ЖЭТФ, 2011, том 140, вып. 6 (12), стр. 1096–1102

MOVING BRANES IN THE PRESENCE OF BACKGROUND TACHYON FIELDS

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Received April 14, 2011

We compute the boundary state associated with a moving Dp-brane in the presence of the open string tachyon field as a background field. The effect of the tachyon condensation on the boundary state is discussed. It leads to a boundary state associated with a lower-dimensional moving D-brane or a stationary instantonic D-brane. The former originates from condensation along the spatial directions and the latter comes from the temporal direction of the D-brane worldvolume. Using the boundary state, we also study the interaction amplitude between two arbitrary Dp_1 and Dp_2-branes. The long-range behavior of the amplitude is investigated, demonstrating an obvious deviation from the conventional form, due to the presence of the background tachyon field.

1. INTRODUCTION

The open string tachyon can be regarded as an instability of branes because open strings introduce quantum excitations of the branes [1]. The tachyon potential has a stationary point where the negative potential energy of the tachyon cancels the tension of the D-brane [2]. This process, which is called tachyon condensation, ends when the brane has completely disappeared. During the condensation process, lower-dimensional branes are produced [3, 4].

On the other hand, we have the boundary state as a quantum state that contains closed string states [5]. It can be used to study D-branes. In the other words, a D-brane couples to all states of the closed string via the boundary state. We can therefore suppose that the exchange of closed strings between two D-branes is responsible for the interaction of branes. For calculating it, we can just connect their corresponding boundary states through the closed string propagator. The coherent state method [6] and the path integral approach [7, 8] have been used to obtain the boundary state. Furthermore, the boundary state in the presence of background fields such as $B_{\mu\nu}$ and $U(1)$ gauge fields in a compact spacetime [9] and in the presence of the tachyon field [10, 11] have been investigated.

Apart from the background $U(1)$ gauge field and the open string tachyon field, which are parallel to the brane worldvolume, the transverse fluctuations of a D-brane are also essential as a dynamical object. Scarcity of this kind of multilateral discussion motivated us to follow this process in this paper in spite of some technical difficulties. Besides, the full brane in the presence of a one-dimensional background tachyon field is usually considered in the literature and the effect of one-stage condensation on that brane is studied. But we here study a D-brane of an arbitrary dimension. Therefore, the Dirichlet boundary conditions are also present. In our setup, the tachyon field has components along all the directions of the D-brane worldvolume. This tachyon profile leads to various condensations and hence a variety of the resulted branes.

Here, using the path integral approach, we calculate the boundary state corresponding to a moving Dp-brane in the presence of a tachyon field. Consequently, we obtain the disk partition function of the closed string. The effect of the tachyon condensation on this partition function is studied. The condensation is applied along the spatial worldvolume directions and gives a partition function associated with a moving lower-dimensional D-brane. The difference from the conventional tachyon condensation (e. g., see [10]) is in the presence of a tachyon-dependent factor in the resultant partition function. That is, although the brane dimension decreases, the effect of the tachyon is not removed by condensation. In this process, the transverse

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fluctuations of a Dp-brane prevent the normal tachyon condensation from occurring. Applying the condensation along the temporal direction of the Dp-brane worldvolume gives an instantonic stationary brane.

The final goal of this paper is to apply the boundary state to obtain the interaction amplitude between two moving D-branes and to study its behavior for large distances between the branes. We observe that due to the inclusion of the open string tachyon background (which is equivalent to considering the instability of the bosonic D-branes), the long-range interaction of the branes tends to zero. This is a consequence of the tachyon rolling toward its minimum potential. We observe that for the interaction of two D-instantons, the conventional long-time interaction amplitude is restored.

2. BOUNDARY STATE AND TACHYON CONDENSATION

To determine the boundary state associated with a moving Dp-brane in the presence of the tachyon field, we begin with an appropriate sigma-model for the string. This action contains the bulk term, a tachyonic term on the boundary, and a velocity term corresponding to the motion of the brane:

$$S_{\text{bulk}} = -\frac{1}{4\pi\alpha'}\int_{\Sigma} d^2\sigma\left(\sqrt{-h}h^{ab}g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu\right), \quad (1)$$

$$S_{\text{boundary}} = \int_{\partial\Sigma} d\sigma (V^i X^0 \partial_\tau X^i + iU_{\alpha\beta}X^\alpha X^\beta), \quad (2)$$

where Σ is the worldsheet of the closed string exchanged between the branes, $\partial\Sigma$ is the boundary of this worldsheet, which can be at $\tau = 0$ or $\tau = \tau_0$ and the d-dimensional spacetime metric is

$$g_{\mu\nu} = (-1, 1, \ldots, 1).$$

In addition, we define

$$V^i = \frac{v^i}{2\pi\alpha'},$$

where v^i is the brane velocity component along the X^i direction. The coupling of the tachyon field to the string via integration over the worldsheet boundary has been discussed in [12]. It occurs squared in the action, i. e., as $T^2(X)$, and in order to produce a Gaussian integral, the tachyon profile must therefore be chosen linear,

$$T(X) = a + u_\mu X^\mu.$$

The constant a has been shifted away in (2). We also consider the symmetric matrix U to have nonzero elements only along the worldvolume of the Dp-brane. The set $\{X^\alpha\}$ specifies the directions along the Dp-brane worldvolume and $\{X^i\}$ shows the directions perpendicular to it.

2.1. The boundary state

We now consider the mode expansion of the coordinates of the closed string

$$X^\mu(\sigma,\tau) = x_0^\mu + 2\alpha' p^\mu \tau + \sqrt{\frac{\alpha'}{2}} \times \\ \times \sum_{m>0} m^{-1/2}(x_m^\mu e^{2im\sigma} + \overline{x}_m^\mu e^{-2im\sigma}), \quad (3)$$

where we define x and $\overline{x}$ as the bosonic mode combinations

$$\begin{aligned} x_m &= a_m e^{-2im\tau} + \widetilde{a}_m^\dagger e^{2im\tau}, \\ \overline{x}_m &= a_m^\dagger e^{2im\tau} + \widetilde{a}_m e^{-2im\tau}, \end{aligned} \quad (4)$$

with

$$a_m^\mu = \frac{i}{\sqrt{m}}\alpha_m^\mu, \quad a_m^{\dagger\mu} = \frac{-i}{\sqrt{m}}\alpha_{-m}^\mu.$$

Similar relations hold for $\widetilde{a}_m^\mu$ and $\widetilde{a}_m^{\dagger\mu}$. If we interpret equations (4) as eigenvalue equations [7], then the corresponding eigenstate is

$$|x,\overline{x}\rangle = \\ = \prod_{m=1}^{\infty} \exp\left(-\frac{1}{2}\overline{x}_m x_m - a_m^\dagger \widetilde{a}_m^\dagger + a_m^\dagger x_m + \overline{x}_m \widetilde{a}_m^\dagger\right) \times \\ \times |\text{vac}\rangle, \quad (5)$$

where contraction with the metric $g_{\mu\nu}$ is applied implicitly. This is a boundary state of the closed string due to the bulk term of the string sigma-model without any boundary interaction. Naturally deforming the action by adding nonvanishing boundary contributions leads to the deformed boundary state

$$|B; S_{\text{boundary}}\rangle = \int [dx\, d\overline{x}] e^{iS_{\text{boundary}}[x,\overline{x}]}|x,\overline{x}\rangle. \quad (6)$$

The boundary actions related to the tachyon, S_T, and the velocity term, S_V, can be written in terms of modes as

$$S_T = i\pi x_0^\alpha U_{\alpha\beta} x_0^\beta + i\pi\alpha' \sum_{m=1}^{\infty} \overline{x}_m^\alpha \frac{U_{\alpha\beta}}{m} x_m^\beta, \quad (7)$$

$$S_V = v^i x_0^0 p^i + i v^i \times$$
$$\times \sum_{m=1}^{\infty} \left(\frac{1}{2} (\overline{x}_m^0 x_m^i + \overline{x}_m^i x_m^0) - \overline{x}_m^0 a_m^i - \widetilde{a}_m^i x_m^0 \right). \quad (8)$$

Here and hereafter, we impose a selected direction X^{i_0} for the motion of the Dp-brane and set $v^{i_0} = v$. Substituting (7) and (8) in (6) and also considering the contribution of the bulk action at the boundary gives the boundary state. The oscillating part of this state is

$$|B_x\rangle^{\rm osc} = \prod_{m=1}^{\infty} \frac{1}{\det R_{(m)}} \times$$
$$\times \exp \left(\sum_{m=1}^{\infty} a_m^\dagger \mathcal{S}_{(m)} \widetilde{a}_m^\dagger \right) |0\rangle, \quad (9)$$

where

$$R_{(m)ab} = -2\Omega_{ab} + \frac{2\pi\alpha'}{m} U_{\alpha\beta} \delta^\alpha{}_a \delta^\beta{}_b,$$
$$\Omega_{ab} = -\frac{1}{2} g_{ab} - \frac{1}{2} v (\delta^0{}_a \delta^{i_0}{}_b + \delta^{i_0}{}_a \delta^0{}_b), \quad (10)$$

and

$$\mathcal{S}_{(m)\mu\nu} = 2(R^{-1}_{(m)})_{ab} \delta^a{}_\mu \delta^b{}_\nu - g_{\mu\nu}. \quad (11)$$

The indices a and b indicate the worldvolume and motion directions (i. e., $a, b \in \{\alpha, i_0\}$). It is seen that when the velocity v and the tachyon matrix U are zero, then

$$(R^{-1}_{(m)})_{ab} = g_{ab}.$$

Hence, boundary state (9) reduces to the state for a Dp-brane where the X^α, $\alpha = 0, \ldots, p$, and the X^i, $i = p+1, \ldots, d-1$, respectively obey the Neumann and Dirichlet boundary conditions [13].

The infinite product in (9) is generated by the path integral. Zeta-function regularization can be used to avoid this divergent quantity [14],

$$\prod_{m=1}^{\infty} \left[\det \left(-2\Omega + 2\pi\alpha' \frac{W}{m} \right) \right]^{-1} =$$
$$= \sqrt{\det(-2\Omega)}\ \det \Gamma \left(1 - \frac{\pi\alpha' W}{\Omega} \right), \quad (12)$$

where the matrix W is defined by

$$W_{ab} = U_{\alpha\beta} \delta^\alpha{}_a \delta^\beta{}_b.$$

The zero-mode part of the boundary state becomes

$$|B_x\rangle^0 = \frac{T_p}{2\sqrt{\det U}} \int dp^\alpha \exp \left(-\frac{1}{4\pi} P^T U^{-1} P \right) \times$$
$$\times \delta(x_0^{i_0} - v x_0^0 - y^{i_0}) \prod_{i' \neq i_0} \delta(x_0^{i'} - y^{i'}) \times$$
$$\times \prod_\alpha |p_L^\alpha = p_R^\alpha\rangle \prod_{i' \neq i_0} |p_L^{i'} = p_R^{i'} = 0\rangle \times$$
$$\times \left| p_L^{i_0} = p_R^{i_0} = \frac{1}{2} v p^0 \right\rangle, \quad (13)$$

where the vector P is defined by

$$P_\alpha = v p^{i_0} \delta^0_\alpha - \frac{1}{2} p_\alpha.$$

The momentum-dependent exponential term appears due to the presence of the momentum components in the zero-mode parts of the boundary actions. Two delta functions indicate the position of the brane. After performing the integration over momenta, the matter part of the boundary state takes the form

$$|B_x\rangle = |B_x\rangle^{\rm osc} |B_x\rangle^0 = T_p \frac{\pi(4\pi)^p}{v^2 + 1/2} \times$$
$$\times \prod_{m=1}^{\infty} \frac{1}{\det R_{(m)}} \exp \left(\sum_{m=1}^{\infty} a_m^\dagger \mathcal{S}(m) \widetilde{a}_m^\dagger \right) \times$$
$$\times \delta(x_0^{i_0} - v x_0^0 - y^{i_0}) \prod_{i' \neq i_0} \delta(x_0^{i'} - y^{i'}) |{\rm vac}\rangle, \quad (14)$$

where

$$|{\rm vac}\rangle = |0\rangle_\alpha |0\rangle_{\widetilde{\alpha}} |p\rangle$$

is written in this form for brevity.

2.2. Partition function and tachyon condensation

Because the partition function is defined as

$$\mathcal{Z} = \int DX e^{iS[X]},$$

it is obvious that there exists a very natural connection between the boundary state and the partition function: the latter is just given by the vacuum amplitude of the boundary state:

$$\mathcal{Z} = \langle {\rm vac} | B; S_{\rm boundary} \rangle. \quad (15)$$

Therefore, the normalization factors in Eq. (14) come from the disk partition function [15], which can also be derived by evaluating the string path integral on a disk:

$$\mathcal{Z}_{\mathrm{disk}} = T_p\,\frac{\pi(4\pi)^p}{v^2+1/2}\,\times \\ \times\prod_{m=1}^{\infty}\left[\det\left(g_{ab} - v(\delta^0{}_a\delta^{i_0}{}_b + \delta^0{}_b\delta^{i_0}{}_a) + \right.\right. \\ \left.\left. + \frac{2\pi\alpha'}{m}U_{\alpha\beta}\delta^\alpha{}_a\delta^\beta{}_b\right)\right]^{-1}. \quad (16)$$

We note that the disk diagram in the closed string theory shows a propagating closed string from the boundary of the disk, which then disappears.

The presence of the open string tachyon field as a background field in our case allows studying the effect of tachyon condensation on the partition function. In our case, where the tachyon profile is linear, studying the tachyon condensation amounts to sending the elements of the tachyon matrix U to infinity [10].

Our tachyon matrix has all elements along the brane worldvolume. We recall that $U_{\alpha\beta}$ is a $(p+1)\times(p+1)$ matrix. Without loss of generality, we let it be a diagonal matrix. We consider condensation of all spatial components of U, which can be done for each component one by one or for all of them at once. After successive condensations along the spatial directions of the Dp-brane $\{X^{\overline{\alpha}}|\bar{\alpha} = 1, 2, \ldots, p\}$, with the limit $U_{\overline{\alpha}\overline{\alpha}}\to\infty$ applied, partition function (16) becomes

$$\mathcal{Z}_{\mathrm{disk}} = T_p\,\frac{\pi(4\pi)^p}{v^2+1/2}\,\left(2\pi\sqrt{\alpha'}\right)^p\sqrt{\det U'}\,\times \\ \times\prod_{m=1}^{\infty}\left(1 - v^2 + \frac{2\pi\alpha'}{m}U_{00}\right)^{-1}, \quad (17)$$

where U' is a new diagonal $p\times p$ tachyon matrix that does not contain the element U_{00}. The zeta function regularization,

$$\prod_{m=1}\left[\det\left(\frac{2\pi\alpha' U'}{m}\right)\right]^{-1} = \left(2\pi\sqrt{\alpha'}\right)^p\sqrt{\det U'},$$

has been used in (17). The relation between the D-brane tensions

$$T_{p-q} = T_p\left(2\pi\sqrt{\alpha'}\right)^q$$

allows interpreting (17) as the partition function related to a moving D0-brane with the effective tension

$$\mathcal{T}_0 = T_0\,\frac{\pi(4\pi)^p}{v^2+1/2}\,\sqrt{\det U'}.$$

This considerable difference from the conventional tachyon condensation [10], comes from the momentum-dependent exponential factor, which exists due to the presence of zero modes in both tachyon and velocity boundary actions. In the absence of the velocity term, there is no momentum dependence in the partition function and the factor $1/\sqrt{\det U}$ that appears from zero modes in the tachyon action cancels the factor $\sqrt{\det U}$ that comes from the tachyon condensation in the infinite determinant. But an additional factor $\sqrt{\det U}$ appears because of the Gaussian integration over momenta and leads to this unusual behavior of the partition function after tachyon condensation.

As the next step, performing tachyon condensation along the X^0-direction in Eq. (17), we eliminate the velocity and obtain a D-instanton with the partition function

$$\mathcal{Z}_{\mathrm{disk}} = T_{(-1)}\,\frac{\pi(4\pi)^p}{v^2+1/2}\,\sqrt{\det U}.$$

In other words, temporal tachyon condensation fixes the D-brane in time as well as eliminates its velocity and fixes it in the space. Generally, temporal condensation on a moving Dp-brane leads to a stationary instantonic Dp-brane (i. e., eliminates the time direction of the worldvolume), and condensation of the spatial components of the tachyon field also reduces the Dp-brane dimension.

Accordingly, after tachyon condensation along any spatial direction of the moving Dp-brane worldvolume, its dimension decreases by one such that after q successive condensations, we have a D$(p-q)$-brane in the presence of a $U_{(p-q+1)\times(p-q+1)}$ tachyon field. The main difference from the usual case is that although the brane dimension decreases, the effect of the tachyon remains in the root factor.

In the next section, by using the boundary state formalism, we compute the interaction amplitude between two D-branes in the closed string channel.

3. INTERACTION OF THE BRANES

Because the conformal invariance is preserved in bulk action (1) and is broken on boundary action (2) (see [16]), the conformal ghosts play a role just in the bulk, and hence their contribution to the boundary state should also be considered. For calculating the interaction amplitude between two D-branes, we return to the previous boundary state (14), but restore the integration over momenta. Those give the total boundary state

$$|B\rangle^{\mathrm{total}} = |B_{\mathrm{gh}}\rangle|B_x\rangle. \quad (18)$$

To find the interaction amplitude between the Dp_1- and Dp_2-branes via exchanges of closed string states, we need the closed string propagator, which is given by a time integral of the closed string Hamiltonian:

$$D = 2\alpha' \int_0^\infty dt e^{-tH},$$

$$H = \alpha' p^\mu p_\mu + 2\sum_{n=1}^{\infty}(\alpha_{-n}.\alpha_n + \tilde{\alpha}_{-n}.\tilde{\alpha}_n) + (d-2)/6.$$

The convention for the indices to be used in the amplitude is as follows. The set $\{\bar{i}\}$ shows the directions perpendicular to both branes except i_0, $\{\bar{u}\}$ is for the directions along both branes except 0, $\{\alpha'_1\}$ is used for the directions along the Dp_1-brane and perpendicular to the Dp_2-brane, and $\{\alpha'_2\}$ indicates the directions along the Dp_2-brane and perpendicular to the Dp_1-brane. Because $\{\alpha_1\}$ and $\{\alpha_2\}$ are arbitrary, the positions of the branes are not fixed, that is, the two branes can be parallel or perpendicular to each other.

3.1. The interaction amplitude

The interaction amplitude is given by the overlap of the two boundary states corresponding to the branes, via the closed string propagator, i. e.,

$$\mathcal{A} = \langle B_1|D|B_2\rangle.$$

After a long calculation, we obtain

$$\begin{aligned}
\mathcal{A} = {} & \frac{\alpha' V_{\bar{u}}}{4(2\pi)^{d_{\bar{i}}}} \frac{T_{p_1}T_{p_2}}{|v_1 - v_2|} \left[\det U_1 \det U_2\right]^{-1/2} \times \\
& \times \prod_{m=1}^{\infty} \left[\det R_{(m)1} \det R_{(m)2}\right]^{-1} \int_0^\infty dt \times \\
& \times \Bigg\{ \prod_{m=1}^{\infty} \left([\det(1 - \mathcal{S}_{(m)1}\mathcal{S}^T_{(m)2} e^{-4mt})]^{-1} (1 - e^{-4mt})^2 \right) \times \\
& \times e^{(d-2)t/6} \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{d_{\bar{i}}} \exp\left(-\frac{1}{4\alpha' t} \sum_{\bar{i}} ({y_1}^{\bar{i}} - {y_2}^{\bar{i}})^2 \right) \times \\
& \times \frac{1}{\sqrt{\det Q \ \det G_1 \ \det G_2}} \times \\
& \times \exp\Bigg(-\frac{1}{4}\Bigg[E^T Q^{-1} E + \sum_{\alpha'_1} [({y_2}^{\alpha'_1})^2 (G_1^{-1})^{\alpha'_1\alpha'_1}] + \\
& + \sum_{\alpha'_2} [({y_1}^{\alpha'_2})^2 (G_2^{-1})^{\alpha'_2\alpha'_2}] \Bigg] \Bigg) \Bigg\}. \qquad (19)
\end{aligned}$$

The matrices Q, G_1, and G_2 and the doublet E are defined in terms of their elements as

$$\begin{aligned}
Q_{11} = {} & \frac{\alpha' t}{(v_2 - v_1)^2}(1 + {v_1}^2)(1 - {v_2}^2) - \\
& - \left[\left(v_1^2 + \frac{1}{2} \right)^2 (U_1^{00})^{-1} \right], \\
Q_{22} = {} & \frac{\alpha' t}{(v_2 - v_1)^2}(1 + {v_2}^2)(1 - {v_1}^2) - \\
& - \left[\left(v_2^2 + \frac{1}{2} \right)^2 (U_2^{00})^{-1} \right], \\
Q_{12} = Q_{21} = {} & \frac{\alpha' t}{(v_2 - v_1)^2}(1 + {v_1}^2) \times \\
& \times (1 + {v_2}^2)(1 - v_1 v_2),
\end{aligned} \qquad (20)$$

$$\begin{aligned}
E_1 &= \frac{i}{v_2 - v_1}[{y_2}^{i_0}(1 + {v_1}^2)^2 - {y_1}^{i_0}(1 + v_1 v_2)], \\
E_2 &= \frac{i}{v_2 - v_1}[{y_1}^{i_0}(1 + {v_2}^2)^2 - {y_2}^{i_0}(1 + v_1 v_2)],
\end{aligned} \qquad (21)$$

and the nonzero elements of the matrix G_1 are

$$\begin{aligned}
G_{1\alpha'_1\alpha'_1} &= -\alpha' t - \frac{1}{4}(U_1^{\alpha'_1\alpha'_1})^{-1}, \\
G_{1\bar{u}\bar{u}} &= -\frac{1}{2}\alpha' t - \frac{1}{4}(U_1^{\bar{u}\bar{u}})^{-1}.
\end{aligned} \qquad (22)$$

With the exchange $1 \leftrightarrow 2$, we obtain the nonzero elements of G_2. We note that there is no sum over the repeated indices α'_1 and $\bar{u}$ in (22).

In the interaction amplitude (19), $V_{\bar{u}}$ is the common worldvolume of the branes and $d_{\bar{i}}$ is the dimension of the directions that are perpendicular to both branes. The infinite product in the second line of (19) shows the effect of the oscillators and conformal ghosts (see Refs. [9, 17] for an analogous effect). The first exponential and its prefactor, which originate from the directions perpendicular to both branes, indicate the damping of the amplitude due to the distance between the branes. The momenta entering the Hamiltonian and the zero mode terms in the boundary state lead to the second exponential and its prefactor. The constant factors behind the time integral somehow show the strength of the interaction, which depends on the brane tensions, their velocities, and the tachyon fields. We note that the regularization of the infinite product in the first line can be done according to (12). Amplitude (19) can also be interpreted as the cylinder partition function for a closed string.

3.2. Long time behavior of the interaction amplitude

An interesting feature of the interaction amplitude is its behavior after sufficiently long times, i. e.,

$$\lim_{t\to\infty} \mathcal{A}.$$

In the ordinary cases (i. e., in the absence of a background tachyon), massless closed string states dominate in this regime. Here, the difference from the conventional interaction amplitudes is in the presence of the matrices Q, G_1, and G_2 and the doublet E, which are functions of time. Therefore, at large separations of the branes in the 26-dimensional spacetime, the closed string tachyon and the massless closed string states (the graviton, the dilaton, and the Kalb–Ramond field) contribute to the interaction amplitude as

$$\begin{aligned}\mathcal{A}_0 &= \lim_{t\to\infty} \mathcal{A} = \\ &= \frac{i(-1)^{(p_1+p_2)/2}\, T_{p_1} T_{p_2}}{4(2\pi)^{d_{\bar{i}}}(1+{v_1}^2)(1+{v_2}^2)} \frac{2^{d_{\bar{u}}+1/2}}{(\alpha')^{(p_1+p_2)/2}} \times \\ &\times [\det U_1 \det U_2]^{-1/2} \prod_{m=1}^{\infty} [\det R_{(m)1} \det R_{(m)2}]^{-1} \times \\ &\times \int dt \left\{ \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar{i}}} \exp\left(-\frac{1}{4\alpha' t}\sum_{\bar{i}} ({y_1}^{\bar{i}} - {y_2}^{\bar{i}})^2\right) \times \right. \\ &\times \left. \lim_{t\to\infty} \left(\frac{e^{4t}}{t^{1+(p_1+p_2)/2}} + \frac{\mathrm{Tr}(\mathcal{S}_{(1)1}\mathcal{S}^T_{(1)2}) - 2}{t^{1+(p_1+p_2)/2}} \right) \right\}, \end{aligned} \quad (23)$$

where $d_{\bar{u}}$ is the dimension of the common worldvolume of the branes. The limit of the exponential and its prefactor in (23) with respect to t is not important for us because they are related to the position of the branes, while the closed string states are independent of these positions. The divergent part in the last line (the first term) corresponds to the tachyonic closed string state. The analog of this divergent term in the absence of the background tachyon field lacks the decelerating coefficient $1/t^{1+(p_1+p_2)/2}$ and is usually omitted in the literature. It is a deficiency of the bosonic string theory, which is to be componented in superstring theory. But the point is that here the time dependence in the denominator slows down this divergence. The other term is related to the contribution of massless states, which also differs from the conventional case, due to the presence of the decelerating factor that makes it rapidly tend to zero in the limit of long time.

There is a remarkable interpretation for this behavior. Taking the open string tachyon into account as a background field means working with unstable D-branes. The consequence of this instability is the tachyon rolling as the system evolves; after a long time, most of the energy that was localized in the tachyon field transfers to the bulk. This is the consequence of decaying of the unstable D-branes into the bulk modes [18]. Therefore, in this picture, the long-time interaction of the D-branes (due to the massless closed string exchange) tends to zero. In other words, after a long enough time, there are no D-branes to interact. The exchange of the closed string tachyon, which is present as a divergent term, also has been moderated in this picture. Although this term tends to infinity anyway, its growth rate is related to the dimension of the branes. Therefore, apart from the tachyonic term that tends to infinity, we can say that the exchange of the massless closed string states causes the D-branes to interact, but their contribution decreases in time due to the instability of the D-branes.

The damping of the interaction amplitude with time depends on the brane dimensions. An interesting exception is a D-instanton. When two D-instantons interact with each other, the factor $1/t^{1+(p_1+p_2)/2}$ reduces to 1 and hence the ordinary long-time amplitude associated with the massless states is restored. In addition, the usual divergent term is related to the tachyonic closed string state. We can therefore say that the general interactive behavior of the D-instantons is unchanged in the presence of an open string background tachyon field.

4. CONCLUSIONS AND SUMMARY

We obtained the boundary state of a closed string, emitted from or absorbed by moving Dp-branes in the presence of the background tachyon field.

The relation between the boundary state and the disk partition function was discussed. The effect of the tachyon condensation on the partition function was studied, which shows a spectacular difference from the conventional condensation. Condensation of the tachyon matrix components along any spatial worldvolume directions leads to a partition function corresponding to a lower dimensional moving D-brane with an effective tension that depends on the condensated components of the tachyon field. However, condensation of U_{00} eliminates velocity and also leads to an instantonic D-brane, which is fixed in time. After complete condensation of the tachyon field, a D-instanton is obtained.

The interaction amplitude between two D-branes with arbitrary dimensions p_1 and p_2 has been calculated. Our calculations are valid for the systems of branes that are parallel or perpendicular to each other. The interaction strength between the branes depends on the brane dimensions, their tensions, their relative configuration, the closed string mode numbers, the tachyon matrices, and the velocities of the branes.

As a special case, in the large-distance interaction of the branes, the contribution of the massless states tends to zero and the divergent part related to the closed string tachyon state considerably slows down. Therefore, the statement that the force associated with the massless states is long range would be valid as long as there is no background tachyon field in the system. This unconventional behavior may be ascribed to the rolling of the tachyon field toward a minimum of its potential. This leads to a closed string vacuum without any D-brane at the end of the process and causes the concept of the interaction of the D-branes to faint. An interesting point is that in the case of the D-instanton interaction, this descension of the long time amplitude jumps to the usual case.

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