, when the left-traveling photon passes through polarizer I, the polarization direction of the right-traveling photon instantaneously collapses to the orientation of polarizer I. Its polarization plane is then rotated by $\pi/2$ when it passes through Ro. Hence, it will be reflected by polarizer II. If the left-traveling photon is reflected by polarizer I, the coincidence rate is zero, irrespective of whether the right-traveling photon is transmitted or reflected. Therefore, the expected coincidence rate is zero in terms of quantum entanglement. According to the hidden-variable theory, measurement on one photon does not affect the other. On the other hand, a roto-optic material does not change the polarization quantum state of a circularly polarized photon. Therefore, the coincidence rate remains unchanged and is always $1/2$. If the hidden variable varies with time, as suggested in [10], then the coincidence rate varies with the position of polarizer II. Similar experiments can be performed for the other three Bell states.

If one does not agree with the assumption of wave packet reduction of the EPR pair and supposes that the roto-optic material does not change the polarization quantum state of the EPR pair, then one obtains the same result as ours. To see whether the roto-optic material can change the polarization quantum state of the EPR pair, we make the above experiment with the $|\phi^+\rangle$ state composed of circularly and linearly polarized photon pairs. Then the question arises: how to obtain this quantum state? When the wave packets of two orthogonally polarized photons overlap at a beam splitter, the $|\psi^+\rangle$ state is generated. Then the two photons are circularly polarized. In the experimental setup in Ref. [23], the rotation direction of one photon is reversed by a mirror, and then the experiment generates the $|\psi^-\rangle$ state. We can then change it into the $|\phi^+\rangle$ state with a half-wave plate and a quarter-wave plate. A quarter-wave plate transforms circular polarization into linear polarization, and in this case the $|\phi^+\rangle$ state is therefore composed of circularly and linearly polarized photon pairs. Similarly, in the experimental setup in Ref. [24], we let the experiment generate the $|\psi^-\rangle$ state by adjusting the birefringent phase shifter. We then use a half-wave plate and a quarter-wave plate to change the $|\psi^-\rangle$ state into the $|\phi^+\rangle$ state. In this case, the $|\phi^+\rangle$ state is composed of circularly and linearly polarized photon pairs. If a roto-optic material is inserted into the optical path without a quarter-wave plate (the photons in this path are circularly polarized), then both theories predict the coincidence rate to be $1/2$. But if a roto-optic material is placed into the optical path with the quarter-wave plate, the expectations of the two theories are different. If the roto-optic material does not change the polarization quantum state, the coincidence rate remains unchanged. According to our theory, the roto-optic material acts as a half-wave plate because it rotates the polarization plane by $\pi/2$, which transforms $|\phi^+\rangle$ into $|\psi^+\rangle$, and we therefore expect the coincidence rate to be $(1/2)\sin^2(a+b)$.

In Wheeler's delayed-choice experiments (see, e. g., [32–34]), which-way measurements are made with a

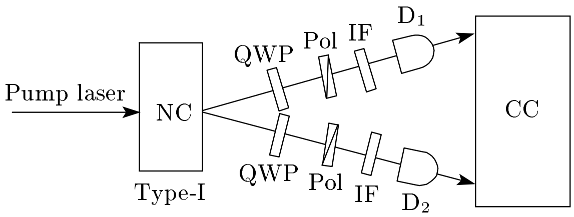

Fig. 4. Generation of the $|\phi^{\pm}\rangle$ state by type-I noncollinear down-conversion: NC is a nonlinear crystal; IF are the interference filters; QWP are quater-wave plates; Pol are polarizers; other notations are as in the upper figures

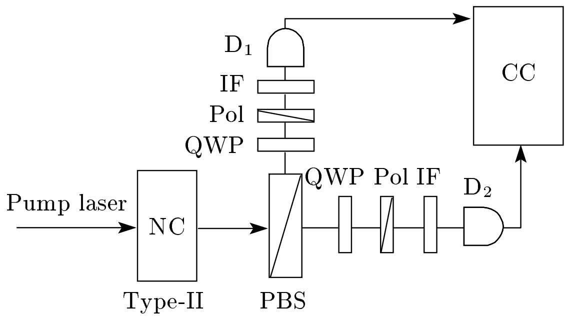

Fig. 5. Generation of the $|\psi^{\pm}\rangle$ state by type-II collinear down-conversion: PBS is a polarizing beam splitter; other notations are the same as in Fig. 4

two-path interferometer that is chosen after a single-photon pulse entered it. The experiments support Bohr's statement that the behavior of a quantum system is determined by the type of measurement, but cannot answer the question of whether measurement on one particle of the EPR pair can affect the other. The above experiments can unambiguously answer it and help understand the EPR paradox (and the Greenberger–Horne–Zeilinger theorem and the Hardy theorem as well), which supposes that a particle quantum state can be predicted with certainty by measuring its partner. The above experiments show that this is not always possible. For example, if we measure photon A with a polarizer and find it to be in the $|H\rangle$ state, then photon B can be in neither the $|H\rangle$ state nor the $|V\rangle$ state. Instead, it can remain in the superposition state, i. e., circular polarization. Only after measurement with a polarizer can we obtain its definite polarization state ($|H\rangle$ or $|V\rangle$ state), and different measurements lead to different results. Hence, the hypothesis of the EPR paradox is not correct.

4.2. Experimental test of the components of polarization-entangled photon pairs

We have supposed above that polarization-entangled Bell states can be composed of circularly polarized photon pairs. To test this assumption, we use a pair of linearly polarized photons generated by type-I noncollinear down-conversion. The experimental setup is shown in Fig. 4. Because the two photons are generated from a single photon, their hidden variables should be correlated. Two quarter-wave plates are inserted into the optical paths to convert the linearly polarized photons into circular polarized ones. If the optical axes of the two quarter-wave plates are parallel, the experiment should generate the $|\phi^{+}\rangle$ state. If the optical axes are oriented orthogonally, i. e., one is set at $45°$ and the other at $-45°$, the rotation directions of the two circularly polarized photons are opposite, then the $|\phi^{-}\rangle$ state should be obtained. A similar experiment can be made with type-II noncollinear down-conversion.

For type-II collinear down-conversion, the hidden variables of the two photons can be regarded as maximally anticorrelated. In this case, a polarizing beam splitter can be used to separate the two orthogonally polarized photons. Then the $|\psi^{\pm}\rangle$ state can be obtained with two quarter-wave plates after the polarizing beam splitter (the $|\psi^{+}\rangle$ state is to be generated when the optical axes of the two quarter-wave plates are parallel). The experimental setup is shown in Fig. 5.

To verify the assumption that circularly and linearly polarized photon pairs can form the Bell state, we remove the quarter-wave plate in the experiment in Fig. 4 or 5, and set the orientation of the polarizer at $\pm 45°$ to the linearly polarized direction of photons, while the other orientation of the polarizer can vary arbitrarily. In this case, we still obtain the $|\phi^{\pm}\rangle$ state in Fig. 4 and the $|\psi^{\pm}\rangle$ state in Fig. 5.

In other down-conversion experiments [21–31], the wave packets of two orthogonally polarized photons overlap at the beam splitter or crystal. The above experiments do not overlap the wave packets of photons and the polarizations of photons are definite. If the Bell states can be generated in this way, then quantum entanglement will not remain a mystery.

The following experiment uses the overlap of multiphoton wave packets to generate the Bell state. The experimental setup is shown in Fig. 6. A beam of linearly polarized laser enters the Mach–Zehnder interferometer, which can be a continuous-wave laser or pulsed laser. A half-wave plate is inserted into one of the arms to rotate the polarization plane by $\pi/2$. If we replace the first (or the second) beam splitter with a polarizing

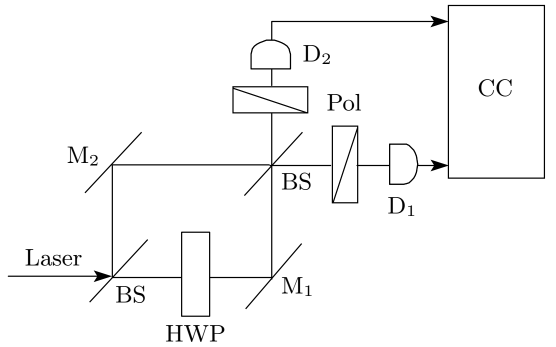

Fig. 6. The $|\phi^{\pm}\rangle$ state obtained by the overlap of multi-photon wave packets: BS is a beam splitter; HWP is a half-wave plate; M_i are the mirrors; other notations are as in upper figures

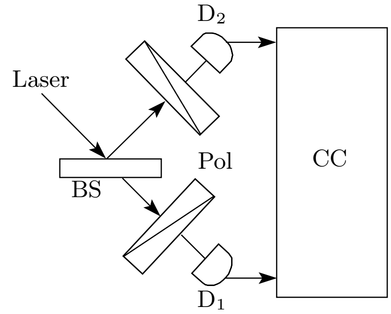

Fig. 7. The simplest way to generate the polarization-entangled Bell state (the notations are the same as in upper figures)

beam splitter, then the half-wave plate can be removed, and the polarization of the input laser should be set at $\pm 45°$. If the relative phase of the photons in the two arms is chosen correctly, the output is circularly polarized. On the other hand, because the photons are coherent or indistinguishable within the coherence length, we can think that the polarization hidden variables of a bunch of photons are correlated within the coherence length. Hence, these photons must behave in the same way when analyzed by a polarizer, i. e., if one photon is transmitted, then all the photons are also be transmitted. In the case where all the photons within the coincidence time window of the detectors are coherent, the Bell state is thus obtained. We note that the experiment adopts the multi-photon wave packet overlap, and therefore, similarly to the experimental results of the overlap of two biphoton wave packets at a beam splitter or a crystal [28–31], we expect the experiment to generate the $|\phi^{\pm}\rangle$ state. A glass plate can be inserted into the other arm or we can scan one of the mirrors to change the relative phase of the photons in the two arms, and the optical path length difference should be shorter than the coherence length of the laser. The key to the experiment is that we must ensure that the polarization quantum states of the photons be identical within the detection time of the detectors; otherwise, the behavior of a bunch of photons would be different. For a continuous-wave laser, the coincidence time window of the photon detectors should be shorter than the coherence time of the laser. For a pulsed laser, on the other hand, the coherence time of photons should be longer than (or equal to) the pulse duration, which can be realized by inserting an interference filter in front of each of the detectors. Compared with other beam-splitter schemes to obtain Bell states, the experiment is much simpler because it does not use down-conversion of the crystal.

In fact, there is the simplest way to generate a polarization-entangled Bell state. We have supposed that the polarization hidden variables of a bunch of photons are correlated within the coherence length, and therefore, if we split a beam of circularly polarized light into two beams and detect them within the coherence time of the laser, then the Bell state should be obtained. The experimental setup is shown in Fig. 7. A 50/50 beam splitter is used to split the circularly polarized laser. The two beams of light are then analyzed by polarizers. We note that if a half-silvered mirror is used as the beam splitter, then different placements of the mirror result in different Bell states. If reflection occurs at the front of the mirror, a phase shift of π accompanies the reflected beam, the rotation directions of the two beams of light are opposite, and the experiment in Fig. 7 should generate $|\phi^{-}\rangle$ state. The $|\phi^{+}\rangle$ state can be obtained in the case of rear surface reflection, because the medium behind the mirror (air) has a lower refractive index than the medium the light is traveling in (glass). If our above prediction is true, then an arbitrary number of correlated photons can be obtained using a couple of beam splitters as desired. For comparison, the current maximum number of entangled photons is eight [35, 36].

Under ideal conditions, all the photons within the coherence length must behave in the same way under the same measuring condition, i. e., they act as one photon. But there exists a major difficulty for the experiments in Figs. 6 and 7. Due to the imperfectness of the correlated photons, the experimental setup and the external circumstances, most of the photons would behave in the same way, but a few of them may not. This would blur the experimental results. To overcome this

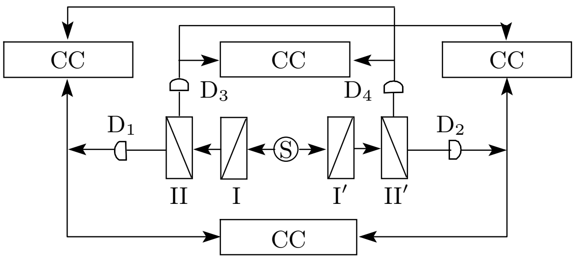

Fig. 8. Two successive polarization measurements on EPR photon pairs: I, I′ and II, II′ are the polarizers; the other notations are as in the previous figures

problem, single-photon detectors D_1 and D_2 may be replaced with photoelectric detectors. We note that in this case, our concern is not the specific values of the intensities of the laser but their relative values compared to the threshold value. When all the photons pass through the polarizer, the detection intensity becomes half that of the input laser. Hence, the threshold value can be set at one-fourth of the input laser intensity. If both luminosities in the two channels are greater (or less) than the threshold value, we obtain a coincident count.

We note that the above experiment differs from that in Ref. [37]. The former detects the polarization correlation, while the latter detects the intensity correlation of the input laser. It is interesting to compare our experiment with that in Refs. [38] and [39], where a PBS is used to separate a beam of monochromatic light (or two orthogonally polarized photons generated by type-II down-conversion) and the relation between intensity correlation (or coincidence count) and the rotation angle of the half-wave plate placed in front of the polarizing beam splitter is recorded. To explain the correlated results that cannot be explained in classical theory, hidden polarization or higher-order polarization is introduced [38–40]. We think that they are equivalent to the polarization hidden variable we introduce here. In fact, if two quarter-wave plates are inserted after the polarizing beam splitter in the experiments in Refs. [38] and [39] to convert linearly polarized photons into circularly polarized photons, as is the case in Fig. 5, the two experiments would also generate the polarized Bell state.

4.3. Successive polarization measurements on EPR photon pairs

If quantum measurement is deterministic, then the experimental result is determined by the measuring conditions and intrinsic properties of a particle, and there are no random disturbances during the measuring process. We can further infer that the collapsed quantum states of a pair of particles with the same quantum state must be the same under the same measuring condition. We now test this assumption. We add another pair of polarizers II and II′ in the transmitted channels in Fig. 2, as shown in Fig. 8. Polarizer I has the same orientation as polarizer I′, and the orientations of polarizers II and II′ are also identical. The source generates circularly polarized $|\phi^+\rangle$ state photon pairs. According to Eq. (9), half of the photon pairs pass through the first pair of polarizers and reach the second pair of polarizers. When they are analyzed again, their behaviors are still correlated, i. e., if one photon is transmitted, then the other is also transmitted. For the second pair of polarizers, we have

$$P_{++} = \cos^2\theta, \quad P_{--} = \sin^2\theta, \quad P_{+-} = P_{-+} = 0,$$

where θ is the angle between the orientations of the two pairs of polarizers. According to quantum theory, a pair of photons is not in an entangled state after the first measurement because their polarizations are definite. In this case, we do not know how to calculate the joint probability in quantum mechanics. But if our predictions are correct, there must exist a conceptual difficulty for quantum mechanics to explain the total correlation of a pair of particles without entanglement, which can be readily understood in a deterministic hidden-variable theory. We note that we can also perform the experiment in the reflected channels of polarizers I and I′, with similar results. The joint measurements between transmitted and reflected channels are not needed because the probabilities must be zero according to Eq. (10). Hence, the experiment is a complete measurement.

Because the collapsed quantum states of a pair of photons after the first measurement are the same, they can be restored into the $|\phi^+\rangle$ state by inserting two quarter-wave plates with parallel-oriented optical axes into the optical paths between the two pairs of polarizers.

We now discuss the coincidence counting results when the orientations of the second pair of polarizers are different. We suppose that the orientation of the first pair of polarizers is along the x axis, and the orientations of the second pair of polarizers are respectively along the directions of a and b. For simplicity, we let a, b, and x lie in one plane, and $\bar{a}$ and $\bar{b}$ be the directions respectively perpendicular to a and b, as shown in Fig. 9.

For circularly polarized photon pairs, the single probabilities $P_+(a)$ and $P_+(b)$ are equal, we can ob-

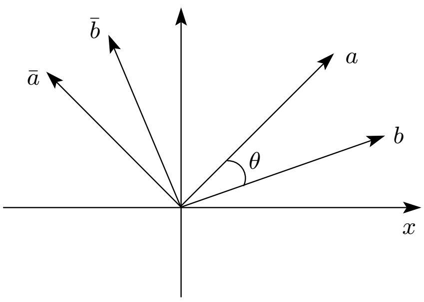

Fig. 9. Orientations of two pairs of polarizers

tain the same joint probability $P_{++}(a,b)$ either by projecting a onto b or by projecting b onto a. For a pair of linearly polarized photons, the single probabilities that the two photons pass through the second pair of polarizers are not equal. Then different projective sequences lead to different results. If we project a onto b, we obtain $P_{++}(a,b) = \cos^2 a \cos^2\theta$, where $\theta = a - b$. If we project b onto a, we obtain $P_{++}(a,b) = \cos^2 b \cos^2\theta$. Because the joint probability cannot be larger than single probabilities, and the latter cannot satisfy this requirement, we choose $P_{++}(a,b) = \cos^2 a\cos^2\theta$ for the moment.

We now consider the expression for $P_{+-}(a,b)$. According to the rule of projecting from one channel with a smaller probability onto another with a larger probability, we obtain

$$P_{+-}(a,b) = \begin{cases} \cos^2 a \sin^2\theta, & \cos^2 a \le \sin^2 b, \\ \sin^2 b \sin^2\theta, & \cos^2 a \ge \sin^2 b. \end{cases}$$

Because the requirement

$$P_{++}(a,b) + P_{+-}(a,b) = P_+(a) = \cos^2 a$$

must be satisfied, and considering the smooth joining of probability formula, we take

$$P_{++}(a,b) = \\ = \begin{cases} \cos^2 a\cos^2\theta, & \cos^2 a \le \sin^2 b, \\ \cos^2 a - \sin^2 b\sin^2\theta, & \cos^2 a \ge \sin^2 b. \end{cases} \tag{14}$$

It can be verified that in addition to satisfying the projective relation in the instances of $\theta = 0$ and $\theta = \pi/2$, Eq. (14) also meets the expectations of $P_{++}(a,b) = \cos^2 a$ for $b = 0$ and $P_{++}(a,b) = 0$ for $a = \pi/2$. It is therefore a reasonable probability formula. With Eq. (14), we can calculate the other three joint probabilities using the relations

$$P_{++}(a,b) + P_{+-}(a,b) = \cos^2 a,$$

$$P_{++}(a,b) + P_{-+}(a,b) = \cos^2 b,$$

$$P_{+-}(a,b) + P_{--}(a,b) = \sin^2 b.$$

In fact, there can exist other projective relations for the calculation of joint probability. When b rotates between 0 and a, the joint probability $P_{++}(a,b)$ can remain unchanged and be always $\cos^2 a$, i.e., the joint probability is the smaller of two single probabilities. This implies that for two dependent events under certain conditions (for example, when a and b lie in the same quadrant), if one event with the smaller probability occurs, then the other event with the larger probability would occur with certainty. Then the four joint probabilities can be written as

$$\begin{aligned} &P_{++}(a,b) = \cos^2 a, \\ &P_{+-}(a,b) = 0, \\ &P_{-+}(a,b) = \cos^2 b - \cos^2 a, \\ &P_{--}(a,b) = \sin^2 b. \end{aligned} \tag{15}$$

It can be seen that in the instance of $\theta = 0$ we obtain the same result as in Eq. (14), i.e.,

$$P_{++}(a,b) = P_+(a) = P_+(b) = \cos^2 a.$$

In other cases, we cannot decide whether Eq. (14) or (15) is correct, which can only be tested by experiment. No matter which formula is correct, we believe that for a deterministic measurement theory, the requirement that the joint probability be equal to the single probabilities must be satisfied in the case $\theta = 0$.

If we suppose that the polarization direction of photon pairs (the x axis in Fig. 9) is distributed uniformly in space and then average over it to obtain the average joint probability, we find that none of the results in Eqs. (14) or (15) agrees with that of quantum mechanics. If the polarization direction of photons is distributed in two orthogonal directions, the result also disagrees with that of quantum mechanics. We do not present the detailed calculation process. We conclude that linearly polarized photon pairs cannot form a Bell state in the general case.

5. DISCUSSION AND CONCLUSION

We show that the true implication of the probability distribution of Bell's correlation function is the probability distribution of the joint measurement outcomes, and it can therefore vary with experimental conditions. In addition, we show that Bell's locality holds neither for two independent events nor for two dependent events. The results of EPR-type experiments can be explained with the projective relation of the quantum state composed of a circularly or linearly polarized photon pair whose hidden variables are maximally correlated or anticorrelated. We also explore the physical meaning of the hidden variable and the measuring process.

The hidden-variable theory does not conflict with the current formalism of quantum mechanics, which can be viewed as holding for the statistic description of behavior of a large number of independent particles but not for the deterministic description of the behavior of individual particles or EPR pairs. Currently, there is no experiment suggested to distinguish between the locality and nonlocality assumptions. Our first experiment is aimed at this purpose, which we think can verify whether collapse of the wave packet of the EPR pair is true. All our expectations for above experiments are based on the assumptions that the local hidden variable exists and the behavior of microscopic particles is also deterministic. But it should be noted that even if all our theoretical expectations are verified by experimental results, we could only abandon the concept of quantum entanglement and Bell's locality assumption. Although the starting point of our theory is local hidden variable, the above experiments cannot adequately prove that local hidden variable does exist. Only when the experimental results cannot be explained by the current theory of quantum mechanics can we say that it is incomplete and a hidden variable should be introduced. More experiments and theoretical analyses are therefore needed in order to solve the problem of hidden variables.

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