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SCREENING OF A CHARGED PARTICLE FIELD IN RAREFIED IONIZED GAS

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A self-consistent field of a charged micron-size particle placed in a rarefied ionized gas is created by both free ions moving along infinite trajectories and trapped ions moving in closed orbits. The character of screening of the particle field is analyzed under dynamic conditions in a nonequilibrium plasma where the temperature (or the mean energy) of electrons greatly exceeds the ion temperature. Under these conditions, trapped ions are generated in a restricted region of the particle field where the transitions between closed ion orbits resulting from resonant charge exchange dominate. This leads to a higher number density of trapped ions compared to that of free ions. The parameters of the self-consistent field of the particle and ions are found when free or trapped ions determine the screening of the particle field, and a similarity law is established for a simultaneous variation of the number density of plasma particles and the particle size. In dusty plasmas of the Solar System, which result from the interaction of the solar wind with dust, formation of trapped ions increases the plasma number density compared to that in the solar wind.

1. INTRODUCTION

If a micron-size particle is placed in an ionized gas, electrons and ions attach to it, and the particle acquires a negative charge because of a higher mobility of electrons in comparison with ions. Charged particles located in a weakly ionized plasma are the basis of a dusty plasma [1–7]. In turn, a charged particle influences the distribution of electrons and ions near it, and this leads to the screening of the particle Coulomb field at a distance from the particle. The character of this screening depends on the ratio between the mean free path λ of electrons and ions in a gas and the particle radius r_0 (for simplicity, we assume the particle to be spherical). If the mean free path of electrons and ions is small compared to the particle size ($\lambda \ll r_0$), then the Debye screening of the particle field [8, 9] occurs that is determined by statistics in the distribution of electrons and ions in the particle field. Of course, it is important for this distribution that electrons and ions are absorbed by the particle [10, 11]. In the other limit case

$$\lambda \gg r_0, \tag{1.1}$$

the screening is determined by the dynamics of ion motion in the particle vicinity. Because electrons penetrate weakly in the region of the particle field, this screening follows from general laws of mechanics [12, 13] for motion of ions in the field of a particle.

In considering the limit case of a rarefied gas in Eq. (1.1), we concentrate on the effect of ion capture in the particle field as a result of the charge exchange process according to the scheme

$$\widetilde{A}^+ + A \to \widetilde{A} + A^+, \tag{1.2}$$

where $\widetilde{A}$ denotes one of the colliding particles. Being guided by this process, we deal with an atomic buffer gas (for example, argon, which is often used as buffer gas). Next, criterion (1.1) has the form

$$N_a \sigma_{res} r_0 \ll 1, \tag{1.3}$$

where N_a is the number density of gas atoms and σ_{res} is the resonant charge exchange cross section. The criterion means that the probability for a free ion that moved in the particle field to take part in process (1.2) is small. But although the probability for a free ion to be captured in a closed orbit of the particle field is small, the lifetime of a trapped ion in a closed orbit is large, and this makes the contribution of captured ions to the particle screening important.

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Thus, as a result of capture of free ions in closed trajectories, an "ion coat" is formed in the vicinity of a charged particle [14], and the field of this particle is screened by both free and trapped ions. The problem is complex because free and trapped ions influence the screening of the particle field, which in turn determines the ion behavior in the particle vicinity. A self-consistent field that is established in the particle vicinity due to the action of the particle and the ions determines the particle charge via the equality of the electron and ion currents to the particle surface. The role of trapped ions in screening the particle field is demonstrated in numerical calculations under certain conditions [15–17], where a self-consistent character of the particle field is taken into account, and some models [18–20] are used for the description of the behavior of trapped ions near a charged particle in a gas. These models are simplified if we take into account that the cross section of resonant charge exchange process (1.2) is independent of the collision velocity [21–23]. This simplifies the complex problem of the behavior of trapped ions in a self-consistent field of a charged particle and allows considering this problem from the standpoint of general laws of collisions of classical particles [12, 13].

2. DISTRIBUTION OF FREE IONS IN AN IONIZED GAS

The problem of a field of a charged particle placed in an ionized gaz is a self-consistent problem, and hence the particle charge Z and the particle field in the vicinity of a charged particle are self-consistent quantities. To separate these problems, we introduce the potential $U(R)$ of the self-consistent field at a distance R from the particle, and find the ion distribution in this field.

We first consider the distribution of free ions in this field. Let dP_i be the probability for an ion to be located at a distance in the range from R to $R + dR$. This probability is proportional to the time dt of ion location in this region, $dP_i \propto dt$, and to the number density of ions, $dP_i \propto N_i(R)$. From the ion equation of the motion [12], we have

$$dt = \frac{dR}{v_R} = \frac{dR}{v\sqrt{1 - \dfrac{\rho^2}{R^2} - \dfrac{U(R)}{\varepsilon}}},$$

where $v_R(R)$ is the normal component of the ion velocity in the particle field at the distance R from the particle center, v is the ion velocity far from the particle, $\varepsilon = Mv^2/2$ is the ion energy far from the particle (M is the ion mass), and ρ is the impact parameter for ion motion relative to the particle. From this, we have the number density of ions

$$N_i \propto \frac{\displaystyle\int \rho\, d\rho\, dP_i}{4\pi R^2 dR}.$$

Normalizing this expression in the case where the interaction is absent, we obtain

$$N_i(R) = N_0 \int\limits_0^{\rho(R)} \frac{\rho\, d\rho}{\sqrt{1 - \dfrac{\rho^2}{R^2} - \dfrac{U(R)}{\varepsilon}}}. \tag{2.1}$$

We use only one half of the trajectory when an ion recedes from the particle. For the free ion motion $U(R) = 0$ and $\rho(R) = R$, this formula gives

$$N_i = N_0.$$

If we divide the ion trajectories into two groups and account for the absence of the removed part for $\rho \le \rho_c$, where ρ_c is the boundary impact parameter for the ion capture by this particle, we obtain the number density of free ions in the particle field in the form [11]

$$N_i(R) = \frac{N_0}{2} \times \\ \times \left[\sqrt{1 - \frac{U(R)}{\varepsilon}} + \sqrt{1 - \frac{\rho_c^2}{R^2} - \frac{U(R)}{\varepsilon}}\right]. \tag{2.2}$$

The boundary impact parameter ρ_c is related to the particle radius r_0 as [12]

$$\rho_c^2 = r_0^2 \left[1 - \frac{U(r_0)}{\varepsilon}\right].$$

We average the ion number density over the Maxwell velocity distribution far from the particle

$$f(\varepsilon) = N_0 \frac{2\varepsilon^{1/2}}{\sqrt{\pi} T_i^{3/2}} \exp\left(-\frac{\varepsilon}{T_i}\right),$$

which is normalized by the relation

$$\int f(\varepsilon)\varepsilon^{1/2} d\varepsilon = N_0,$$

where T_i is the ion temperature expressed in energy units. In the space region, where $|U(R)| \gg T_i$ (the potential energy $U(R)$ is negative), we have

$$N_i(R) = N_0 \left[\sqrt{\frac{|U(R)|}{\pi T_i}} + \right. \\ \left. + \sqrt{\frac{|U(R)| - |U(r_0)| r_0^2/R^2}{\pi T_i}}\right]. \tag{2.3}$$

In the particle vicinity, this formula gives

$$N_i(R) = N_0\sqrt{\frac{|U(r_0)|}{\pi T_i}}, \quad R - r_0 \ll r_0,$$
$$N_i(R) = N_0\sqrt{\frac{4|U(R)|}{\pi T_i}}, \quad R \gg r_0. \tag{2.4}$$

In the other limit case $U(R) \ll T_i$, which corresponds to large ion distances from the particle, $R \to \infty$, we have

$$N(R) = N_0.$$

Accounting for this limit in formula (2.3) and retaining the form of formula (2.2), we generalize formula (2.3) to

$$N_i(R) = \frac{N_0}{2}\left[\sqrt{1+\frac{4|U(R)|}{\pi T_i}} + \right.$$
$$\left. + \sqrt{1+\frac{4[|U(R)|-|U(r_0)|r_0^2/R^2]}{\pi T_i}}\right]. \tag{2.5}$$

In the region $R \gg r_0$, this formula takes the form

$$N_i(R) = N_0\sqrt{1+\frac{4|U(R)|}{\pi T_i}}. \tag{2.6}$$

Thus, in the region where the particle field is present, ions are characterized by an increased number density in comparison with the region where this field is absent. We note that because the number density of ions N_i in the particle field exceeds the equilibrium number density N_0 and the number density of electrons is less than the equilibrium number density, screening of the particle field is determined just by ions.

3. WEAK SCREENING OF THE PARTICLE FIELD IN A RAREFIED PLASMA

We first consider the case of a rarefied plasma where the electrons and ions do not screen the Coulomb field of a charged particle. This limit case allows establishing certain peculiarities of the particle interaction with the surrounding plasma. We note that the case of a micron-size particle under consideration corresponds to the criterion

$$\frac{e^2}{r_0} \ll T_i. \tag{3.1}$$

In particular, for $T_i = 300$ K, this criterion has the form $r_0 \gg 60$ nm. In the limit case of a low density of electrons and ions of the plasma in which the particle is located, the potential energy of the particle field $U(R)$ at a distance R from the particle is given by

$$U(R) = \frac{Ze^2}{R}, \tag{3.2}$$

where Z is the particle charge expressed in units of the electron charge. Assuming criterion (3.1) to be valid, we obtain that the particle charge is large, and one electron or ion attached to the particle does not change the particle field much. This allows considering the process of particle charging to be continuous and finding the particle charge Z from the equality of electron and ion currents at the particle surface, which gives (see, e.g., [4, 24])

$$|Z| = \frac{r_0 T_e}{2e^2}\ln\left(\frac{T_e M}{T_i m_e}\right), \tag{3.3}$$

where T_e is the electron temperature, and M and m_e are the ion and electron masses.

We now derive the criterion for the number density N_0 of charged plasma particles to be low and the screening of the particle Coulomb field to be weak. We introduce the size R_0 of the region where the particle field acts on ions by the relation

$$|U(R_0)| = T_i, \tag{3.4}$$

which in the case of the Coulomb field has the form

$$R_0 = \frac{|Z|e^2}{T_i} = r_0 X, \quad X = \frac{T_e}{2T_i}\ln\left(\frac{T_e M}{T_i m_e}\right),$$
$$|Z| = \frac{T_i R_0}{e^2}. \tag{3.5}$$

This gives

$$R_0 \gg r_0.$$

In the absence of screening of the particle charge, it follows from (2.4) and (2.6) that the number density of free ions near the particle surface and in its field is given by

$$N_i(r_0) = N_0\sqrt{\frac{2R_0}{\pi r_0}}, \quad N_i(R) = N_0\sqrt{1+\frac{4R_0}{\pi R}},$$
$$R \gg r_0. \tag{3.6}$$

Because plasma charges are separated in the region $R < R_0$, this region is responsible for the particle field screening. Then the criterion of weak particle field screening takes the form of the condition that the total ion charge in this region be small compared with the particle charge, that is,

$$N_i(R_0)R_0^3 \sim N_0 R_0^3 \ll 1.$$

This criterion has the form

$$N_0 r_0^2 \frac{e^2}{T_e} \ll \frac{1}{X^2}. \qquad (3.7)$$

In particular, if we take argon plasma with $T_e = 1$ eV, $T_i = 400$ K, and the particle size $r_0 = 1$ μm, criterion (3.7) becomes $N_0 \ll 5 \cdot 10^8$ cm^{-3}. For these parameters, we have $|Z| = 5 \cdot 10^3$, $X = 210$, and $R_0 = 210$ μm. In addition, the ratio of the ion number densities near the particle surface and far from the particle is approximately 12 in this example, i. e., differs by an order of magnitude. This example shows that both cases of weak and strong screening are possible for micron-size particles located in a real gas discharge plasma.

We note that the electron temperature T_e used in the above formulas corresponds to the Maxwell energy distribution for electrons. In a gas discharge plasma with low concentration of electrons and ions, the Maxwell distribution over electron energies does not hold, and it is convenient to describe the electron energy distribution by the Townsend characteristic temperature [25]

$$T_{ef} = \frac{eD_e}{K_e},$$

where D_e and K_e are the diffusion coefficient and the mobility of electrons located in a gas in an external field. In this case, formula (3.3) for the particle charge remains valid if the electron temperature T_e is replaced with the characteristic electron temperature T_{ef} [10]. In this manner, we can describe interaction of a particle with a surrounding ionized gas located in an external field.

The ionized gas under consideration satisfies the rarefaction criterion that takes the form

$$N_a R_0 \sigma^* \ll 1$$

instead of criterion (1.3). Here, σ^* is the ion–atom diffusion scattering cross section, which is assumed to be independent of the collision velocity. Because this ion scattering proceeds through the resonant charge exchange process, where an ion and an atom move along straight trajectories, we have $\sigma^* = 2\sigma_{res}$ [26], and hence criterion (1.3) takes the form

$$2N_a R_0 \sigma_{res} \ll 1. \qquad (3.8)$$

In this case, the electron and ion currents originate at distances $\sim \lambda$ from the particle, where the interaction of ions with the particle field is negligible. In particular, in the above example of a particle of the radius 1 μm in argon, the cross section of resonant charge exchange involving an argon atom and its ion is $\sigma_{res} = 83$ Å^2 at the collision energy 0.01 eV [27]. Criterion (3.8) for the argon pressure p then becomes

$$p \ll 0.1 \text{ Torr}.$$

If criterion (3.7) is violated and screening of the particle field becomes important for ions, the general strategy of determining the particle charge from the equality of electron and ion currents holds. Instead of (3.3), we then have the following relation from the equality of electron and ion currents:

$$|U(r_0)| = \frac{T_e}{2} \ln\left(\frac{T_e M}{T_i m_e}\right). \qquad (3.9)$$

4. ION CAPTURE IN CLOSED ORBITS AROUND PARTICLES

Along with ion trajectories that correspond to ion capture by the particle surface and free trajectories with ion removal, capture of ions in closed trajectories is important for screening of the particle field [14]. These trajectories are different for the Coulomb and screened Coulomb fields [12, 13]. In the case of the Coulomb field, an ion moves in the same elliptic orbit after each period, while for the screened Coulomb field, the ion elliptic orbit rotates after each period. In analyzing the character of ion capture in an elliptic orbit, determined by resonant charge exchange process (1.2), we take into account that the resonant charge exchange cross section σ_{res} is independent of the collision velocity [21, 22]. Hence, a forming ion acquires the velocity of the atom from which it is formed. The role of trapped ions in the screening of the field of a charged particle located in a dusty plasma is demonstrated in numerical calculations under certain conditions [2, 15–17, 28–30]. Below, we consider the character of ion capture in closed orbits and examine peculiarities of this process, guided by atomic ions traveling in a parent atomic gas in the case where transition of an ion in a closed orbit starts from resonant charge exchange process (1.2). Because at thermal collision energies the resonant charge exchange cross section exceeds the elastic ion–atom cross section by almost an order of magnitude, we assume that the colliding ion and the atom move along straight trajectories. We find the conditions of ion capture in a closed orbit, assuming the resonant charge exchange act to be at a distance R from the particle, with an energy ε of the atom involved in the resonant charge exchange process. The parameters of a forming closed orbit are given in Fig. 1, and

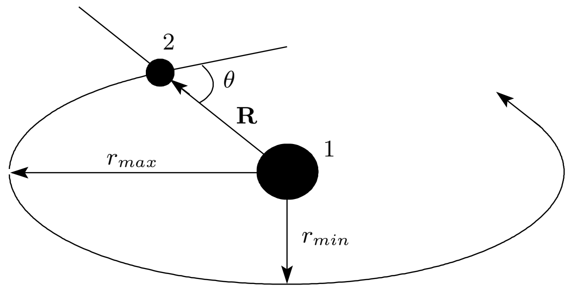

Fig. 1. Parameters of motion of a trapped ion in a closed trajectory: r_{min} and r_{max} are the minimum and maximum separations from the particle, $\mathbf{R}$ is the coordinate of the point where the resonant charge exchange event occurs, and θ is the angle between the direction of ion motion after the charge exchange and the vector $\mathbf{R}$; 1 — particle, 2 — atom converted into an ion

we assume the isotropic distribution of atoms over angles θ between the atom velocity direction and the axis joining the particle and the atom at the point of the resonant charge exchange. Evidently, transition into a closed orbit requires the following condition for the energy of the atom converted into an ion at a distance R from the particle:

$$U(R) \geq \varepsilon. \qquad (4.1)$$

Assuming the energy distribution function of ions to be identical to that of atoms far from the charged particle, we obtain that the capture of ions into a bound orbit occurs in the region of strong ion–particle interaction

$$R < R_0,$$

where

$$R_0 = \frac{|Z|e^2}{\varepsilon}, \qquad (4.2)$$

similarly to definition (3.4). This means that formation of trapped ions proceeds in the range of action of the particle field, and criterion (4.1) requires that

$$r_{max} < R_0$$

for the ion trajectory given in Fig. 1. Therefore, formation of a trapped ion requires that the resonant charge exchange process and the trajectory of a forming ion be located in the range of action of the particle field given by formula (3.4). According to criterion (4.1), a forming ion remains in the region of the particle field and does not recede to infinity.

The second criterion of formation of a trapped ion requires that the minimum separation r_{min} of the ion and the particle exceed the particle radius r_0,

$$r_{min} \geq r_0.$$

If this criterion is violated, the forming ion is captured by the particle. To apply this criterion, we use the energy and momentum conservation laws for ion motion in the particle field [12]. The total energy E of a trapped ion, if it is formed at a distance R from the particle and has the kinetic energy ε, is conserved in the course of ion motion and has the following form when the ion is located at a distance r from the particle:

$$E = U(R) + \varepsilon = \frac{Mv_R^2}{2} + U(r) + \frac{L^2}{2Mr^2}. \qquad (4.3)$$

Here, M is the ion mass, v_R is the ion velocity component in the direction of the particle, ε is the kinetic energy of the atom before the resonant charge exchange, corresponding to the initial ion energy, and $L^2/2Mr^2$ is the ion centrifugal energy, where the ion momentum L with respect to the particle center is

$$L = Mv_\tau R = MvR\sin\theta, \qquad (4.4)$$

with v_τ being the tangent velocity component at the point of charge exchange, v the initial atom velocity direction, and θ the angle between the atom velocity and this line to the point of the charge exchange act (see Fig. 2). We assume that the formed ion acquires the atom velocity as a result of the resonant charge exchange process because the colliding ion and atom move along straight trajectories.

From formulas (4.3) and (4.4), we find the relation between the minimum separation distance r_{min}, at which the normal velocity component is zero, $v_R = 0$, and the angle θ between the atom velocity direction and the line joining the atom and the particle centers at the point R of the resonant charge exchange event,

$$\sin\theta = \frac{r_{min}}{R}\sqrt{\frac{\varepsilon + U(R) - U(r_{min})}{\varepsilon}}.$$

From this, we find the angle θ_0 at which the minimum separation distance is $r_{min} = r_0$ and the ion capture occurs:

$$\sin\theta_0 = \frac{r_0}{R}\sqrt{\frac{\varepsilon + U(R) - U(r_0)}{\varepsilon}}. \qquad (4.5)$$

If the right-hand side of this relation exceeds unity, all the trajectories of forming ions terminate on the particle surface, i. e., the forming ion is captured by the particle and its transition to a closed orbit is impossible. Correspondingly, the ion capture into a closed orbit is possible under the condition

$$\frac{R^2}{r_0^2} \leq 1 + \frac{U(R) - U(r_0)}{\varepsilon}. \qquad (4.6)$$

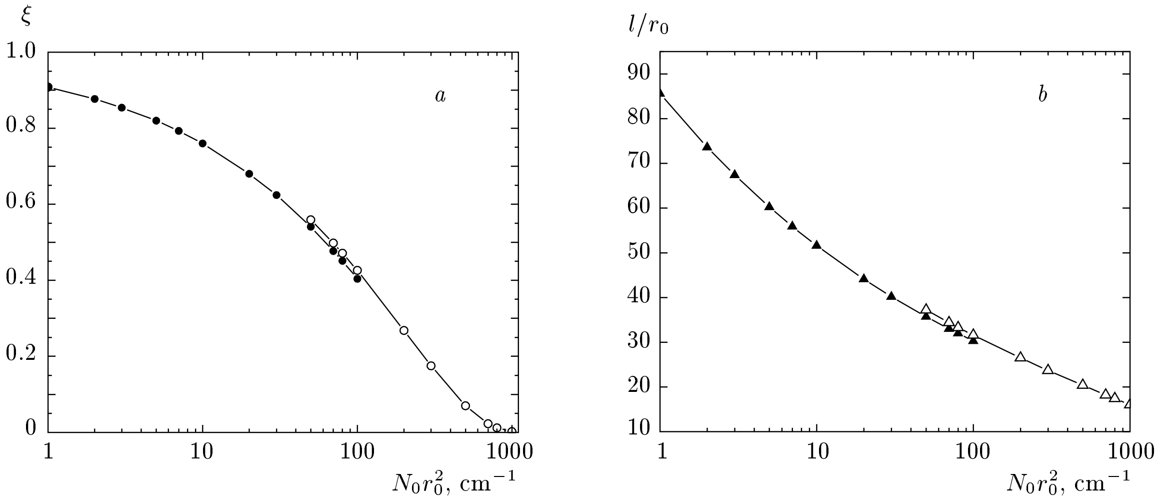

Fig. 2. The part of the screening charge owing to trapped ions (a) and the reduced size l/r_0 of the range of the self-consistent field of a particle and ions (b) as a function of the reduced number density of the surrounding rarefied ionized gas with the parameters $T_e = 1$ eV and $T_i = 400$ K. Filled symbols correspond to the case where trapped ions dominate in the particle field shielding, open symbols relate to the opposite case

We consider the range of distances for the resonant charge exchange event such that

$$|U(r_0)| \gg |U(R)| \gg \varepsilon. \tag{4.7}$$

Under this criterion, the forming ion cannot escape to the infinity and the ion capture in a closed orbit is possible. If we assume a range l of the particle field $l \gg r_0$, we obtain that the potential on the particle surface is equal to that in the absence of screening,

$$U(r_0) = -\frac{|Z|e^2}{r_0}.$$

With criterion (4.7), formula (4.5) then gives

$$\sin\theta_0 = \frac{\sqrt{r_0 R_0}}{R} \tag{4.8}$$

with R_0 defined in accordance with (4.2). It follows that the ion transition to a closed orbit is possible if the resonant charge exchange occurs at large distances R from the particle, that is,

$$R \geq \sqrt{r_0 R_0}.$$

We introduce the respective probabilities $P_{tr}(R,\varepsilon)$ and $p_{tr}(R,\varepsilon)$ for a free and bound ion to occupy a closed orbit after the resonant charge exchange act under conditions (4.7). For $\theta \leq \theta_0$, where θ_0 is given by (4.8), the ion is captured on the particle surface, and for $\theta > \theta_0$, the ion is captured in a closed orbit. Hence, the probability for an ion to be captured in a closed orbit is given by

$$P_{tr}(R,\varepsilon) = p_{tr}(R,\varepsilon) = \int\limits_0^{\cos\theta_0} d\cos\theta = \cos\theta_0 = \\ = \sqrt{1 - \frac{r_0 R_0}{R^2}}, \quad R \geq \sqrt{r_0 R_0}. \tag{4.9}$$

In the above analysis, we ignore the possibility that the ion recedes to infinity after the resonant charge exchange event because of criterion (4.7).

We also consider the possibility for a bound ion to become free as a result of the resonant charge exchange at a point R that allows the ion to go to infinity. We use formula (4.5), which is valid for both the minimum r_{min} and the maximum r_{max} distance from the particle if the ion moves in a closed orbit. If we ignore screening of the particle field, i.e., take the Coulomb field

$$U(r) = -\frac{|Z|e^2}{r},$$

then we have a relation between the angle θ given in Fig. 1 and the maximum ion distance r_{max} from the particle,

$$\sin\theta = \frac{r_{max}}{R}\sqrt{1 - \frac{R_0}{R} + \frac{R_0}{r_{max}}}.$$

Because the radicand in this expression is positive, we have

$$R < r_{max} < \frac{R_0 R}{R_0 - R}.$$

This implies that the ion can leave an elliptic orbit if the resonant charge exchange event occurs at a point R such that

$$R \geq \frac{R_0}{2}.$$

We note that averaging over the ion kinetic energies ε at the point R gives a wider range of ion distances from the particle, $R \sim R_0$, and hence the resonant charge exchange at these points may lead to an ion release. We thus obtain that an ion is captured at a point R in an elliptic orbit with

$$\sqrt{r_0 R_0} < R < R_0$$

and the number density of trapped ions has a maximum in the range

$$R_0/2 < R < R_0$$

where the ion can leave the closed orbit as a result of resonant charge exchange. In generalizing this result to the case where the particle field acts within a range $r \leq l$, we account for the probability that the ion recedes to infinity by the factor $(1 - R/l)$, and formula (4.9) for the probability of the ion capture in a closed orbit becomes

$$P_{tr}(R,\varepsilon) = p_{tr}(R,\varepsilon) = = \sqrt{1 - \frac{r_0 R_0}{R^2}}\left(1 - \frac{R}{l}\right), \quad l \geq R \geq \sqrt{r_0 R_0}. \quad (4.10)$$

We note that the assumption on the separation of the processes of ion capture and ion recession to infinity as a result of the resonant charge holds if $R_0 \gg r_0$. We used this criterion in the foregoing.

5. TRAPPED IONS IN A PARTICLE FIELD FOR LOW-DENSITY PLASMA

The number density of trapped ions N_{tr} follows from the balance equation

$$N_a \sigma_{res} N_i P_{tr} v_i = N_a \sigma_{res} N_{tr} v_{tr} (1 - p_{tr}), \quad (5.1)$$

where N_i is the number density of free ions, v_i is the relative velocity for a free ion and an atom involved in the resonant charge exchange, v_{tr} is the relative velocity of a trapped ion and the atom, P_{tr} is the probability to form a trapped ion as a result of resonant charge exchange involving a free ion, and p_{tr} is the probability for a trapped ion to remain in a closed trajectory after the resonant charge exchange. If we assume that

$$v_i \sim v_{tr}, \quad P_{tr} \sim 1, \quad p_{tr} < 1,$$

then Eq. (5.1) implies at least the same order of magnitude for the number density of trapped ions compared with the number density of free ions in the range of the particle field. Moreover, if p_{tr} is close to unity, then the number density of trapped ions N_{tr} exceeds that of free ions N_i. Therefore, although the resonant charge exchange probability is small for a free ion moving along an open trajectory, the lifetime of trapped ions is large compared with the flight time of free ions in the range of action of the particle field. Hence, the number density of trapped ions may be comparable to or exceed the number density of free ions. Therefore, trapped ions are important in screening the particle field.

We now determine the number density of trapped ions on the basis of balance equation (5.1) under conditions (4.7). Inside the region $|U(R)| \gg \varepsilon$, the velocity of a free ion is

$$v_i = \sqrt{\frac{2|U(R)|}{M}}.$$

If the particle field is nearly Coulomb in this region, then according to the virial theorem [31], the average kinetic energy is $|U(R)|/2$ and the velocity is

$$v_{tr} = \sqrt{\frac{|U(R)|}{M}},$$

which gives

$$\frac{v_i}{v_{tr}} = \sqrt{2},$$

and balance equation (5.1) yields the number density of trapped ions

$$N_{tr}(R) = N_i(R)\frac{P_{tr}\sqrt{2}}{1 - p_{tr}}.$$

Using formula (4.9) for the probability for an ion to leave an infinite or closed orbit as a result of resonant charge exchange, we reduce this relation to the form

$$N_{tr}(R) = N_i(R)\frac{R^2\sqrt{2}}{r_0 R_0}\sqrt{1 - \frac{r_0 R_0}{R^2}} \times \times \left(1 + \sqrt{1 - \frac{r_0 R_0}{R^2}}\right), \quad R_0 \gg R \geq \sqrt{R_0 r_0}. \quad (5.2)$$

If we use formula (3.6) for the number density of free ions and generalize this formula to the entire range of the particle field $R \leq l$ similarly to (4.10), then

$$N_{tr}(R) = N_0\sqrt{1+\frac{4U(R)}{\pi T_i}\,\frac{R^2\sqrt{2}}{r_0R_0}\sqrt{1-\frac{r_0R_0}{R^2}}}\times \\ \times\left(1+\sqrt{1-\frac{r_0R_0}{R^2}}\right)\left(1-\frac{R}{l}\right), \\ l \ge R \ge \sqrt{R_0r_0}. \quad (5.3)$$

Far from the threshold of creation of trapped ions, this formula gives

$$N_{tr}(R) = N_0\sqrt{1+\frac{4U(R)}{\pi T_i}\,\frac{2R^2\sqrt{2}}{r_0R_0}}\left(1-\frac{R}{l}\right), \\ l \ge R \ge \sqrt{R_0r_0}. \quad (5.4)$$

In particular, in the case of weak screening of the particle field, when this is the Coulomb field and $l = R_0$, formula (5.4) gives

$$N_{tr}(R) = \frac{4\sqrt{2}}{\sqrt{\pi}}\,\frac{N_0R^{3/2}}{r_0\sqrt{R_0}}\left(1-\frac{R}{R_0}\right), \\ R_0 \ge R \gg \sqrt{R_0r_0}. \quad (5.5)$$

From (5.5), the maximum number density of trapped ions that corresponds to the distance $R_{max} = 0.6R_0$ from the particle is

$$N_{tr}(R) = 0.59N_0\frac{R_0}{r_0}. \quad (5.6)$$

In particular, using this formula in the above example of an argon dusty plasma with the number density $N_0 = 1\cdot 10^{10}$ cm^{-3} far from the particle, we find the maximum number density of trapped ions $N_{max} \sim 10^{12}$ cm^{-3} if we ignore the particle field screening.

We now evaluate the ion charge located in the range of action of the particle field in the limit of low number density of ions and find the criterion for a weak screening of the particle field when it has the Coulomb form. From (5.5), we find the ion charge q in the sphere of a radius R_0 as

$$q(R_0) = e\int_{r_0}^{R_0} N_{tr}(r)\cdot 4\pi r^2 dr = \frac{64\sqrt{2\pi}}{99}\,\frac{eN_0R_0^4}{r_0}. \quad (5.7)$$

From this, we obtain the criterion $q \ll |Z|e$ of the weakness of the particle charge screening

$$N_0 \ll \frac{0.6|Z|r_0}{R_0^4}. \quad (5.8)$$

In particular, in the above example of an argon plasma with $T_e = 1$ eV, $T_i = 400$ K, and the particle size $r_0 = 1$ μm, we have $R_0 \approx 210$ nm, and criterion (5.8) gives $N_0 \ll 3\cdot 10^5$ cm^{-3}, which means the importance of screening of the particle field by a laboratory surrounding plasma for micron-size particles.

Guided by the case of a particle in a gas discharge plasma, we find an increased number density of ions in the range of the particle field in comparison with the number density of electrons and ions far from the particle. On contrary, the number density of electrons in this region is lower than that far from the particle. Therefore, screening of the particle field is determined by plasma ions, and their redistribution in the particle field creates a self-consistent field. At moderate number densities of ions, this field and the screening are mostly determined by trapped ions, but as the number density of plasma ions increases, the contribution of trapped ions to the screening of the particle field decreases. In what follows, we consider the character of this screening when it is significant.

6. SELF-CONSISTENT FIELD OF A PARTICLE AND FREE IONS

We now consider the self-consistent problem for a plasma in the vicinity of a charged particle. Then screening of the charged particle is determined by the number density of ions $N_i(R)$ near the particle; the number density of ions in the range of the particle field in turn depends on the screened field potential. From the Gauss theorem [32, 33], the electric field strength $E(R)$ is given by

$$E(R) = \frac{e}{R^2}[|Z| - q(R)], \quad z(R) = |Z| - q(R), \\ q(R) = \int_{r_0}^{R} N_i(r)\cdot 4\pi r^2 dr, \quad (6.1)$$

where $z(R)$ is the current charge inside the sphere of a radius R, $q(R)$ is the part of this charge that is the ion charge in this sphere, and $N_i(R)$ is the total number density of free and trapped ions. According to definition (6.1) of the current charge $z(R)$, we have the equation

$$\frac{dz(R)}{dR} = -4\pi R^2 N_i(R). \quad (6.2)$$

From (6.1), the particle field potential is given by

$$U(R) = \int_R^l \frac{z(r)e^2}{r^2}\,dr, \quad (6.3)$$

where l is a distance measuring the range of the particle field, defined as

$$z(l) = 0. \tag{6.4}$$

We consider the case where the particle field screening is mostly determined by free ions; this is the case of a high number density of the surrounding plasma when trapped ions do not take part in screening the particle field. According to formula (3.6), relation (6.1) for $z(R)$ then becomes

$$z(R) = |Z| - N_0 \int_{r_0}^{R} \sqrt{1 + \frac{4U(r)}{\pi T_i}} \cdot 4\pi r^2 dr, \tag{6.5}$$

where N_0 is the ion number density far from the particle and T_i is the ion temperature in the surrounding plasma. We thus obtain two equations (6.3) and (6.5) that relate the potential $U(R)$ of a self-consistent field of the particle and the ions to the effective charge $z(R)$ of the particle and the ions.

Below, we give an approximate solution of the set of equations (6.3) and (6.5). Taking the integral for $U(R)$ by parts, we represent the potential of the particle field in the form

$$U(R) = \frac{z(R)}{R} - \Delta U, \qquad \Delta U = \int_R^l \frac{[z(R) - z(r)]e^2}{r^2}\, dr. \tag{6.6}$$

In the first approximation, we ignore the second term in this expression for the particle potential. We thus decouple Eqs. (6.3) and (6.5), whence

$$\frac{dz}{dR} = -4\pi R^2 N_0 \sqrt{\frac{4ze^2}{\pi T_i R}}.$$

Solving this equation with the initial condition $z(r_0) = |Z|$, we obtain

$$z = \left(\sqrt{|Z|} - \frac{8\sqrt{\pi}}{5} N_0 R^{5/2} \sqrt{\frac{e^2}{T_i}}\right)^2 = |Z| \left[1 - \left(\frac{R}{l}\right)^{5/2}\right]^2. \tag{6.7}$$

The size l of the particle field range follows from Eq. (6.3) and is given by

$$l = \frac{0.66}{N_0^{2/5}} \left(\frac{|Z| T_i}{e^2}\right)^{1/5} = \frac{0.66 |Z|^{2/5}}{N_0^{2/5} R_0^{1/5}}. \tag{6.8}$$

In particular, for the number density of electrons and ions $N_0 = 10^{10}$ cm^{-3} of an argon plasma with the above parameters ($T_e = 1$ eV, $T_i = 400$ K, and the particle radius $r_0 = 1$ μm), it follows that $l = 43$ μm; for $N_0 = 10^9$ cm^{-3}, we obtain $l = 108$ μm assuming that free ions determine the screening of the particle Coulomb field.

It is more correct to define the size l of the particle field range by analogy with formula (3.4), which now has the form

$$eU(l) = T_i \tag{6.9}$$

instead of Eq. (6.3). With the above parameters, this formula gives $l = 40$ μm for the number density of electrons and ions $N_0 = 10^{10}$ cm^{-3} and $l = 99$ μm for $N_0 = 10^9$ cm^{-3}. We see that as regards the screening of the particle field by free ions, the definitions given by formulas $z(l) = 0$ and (6.7) are practically identical.

We can estimate the accuracy of ignoring the second term in formula (6.6). Evidently, this is accurate near the particle, where the screening is weak. But the second term in (6.6) may decrease the particle potential $U(R)$ by a factor of 2 if $R \sim l$. However, this approximation is transparent and allows us to analyze the peculiarities of the self-consistent particle field, and we keep it.

We use formula (3.9) for determining the particle charge under the condition $l \gg r_0$. Taking

$$U(r_0) = \int_{r_0}^{l} E(R)\, dR$$

and using formula (6.1) for the electric field strength of the self-consistent field, we again obtain formula (3.3) for the particle charge Z. The reason for the coincidence of the particle charges with and without the screening of the particle field is that fluxes of electrons and ions are created far from the particle surface, where the surrounding plasma is quasineutral, and the particle charge follows from the equality of the electron and ion fluxes. Therefore, the particle charge is independent of the particle field screening if the particle is located in a rarefied plasma.

We also note that the gas rarefaction condition changes in the case of a particle field screening because the range of the particle field then decreases. Specifically, the criterion of gas rarefaction in this case has the form

$$2N_a \sigma_{res} l \ll 1$$

instead of rough criteria (1.3) and (3.8). It changes the condition of plasma rarefaction in the above example and leads to $p \ll 1$ Torr.

7. SELF-CONSISTENT FIELD OF A PARTICLE AND TRAPPED IONS

In analyzing the screening of the field of a charged particle by the surrounding plasma, we find that it is determined by plasma ions. Indeed, even at distances R from the particle where $U(R) \sim T_e$, we obtain that the electron number density N_e is of the order of that far from the particle ($N_e \sim N_0$). But at distances such that $U(R) \gg T_i$, the number density of ions N_i significantly exceeds the equilibrium value ($N_i \gg N_0$). Next, dividing ions into free and trapped ones, we can evaluate the number density of free ions and the particle field screening by free ions more or less accurately, but these parameters are mere estimates for trapped ions. Assuming that free or trapped ions separately determine the particle field screening, we separately find the size of the self-consistent field range when this field results from the particle–ion interaction.

Assuming that trapped ions determine the particle field screening, we now repeat the analysis in the previous section with the number density of free ions replaced with that of trapped ions. Solving the equation for the charge z inside a sphere with radius R, we obtain (instead of formula (6.7) for the screening by free ions)

$$z = \left[\sqrt{|Z|} - \frac{16\sqrt{\pi}}{9}\frac{N_0 R^{5/2}}{r_0 R_0}\sqrt{\frac{e^2}{T_i}} \times \right.$$
$$\left. \times \left(1 - \frac{R}{l}\right)\Phi(R)\right]^2 = |Z|\left[1 - \left(\frac{R}{l}\right)^{9/2}\right]^2, \quad (7.1)$$

where

$$\Phi(R) = \frac{1}{2}\sqrt{1 - \frac{r_0 R_0}{R^2}}\left(1 + \sqrt{1 - \frac{r_0 R_0}{R^2}}\right),$$
$$l = 1.05\left(\frac{|Z| r_0 \sqrt{R_0}}{N_0 \Phi(9l/11)}\right)^{2/9}. \quad (7.2)$$

We assume a weak dependence $\Phi(R)$ and evaluate this function at the distance where the integrand has a maximum. Accounting for a difference $\Phi(R)$ from unity gives a small correction to the result. Using formula (7.2) in the above example of an argon dusty plasma ($T_e = 1$ eV, $T_i = 400$ K, and $r_0 = 1$ μm), we obtain $l = 36$ μm for the number density of electrons and ions $N_0 = 10^{10}$ cm^{-3}, and $l = 59$ μm for $N_0 = 10^9$ cm^{-3}. Comparing these values with those for screening by free ions in the previous section, we conclude that trapped ions dominate in the screening of the particle field if the number density of plasma electrons and ions is below $N_0 = 10^{10}$ cm^{-3}.

In considering the case where trapped ions dominate in screening the particle field and the self-consistent field of the charged particles and ions is given by formula (7.2), we also give the expressions for the number density of free and trapped ions in the particle field according to formulas (2.6) and (5.4). For the potential of the self-consistent field, we use simplified formula (6.3) in the form

$$U(R) = \frac{z(R)e^2}{R},$$

and take the current charge $z(R)$ of a sphere of radius R to be given by formula (7.1). In this case, the number densities of free $N_i(R)$ and trapped $N_{tr}(R)$ ions at a distance R from the particle are

$$N_i(R) = N_0\sqrt{1 + \frac{4R_0}{\pi R}\left[1 - \left(\frac{R}{l}\right)^{9/2}\right]^2},$$
$$N_{tr}(R) = N_i(R)\frac{2R^2\sqrt{2}}{r_0 R_0}\Phi(R)\left(1 - \frac{R}{l}\right). \quad (7.3)$$

We also give the expressions for the number density of free and trapped ions in the particle field in the opposite case where (2.6) and (5.4) describe the number densities of free and trapped ions and simplified formula (6.3) holds for the self-consistent field potential energy

$$U(R) = \frac{z(R)e^2}{R},$$

but the current charge $z(R)$ of a sphere of radius R is given by (6.7). Instead of (7.3), we then obtain

$$N_i(R) = N_0\sqrt{1 + \frac{4R_0}{\pi R}\left[1 - \left(\frac{R}{l}\right)^{5/2}\right]^2},$$
$$N_{tr}(R) = N_i(R)\frac{2R^2\sqrt{2}}{r_0 R_0}\Phi(R)\left(1 - \frac{R}{l}\right). \quad (7.4)$$

We now formulate a general algorithm for determining the parameters of a self-consistent field with the screening by both free and trapped ions taken into account. We define the screening charges by free Q_i and trapped Q_{tr} ions as

$$Q_i = \int_{r_0}^{l} 4\pi N_i(R) R^2 dR,$$
$$Q_{tr} = \int_{\sqrt{r_0 R_0}}^{l} 4\pi N_i(R) R^2 dR. \quad (7.5)$$

Evidently, according to the definition of the size l of the particle field range, we have

$$Q = Q_i + Q_{tr} = |Z|. \quad (7.6)$$

This is the equation for the size of the particle field range. Simultaneously, we determine the part of the screening charge

$$\xi = \frac{Q_{tr}}{Q_i + Q_{tr}} \quad (7.7)$$

that is created by trapped ions. In evaluating these parameters of the particle field screening, we use formulas (7.3) or (7.4) for the number densities of free and trapped ions. We note that in the range of competition of these screening channels, the difference of the results for these versions is not significant. In particular, in above example of a dusty argon plasma ($T_e = 1$ eV, $T_i = 400$ K, and $r_0 = 1$ μm) at $N_0 = 10^{10}$ cm^{-3}, the contribution of trapped ions to the particle screening is $\xi = 0.50$ or $\xi = 0.53$ if we respectively use formula (7.3) or (7.4), and the respective size l of the particle field range is 28 μm or 29 μm.

We now analyze the dependence of the effects under consideration on the particle radius r_0. If we let l_{free} denote the size of the range of the self-consistent field where the particle field is screened by free ions and l_{trap} denote the size of the particle field screened with trapped ions, then it follows from (6.8) and (7.2) that

$$\frac{l_{free}}{r_0} = \frac{A}{(N_0 r_0^2)^{2/5}}, \frac{l_{trap}}{r_0} = \frac{B}{(N_0 r_0^2)^{2/9}}, \quad (7.8)$$

where

$$A = 0.66 \frac{(|Z|/r_0)^{2/5}}{(R_0/r_0)^{1/5}},$$
$$B = \frac{1.05(|Z|/r_0)^{2/9}(R_0/r_0)^{1/9}}{\Phi}. \quad (7.9)$$

It can be seen that the parameters A and B are independent of r_0. As a result, we obtain a similarity law according to which the size parameters of the self-consistent field vary proportionally to the particle radius if the product $N_0 r_0^2$ is kept constant. In particular, it follows that for the above Ar plasma parameters at the particle radius $r_0 = 10$ μm, the contribution of free and trapped atoms is identical at $N_0 \approx 1 \cdot 10^8$ cm^{-3}, and trapped ions disappear at $N_0 \approx 10^9$ cm^{-3}.

In Fig. 2*a*, we give the dependence of the part of the screening charge ξ created by trapped ions on the reduced number density $N_0 r_0^2$ of the surrounding plasma. The dependence of the reduced size of the particle field on this parameter is shown in Fig. 2*b*. Figure 3 contains the reduced number densities of free $N_i r_0^2$ and trapped $N_{tr} r_0^2$ ions as functions of the reduced distance R/r_0 from the particle. These number densities correspond to the reduced number density of the surrounding plasma $N_0 r_0^2 = 100$ cm^{-1}, at which the contribution of trapped ions to the particle screening is approximately 40 %.

Comparing the results of two versions allows estimating the accuracy of our analysis as ~ 10 % for the number density of trapped ions. In particular, according to the first version (formulas (7.3)), the contribution of free and trapped ions to the particle screening is identical at the reduced number density of plasma ions $N_0 r_0^2 = 63$ cm^{-1}, whereas the second version (formulas (7.4)) gives $N_0 r_0^2 = 69$ cm^{-1}. The size of the particle field range is $l/r_0 = 30$ and 31 according to these versions. The difference between the number densities of ions (free and trapped) in the basic region of screening of the particle field according to these versions is approximately 10 %, as follows from the data in Fig. 3 at the reduced number density of the surrounding plasma $N r_0^2 = 100$ cm^{-1}, at which the first version is more preferable.

We note one more aspect of this phenomenon. We consider the case $R_0 \gg r_0$, which allows us to divide the ion trajectories into those from which an ion attaches to the particle or goes into the surrounding plasma. Next, the nearest elliptic ion orbit is located far from the particle. All this according to formula (3.5) is possible only for a nonequilibrium plasma where $T_e \gg T_i$, as it is realized in a gas discharge plasma containing dust particles. In an equilibrium plasma ($T_e = T_i$), we have $R_0/r_0 \equiv X \approx 5$–6, and the above effects are mixed.

As follows from the above analysis, trapped ions are important in a rarefied plasma. It is typical for an astrophysical plasma that fluxes may contain dust particles along with plasma [34]. Below, we consider two examples of an astrophysical plasma with dust particles. The first example relates to Saturn rings. In particular, the E-ring and F-rings of Saturn contain ice particles of a size ranging from 0.5 μm to 10 μm [35, 36] and a typical number density of ice particles is 30 cm^{-3} [36]. The sources of ice particles are the neighboring satellites: Enceladus [37] for E-ring, and

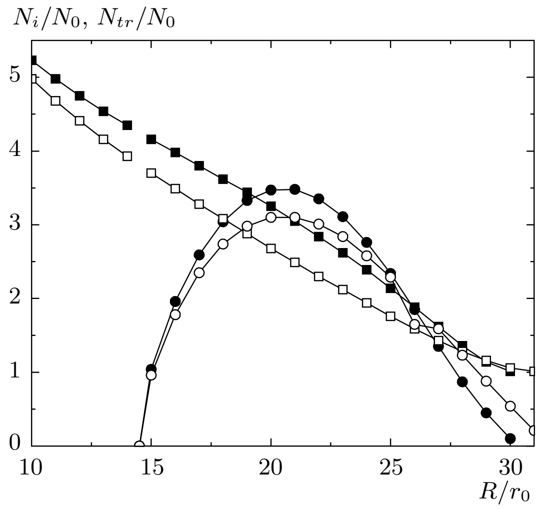

Fig. 3. The number densities of free (squares) and trapped (circles) ions in a self-consistent field of the particle at the reduced number density of a surrounding ionized gas $N_0 r_0^2 = 100\ \mu\text{m}^{-1}$ with the parameters $T_e = 1$ eV and $T_i = 400$ K. Closed symbols correspond to formula (7.3) and open symbols relate to formula (7.4)

Prometheus and Pandora for the F-ring [38]. The number density of the plasma that results from the interaction of the Saturn magnetic field and solar wind [39] is $N_0 = 30$–100 cm^{-3} [40] and the electron temperature is 10–100 eV [40], and the magnetic field has a significant effect on the properties of this plasma. Ions of various types exist in a ring plasma, and basic sorts of ions are OH^+ and H_2O^+ with temperatures of the order of 10^3 K. Taking $r_0 = 1\ \mu$m, $N_0 = 10^2$ cm^{-3}, $T_e = 30$ eV, and $T_i = 10^3$ K, we obtain the particle charge $|Z| = 2\cdot 10^5$ according to formula (3.3); from (3.5), $R_0 \approx 0.3$ cm, which is comparable to the distance between nearest particles because their number density is $N_p \approx 30$ cm^{-3} [36]. This value is also comparable to the size of the particle field range $l \sim R_0$, which is evidence of a partial screening of the particle field by plasma ions.

A subsequent analysis of the character of the particle field leads to a contradiction. Indeed, the above formulas are based on the assumption that attachment of ions and electrons to the particle does not change the parameters of the surrounding plasma. If such an equilibrium is established, each particle takes $\sim |Z|$ ions in its field, and because the number density of particles is $N_p \approx 30$ cm^{-3} [36], the mean number density of trapped ions must be $N_i \approx 10^6$ cm^{-3} under equilibrium of electron and ion fluxes. But this value exceeds the observational value by several orders of magnitude. This means that the equilibrium that we used for a laboratory dusty plasma is violated for this plasma. Indeed, the source of a plasma is the solar wind with the number density of fast electrons and protons 0.1 cm^{-3}. These atomic particles decelerate in this dusty plasma and ions. As a result, particles are charged negatively, and ions are partially captured by these charged particles and may be transformed into other ion types. But fluxes of electrons and protons to an individual particle are limited, and hence the observational number density of the plasma near the particle is lower than that in the case of a stronger plasma source. This example shows that the property of a certain astrophysical plasma is determined by some processes in which the interaction of particles with plasma fluxes plays an important role.

We consider the Solar System plasma from another standpoint. The basis of this plasma is the solar wind—a flux of plasma emitted by the Sun corona [41, 42]. In formation of a dusty plasma, this plasma flux encounters dust generated by a cold condensed system. As a result of mixing, a dusty plasma is formed, and the number density of charged particles in this plasma exceeds that in the solar wind by several orders of magnitude. Above, we verified this for the dusty plasma of Saturn rings, and the same occurs for other types of dusty plasma realized in the Solar System, in particular, for the comets [43, 44]. Comet tails result from the interaction of solar wind with the dusty plasma of comets, and although the magnetic properties are important for the properties of this plasma [45], processes in the field of charged particles of this plasma proceed as described above. Correspondingly, the long lifetime of trapped ions greatly increases the number density of the plasma of a comet tail. This number density is 10^3–10^4 cm^{-3} [42, 46, 47] and exceeds that in the solar wind, while the electron temperature $T_e \sim 10^4$ K [42, 47–49] corresponds to that in the solar wind. One of the reason for an increase in the plasma density in the dusty plasma of comet tails is formation of trapped ions in the field of dust particles with a relatively long lifetime.

8. CONCLUSION

If a micron-size particle is placed in a weakly ionized plasma, electrons and ions attach to the particle surface and create a negative particle charge because the electron mobility is higher than that of ions. As a result, plasma charges are separated in the particle field, and the ions that penetrate in the particle field screen it. Along with free ions, which screen the particle field in the course of their flight in the particle field, trapped ions, which move along closed trajectories, contribute to the particle field screening. Trapped ions exist in the range of action of the particle field at distances not close to the particle. The number density of trapped ions may significantly exceed that of free ions because charge exchange of trapped ions with gas atoms keeps ions in closed trajectories in some region of the particle field. The more rarefied the surrounding plasma is, the larger contribution to the particle screening is made by trapped ions. For a typical dusty plasma based on a gas discharge plasma, the contribution to the particle screening due to free ions exceeds or is comparable to that due to trapped ions.

In addition, the particle charge for a rarefied buffer gas is independent of the degree of the particle field screening because fluxes are created in regions beyond the particle field range, and the equality of the electron and ion fluxes at the particle surface determines its charge. This equilibrium is violated in a dusty astrophysical plasma because the plasma fluxes directed to dust particles are small and attachment of electrons and ions to dust particles affects the parameters of the surrounding plasma. Nevertheless, interaction of this plasma with dust particles, especially the formation of trapped ions with a long lifetime, is important for properties of such a dusty plasma.

Comparing this approach and results with studies of trapped ions in a self-consistent field of a particle located in a plasma, we note a nonrealistic character of the resonant charge process in the models in Refs. [18–20]. We took the Sena effect into account in this process [21–23], assuming that an ion and an atom move along straight trajectories in the course of this process, and hence a forming ion acquires the velocity of the incident atom at the point where this process occurs. This allowed us to precisely evaluate the parameters of trapped ions in the region where the potential of the particle self-consistent field significantly exceeds the atom kinetic energy and continue the results to regions where this criterion does not hold. In particular, we thus found that the number density of trapped ions in some region may significantly exceed that of free ions.

Of course, computer simulation of this problem in the molecular dynamics framework gives, in principle, an accurate description of the behavior of trapped ions also on the boundary of the particle field range. But this method requires accounting for a large number of ions involved in the creation of the self-consistent particle field. In particular, in the conditions of the above example, approximately $\sim 5 \cdot 10^3$ ions take part in the screening of the particle field, and because the lifetime of trapped ions is large compared to the flight time of free ions, it is necessary to include 10^5–10^6 ions into consideration. Although modern computers allow solving this problem, it is still arduous enough, and therefore a number of assumptions are used in some numerical evaluations [15–17, 28–30], which cannot be grounded. We also note that our analytic approach allows considering the problem for an arbitrary shape of the potential of the self-consistent particle field, which simplifies the problem. As a result, our semianalytic method allows accurately describing the behavior of trapped ions in the internal region of the particle field and exhibiting the character of formation of the particle self-consistent field in a wide range of parameters.

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