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TUNNELING CONDUCTANCE OF THE GRAPHENE SNS JUNCTION WITH A SINGLE LOCALIZED DEFECT

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Using the Dirac–Bogoliubov–de Gennes equation, we study the electron transport in a graphene-based superconductor–normal(graphene)–superconductor (SNS) junction. We consider the properties of tunneling conductance through an undoped strip of graphene with heavily doped superconducting electrodes in the dirty limit $l_{def} \ll L \ll \xi$. We find that the spectrum of Andreev bound states is modified in the presence of a single localized defect in the bulk. The minimum tunneling conductance remains the same, and this result is independent of the actual location of the imperfection.

1. INTRODUCTION

Graphene — a monolayer of graphite — is formed by carbon atoms on a two-dimensional honeycomb lattice. In graphene, due to its unique band structure with the valence and conductance bands touching at two inequivalent Dirac points (often referred to as K and K') of the Brillouin zone, the electrons around the Fermi level obey the massless relativistic Dirac equation, which results in a linear energy dispersion relation. Recent exciting developments in transport experiments on graphene have stimulated theoretical studies of superconductivity phenomena in this material, which has been recently fabricated [1, 2]. A number of unusual features of the superconducting state have been predicted, which are closely related to the Dirac-like spectrum of normal state excitations [3, 4]. In particular, the unconventional normal electron dispersion has been shown to result in a nontrivial modification of the Andreev reflection and Andreev bound states in Josephson junctions with superconducting graphene electrodes [5, 6].

Other interesting consequences of the existence of Dirac-like quasiparticles can be understood by studying superconductivity in graphene [7–11]. It has been suggested that superconductivity can be induced in a graphene layer in the presence of a superconducting electrode near it via proximity effect [12–14].

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In this work, we study the Josephson effect and find bound states in graphene for a tunneling SNS junction in the presence of a single localized defect [15]. We concentrate on the SNS junction with a normal region thickness $L \ll \xi$, where ξ is the superconducting coherence length, and width W, which has an applied gate voltage U across the normal region [16, 17]. In the dirty limit $l_{def} \ll L \ll \xi$ considered in [18], we investigate the tunneling conductance in an SNS junction in the presense of a single localized defect and find that Andreev levels are modified, while the minimum tunneling conductance remains the same [18–20].

2. TUNNELING RESONANT CONDUCTANCE OF THE GRAPHENE SUPERCONDUCTOR/NORMAL/SUPERCONDUCTOR JUNCTION WITH A SINGLE LOCALIZED DEFECT

We consider an SNS junction with a single localized defect in a graphene sheet of width W lying in the xy plane and extending from $x = -L/2$ to $x = L/2$; the superconducting region occupies the range $|x| > L/2$ (see Fig. 1). The SNS junction can then be described by the Dirac–Bogoliubov–de Gennes (DBdG) equations [21]

$$\begin{pmatrix} H_s - E_F + U & \Delta \\ \Delta^* & E_F - U - H_s \end{pmatrix} \psi_s = \epsilon \psi_s,$$

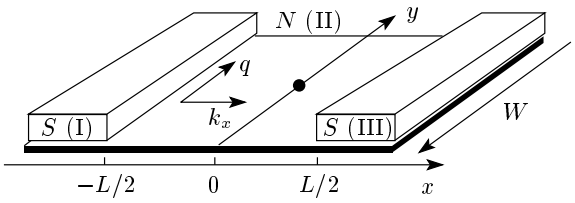

Fig. 1. An undoped graphene ribbon is contacted by two superconducting leads. Charge carriers tunnel from one lead to another via multiple tunneling states formed in the graphene strip. A defect is placed inside the strip

where $\psi_s = (\psi_{As}, \psi_{Bs}, \psi^*_{A\overline{s}}, -\psi^*_{B\overline{s}})$ and $\psi = (u_1, u_2, v_1, v_2)$ are 4-component wave functions for the electron and hole spinors, the index s ranges over K or K' for electrons or holes near the K and K' points, $\overline{s}$ takes the value $K(K')$ for $s = K(K')$, E_F denotes the Fermi energy, A and B denote the two inequivalent sites in the hexagonal lattice of graphene, and the Hamiltonian H_s is given by

$$H_s = -i\hbar v_F[\sigma_x\partial_x + \mathrm{sign}(s)\sigma_y\partial_y]. \qquad (1)$$

In Eq. (1), v_F denotes the Fermi velocity of the quasiparticles in graphene and sign(s) takes values $\pm$ for $s = K(K')$. The 2×2 Pauli matrices σ_i act on the sublattice index. The excitation energy $\epsilon > 0$ is measured relative to the Fermi level (set at zero). The electrostatic potential U and pair potential Δ have step-function profiles, as in the case of the semiconducting two-dimensional electron gas [22–24],

$$U(x) = \begin{cases} -U, & x < -L/2, \\ 0, & |x| < L/2, \\ -U, & x > L/2, \end{cases}$$

$$\Delta(x) = \begin{cases} \Delta_0 \exp(i\phi/2), & x < -L/2, \\ 0, & |x| < L/2, \\ \Delta_0 \exp(-i\phi/2), & x > L/2. \end{cases}$$

The reduction of the order parameter $\Delta(x)$ in the superconducting region on approaching the SN interface is neglected; that is, we approximate the parameter $\Delta(x)$ as is indicated above. As discussed in [25], this approximation is justified if the length and width of the weak link are much smaller than ξ. There is no lattice mismatch at the NS interface, and hence the honeycomb lattice of graphene is unperturbed at the boundary, the interface is smooth and impurity free.

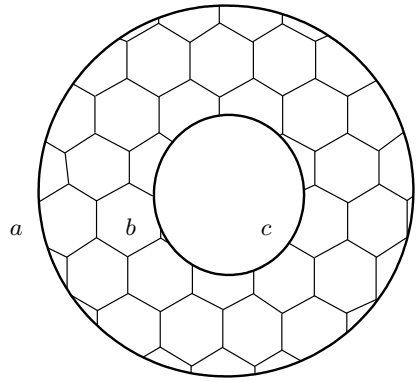

Fig. 2. Normal region II is divided into three areas a, b, c. The DBdG equations are solved for area b with the "infinite mass" boundary conditions induced by $V(r) \to \infty$ in areas a and c

Solving the DBdG equations, we obtain the wave functions in the superconducting and the normal regions. In region I (III), for the DBdG quasiparticles moving in the $\pm x$ direction with a transverse momentum $k_y = q$ and energy ϵ, the wave functions are given by

$$\Psi^+ = \exp(iqy + ik_s x + \kappa m x) \times \times \begin{pmatrix} \exp(-im\beta) \\ \exp(i\gamma - im\beta) \\ \exp(-im\phi/2) \\ \exp(i\gamma - im\phi/2) \end{pmatrix},$$

$$\Psi^- = \exp(iqy - ik_s x + \kappa m x) \times \times \begin{pmatrix} \exp(im\beta) \\ \exp(-i\gamma + im\beta) \\ \exp(-im\phi/2) \\ \exp(-i\gamma - im\phi/2) \end{pmatrix}.$$

The parameters are defined by $\beta = \arccos(\epsilon/\Delta_0)$, $\gamma = \arcsin[\hbar v_F q/(U_0 + E_F)]$, $k_s = \sqrt{(U_0 + E_F)^2/(\hbar v_F)^2 - q^2}$, and $\kappa = (U_0 + E_F)\Delta_0 \sin(\beta)/(\hbar^2 v_F^2 k_s)$; $m = \pm$ denotes region I (III), with $m = +$ for I and $m = -$ for III. We also assume that the Fermi wavelength λ'_F in the superconducting region is much smaller than the wavelength λ_F in the normal region and that $U_0 \gg E_F, \epsilon$. Because $|q| \le E_F/\hbar v_F$, this regime of a heavily doped superconductor corresponds to the limit $\gamma \to 0$, $k_s \to U_0/\hbar v_F$, $\kappa \to (\Delta_0/\hbar v_F)\sin\beta$.

Region II consists of three areas: a, b, c (see Fig. 2). We solve the DBdG equations for area b; area c is

where we place the defect and area a is to be extended and matched with the superconducting regions. The two valleys $s\pm$ decouple, and we can solve the equations separately for each valley, $H_s\psi_s = (\epsilon + sE_F)\psi_s$, $H_s = H_0 + sV(r)\sigma_z$. The term proportional to σ_z in the Hamiltonian is a mass term confining the Dirac electrons to area b.

We rewrite the Hamiltonian in cylindrical coordinates. Because H_s commutes with $J_z = l_z + \frac{1}{2}\sigma_z$, its electron eigenspinors ψ_e are eigenstates of J_z [26],

$$\Psi_e(r,\alpha) = \begin{pmatrix} \exp(id(n-1/2)\alpha)J_{d(n-1/2)}(k(\epsilon)r) \\ \exp(id(n+1/2)\alpha)J_{d(n+1/2)}(k(\epsilon)r) \end{pmatrix}$$

with eigenvalues n, where n is a half-odd integer, $n = d\frac{1}{2}, d\frac{3}{2}, \ldots$, and $J_{d(n-1/2)}(k(\epsilon)r)$ is the Bessel function of the order $n-1/2$. In the xy plane, d denotes the moving direction of the corresponding quasiparticle, $d = +$ for the quasiparticle moving toward $x = L/2$ and $d = -$ for the quasiparticle moving toward $x = -L/2$.

In what follows, we are interested in finding zero-energy states [14]. In this case, the DBdG equations have a general symmetry under changing the sign of energy,

$$\epsilon \to -\epsilon, \quad i\widehat{\sigma}_y\widehat{u}^* \to \widehat{v}, \quad i\widehat{\sigma}_y\widehat{v}^* \to -\widehat{u}, \tag{2}$$

where we set $\widehat{u} = (u_1, u_2)$ and $\widehat{v} = (v_1, v_2)$. Therefore, for a set of zero modes $(\widehat{u}_i,\widehat{v}_i)$ labeled by a certain index i, we should have

$$\widehat{v}_i = i\widehat{\sigma}_y\widehat{u}_j^*, \quad \widehat{u}_i = -i\widehat{\sigma}_y\widehat{v}_j^*. \tag{3}$$

In the same manner as for electrons, the hole spinors have the form

$$\Psi_h(r,\alpha) = \\ = \begin{pmatrix} -\exp(id(n+1/2)\alpha')J_{d(n+1/2)}(k'(\epsilon)r) \\ \exp(id(n-1/2)\alpha')J_{d(n-1/2)}(k'(\epsilon)r) \end{pmatrix},$$

where

$$\begin{aligned} \alpha(\epsilon) &= \arcsin[\hbar v_F q/(\epsilon + E_F)], \\ \alpha'(\epsilon) &= \arcsin[\hbar v_F q/(\epsilon - E_F)], \end{aligned} \tag{4}$$

$$\begin{aligned} k(\epsilon) &= (\hbar v_F)^{-1}(\epsilon + E_F)\cos\alpha, \\ k'(\epsilon) &= (\hbar v_F)^{-1}(\epsilon - E_F)\cos\alpha. \end{aligned} \tag{5}$$

The angle $\alpha \in (-\pi/2, \pi/2)$ is the incidence angle of the electron (with a longitudinal wave vector k), and α' is the reflection angle of the hole (with a longitudinal wave vector k') [27, 28]. To obtain an analytic approximation of the spectrum, we use the asymptotic form of the Bessel functions for large r. This is indeed the desired limit because $rk(\epsilon) \approx r_{def}k(\epsilon) \propto r_{def}/L \ll 1$, where r_{def} is the defect radius (the radius of area c), for all eigenvalues $n = d/2$. In this limit, we impose the "infinite mass" boundary conditions at $y = 0, W$, for which $q_n = (n+1/2)\pi/W$ in area b with $V(r) \to \infty$ in areas a and c. Half-odd integer values of n reflect the Berry phase π of a closed size of a single localized defect in graphene.

To obtain the subgap ($\epsilon < \Delta_0$) Andreev bound states, we now impose the boundary conditions at graphene. The wave functions in the superconducting and normal regions can be constructed as

$$\Psi_{\rm I} = a_1\psi_{\rm I}^{+} + b_1\psi_{\rm I}^{-}, \quad \Psi_{\rm III} = a_2\psi_{\rm I}^{+} + b_2\psi_{\rm I}^{-}, \tag{6}$$

$$\Psi_{\rm II} = a\psi_{\rm II}^{e+} + b\psi_{\rm II}^{e-} + c\psi_{\rm II}^{h+} + d\psi_{\rm II}^{h-}, \tag{7}$$

where $a_1(b_1)$ and $a_2(b_2)$ are the amplitudes of right and left-moving DBdG quasiparticles in region I (III), and $a(b)$ and $c(d)$ are the amplitudes of right (left) moving electrons and holes in the normal region [5]. These wave functions must satisfy the boundary conditions

$$\begin{aligned} \Psi_{\rm I}|_{x=-L/2} &= \Psi_{\rm II}|_{x=-L/2}, \\ \Psi_{\rm II}|_{x=L/2} &= \Psi_{\rm III}|_{x=L/2}. \end{aligned} \tag{8}$$

Because the wave vector k_y parallel to the NS interface and different wave vectors in the y-direction are not coupled, we can solve the problem for a given $k_y = q$ and consider each transverse mode separately. In the leading order in the small parameter $\Delta_0 L/\hbar v_F$, we can substitute $\alpha(\alpha') \to \alpha(0)$, $k(\epsilon)(k'(\epsilon)) \to k(0)$. After some algebra, we obtain the equation

$$\cos 2\beta \left\{ \frac{\sin^2(kL) - \cos^2(kL)\sin^2\alpha}{\cos^2(kL)\cos^2\alpha - 1} \right\} - \\ - \sin 2\beta \left\{ \frac{\cos(kL)\sin(kL)\sin\alpha}{\cos^2(kL)\cos^2\alpha - 1} \right\} = \cos\phi. \tag{9}$$

It differs from the equation obtained for an SNS junction without a single defect. Dropping the second term in Eq. (9) immediately yields a reduction of the equation applicable in the case of weak SNS junctions [14]. The solution of Eq. (9) is a single bound state per mode,

$$\epsilon_n = \Delta_0 \left[\frac{2}{A^2 + B^2} \times \right. \\ \left. \times \left(-2CA^2 - B^2 + B\sqrt{1 - 4CA^2(C+1)} \right) \right]^{1/2}, \tag{10}$$

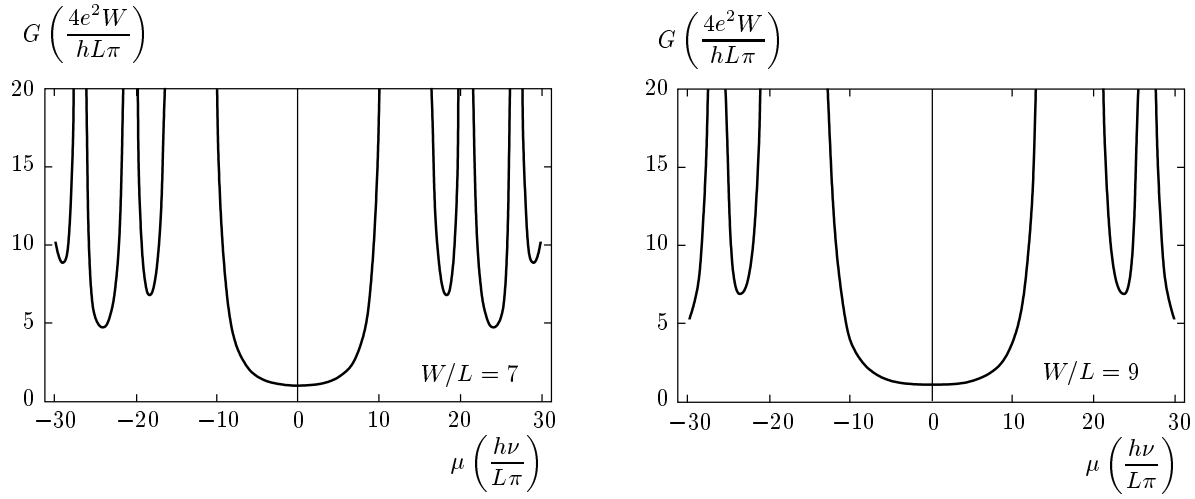

Fig. 3. Tunneling conductance of the graphene SNS junctions with a single localized defect versus Fermi energy, calculated from Eq. (13). The tunneling conductance exhibits oscillatory behavior

where

$$A = \frac{\sin^2(k_nL) - \cos^2(k_nL)\sin^2\alpha}{\cos^2(k_nL)\cos^2\alpha - 1}, \qquad B = \frac{\cos(k_nL)\sin(k_nL)\sin\alpha}{\cos^2(k_nL)\cos^2\alpha - 1}, \tag{11}$$

$$C = \frac{1}{2} + \frac{1}{\tau_n}\left(\frac{1}{2} - \sin^2\frac{\phi}{2}\right), \qquad \tau_n = \frac{\cos^2(k_nL)\cos^2\alpha - 1}{\cos^2(k_nL)\cos^2\alpha - \cos(2k_nL)}. \tag{12}$$

We do not have a simple analytic expression for the ϕ-dependence, but we obtained modified Andreev levels in the presense of a single localized defect in the bulk. The conductance of a graphene strip is expressed through the transmission probability by the Landauer formula,

$$G = g_0 \sum_{n=0}^{n(\mu)} \tau_n, \quad g_0 = \frac{4e^2}{h}, \tag{13}$$

where $n(\mu) \gg 1$ is given by

$$n(\mu) = \mathrm{Int}\left(\frac{k_nW}{\pi} + \frac{1}{2}\right).$$

Substituting the transmission probability in Eq. (13) gives the conductance versus the Fermi energy (see Fig. 3). The result for the minimal conductivity agrees with other calculations [29–31], which start from an unbounded disordered system and then take the limit of infinite mean free path l. There is no geometry dependence if the limits are taken in that order.

3. SUMMARY

We have shown that the tunneling conductance of the graphene SNS junction with a single localized defect has a nonzero minimal value if the Fermi level is tuned to the point of zero carrier concentration. We have demonstrated that the tunneling conductance exhibits oscillatory behavior. The Andreev levels are modified, the minimum tunneling conductance remains the same, and this result is independent of the actual location of the imperfection. The analysis of tunneling conductance of SNS junctions with multiple defects will be considered elsewhere.

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