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FIELD-LIKE SPIN-TRANSFER TORQUE IN A CHIRAL HELIMAGNET

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Based on the microscopic model of sd coupling between free electrons and local moments, we present a quantum calculation of a nonadiabatic spin-transfer torque in a chiral helimagnet.

1. INTRODUCTION

The mutual influence of charge transport and spin dynamics is a central issue of spintronics studies. A topical issue is the current-driven manipulation of magnetization based on the spin-transfer torque (STT) mechanism, when the magnetization dynamics is controlled by spin-polarized conduction electrons via the spin angular momentum transfer [1]. Recently, considerable interest arose in the current-driven magnetization dynamics of complex magnetic textures, such as higher-dimensional domain-wall models [2], spin spirals [3], and helical spin-density waves [4]. In contrast to layered systems, the STT in the last two systems is a bulk effect.

Theoretical studies of the STT either use the modified classical Landau–Lifshitz–Gilbert equations [5] or use the generalized charge transport theory to account for spin currents and spin relaxation [6], or are based on a microscopic derivation of the STT terms in the framework of the sd model [7]. It is now well established that the STT consists of two components: the adiabatic in-plane part (Slonczewski torque) and the nonadiabatic perpendicular part (the field-like torque). The first constituent describes the adiabatic process of nonequilibrium conduction electrons. The field-like torque is related to the spatial mistracking of spins between conduction electrons and the local magnetization. Such a division has been confirmed by recent current-switching and spin-transfer-driven ferromagnetic resonance measurements in magnetic tunnel junctions [8].

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In this paper, we present microscopic calculations of the nonadiabatic STT in a chiral helimagnet, where the incommensurate magnetic order is supported by the antisymmetric Dzyaloshinskii–Morya (DM) exchange. A specific ground state of the system known as the soliton lattice (SL) [9], which is realized in an external magnetic field applied perpendicularly to the helical axis, is of special interest. Recently, we showed that the SL involves an intrinsic sliding mode, which may be detectable experimentally [10]. Apparently, the STT may serve as a possible mechanism of the SL movement.

The microscopic calculations of the STT are based on the simple sd model, which captures most of the physics of the interplay between the spin-polarized transport of itinerant electrons and the magnetization dynamics of local moments [11]. The electrons carrying a current and coupled to local moments are treated within the nonequillibrium (Keldysh) Green's function formalism [12]. For simplicity, we consider an effective one-dimensional model, assuming that electrons travel along the chiral z axis of the soliton lattice. The model implies a ferromagnetic order of local moments in the perpendicular xy plane, and the electron hopping along the chiral axis is much greater than the hopping in perpendicular directions.

2. CONDUCTION ELECTRONS

The Hamiltonian of free electrons is

$$\mathcal{H}_0 = \frac{t}{2}\sum_{i\sigma}\left(c^{\dagger}_{i+1\,\sigma}c_{i\,\sigma} + c^{\dagger}_{i\sigma}c_{i+1\sigma}\right) - \mu\sum_{i\sigma}c^{\dagger}_{i\sigma}c_{i\sigma}, \quad (1)$$

where t is the hopping integral between the ith and $(i+1)$th sites located along the z axis, $c^\dagger_{i\sigma}$ ($c_{i\sigma}$) is the creation (annihilation) operator of the ith-site electron with the spin quantum number σ, and μ denotes the chemical potential.

The sd interaction with the exchange coupling J between the itinerant electron spins $\mathbf{s}_i$ and the localized moments $\mathbf{S}_i$ is

$$H_{sd} = -J\sum_i \mathbf{S}_i\cdot\mathbf{s}_i = -\frac{J}{2}\sum_{i\sigma\sigma'}(\mathbf{S}_i\cdot\boldsymbol{\sigma})_{\sigma\sigma'}\,c^\dagger_{i\sigma}c_{i\sigma'}, \quad (2)$$

where $\boldsymbol{\sigma}$ is the vector of Pauli matrices.

The dynamics of magnetization is slow compared to that of conduction electrons, and the spin of itinerant carriers approximately follows the direction of the local moment. To describe the electron transport in such a slowly varying magnetization field, it is convenient to use a local frame of the electron.

The electron operators in the new frame are introduced via the local gauge transformation $a_i = \hat{U}_i c_i$, $a^\dagger_i = c^\dagger_i \hat{U}^{-1}_i$, where the rotation matrix

$$\hat{U}_i = \cos(Qz_i/2) + i\hat{\sigma}^z\sin(Qz_i/2)$$

at the position z_i depends on the wave number (Floquet index) of the soliton lattice. For simplicity, we consider the regime of a weak external magnetic field, when $Qa \approx D/\mathcal{J}$, where a is the lattice constant, D is the DM coupling strength, and $\mathcal{J}$ is the symmetric exchange integral inside the SL (see, e. g., Ref. [10]). In this regime, the texture of localized moments is well approximated by $\mathbf{n}_0 \approx (\cos(Qz), \sin(Qz), 0)$. The approximation becomes exact at zero magnetic field, where the case of a left-handed ($Q<0$) or right-handed ($Q>0$) spiral is realized.

After the local gauge transformation, the kinetic energy term transforms into

$$\mathcal{H}_0 = \sum_{k\sigma}(\varepsilon_{k\sigma}-\mu)a^\dagger_{k\sigma}a_{k\sigma}, \quad (3)$$

where the spin-up and the spin-down dispersions are

$$\varepsilon_{k\uparrow} = t\cos\left[\left(k-\frac{Q}{2}\right)a\right], \quad \varepsilon_{k\downarrow} = t\cos\left[\left(k+\frac{Q}{2}\right)a\right].$$

Here, the Fourier transformation

$$a_{i\sigma} = L^{-1/2}\sum_k e^{ikz_i}a_{k\sigma}, \quad a^\dagger_{i\sigma} = L^{-1/2}\sum_k e^{-ikz_i}a^\dagger_{k\sigma}$$

is introduced, and L is the length of the system along the z axis. To obtain result (3), the identity

$$\hat{U}_i\hat{U}^{-1}_j = \cos\frac{Q(z_i-z_j)}{2} + i\hat{\sigma}^z\sin\frac{Q(z_i-z_j)}{2}$$

was used.

In the new rotating frame, the local moment $\mathbf{n}_0$ is directed along the x axis and the sd interaction acquires the form

$$\mathcal{H}_{sd} = -\frac{JS}{2}\sum_k a^\dagger_k\hat{\sigma}^x a_k, \quad (4)$$

where the relation $\hat{U}_i\,(\mathbf{n}_0(z_i)\cdot\boldsymbol{\sigma})\,\hat{U}^{-1}_i = \hat{\sigma}^x$ was used. Hereinafter, the notation $a_k = (a_{k\uparrow}, a_{k\downarrow})$ is introduced.

The total field-like spin torque on $\mathbf{n}_0$ is given by [8]

$$\mathfrak{T} = a^{-1}\int_0^L dz\,[\mathbf{n}_0(z)\times\mathbf{s}(z)]_z\,.$$

Using

$$\hat{U}_i\,[\mathbf{n}_0(z_i)\times\boldsymbol{\sigma}]_z\,\hat{U}^{-1}_i = \hat{\sigma}^y$$

allows representing the torque as

$$\mathfrak{T} = \frac{1}{2}\sum_k a^\dagger_k\hat{\sigma}^y a_k. \quad (5)$$

Obviously, the STT is related to spin accumulation in the subsystem of itinerant carriers perpendicular to the local moment. We note that the transverse s^y component does not emerge in an isolated system. The spin accumulation is a feature of the nonequilibrium state created by the applied electric field, when the electric current passes through the helimagnet.

Below, we present a formulation of the nonadiabatic coupling between the conduction electrons and the local moments obtained within the equation-of-motion (EOM) approach of the nonequilibrium Green's function method [13]. The approach uses the lowest-order Born approximation that is valid for a relatively weak sd interaction.

3. NONEQUILIBRIUM GREEN'S FUNCTIONS

In the EOM method, the Hamiltonian of the system is split into two parts, $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$, where $\mathcal{H}_0$ describes noninteracting electrons and is given by Eq. (3). The term $\mathcal{H}_1 = \mathcal{H}_{sd}$ represents the sd interaction of electrons with the local moments (see Eq. (4)), and it is initilally supposed that $\mathcal{H}_1 = 0$ at $t = -\infty$. When $\mathcal{H}_1$ is adiabatically switched on, the sd interaction starts to affect the electron transport.

Our goal is to compute the components of the spin accumulation

$$s^\alpha_k(t) = \frac{\left\langle a^\dagger_k(t)\hat{\sigma}^\alpha a_k(t)\right\rangle}{2}, \quad \alpha = x, y, z$$

averaged over a nonequillibrium state. To perform the calculations, the contour-ordered Green's function

$$G_{k\sigma,k\sigma'}(t,t') = -i\left\langle T_{\mathcal{C}} a_{k\sigma}(t) a^{\dagger}_{k\sigma'}(t')\right\rangle$$

defined on the complex time plane is needed, where t and t' are defined on the Keldysh contour $\mathcal{C}$, and $T_{\mathcal{C}}$ denotes $\mathcal{C}$-contour ordering. Because the one-directional time axis is changed into a loop with two branches $\mathcal{C}^{\pm}$, the Green's function contains information on four Green's functions depending on the relative positions of t and t' in the loop:

$$G^{T}_{k\sigma,k'\sigma'}(t,t') = -i\left\langle T a_{k\sigma}(t) a^{\dagger}_{k'\sigma'}(t')\right\rangle, \quad \text{time-ordered}, \quad t,t' \in \mathcal{C}^{+}; \tag{6}$$

$$G^{\tilde{T}}_{k\sigma,k'\sigma'}(t,t') = -i\left\langle \tilde{T} a_{k\sigma}(t) a^{\dagger}_{k'\sigma'}(t')\right\rangle, \quad \text{anti-time-ordered}, \; t,t' \in \mathcal{C}^{-}; \tag{7}$$

$$G^{>}_{k\sigma,k'\sigma'}(t,t') = -i\left\langle a_{k\sigma}(t) a^{\dagger}_{k'\sigma'}(t')\right\rangle, \quad \text{“greater” component}, \quad t \in \mathcal{C}^{-}, \; t' \in \mathcal{C}^{+}; \tag{8}$$

$$G^{<}_{k\sigma,k'\sigma'}(t,t') = i\left\langle a^{\dagger}_{k'\sigma'}(t') a_{k\sigma}(t)\right\rangle, \quad \text{“lesser” component}, \; t \in \mathcal{C}^{+}, \; t' \in \mathcal{C}^{-}. \tag{9}$$

Here, $T(\tilde{T})$ is the usual (anti)time-ordering operator. The usual retarded G^R and advanced G^A Green's functions (lower indices are omitted) are expressed in terms of these time-loop-ordered Green's functions as

$$G^{R}(t,t') = \theta(t-t')\left[G^{>}(t,t') - G^{<}(t,t')\right], \tag{10}$$

$$G^{A}(t,t') = \theta(t'-t)\left[G^{<}(t,t') - G^{>}(t,t')\right]. \tag{11}$$

Due to the well-known relations

$$G^{T}(t,t') = \theta(t-t')G^{>}(t,t') + \theta(t'-t)G^{<}(t,t'), \tag{12}$$

$$G^{\tilde{T}}(t,t') = \theta(t'-t)G^{>}(t,t') + \theta(t-t')G^{<}(t,t'), \tag{13}$$

$$G^{T}(t,t') + G^{\tilde{T}}(t,t') = G^{>}(t,t') + G^{<}(t,t'), \tag{14}$$

where $\theta(t)$ is the step function, only two of the time-loop-ordered Green's functions are independent.

According to the general formalism [13], the EOM for the nonequilibrium time-loop Green's function

$$\langle\langle A(t_a)B(t_b)\rangle\rangle = -i\mathrm{Tr}\left\{\rho_0 T_{\mathcal{C}}\left[A(t_a)B(t_b)\right]\right\}$$

is given by

$$\langle\langle A(t_a)B(t_b)\rangle\rangle = g_a(t_a-t_b)\left\langle\left[A(t_b),B(t_b)\right]_{\pm}\right\rangle + \int\limits_{\mathcal{C}} dt\, g_a(t_a-t)\left\langle\langle\left[A(t),H_1(t)\right],B(t_b)\right\rangle\right\rangle, \tag{15}$$

where the upper (lower) sign is for fermions (bosons). In Eq. (15), ρ_0 stands for the equilibrium density matrix, and the single-particle time-loop Green's function g_a is defined for different relative orders of t_1 and t_2 on the Keldysh contour as follows:

$$ig_a(t_2-t_1) = \begin{cases} \dfrac{\tilde{f}_a(t_2-t_1)}{1\pm F_a}, & t_2 >_{\mathcal{C}} t_1, \\ \mp\dfrac{F_a\tilde{f}_a(t_2-t_1)}{1\pm F_a}, & t_2 <_{\mathcal{C}} t_1. \end{cases} \tag{16}$$

The coefficients F_a and $\tilde{f}_a$ are obtained from the relations $A(t_a)\rho_0 = F_a\rho_0 A(t_a)$ and $A(t_a) = \alpha_1\ldots\alpha_l\tilde{f}(t_a)$, where α is either a creation or an annihilation operator. Following the Langreth theorem, the integration over the loop in Eq. (15) can be changed to the integration along the real time axis [12].

4. SPIN ACCUMULATION IN THE NONEQUILIBRIUM STATE

The electron spin density in the nonequilibrium state is defined through the “lesser” Green's function

$$\left\langle a^{\dagger}_{k}\sigma^{\alpha}a_{k}\right\rangle = -i\mathrm{Tr}\left\{\hat{G}^{<}_{k,k}(t,t)\hat{\sigma}^{\alpha}\right\}, \quad \alpha = x,y,z. \tag{17}$$

Using the Langreth method for Eq. (15), we obtain the EOM for the “lesser” component

$$G^{<}_{k\sigma,k'\sigma'}(t,t') = g^{<}_{k\sigma}(t-t')\delta_{kk'}\delta_{\sigma\sigma'} - \frac{JS}{2}\times \times\sum_{\sigma_1}\sigma^{x}_{\sigma\sigma_1}\int\limits_{-\infty}^{\infty} dt_1 g^{R}_{k\sigma}(t-t_1)G^{<}_{k\sigma_1,k'\sigma'}(t_1,t') - - \frac{JS}{2}\sum_{\sigma_1}\sigma^{x}_{\sigma\sigma_1}\int\limits_{-\infty}^{\infty} dt_1 g^{<}_{k\sigma}(t-t_1)\times \times G^{A}_{k\sigma_1,k'\sigma'}(t_1,t'), \tag{18}$$

where we set $A(t)\equiv a_{k\sigma}(t)$, $B(t')\equiv a^{\dagger}_{k'\sigma'}(t')$.

Using the equilibrium density matrix

$$\rho_0 = \frac{\exp\left(-\beta\sum\limits_{k\sigma}(\varepsilon_{k\sigma}-\mu)a^{\dagger}_{k\sigma}a_{k\sigma}\right)}{\mathrm{Tr}\exp\left(-\beta\sum\limits_{k\sigma}(\varepsilon_{k\sigma}-\mu)a^{\dagger}_{k\sigma}a_{k\sigma}\right)},$$

where $\beta = 1/T$ is the inverse temperature, we recover the coefficient $F_{k\sigma} = e^{-\beta\varepsilon_{k\sigma}}$ and the single-particle time-loop Green's function

$$i\,g_{k\sigma}(t_2 - t_1) = \\ = \begin{cases} (1 - f_{k\sigma})\; e^{-i(\varepsilon_{k\sigma}-\mu)(t_2-t_1)}, & t_2 >_C t_1, \\ -f_{k\sigma}\, e^{-i(\varepsilon_{k\sigma}-\mu)(t_2-t_1)}, & t_2 <_C t_1, \end{cases} \tag{19}$$

where $f_{k\sigma} = \left(e^{\beta(\varepsilon_{k\sigma}-\mu)} + 1\right)^{-1}$ is the Fermi–Dirac distribution function.

Equation (19) yields the results

$$\begin{gathered} g^R_{k\sigma}(t_2 - t_1) = -i\theta(t_2 - t_1)e^{-i(\varepsilon_{k\sigma}-\mu)(t_2-t_1)}, \\ g^A_{k\sigma}(t_2 - t_1) = i\theta(t_1 - t_2)e^{-i(\varepsilon_{k\sigma}-\mu)(t_2-t_1)}, \\ g^>_{k\sigma}(t_2 - t_1) = -i(1 - f_{k\sigma})e^{-i(\varepsilon_{k\sigma}-\mu)(t_2-t_1)}, \\ g^<_{k\sigma}(t_2 - t_1) = i f_{k\sigma}e^{-i(\varepsilon_{k\sigma}-\mu)(t_2-t_1)}, \end{gathered} \tag{20}$$

for the retarded, advanced, "greater", and "lesser" components of the Green's function.

In the first-order Born approximation, the "lesser" component obtained from Eq. (18) is given by

$$G^<_{k\sigma;k'\sigma'}(t_a, t_b) \approx i f_{k\sigma}\, e^{-i(\varepsilon_{k\sigma}-\mu)(t_a-t_b)}\delta_{kk'}\delta_{\sigma\sigma'} - \\ -i\frac{JS}{2}\left(f_{k\sigma}e^{-i(\varepsilon_{k\sigma}-\mu)(t_a-t_b)} - f_{k\sigma'}e^{-i(\varepsilon_{k\sigma'}-\mu)(t_a-t_b)}\right)\times \\ \times \frac{\delta_{kk'}\sigma^x_{\sigma\sigma'}}{\varepsilon_{k\sigma} - \varepsilon_{k\sigma'} - i0}. \tag{21}$$

This produces the 2×2 matrix at equal time arguments

$$\hat{G}^<_{k,k}(t, t) \approx \\ \approx \begin{pmatrix} i f_{k\uparrow} & -i\dfrac{JS}{2}\,\dfrac{f_{k\uparrow} - f_{k\downarrow}}{\varepsilon_{k\uparrow} - \varepsilon_{k\downarrow} - i0} \\ -i\dfrac{JS}{2}\,\dfrac{f_{k\downarrow} - f_{k\uparrow}}{\varepsilon_{k\downarrow} - \varepsilon_{k\uparrow} - i0} & i f_{k\downarrow} \end{pmatrix}. \tag{22}$$

The spin accumulation components are easily obtained from Eq. (17) as

$$\left\langle a^\dagger_k \hat{\sigma}^x a_k \right\rangle = -JS\,\mathcal{P}\frac{f_{k\uparrow} - f_{k\downarrow}}{\varepsilon_{k\uparrow} - \varepsilon_{k\downarrow}}, \tag{23}$$

$$\left\langle a^\dagger_k \hat{\sigma}^y a_k \right\rangle = \pi JS\,(f_{k\uparrow} - f_{k\downarrow})\,\delta\,(\varepsilon_{k\uparrow} - \varepsilon_{k\downarrow}), \tag{24}$$

$$\left\langle a^\dagger_k \hat{\sigma}^z a_k \right\rangle = f_{k\uparrow} - f_{k\downarrow}, \tag{25}$$

where $\mathcal{P}$ means the principal value. Substituting these results in Eq. (5), we obtain the final result for the nonadiabatic STT

$$\mathfrak{T} = \frac{\pi JS}{2}\sum_k (f_{k\uparrow} - f_{k\downarrow})\,\delta\,(\varepsilon_{k\uparrow} - \varepsilon_{k\downarrow}). \tag{26}$$

To estimate the torque, we use the relaxation time approximation [4, 6] for the distribution function $f_{k\sigma}$ in the presence of an electric field $\mathbf{E}$ directed along the z axis

$$f_{k\sigma} \approx f^{(0)}(\varepsilon_{k\sigma}) - eE\tau v_{k\sigma}\frac{\partial f^{(0)}(\varepsilon_{k\sigma})}{\partial\varepsilon_{k\sigma}}, \tag{27}$$

where τ is the electron relaxation time, $v_{k\sigma} = \hbar^{-1}\partial\varepsilon_{k\sigma}/\partial k$ is the electron band velocity, and $f^{(0)}(\varepsilon_{k\sigma}) = (e^{\beta(\varepsilon_{k\sigma}-\mu)} + 1)^{-1}$ is the equilibrium Fermi–Dirac distribution function.

Substituting (27) in Eq. (26) and replacing summation with integration over the Brillouin zone, we obtain

$$\mathfrak{T} = -JSL\frac{eE\tau}{4}\int\limits_{-\pi/a}^{\pi/a} dk\left(v_{k\uparrow}\frac{\partial f^{(0)}(\varepsilon_{k\uparrow})}{\partial\varepsilon_{k\uparrow}} - \right. \\ \left. - v_{k\downarrow}\frac{\partial f^{(0)}(\varepsilon_{k\downarrow})}{\partial\varepsilon_{k\downarrow}}\right)\delta(\varepsilon_{k\uparrow} - \varepsilon_{k\downarrow}). \tag{28}$$

The δ-function implies the condition $\varepsilon_{k\uparrow} = \varepsilon_{k\downarrow}$, which selects definite points in the reciprocal space, i. e., $k = 0, \pi/a$ in our case. (We bear in mind that the points $k = \pm\pi/a$ are equivalent due to the periodicity.) This provides the transformation

$$\delta(\varepsilon_{k\uparrow} - \varepsilon_{k\downarrow}) = \frac{1}{2|t|a\sin(Qa/2)}\left[\delta\left(k - \frac{\pi}{a}\right) + \delta\,(k)\right].$$

Together with the relation

$$\frac{\partial f^{(0)}(\varepsilon_{k\sigma})}{\partial\varepsilon_{k\sigma}} = -\frac{1}{4T}\,\mathrm{ch}^{-2}\left(\frac{\varepsilon_{k\sigma} - \mu}{2T}\right),$$

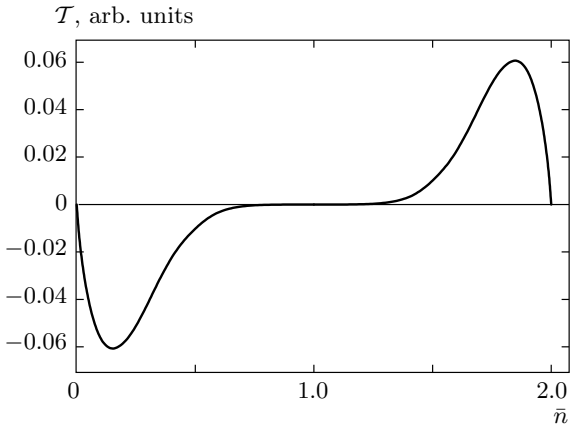
this yields

$$\mathfrak{T} = JSL\frac{t}{|t|}\frac{eE\tau}{16\hbar T}\times \\ \times\left[\mathrm{th}^2\left(\frac{\varepsilon_0 + \mu}{2T}\right) - \mathrm{th}^2\left(\frac{\varepsilon_0 - \mu}{2T}\right)\right], \tag{29}$$

where $\varepsilon_0 = t\cos(Qa/2)$. The chemical potential μ can be found from the given electron concentration per site $0 \le \bar{n} \le 2$ as

$$\frac{1}{L}\sum_{k\sigma} f^{(0)}(\varepsilon_{k\sigma}) = \bar{n}.$$

The dependence of $\mathfrak{T}$ on the electron concentration $\bar{n}$ is presented in the Figure. Apparently, a change in the concentration of free carriers causes a change of sign of the spin-transfer torque.



Dependence of the spin-transfer torque $\mathfrak{T}$ on the electron concentration per site $\bar{n}$

5. CONCLUSIONS

Based on the microscopic sd model, we analyzed the nonadiabatic STT in a chiral helimagnet with a zero or weak perpendicular magnetic field using the nonequilibrium (Keldysh) Green's function method. Ths electric current that passes through a chiral helimagnet generates spin accumulation with a component transverse to the local magnetization and creates a field-like STT. The torque changes sign with the sd exchange coupling and the carrier concentration. In contrast to domain-wall configurations, where the nonadiabatic torque arises for short domain walls [3], this phenomenon is a bulk effect in a chiral helimagnet.

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