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ENGINEERING OF SCHRÖDINGER CAT STATES BY A SEQUENCE OF DISPLACEMENTS AND PHOTON ADDITIONS OR SUBTRACTIONS

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A method to generate Schrödinger cat states in free propagating optical fields based on the use of displaced states (or displacement operators) is developed. Some optical schemes with photon-added coherent states are studied. The schemes are modifications of the general method based on a sequence of displacements and photon additions or subtractions adjusted to generate Schrödinger cat states of a larger size. The effects of detection inefficiency are taken into account.

1. INTRODUCTION

It is widely recognized that nonclassical states of light are a valuable resource for numerous applications. It is often desirable to generate nonclassical states in traveling optical modes, as opposed to ones in experiments in a cavity, where the generated state can be probed only indirectly. Many ingenious schemes have been proposed and experimentally demonstrated to generate single-photon states [1] and various multiphoton entangled states such as the Greenberger–Horne–Zeilinger (GHZ) states [2], cluster states [3], and the so-called NOON ($|\mathrm{N}\rangle_1|\mathrm{O}\rangle_2 \pm |\mathrm{O}\rangle_1|\mathrm{N}\rangle_2$) states [4]. Considerable attention has also been devoted to continuous-variable states. It is well known that if the postulates of quantum mechanics are applied to macroscopic systems, then there must exist, for example, a linear superposition of dead-cat and alive-cat states (the Schrödinger cat paradox) [5]. It is not very easy to actually imagine a superposition of alive and dead cats. Therefore, a search for approximations of the object that Schrödinger had in view looks logical. Quantum mechanics allows two coherent states with the amplitudes equal in modulus but with different sign to be in a superposition state [6], which is considered an implementation of the Schrödinger cat paradox [5] for optics or the same coherent state (CS) superposition (Schrödinger-cat-like states, or simply cat states). It is well known that two CSs with opposite amplitudes may give macroscopically distinguishable outcomes by a homodyne measurement [7], especially if their amplitudes are well separated. The CS superpositions show nonclassical properties such as interference patterns in phase space and negative values of the Wigner functions [8].

The two-mode version of CS superposition, namely, an entangled CS, is a valuable resource of entanglement [9–15]. Once a single-mode CS superposition is produced, it is relatively easy to generate an entangled CS by passing the CS superposition through a balanced beam splitter (another input port of the beam splitter being empty). The power of the entangled CSs, as the resource of entanglement for quantum information processing with CSs, lies in the fact that all the four Bell entangled CSs can be discriminated in a deterministic way only using a beam splitter and photon counting [10–13], which is obviously not the case for the single-photon-based approach. Even though there is a nonzero failure probability in distinguishing Bell entangled CSs due to a nonzero overlap,

$$P_f = \frac{2\exp(-2|\alpha|^2)}{1+\exp(-4|\alpha|^2)},$$

where α is the amplitude or the size of the entangled CS, the failure probability P_f is small for an appropriate choice of α and the failure event is known from the result whenever it occurs. For example, the failu-

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re probability P_f is approximately 28 % for $\alpha = 0.8$, near 14 % for $\alpha = 1$, and only of the order of 10^{-4} for $\alpha = 2$. Thus, the entangled CS or CS superposition of larger amplitudes is interesting both from the fundamental standpoint (the Schrödinger cat paradox) and as regards possible applications. Entangled CSs with small amplitudes $\alpha < 1$ can be used for tests of Bell inequalities by continuous variable states [16–21] using various types of measurements.

We note that there are experimental demonstrations of cat states in a cavity and in a trap [22, 23]. We also mention ideas to use trapped ions [24] and optical solitons [25] to realize the cat states. But just free-traveling cat states, whose generation is a nontrivial problem, are needed for the quantum information processing. It has been known theoretically that a cat state can be generated from a CS by a nonlinear interaction in a Kerr medium [26]. The authors of Refs. [27, 28] noted that an optical fiber about 3000 km in length is needed for an optical frequency $\omega \approx 5 \cdot 10^{14}$ rad/s to generate a CS superposition with the currently available Kerr nonlinearity using two-mode nonlinear interactions and a Mach–Zehnder interferometer. A nonlinear cell about 1500 km in length is required when single-mode interactions are used [29]. Although it is possible to make such a long optical fiber in principle, the decoherence effect during the propagation becomes too large. A signal loses half of its energy over about 15 km of propagation through a typical commercial fiber.

Currently, all proposals to generate free-traveling cat states are based on the use of squeezed vacuum and squeezed number states at different interpretations. For example, the setup in [30] requires a source of squeezed vacuum states, beam splitters, strong coherent beams, photodetectors with single-photon sensitivity, and a final squeezer. Because photon number resolving detectors are required, the scheme in [30] can be hardly realized with the current technology level. With time, the optical schemes have become more refined [31–36] and better approximating realistic experimental situation. For example, the successful observation in [31, 32] that squeezed single photons can approximate odd CS superpositions with small amplitudes and that their amplitudes can be nondeterministically amplified using realistic resources, gave rise to a whole new direction related to adding and subtracting photon(s) from a squeezed vacuum. Consequently, there is a large number of theoretical ideas [37–40] based on single-photon-subtracted squeezed states, which are close to the CS superpositions with small amplitudes, and their experimental realization [41–45]. For example, a squeezed CS superposition in a free propagating field with a limited amplitude ($\alpha \approx 1.6$) and small fidelity (near 50 %) with CS superposition (not squeezed) was experimentally realized based on the idea of homodyne conditioning on the photon number state [45]. The result in [45] is considered the best experimental achievement in the area. Another remarkable experimental result by subtracting up to three photons from a squeezed vacuum was recently presented in [46].

The problem of generating a free-traveling CS superposition of large amplitudes $\alpha \geq 2$ needed for quantum information processing remains open and challenging. In this paper, we develop a method based on displacements, photon additions, and subtractions to generate CS superpositions of larger amplitudes. Our motivation, in particular, is inspired by the proposal in [47], where it was shown that an arbitrary single-mode state can be engineered starting from the vacuum by applying a sequence of displacements and single-photon additions. Another reason to consider the problem is to relate displaced photon number states and CS superpositions. We analyze several optical schemes. One of them involves pairs of photon-added CSs to combine them on a beam splitter to generate truncated versions of CS superpositions. Another treatment consists in a sequence of displacements, photon additions and subtractions. This approach allows introducing a displaced version of CS superpositions, which are different from squeezed CS superpositions [45]. The analysis is done in terms of the photon number states.

2. GENERATION OF A CS SUPERPOSITION WITH THE HELP OF DISPLACED PHOTON-ADDED CSs

We introduce the following definitions to be used throughout the paper. We define a CS superposition (CSS) (or, equivalently, the Schrödinger cat state) as

$$|\mathrm{CSS}_\varphi(\alpha)\rangle = N_\varphi(\alpha)\left(|0,-\alpha\rangle + e^{i\varphi}|0,\alpha\rangle\right) \qquad (1)$$

with the normalization factor

$$N_\varphi(\alpha) = 1/\sqrt{2\left[1+\cos\varphi\exp(-2|\alpha|^2)\right]},$$

where $|0,\alpha\rangle = \hat{D}(\alpha)|0\rangle$ is a CS with amplitude α ($|n,\alpha\rangle = \hat{D}(\alpha)|n\rangle$ is a displaced n-photon-number state) with $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ being the displacement operator, a ($a^\dagger$) the bosonic annihilation (creation) operator, and $|n\rangle$ a photon number state. We refer to α as the size or the amplitude of the CS superposition. Because $|\mathrm{CSS}_+(\alpha)\rangle = |\mathrm{CSS}_{\varphi=0}(\alpha)\rangle$ ($|\mathrm{CSS}_-(\alpha)\rangle = |\mathrm{CSS}_{\varphi=\pi}(\alpha)\rangle$) contains an even (odd)

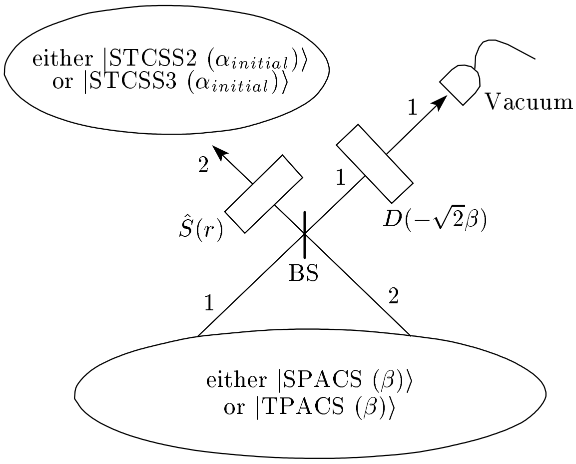

Fig. 1. Schematic representation of the conditional preparation of $|\mathrm{STCSS2}(\alpha_{initial})\rangle$ and $|\mathrm{STCSS3}(\alpha_{initial})\rangle$ states with the help of displaced-photon-added CSs: BS — beam splitter; 1 and 2 are the spatial modes

number of photons, such states are called even (odd) CS superpositions.

We use the optical scheme in Fig. 1, and assume that we have a pair of normalized single-photon-added CSs (SPACS)

$$|\mathrm{SPACS}(\beta)\rangle = N_{\mathrm{SPACS}}(\beta)a^\dagger|0,\beta\rangle = \\ = \frac{|1,\beta\rangle + \beta^*|0,\beta\rangle}{\sqrt{1+|\beta|^2}}, \quad (2)$$

where β is the amplitude of such CSs and N_{SPACS} is a normalization factor. The single-photon-added CS is conditionally generated by using a nondegenerate parametric amplifier described by the interaction Hamiltonian

$$H = i\hbar\chi(a_1^\dagger a_2^\dagger - a_1 a_2),$$

where the coupling coefficient χ is related to the nonlinear second-order susceptibility tensor $\chi^{(2)}$. For the input $|\Psi_{in}\rangle = |0,\beta\rangle_1|0\rangle_2$ to the Hamiltonian, we have the output

$$|\Psi_{out}\rangle = \exp(-iHt/\hbar)|\Psi_{in}\rangle \approx \\ \approx \left[1 + g\left(a_1^\dagger a_2^\dagger - a_1 a_2\right)\right]|\Psi_{in}\rangle$$

in the case of a low parametric gain $g \ll 1$, where $g = \chi t$ (t is the interaction time), if a single photon is registered in the second output mode [48]. A beam splitter with the unitary matrix

$$B = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

is used in Fig. 1 to combine two states (2).

A state of a subsystem of a correlated bipartite system "collapses" into a particular state if another subsystem is measured and the result of the measurement is known. We need to distinguish between the displaced-vacuum, the single-photon, and two-photon states to produce the desired state. For this, we use a displacement operator $D\left(-\sqrt{2}\beta\right)$ in the first mode, succeeded by a detector being on/off observable (Fig. 1). A beam splitter can be described by the displacement operator $D\left(-\sqrt{2}\beta\right)$, where $-\sqrt{2}\beta = \xi\sqrt{1-T^2}$, for high transmittivity $T \to 1$ and a strong ancillary coherent field $|0,\xi\rangle$ with an amplitude ξ. To be more accurate, the density matrix is generated in this case, and the density matrix is well approximated by the input wave function shifted by some value. A pure truncated CS superposition (TCSS) containing two terms,

$$|\mathrm{TCSS2}(\alpha_{initial})\rangle = \frac{|0\rangle + \alpha_{initial}^2|2\rangle/\sqrt{2}}{\sqrt{1+|\alpha_{initial}|^2/2}}, \quad (3)$$

where $\alpha_{initial}^2 = -1/\beta^{*2}$ and β is purely imaginary, $\beta = i|\beta|$, is then generated if the detector registers nothing (the vacuum) in the case of the unit detector efficiency $\eta = 1$. The detector efficiency is less than 1 in real experiments, which gives rise to an ensemble of quantum states described by some density matrix ρ. The fidelity can serve as a measure of the distance between quantum states. The fidelity is defined by [49]

$$F = \sqrt{\langle\mathrm{CSS}_+(\alpha_{initial})|\rho|\mathrm{CSS}_+(\alpha_{initial})\rangle}.$$

Figure 2*a* shows the fidelity between an output state ρ and an even CS superposition as a function of $\alpha_{initial}$ for different detector efficiencies η.

We now extend the treatment above to involve a pair of two-photon-added CSs (TPACS) defined as

$$|\mathrm{TPACS}(\beta)\rangle = f(\beta)a^{\dagger 2}|0,\beta\rangle = \\ = \frac{\beta^{*2}|0,\beta\rangle + 2\beta^*|1,\beta\rangle + \sqrt{2}\,|2,\beta\rangle}{\sqrt{2+4|\beta|^2+|\beta|^4}}. \quad (4)$$

It can be shown that the output of the beam splitter in Fig. 1 is a truncated CS superposition that contains three terms $|\mathrm{TCSS3}(\alpha_{initial})\rangle$, where $\alpha_{initial}^2 = -2/\beta^{*2}$ and $\beta = i|\beta|$. The corresponding fidelity of the generated $|\mathrm{TCSS3}(\alpha_{initial})\rangle$ state with an even CS superposition is plotted in Fig. 2*b*. The corresponding success probabilities to generate $|\mathrm{TCSS2}(\alpha_{initial})\rangle$ and $|\mathrm{TCSS3}(\alpha_{initial})\rangle$ states are plotted in Fig. 3 for different detection efficiencies. A typical method to increase

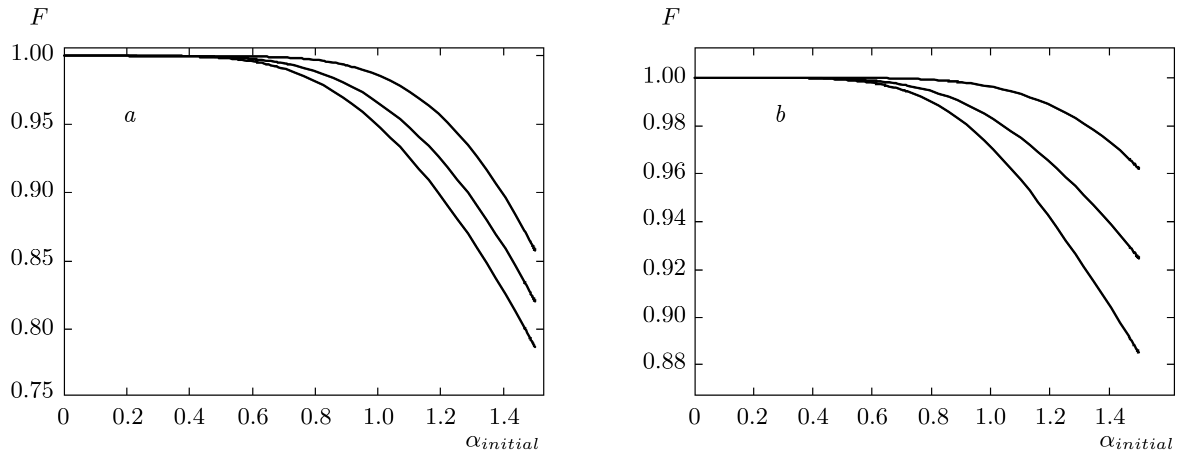

Fig. 2. The fidelity between the even CS superposition and $|\mathrm{TCSS2}\rangle$ (a) and $|\mathrm{TCSS3}\rangle$ (b) states versus the amplitude. The curves (from top down) are obtained for the detector efficiencies $\eta = 1$ (the ideal case), $\eta = 0.9$, and $\eta = 0.8$

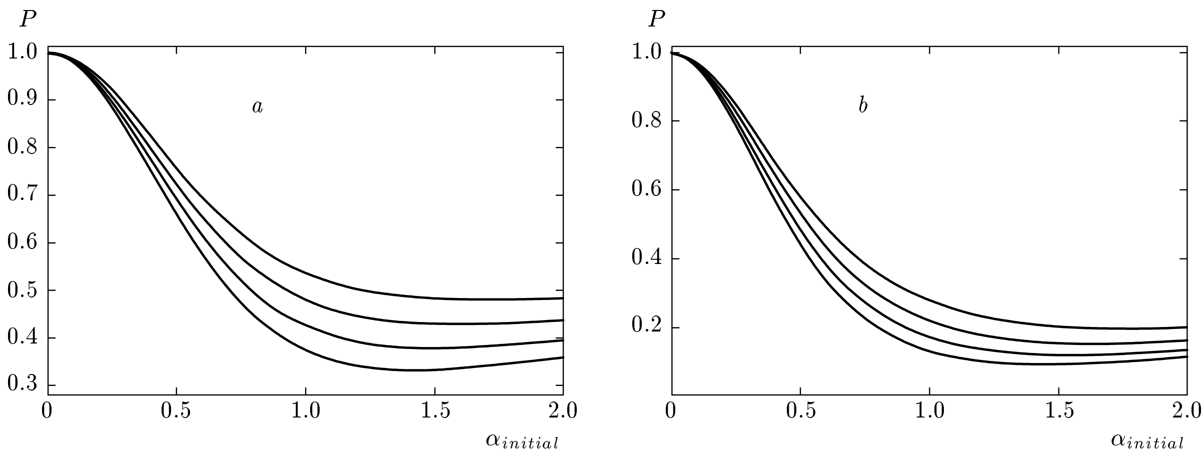

Fig. 3. The success probabilities to produce $|\mathrm{TCSS2}\rangle$ (a) and $|\mathrm{TCSS3}\rangle$ (b) states versus the amplitude. The curves (from top down) are obtained for the detector efficiencies $\eta = 1$ (the ideal case), $\eta = 0.9$, $\eta = 0.8$, and $\eta = 0.7$

the size of the generated CS superposition [45] is to use the single-mode squeezing operator given by

$$\hat{S}(r) = \exp\left[\frac{r}{2}(a^{\dagger 2} - a^2)\right], \tag{5}$$

where r is the squeezing parameter. This operator reduces quantum noise of a vacuum state in the phase quadrature by the factor $\exp(-2r)$, while increasing quantum noise by the factor $\exp(2r)$ in the amplitude quadrature. Figure 1 shows the squeezing operator applied to either the post-selected $|\mathrm{TCSS2}(\alpha_{initial})\rangle$ or the $|\mathrm{TCSS3}(\alpha_{initial})\rangle$ state. The resultant state, which is a squeezed truncated CS superposition (STCSS2), can be expanded in terms of photon-number states as

$$\begin{aligned}
&|\mathrm{STCSS2}(\alpha_{initial})\rangle = \hat{S}(r)|\mathrm{TCSS2}(\alpha_{initial})\rangle = \\
&= \frac{1}{\sqrt{1+|\alpha_{initial}|^4/2}} \times \\
&\times \Bigg\{ \left(\frac{1}{\sqrt{\operatorname{ch} r}} - \frac{\alpha_{initial}^2}{\sqrt{2}}\,\frac{\operatorname{th} r}{\sqrt{2\operatorname{ch} r}}\right)|0\rangle + \\
&+ \sum_{n=0}^{\infty}\left[\frac{\sqrt{2(n+1)!}}{2^{n+1}(n+1)!}\,\frac{\operatorname{th}^{n+1} r}{\sqrt{\operatorname{ch} r}}\right] + \\
&+ \frac{\alpha_{initial}^2}{\sqrt{2}}\left(\frac{\sqrt{2(n+1)!}}{n!2^n\sqrt{2}}\,\frac{\operatorname{th}^n r}{\sqrt{\operatorname{ch}^5 r}} - \right. \\
&\left. - \frac{\sqrt{2(n+1)!}}{2^{n+1}(n+1)!}\,\frac{\operatorname{th}^{n+2} r}{\sqrt{2\operatorname{ch} r}}\right)\Bigg\}|2(n+1)\rangle.
\end{aligned} \tag{6}$$

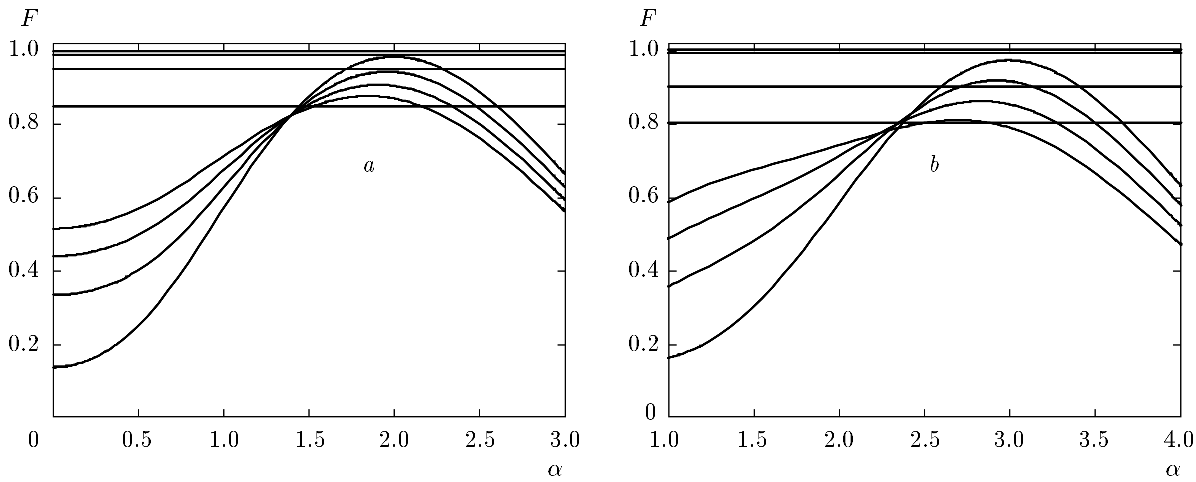

Fig. 4. An example of the amplifying action of the squeezing operator on the conditional state in Fig. 1. From top down: (a) the fidelity between $|\mathrm{STCSS2}(\alpha_{initial} = 1.6)\rangle$ and (b) $|\mathrm{STCSS3}(\alpha_{initial} = 1.8)\rangle$ states with an even $|\mathrm{CSS}(\alpha)\rangle$ state versus the amplitude with the amplifying squeezing parameter $r = 0.6$ for the detector efficiencies $\eta = 1$ (the ideal case), $\eta = 0.9$, $\eta = 0.8$, and $\eta = 0.7$. The horizontal lines are for eye guide

State (6) contains only even photon numbers and has the coefficients decaying exponentially as n increases. The fidelity

$$F = F(\alpha, r, \alpha_{initial}) = = \sqrt{\langle \mathrm{CSS}_+(\alpha)|\rho(r,\alpha_{initial})|\mathrm{CSS}_+(\alpha)\rangle}$$

of state (6) with an even $|\mathrm{CSS}(\alpha)\rangle$ state, where $\rho(r,\alpha_{initial})$ is the output density matrix, is calculated numerically. Some sample values for perfect detection are

$$F(\alpha = 1.5, r = 0.3763, \alpha_{initial} = 1.2648) = 0.9979$$

and

$$F(\alpha = 1.6, r = 0.418984, \alpha_{initial} = 1.32753) = = 0.99653.$$

Detection imperfections essentially affect the output state by decreasing its fidelity. The corresponding results are given in Fig. 4, which show that highly efficient detectors are required for our purpose. If the detection efficiency is 90 % [50], which is a reasonable value for avalanche photodetectors, then we can achieve $F > 0.95$. Currently, highly efficient detectors have relatively high dark-count rates, while less efficient detectors have very low dark-count rates. Therefore, highly efficient detectors [50] are preferable for generation of the truncated CS superpositions, because click events are discarded in Fig. 1. We assumed that the initial prearranged states, either $|\mathrm{SPACS}(\beta)\rangle$ or $|\mathrm{TPACS}(\beta)\rangle$, are pure. Current technology (down converters) cannot produce pure single-photon-added CSs; the states always involve an admixture of the vacuum,

$$\rho_1 = p|\mathrm{SPACS}(\beta)\rangle\langle\mathrm{SPACS}(\beta)| + (1-p)|0\rangle\langle 0|,$$

where p is the state production efficiency. Therefore, silicon avalanche photodiodes operating at visible wavelengths, with a relatively high efficiency and small dark count rate have, to be used to initially create either $|\mathrm{SPACS}(\beta)\rangle$ or $|\mathrm{TPACS}(\beta)\rangle$ state. Even if the detector misses an incoming photon, the discarded event influences only probability distributions. In practice, the input density matrix $\rho_1 \otimes \rho_2$ (Fig. 1) has three terms: the term with p^2 is the desired one, but $p(1-p)$ and $(1-p)^2$ are undesired error terms. It can be shown that the influence of the undesired terms can be decreased (purification effect) in Fig. 1. We show this in the example of the term proportional to $(1-p)^2$. After the displacement $D\left(-\sqrt{2}\,\beta\right)$ in Fig. 1, the state proportional to $(1-p)^2$ is $|0, -\sqrt{2}\,\beta\rangle\langle 0, -\sqrt{2}\,\beta|$ and the probability to register the vacuum for the state is $\exp(-2|\beta|^2) < 1$, which reduces the contribution of the term proportional to $(1-p)^2$.

3. SCHRÖDINGER CAT STATES AS A SEQUENCE OF DISPLACEMENTS AND PHOTON ADDITIONS AND SUBTRACTIONS

We conclude from the foregoing that the method developed above is very sensitive to detector imperfections. Therefore, we use the idea with displaced-photon-number states and give it an interpretation different from that in Fig. 1. In this case, the CS superposition engineering starts with a coherent state $|0, \alpha_1\rangle$ subject to a combination of displacements and photon additions as (with the normalization factor omitted)

$$\begin{aligned} & a^\dagger D(-2\alpha_1) a^\dagger |0, \alpha_1\rangle = \\ & = a^\dagger D(-2\alpha_1) D(\alpha_1) D^\dagger(\alpha_1) a^\dagger D(\alpha_1)|0\rangle = \\ & = a^\dagger D(-\alpha_1)(a^\dagger + \alpha_1^*)|0\rangle = \\ & = D(-\alpha_1) D^\dagger(-\alpha_1) a^\dagger D(-\alpha_1)(a^\dagger + \alpha_1^*)|0\rangle = \\ & = D(-\alpha_1)(a^\dagger - \alpha_1^*)(a^\dagger + \alpha_1^*)|0\rangle = \\ & = D(-\alpha_1)(a^{\dagger 2} - \alpha_1^{*2})|0\rangle = \\ & = D(-\alpha_1)\left(\sqrt{2}|2\rangle - a_1^{*2}|0\rangle\right) = \\ & = D(-\alpha_1)|\alpha_1|^2\left(|0\rangle + \alpha_{initial}^2|2\rangle/\sqrt{2}\right), \end{aligned} \quad (7)$$

where $\alpha_1 = i|\alpha_1|$ and $\alpha_{initial}^2 = 2/|\alpha_1|^2$. A possible extension is given by

$$\begin{aligned} & a^\dagger D(-2\alpha_2) a^\dagger D(\alpha_1+\alpha_2) a^\dagger D(-2\alpha_1) a^\dagger |0, \alpha_1\rangle = \\ & = a^\dagger D(-2\alpha_2) a^\dagger D(\alpha_1+\alpha_2) D(-\alpha_1)(a^{\dagger 2} - \alpha_1^{*2})|0\rangle = \\ & = D(-\alpha_2)(a^{\dagger 2} - \alpha_2^{*2})(a^{\dagger 2} - \alpha_1^{*2})|0\rangle = \\ & = D(-\alpha_2)\left(\sqrt{4!}|4\rangle - \sqrt{2}(\alpha_1^{*2}+\alpha_2^{*2})|2\rangle + \alpha_1^{*2}\alpha_2^{*2}|0\rangle\right) = \\ & = |\alpha_1|^2|\alpha_2|^2 D(-\alpha_2) \times \\ & \times \left(|0\rangle + \alpha_{initial}^2|2\rangle/\sqrt{2!} + \alpha_{initial}^4|4\rangle/\sqrt{4!}\right), \end{aligned} \quad (8)$$

where

$$\alpha_1 = i|\alpha_1|, \quad \alpha_2 = i|\alpha_2|,$$

$$\alpha_{initial}^2 = 2\left(3 \pm \sqrt{3}\right)/|\alpha_2|^2,$$

$$\alpha_{initial}^4 = 24\left(2 \pm \sqrt{3}\right)/|\alpha_2|^4.$$

States (7) and (8) are simply the displaced versions of a truncated even CS superposition, respectively involving the two first and the three first terms. A parametric down-converter produces twin photons in two different modes. A photon counting at one of the modes heralds that a photon has been added ($a^\dagger$) to the input field that passes through the down-converter. The displacement operator with some amplitude is based on the beam splitter with transmittivity close to 1 and a large-amplitude ancillary coherent field as described above. Thus, successive two and four clicks in ancillary modes of two down-converters result in respective states (7) and (8). The same idea can be used to generate an odd CS superposition if a displaced single-photon input state [1] is used. Indeed, we have

$$\begin{aligned} & a^\dagger D(-2\alpha_1) a^\dagger |1, \alpha_1\rangle = D(-\alpha_1)(a^\dagger - \alpha_1^*)(a^\dagger + \alpha_1^*)|1\rangle = \\ & = -\alpha_1^{*2}\left(|1\rangle + \frac{3!|3\rangle}{|\alpha_1|^2\sqrt{3!}}\right) = \\ & = -\alpha_1^{*2}\left(|1\rangle + \frac{\alpha_{initial}^3|3\rangle}{\sqrt{3!}}\right), \end{aligned} \quad (9)$$

if $\alpha_1 = i|\alpha_1|$ and $\alpha_{initial}^2 = 6/|\alpha_1|^2$. The corresponding extension is given by

$$\begin{aligned} & a^\dagger D(-2\alpha_2) a^\dagger D(\alpha_1+\alpha_2) a^\dagger D(-2\alpha_1) a^\dagger |1, \alpha_1\rangle = \\ & = D(-\alpha_2) D^\dagger(-\alpha_2) a^\dagger D(-\alpha_2) D^\dagger(\alpha_2) a^\dagger D(\alpha_2) e^{i\psi} = \\ & = D^\dagger(-\alpha_1) a^\dagger D(-\alpha_1) D^\dagger(\alpha_1) a^\dagger D(\alpha_1)|1\rangle = \\ & = e^{i\psi} D(-\alpha_2)(a^{\dagger 2} - \alpha_2^{*2})(a^{\dagger 2} - \alpha_1^{*2})|1\rangle = \\ & = e^{i\psi}\alpha_1^{*2}\alpha_2^{*2} D(-\alpha_2) \times \\ & \times \left(|1\rangle - \frac{3!(a_1^{*2} + \alpha_2^{*2})|3\rangle}{\alpha_1^{*2}\alpha_2^{*2}\sqrt{3!}} + \frac{5!|5\rangle}{\alpha_1^{*2}\alpha_2^{*2}\sqrt{5!}}\right) = \\ & = e^{i\psi}\alpha_1^{*2}\alpha_2^{*2} D(-\alpha_2) \times \\ & \times \left(|1\rangle \frac{\alpha_{initial}^2|3\rangle}{\sqrt{3!}} + \frac{\alpha_{initial}^4|5\rangle}{\sqrt{5!}}\right), \end{aligned} \quad (10)$$

where

$$x = |y|e^{i\varphi}\frac{2 \pm i\sqrt{5}}{3}, \quad x = \frac{1}{\alpha_1^{*2}}, \quad y = |y|e^{i\varphi} = \frac{1}{\alpha_2^{*2}},$$

$$\alpha_{initial}^2 = -2|y|e^{i\varphi}\left(5 \pm i\sqrt{5}\right),$$

$$\alpha_{initial}^4 = 120|y|^2, \quad e^{i\varphi} = -\frac{5 \mp i\sqrt{5}}{\sqrt{30}},$$

$$e^{2i\varphi} = \frac{2 \mp i\sqrt{5}}{3},$$

and ψ is the total phase shift. The generated states (9) and (10) are the displaced truncated odd CS superpositions with two and three terms, respectively.

Hence, two additions and one displacement are required to generate either a displaced even or a displaced odd CS superposition (Eqs. (7) and (9)), while four additions and three displacements are used to generate states (8) and (10). Displaced versions of the CS superposition are analogs of the squeezed CS superposition considered in the preceding section. The method is less sensitive to detector imperfections. Indeed, detection with a low dark-count rate described above can be used to decrease the influence of dark counts. The probability to generate down-converted twin photons prevails over the probability to generate higher-order correlated photons, which decreases the effect of imperfections on the output state. We note that a possible complication of the optical scheme for generation of a displaced CS superposition (DCSS) may be overcome if the corresponding inputs given by a superposition of displaced-photon-number states are used. We consider this in the example of application of the subtraction operator to the displaced state

$$aN\left[a\left(|0,\alpha_1\rangle+\frac{\alpha^2|2,\alpha_1\rangle}{\sqrt{2!}}+\frac{\alpha^4|4,\alpha_1\rangle}{\sqrt{4!}}\right)+\right.$$
$$\left.+\,b\left(\alpha|1,\alpha_1\rangle+\frac{\alpha^3|3,\alpha_1\rangle}{\sqrt{3!}}\right)\right]=ND(\alpha_1)(a+\alpha_1)\times$$
$$\times\left[a\left(|0,\alpha_1\rangle+\frac{\alpha^2|2,\alpha_1\rangle}{\sqrt{2!}}+\frac{\alpha^4|4,\alpha_1\rangle}{\sqrt{4!}}\right)+\right.$$
$$\left.+\,b\left(\alpha|1,\alpha_1\rangle+\frac{\alpha^3|3,\alpha_1\rangle}{\sqrt{3!}}\right)\right]=-ND(\alpha_1)\frac{\alpha(a^2-b^2)}{b}\times$$
$$\times\left(|0\rangle+\frac{\alpha^2|2\rangle}{\sqrt{2}}+\frac{a^2}{a^2-b^2}\frac{\alpha^4|4\rangle}{\sqrt{4!}}\right),\qquad(11)$$

where $\alpha_1=-\alpha a/b$ and N is a normalization factor. We again produce a displaced truncated even CS superposition (with three terms (8)) if $a\gg b$ by application of only one subtraction operator, which can be conditionally done by a beam splitter with high transmittivity that reflects only a negligible part of the incident beam. We note that only if we choose $\alpha_1=-\alpha b/a$ can we generate a displaced odd CS superposition (9) under the condition $a\gg b$. Properties of displaced CS superpositions deserve separate consideration. We only note that even truncated versions of CS superpositions (if we apply the displaced operator with the opposite amplitude) may be useful in various applications (hence, Fig. 5 shows the fidelity of CS superpositions with states (7)–(11) without displacement).

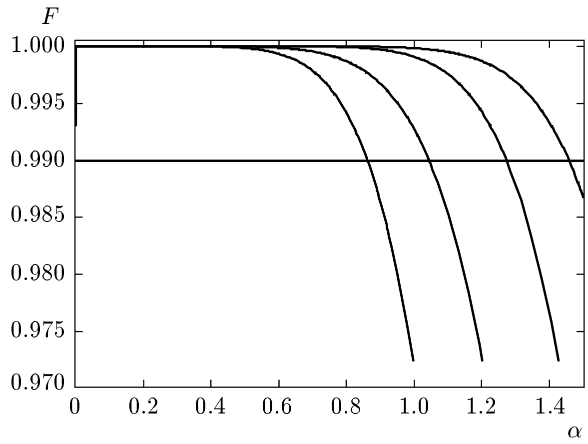

Fig. 5. The fidelity between nonsqueezed and nondisplaced states (7), (9), (8), and (10) (from left to right) and even and odd CS superpositions. Nondisplaced versions of CS superpositions (7)–(11) approximate small-size CS superpositions well

4. CONCLUSION

The interest in continuous variable states and, in particular, in Schrödinger cat states only increases. In particular, it is related to a possible application of the free-propagating Schrödinger cat states to quantum computing [11, 13]. Squeezed Schrödinger cat states and methods of their generation based on different modifications of additions and subtractions from a squeezed vacuum (or squeezed photon number states) are well studied and understood [30–46]. We were interested in relating Schrödinger cat states to displaced (not squeezed) photon-number states. As a consequence, we began our analysis with either a CS or displaced photon-number CSs. We also introduced the notion of displaced Schrödinger cat states by analogy with squeezed Schrödinger cat states [45]. Some set of different optical schemes to connect displaced states with a free-propagating CS superposition is considered and analyzed. The scheme in Fig. 1 may be useful for generation of even CS superpositions of a small size up to $\alpha\approx1.2$ (Fig. 2) [45] even without squeezing amplification (Fig. 4) to avoid complicating the scheme. The calculations for the scheme in Fig. 1 were done taking detection imperfections into account that substantially deteriorate the quality of the generated CS superposition due to generation of a mixture of states at the output.

To avoid the problem with detection imperfections, we considered other possible optical schemes, which consist of some sequences of photon additions and displacements. This allows realizing displaced Schrödinger cat states (both even and odd) of a larger size at the cost of complications of the optical schemes. Another possibility to produce larger-size CS superpositions is to use the distillation procedure (Eq. (11)) with one subtraction operator if the input is the corresponding wave function. We note that the treatment in Sec. 2 is simply some modification of the methods of photon additions. We can see from Fig. 5 that the generated CS superposition of a larger size $\alpha>1.2$ is produced and we do not need to address squeezing amplification. In contrast to previous schemes, our method requires a $\chi^{(2)}$ nonlinearity only for photon additions. Displacement can be performed only using a beam splitter. The treatment with displaced states (or displacement operators) deserves further consideration to adjust it to realistic cases and presents best recommendations for experimentalists interested in generation of larger-size CS superpositions.

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