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SPONTANEOUS SYMMETRY BREAKING IN A SYSTEM OF STRONGLY INTERACTING MULTICOMPONENT FERMIONS (ELECTRONS WITH SPIN AND CONDUCTING NANOTUBES)

V. V. Afonin [a]*, ***V. L. Gurevich*** [a], ***V. Yu. Petrov*** [b]

[a] *Ioffe Institute, Russian Academy of Sciences*
194021, Saint Petersburg, Russia

[b] *Konstantinov Saint Petersburg Nuclear Physics Institute, Russian Academy of Sciences*
18830, Gatchina, Leningrad region, Russia

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We calculate the ground-state wave functions for a system of multicomponent strongly interacting fermions. We show that it is a state with spontaneously broken chiral symmetry that discribes a phase with a finite density of chiral complexes. The number of particles constituting a complex depends on the number of fermion components. For example, in the case of two-component electrons (spin), the condensate is built of four-particle complexes consisting of two "right" electrons and two "left" holes with the opposite spins.

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1. INTRODUCTION AND DISCUSSION OF RESULTS

Advances in semiconductor technology have renewed the interest in the properties of one-dimensional ($1D$) electron systems. It is well known that the electron–electron (e–e) interaction alters the properties of $1D$ systems qualitatively. For a better understanding of the problem, the physical question of primary importance must be clarified: what is the nature of the ground state of this system? The answer is usually sought in the studies of the ground state of $1D$ interacting fermions using the "density–density" correlation functions. However, this information is not direct and the results have to be interpreted. The anomalies obtained in correlation functions (see, e.g., [1]) were usually interpreted as an example of the Peierls instability [2, 3] (the part oscilating with $2p_F$) and as a marginal Wigner crystal [4] (oscillating with $4p_F$).

In Ref. [5], the ground-state wave function of spinless fermions was constructed for the exactly soluble Tomonaga–Luttinger model. It has been shown that at sufficiently low temperatures, the system should be in the state that has nothing in common with a system undergoing the Peierls transition. It is a state with a spontaneously broken chiral symmetry. Hence, the electron system is a system with a long-range order. In the limit of infinitely strong interaction, at low temperatures, a finite-density condensate is formed. It consists of neutral (exciton-like) pairs of a right electron and a left hole or vice versa. The uniqueness of a $1D$ system requires considering a second-order phase transition in a channel of a finite length $L_\parallel$. The point is that the phase transition temperature vanishes as $1/L_\parallel$, as it should. At the same time, we could not use the standard limit $L_\parallel \to \infty$ from the very beginning because the normalizing coefficient of the ground-state wave function tends to zero in this limit. (This phenomenon is well known as the "orthogonality catastrophe", see [6].) Hence, in passing to this limit, the result depends, on the one hand, on the method and on the other hand, on the step at which the limit was taken. (This point is carefully discussed in our previous paper [5].) We note that the phase transition temperature need not be too small. For $L_\parallel \sim 10^{-4}$ cm, it should be about 1 K.

The multi-component fermions in $1D$ systems were extensively discussed in the literature. In a number of papers [2, 3], separation of the spatial and spin degrees

*E-mail: vasili.afonin@mail.ioffe.ru

of freedom was considered. In the present paper, we discuss the form of the ground-state wave function for such a system. Because the variables separate, we can expect that the ground-state wave function is a direct product of two factors, with one of them, describing the spinless component, coincident with the ground-state wave function of spinless fermions. In the present paper, we show that an entirely different situation is actually realized. Namely, the most correlated state is the state where the total spin vanishes. However, for n-component electrons, this state consists not of pairs (as in the case of spinless fermions) but of point-like neutral complexes containing $2n$ particles and having the chirality $\pm 2n$.

In the ordinary electron system, $n = 2$, there are the complexes consisting of two right electrons and two left holes with opposite spins. Therefore, in accordance with Ref. [10], the ground state of the system is spinless. For the conducting nanotubes, $n = 4$ (Refs. [7–9]), and the complexes consist of eight particles. The complexes with a smaller number of particles can have a nonzero spin, but their correlation is much weaker. For example, for $n = 2$, the spin phase can be realized only as a Berezinskii–Kosterlitz–Thouless (BKT) phase, and in the limit of infinitely strong interaction, the complex density tends to zero as $1/\sqrt{L_\parallel}$. By contrast, the spinless phase has a finite density in this limit.

This situation is typical of many field-theory models with an Adler anomaly (this is the case for the Luttinger model as well). It is known that in such models, the new fermion interaction ("'t Hooft interaction" [11]) can appear with a vortex that is a product of all the components of fermions. In many cases, the 't Hooft interaction leads to a spontaneous breakdown of the chiral invariance with $2n$ fermion order parameters. In particular, this is the case for the multicomponent Schwinger model [12] and, as we see in what follows, for the Luttinger model in the limit of infinitely strong interaction.

For the last model, the most correlated state is built out of complexes each of which has the maximum possible number of the degrees of freedom. As a result, the state has a highest phase volume and appears to be the most correlated one. This state differs qualitatively from a marginal Wigner crystal. Instead of an almost first-order phase transition, an almost second-order phase transition occurs. To manifest the breakdown of the chiral symmetry in the Luttinger liquid, we have exactly calculated the wave function of the ground state in this model, Eq. (27), and explicitly demonstrated that its symmetry is less than the original symmetry of the Hamiltonian. (This is the definition of spontaneous symmetry breaking). These are basically the main results in this paper.

For a one-component fermion system, point-like complexes with more than two particles as well as collisions of such complexes are forbidden by the Pauli principle. This is not the case for multicomponent fermions. For this reason, the ground-state wave function in Eq. (27) is much more cumbersome and the calculations are more involved. Therefore, we present the results only for a short-range potential in the limit of infinitely strong interaction. At the same time, the physical picture in our case is quite similar to that of one-component fermions. For instance, had we taken corrections in the reciprocal strength of the interaction into account, we would have also obtained a BKT phase for spinless complexes. With the strength of the interaction increasing, it would be gradually transformed into a state of a definite condensate density.

This paper is organized as follows. In Sec. 1, we give a brief review of our results. Section 2 contains a discussion of the main difference between the multicomponent problem and the spinless one in regard to the theoretical description and most essential steps of the calculation. In Sec. 3, we give arguments concerning the applicability of our theory to nanotubes. The Appendix is devoted to a derivation of some intermediate results.

2. DESCRIPTION OF THE APPROACH AND THE DERIVATION OF MAIN RESULTS

Our starting point is the usual Tomonaga–Luttinger Hamiltonian (see, e.g, Refs. [3, 13]) for a system of interacting electrons with backscattering ignored. In the case where the interaction does not change the electron spin, the Hamiltonian can be expressed through the density of the right (R) and left (L) electrons

$$\varrho\,(x)_\alpha = \varrho_{R,\alpha}\,(x) + \varrho_{L,\alpha}\,(x)$$

(the spin index α equals $\pm$ for the spin $\pm 1/2$ respectively) as

$$H = \sum_\alpha \int dx \left[\hat\Psi^\dagger_{R,\alpha}\,(x)\, v_F\,(-i\partial_x)\,\hat\Psi_{R,\alpha}\,(x) + \right.$$
$$\left. + \hat\Psi^\dagger_{L,\alpha}\,(x)\, v_F\, i\partial_x\, \hat\Psi_{L,\alpha}\,(x)\right] +$$
$$+ \int dx\, dy\, \varrho\,(x)\, V\,(x-y)\, \varrho\,(y)\,, \quad (1)$$

where v_F is the Fermi velocity,

$$\hat{\Psi}_\alpha(x) = \exp(ip_F x)\,\hat{\Psi}_{R,\alpha}(x) + \exp(-ip_F x)\,\hat{\Psi}_{L,\alpha}(x), \quad (2)$$

and ϱ is the total density. For simplicity, we assume that the interaction is spin-independent (it is not too essential for calculations). We introduce the electron and hole operators in the usual way

$$\hat{\Psi}_{(R,L)\alpha}(x) = \int_0^\infty \frac{dp}{2\pi}\left(\exp(\pm ipx)\,\hat{a}_{(R,L)\alpha}(p) + \exp(\mp ipx)\,\hat{b}^\dagger_{(R,L)\alpha}(p)\right) = \hat{a}_{(R,L)\alpha}(x) + \hat{b}^\dagger_{(R,L)\alpha}(x), \quad (3)$$

where $\hat{a}_{R,+}(x)$ is the operator of annihilation of an electron with spin $+1/2$ and $\hat{b}_{R,+}(x)$ is the operator of annihilation of a hole with spin $-1/2$. To write this interaction via a functional integral, it is necessary to introduce one Bose field Φ and apply a version of the Hubbard–Stratonovich identity [14]:

$$\exp\left[\frac{i}{2}\int_0^T dt\int_{-\infty}^\infty \frac{dp}{2\pi}V(p)\,\varrho(p,t)\,\varrho(-p,t)\right] = \frac{1}{\mathcal{N}}\int \mathcal{D}\Phi\exp\left[\frac{i}{2}\int_0^T dt\int_{-\infty}^\infty \frac{dp}{2\pi}\times\right.$$
$$\times\,\Phi(p,t)\,\Phi(-p,t)\,V^{-1}(p) - \frac{i}{2}\int_0^T dt\int_{-\infty}^\infty \frac{dp}{2\pi}\times$$
$$\left.\times\,(\varrho(p,t)\,\Phi(-p,t) + \varrho(-p,t)\,\Phi(p,t))\right], \quad (4)$$

where $V(p)$ is the Fourier transform of the e–e interaction and the normalization factor is

$$\mathcal{N} = \int \mathcal{D}\Phi\times \exp\left[\frac{i}{2}\int_0^T dt\int_{-\infty}^\infty \frac{dp}{2\pi}\Phi(p,t)\,\Phi(-p,t)\,V^{-1}(p)\right]. \quad (5)$$

This identity shows that the theory with the e–e interaction is equivalent to the theory of noninteracting electrons in an external field Φ. (Had we considered a theory with a spin-dependent interaction, we would have to introduce two independent Bose fields. The calculation would be more cumbersome, but the physical picture would be the same.)

Calculation of the ground-state wave function is given in detail in Ref. [5]. It is based on the calculation of the evolution operator for the electrons

$$S(T) = \sum_{m,n}|n\rangle\langle n|\exp(-iHT)|m\rangle\langle m|, \quad (6)$$

where $|n\rangle$ are the exact wave functions of the Hamiltonian in the secondary quantization representation and T is the observation time. $S(T)$ determines the evolution of an arbitrary initial wave function ($\langle m|$) from the instant $t = 0$ up to final states (at $t = T$). (Henceforth, we imply that the Schrödinger representation for operators with time-dependent wave functions is used.)

Equation (6) suggests the general method to obtain the wave functions. We first calculate the evolution operator and present it as a sum of time-dependent exponentials. The coefficients in front of these exponentials are products of the exact wave functions and their complex conjugates. To extract the ground-state wave function, we then take the limit $T \to \infty$ (we add an infinitesimal imaginary part to the energy). Proceeding to the Euclidean time $T \to -i/\Theta$, we see that the evolution operator determines the density matrix for the equilibrium system at a nonzero temperature Θ.

As in the case of spinless electrons (see [5] for the details), the evolution operator for the electrons in an external field can be represented as

$$\hat{S}(\Phi) = \exp(\mathcal{S}_0 + \ln[\,\mathrm{Det}\,\Phi(T)\,])|F\rangle\langle F|. \quad (7)$$

Before the integration over the fields Φ, the equation for $\hat{S}(\Phi)$ undergoes some changes in comparison with the spinless case. They amount to the appearance of a factor n, the number of components of the electron wave function, in the equation describing the quantum fluctuations in the electron system under the action of the field $\Phi(T)$ (for the spin case $n = 2$):

$$\ln[\mathrm{Det}\,\Phi(T)] = -\frac{n}{4\pi}\int_0^T dt\,dt_1\times \times\int_{-\infty}^\infty \frac{dp}{2\pi}\Phi(-p,t)\,\Phi(p,t_1)\,|p|\exp[-i|p|v_F|t-t_1|]. \quad (8)$$

The operator structure of Eq. (7) is determined by the second part of the action, $\mathcal{S}_0$. Here, we should take the spin index α into account,

$$\mathcal{S}_0 = \\ = \sum_{i=R,L;\alpha} \int dx\, dx' \left[\hat{b}_{i,\alpha}(x')\, G_i^0(x',0;x,\epsilon)\, \hat{a}_{i,\alpha}(x) + \right. \\ + \hat{a}^\dagger_{i,\alpha}(x')\, G_i^0(x',T;x,T-\epsilon)\, \hat{b}^\dagger_{i,\alpha}(x) - \\ - \hat{a}^\dagger_{i,\alpha}(x')\, G_i^0(x',T;x,0)\, \hat{a}_{i,\alpha}(x) - \\ \left. - \hat{b}_{i,\alpha}(x')\, G_i^0(x',0;x,T)\, \hat{b}^\dagger_{i,\alpha}(x)\right], \quad (9)$$

where G_i^0 is the spin-independent free-particle Green's function

$$G^0_{R,L}(x,t;x_1,t_1) = \frac{1}{2\pi i} \times \\ \times \left[v_F(t-t_1) \mp (x-x_1) - i\delta\,\mathrm{sign}(t-t_1)\right]^{-1}. \quad (10)$$

Behavior of the multicomponent fermions in the external field is quite similar to that in the one-component case. An essential complication, however, occurs after the integration of the operator $\hat{S}(\Phi)$ over Φ with weight (4).

Two points should be indicated.

1) The coefficient n in Eq. (8) enters the equation for the action. Due to this coefficient, after calculation of the integral over Φ, the analytic structure of the resulting expression becomes much more complicated. As a result, integrals defining wave functions of multiparticle complexes acquire cuts instead of simple poles as in the one-component case. This leads to rather cumbersome complex wave functions. In particular, the complexes are nonlocal.

2) The property of nonlocality results in an essential increase in the number of various electron states. A number of electron states are forbidden for one-component electrons due to the Pauli principle. In contrast to this, in the multicomponent case, the number of connected diagrams becomes infinite. This makes the expression for the ground-state wave function rather cumbersome. However, the very fact of the existence of the chiral phase for the infinitely strong interaction persists.

Because of a non-Gaussian form of the final functional integral, it is impossible to perform the integration in Eq. (8) over $\Phi(x,t)$ in a closed form, but it is possible to obtain an arbitrary term of the evolution operator by expanding it in $(\mathcal{S}_0)^n$. This suffices for obtaining the ground-state wave function, because the relevant integral is of a Gaussian type and can be easily calculated. After the final integration over Φ, the final recipe of calculation of the evolution operator can be written as a sum of the terms (see [5], where similar calculation were done very carefully)

$$\mathcal{S}_0^{(n)}\left(\hat{a}_{(R,L)}, \hat{b}_{(R,L)}, \dots\right) \exp\left(\mathcal{S}_n^{eff}\right) |F\rangle\langle F|. \quad (11)$$

Here, $|F\rangle$ is the Fermi "sphere" and the term $\mathcal{S}_0^{(n)}$ (one term in the entire sum $(\mathcal{S}_0)^n$) determines the operator structure of the wave functions, i.e., a possible particle configuration due to the e–e interaction. Equation (11) is a sort of symbolic expression. Indeed, the analytic equation for the effective action $\mathcal{S}_n^{eff}$ in the nth term of the expansion depends explicitly on the particle configuration in the preexponential factor $\mathcal{S}_0^{(n)}$. Naturally, it is different for different terms. We note that evolution operator (6) is determined such that the initial state expressed through the electron and hole annihilation operators and the final state determined through the creation operators are given at different times. This means that in calculating the evolution operator, we should regard the operators $\hat{a}^\dagger_{R,L}(x)$ and $\hat{a}_{R,L}(y)$ as anticommuting.

In what follows, we give a prescription to write the effective action $\left(\mathcal{S}_n^{eff}\right)$ for a given electron–hole configuration. To write the expression for $\mathcal{S}_n^{eff}$, we introduce the following notation for the coordinates of the electron–hole creation–annihilation operators.

1. We let x (y) denote the coordinates of the right (left) particles.

2. We put a tilde on the coordinates related to annihilation operators (the initial state): the coordinates of creation operators (the final state) have no tilde.

3. We prime the hole coordinates.

The effective action differs from the action for the one-component fermions only by a factor and in the limit of strong interaction

$$\frac{V(p)}{\pi v_F} \gg 1$$

is equal to

$$\mathcal{S}_{eff} = -\frac{\pi}{nL} \sum_{m\neq 0} \frac{1}{|p_m|} \left[\mathcal{R}_f(-p, x_1 \dots) \times \right. \\ \left. \times\, \mathcal{R}_f(p, x_1 \dots) + \mathcal{R}_i(-p, \tilde{x}_1 \dots)\, \mathcal{R}_i(p, \tilde{x}_1 \dots)\right] - \\ - \frac{2\pi}{nL} \sum_{m\neq 0} \frac{1}{|p_m|} \exp\left(\frac{-|p_m| v_F}{\Theta}\right) \times \\ \times\, \mathcal{R}_f(-p, x_1 \dots)\, \mathcal{R}_i(p, \tilde{x}_1 \dots). \quad (12)$$

The extra factor n is the number of the fermion components. It occurs because in the Luttinger model, the excitation spectrum is [15]

$$\omega_p = |p| v_F \sqrt{1 + \frac{nV(p)}{\pi v_F}}. \quad (13)$$

Equation (12) is valid in the temperature region

$$\Theta_{chiral} \ll \Theta \ll \Theta_c = \omega\left(p_{min}\right).$$

The right-hand side of the last inequality is the energy of excitations with a minimal momentum (for periodic boundary conditions $p_{min} = 2\pi/L_{\|}$) and $\Theta_{chiral} = |p_{min}|v_F$ is the degeneracy temperature of the ground state. In this temperature region, the energy difference between the states with different chiralities can be neglected. This means that the ground states with a different chirality become degenerate.

The origin of this inequality has been discussed in detail in [5]. It is not sensitive to the number of wave function components. For the temperatures $\Theta \ll \Theta_{chiral}$, the last term in Eq. (12) should be omitted. Then the corresponding equation is also valid for the lowest temperatures.

The functions $\mathcal{R}_{i,f}\left(p, \tilde{x}_1 \dots\right)$ in Eq. (12) depend explicitly on the electron $(x \dots)$ and hole $(\tilde{x} \dots)$ coordinates in the preexponential factor in Eq. (11). These functions are given by

$$\begin{aligned}
\mathcal{R}_f\left(p, x \dots\right) &= \sum_{x \dots; x' \dots; y \dots; y' \dots} \theta\left(p\right)\left[\exp\left(ipx\right) - \right. \\
&\left. - \exp\left(ipx'\right)\right] + \theta\left(-p\right)\left[\exp\left(ipy\right) - \exp\left(ipy'\right)\right], \\
\mathcal{R}_i\left(p, \tilde{x} \dots\right) &= \sum_{\tilde{x} \dots; \tilde{x}' \dots; \tilde{y} \dots; \tilde{y}' \dots} \theta\left(-p\right)\left[\exp\left(ip\tilde{x}\right) - \right. \\
&\left. - \exp\left(ip\tilde{x}'\right)\right] + \theta\left(p\right)\left[\exp\left(ip\tilde{y}\right) - \exp\left(ip\tilde{y}'\right)\right].
\end{aligned} \tag{14}$$

To obtain the complete expression for the ground-state wave function, we have to consider all the complexes and separate their connected parts. This is not necessary, however, because according to the general theorem, the complete wave function is an exponential of the connected complexes [16][1)].

For one-component fermions, there is only one possible two-particle connected complex. This is not so in the multi-component case, where many of the scattering channels are possible and hence the number of connected diagrams is infinite. In principle, the exact wave function of any given complex can be calculated by taking the Gaussian integral over Φ. Unfortunately, this is not enough to present the whole wave function of the system in a closed form, but we do not actually need it to prove the existence of symmetry breaking. To verify this, it is sufficient to prove that the wave function symmetry is less than that of the Hamiltonian. For this, we analyze the simplest connected diagrams resulting in a spontaneous breaking of the symmetry of the Hamiltonian. The other terms either have the symmetry of the Hamiltonian or describe the scattering of the simplest correlated complexes and also violate the chiral symmetry.

In what follows, we restrict ourself to the case of a short-range interaction ($V(p) = V_0$) in the limit $nV_0/\pi v_F \gg 1$. Now we embark on analysis of the simplest diagrams of the evolution operator for the electrons having a spin. We begin with the temperature region $\Theta_{chiral} \gg \Theta$. In this case, the action and therefore the evolution operator factor, and hence we can explicitly consider the ground-state wave function $|\Omega\rangle$. (It is actually the ground state with the lowest energy, and is therefore realized at $\Theta = 0$.) The simplest nontrivial diagram we should be interested in is

$$\begin{aligned}
&\int \frac{dx_+ dx'_+ dx_- dx'_-}{\left(2\pi i\right)^2} \frac{dy_+ dy'_+ dy_- dy'_-}{\left(2\pi i\right)^2} \times \\
&\times \frac{\hat{a}^\dagger_{R,+}\left(x_+\right)\hat{b}^\dagger_{R,+}\left(x'_+\right)}{x'_+ - x_+ - i\delta} \frac{\hat{a}^\dagger_{R,-}\left(x_-\right)\hat{b}^\dagger_{R,-}\left(x'_-\right)}{x'_- - x_- - i\delta} \times \\
&\times \frac{\hat{a}^\dagger_{L,+}\left(y_+\right)\hat{b}^\dagger_{L,+}\left(y'_+\right)}{y_+ - y'_+ - i\delta} \frac{\hat{a}^\dagger_{L,-}\left(y_-\right)\hat{b}^\dagger_{L,-}\left(y'_-\right)}{y_- - y'_- - i\delta} \times \\
&\times \exp[\mathcal{S}^f_{eff}\left(x_+, \dots\right)]|F\rangle.
\end{aligned} \tag{15}$$

We see in what follows that the terms with a smaller number of operators give a weaker correlation than Eq. (15).

The action for this electrons–hole configuration is

$$\begin{aligned}
&\mathcal{S}^f_{eff}\left(x_\alpha, \dots\right) = \\
&= \frac{1}{2}\ln\frac{\prod_{\alpha,\alpha' \dots}\left(x_\alpha - y_\beta + i\delta\right)\left(x'_{\alpha'} - y'_{\beta'} + i\delta\right)}{\prod_{\alpha,\alpha' \dots}\left(x'_\alpha - y_\beta + i\delta\right)\left(x_{\alpha'} - y'_{\beta'} + i\delta\right)}.
\end{aligned} \tag{16}$$

In fact, it differs from the corresponding expression for one-component fermions by the factor 1/2 and by a greater number of the independent variables. The factor 1/2 does not permit calculating the integral in the problem explicitly because it involves cuts instead of the simple poles that occur in the one-component case.

Nevertheless, it is possible to recognize the spontaneous breakdown of the chiral symmetry in our system. For this, several steps are necessary. First, we have to analyze what new bound complexes appear as a result of the interaction. We have to take an arbitrary connected diagram and try to separate complexes with a smaller number of particles out of it. To do this, we have to consider all particles in one complex as being close to one another, whereas the distances between

[1)] This theorem is, in fact, a purely combinatorial statement. In field theory, we apply it mostly to Green's functions. In statistical physics, it is known as the first Mayer theorem (G. E. Uhlenbek, G. W. Ford, and E. W. Montroll, *Lectures in Statistical Mechanics*, American Mathematical Society, Providence (1963)).

different complexes are large. If the full wave function turns into a product of two wave functions in this limit, one of them depending only on the coordinates of the first complex and the other depending only on coordinates of the second one, then the complexes can be considered new "free particles"[2] because the probability to find one such complex is independent of the position of the other. In other words, we should represent a term of the expansion of the evolution operator we consider as a product of the form

$$\int \frac{dx_+ dy_+}{2\pi i} \dots \hat{a}^\dagger_{R,+}(x_+) \dots K(x_+, \dots, y_+ \dots;) \times \\ \times a^\dagger_{L,+}(y_+) \dots$$

Next, we should verify that if the complex with coordinates $(x_+, \dots)$ is moved away from the complex $(y_+ \dots)$ over the distance of the order of $L_\parallel$, then the amplitude $K(x_+, \dots, y_+, \dots)$ tends not to zero (as is usually the case with scattering amplitudes) but to the factored product $k(x_+, \dots) k_1(y_+, \dots)$, where each factor depends on the variables of the respective group. This means that the two complexes are formed as a result of the interaction. If the intercomplex distance is large enough, their contribution to the wave function can be represented as

$$\int \frac{dx_+}{\sqrt{2\pi i}} \dots k(x_+ \dots) \hat{a}^\dagger_{R,+}(x_+ \dots) \times \\ \times \int \frac{dy_+}{\sqrt{2\pi i}} k_1(y_+ \dots) \hat{a}^\dagger_{L,+}(y_+ \dots) |F\rangle.$$

The remaining part of K (which is $K - kk_1$) is a connected diagram that describes the intercomplex scattering under the condition that it tends to zero as the distance between the complexes increases. The theorem of logarithmic connectedness [16] guarantees that the same connected complexes appear in all orders with the correct combinatorial coefficients and the final answer is an exponential of connected complexes. In particular, the first-order term

$$\int \frac{dx_+}{\sqrt{2\pi i}} \dots k(x_+, \dots) \hat{a}^\dagger_{R,+}(x_+) \dots |F\rangle$$

should appear in the expansion of the evolution operator directly unless it is forbidden by some conservation law (e.g., the chirality conservation for the lowest-temperature case in our model). In this case, we have to use the projector on the proper state as in Eq. (24) below. It permits excluding the states forbidden by a conservation law.

[2] We note that the analogy with a bound state is quite limited. It would be more correct to write about a correlation in the momentum space.

Whether a symmetry breaks down depends on the symmetry of the complexes $\hat{a}^\dagger_{R,+}(x_+) \dots$ If they are less symmetric than the initial Hamiltonian, the symmetry is broken. As a result, it is possible to introduce a nonvanishing order parameter in the less symmetric phase (in the more symmetric phase, where the complexes do not exist, the order parameter vanishes). More precisely, taking fluctuations of the low-symmetry phase in the phase with a nonbroken symmetry into account, we can see that the order parameter should not increase with $L_\parallel$ in the high-symmetry phase. (Our definition of the order parameter is given below in Eq. (29).) The fluctuations result in well-known effects such as the Aslamasov–Larkin one [17]. According to Landau (see [18]), the appearance of the order parameter is the definition of the second-order phase transition. But if the symmetries of all the connected complexes and of the Hamiltonian are the same, there are long-range correlations without a spontaneous symmetry breaking.

By analogy with the theory of one-component fermions, we can assume that the simplest connected diagram originates from the term

$$\int \frac{dx_+ dx'_+}{2\pi i} \frac{dy_- dy'_-}{2\pi i} \frac{\hat{a}^\dagger_{R,+}(x_+) \hat{b}^\dagger_{R,+}(x'_+)}{x'_+ - x_+ - i\delta} \times \\ \times \frac{\hat{a}^\dagger_{L,-}(y_-) \hat{b}^\dagger_{L,-}(y'_-)}{y_- - y'_- - i\delta} \times \\ \times \exp[\mathcal{S}^f_{eff}(x_+, \dots)] |F\rangle. \quad (17)$$

However, because of the factor n^{-1}, the corresponding contribution to the action is

$$\mathcal{S}^f_{eff}(x_+, \dots) = \\ = \frac{1}{2} \ln \frac{(x_+ - y_- + i\delta)(x'_+ - y'_- + i\delta)}{(x'_+ - y_- + i\delta)(x_+ - y'_- + i\delta)}. \quad (18)$$

The bound chiral complexes are determined by the singularities of the integrand at $|x'_+ - y_-| \sim d$, $|x_+ - y'_-| \sim d$, and $|x_+ - y_-| \sim L_\parallel$ (where d is the width of the conductor). As a result, the contribution we are interested in is of the order of

$$\int dx_+ dy_- \hat{a}^\dagger_R(x_+) \hat{b}^\dagger_L(x_+) \hat{a}^\dagger_L(y_-) \times \\ \times \hat{b}^\dagger_R(y_-) \frac{d}{|x_+ - y_-|} |F\rangle.$$

This quantity tends to 0 as $|x_+ - y_-| \to L_\parallel \to \infty$, but more slowly than for a free particle[3].

[3] This means that even in the strong interaction limit, the spin phase can exist as a Kosterlitz–Thouless phase with the Thouless constant of the order of 1/2.

We now show that the most correlated state can be obtained from Eq. (15). It can be split into two four-particle complexes, each with zero spin, having the chirality charges ± 4: $\hat{a}^\dagger_{R,+}\hat{a}^\dagger_{R,-}\hat{b}^\dagger_{L,+}\hat{b}^\dagger_{L,-}$ and $\hat{a}^\dagger_{L,+}\hat{a}^\dagger_{L,-}\hat{b}^\dagger_{R,+}\hat{b}^\dagger_{R,-}$. (We assign the chirality $+1$ to a right electron and a left hole and -1 to their counterparts.) The amplitude K in Eq. (15) factores and does not tend to 0 as $L_\parallel \to \infty$. Indeed, the C-factor in the integrand is

$$K(x_+,\dots) = \frac{1}{x'_+ - x_+ - i\delta}\,\frac{1}{x'_- - x_- - i\delta}\times$$
$$\times\,\frac{1}{y_+ - y'_+ - i\delta}\,\frac{1}{y_- - y'_- - i\delta}\times$$
$$\times\,\frac{\sqrt{\prod_{\alpha,\alpha'\dots}(x_\alpha - y_\beta + i\delta)\left(x'_{\alpha'} - y'_{\beta'} + i\delta\right)}}{\sqrt{\prod_{\alpha,\alpha'\dots}(x'_\alpha - y_\beta + i\delta)\left(x_{\alpha'} - y'_{\beta'} + i\delta\right)}}. \quad (19)$$

We now consider the integration regions $x_+ \sim x_- \sim y'_- \sim y'_+$ and $x'_+ \sim x'_- \sim y_+ \sim y_-$ assuming that the distances between these groups of variables are of the order of $L_\parallel$. Then the amplitude K tends to

$$V_{+4}(x_+,\dots)\,V_{-4}(x'_+,\dots) = 1/\sqrt{(x_+ - y'_+ + i\delta)(x_+ - y'_- + i\delta)(x_- - y'_+ + i\delta)(x_- - y'_- + i\delta)}\times$$
$$\times\,1/\sqrt{(x'_+ - y_+ + i\delta)(x'_+ - y_- + i\delta)(x'_- - y_+ + i\delta)(x'_- - y_- + i\delta)}. \quad (20)$$

This means that each amplitude V depends on the variables belonging either to the first or to the second group. This property of the amplitude allows indentifying the full equation for the evolution operator of the connected complexes and the amplitude of intercomplex scattering that tends to zero for large intercomplex distances.

This term of the expansion, besides the chiral complexes, has also neutral complexes with zero chirality, $\hat{a}^\dagger_{R,+}\hat{b}^\dagger_{R,+}\hat{a}^\dagger_{L,-}\hat{b}^\dagger_{L,-}$ and $\hat{a}^\dagger_{R,-}\hat{b}^\dagger_{R,-}\hat{a}^\dagger_{L,+}\hat{b}^\dagger_{L,+}$. They do not violate the symmetry of the Hamiltonian. But they should be isolated in order that the remaining scattering amplitude tend to zero in the whole region of the variables. This permits interpreting it as the intercomplex scattering amplitude. The zero-chirality complexes are not important for the existence of a phase transition. However, they should be taken into account in the calculation of matrix elements because they are not small. To check that they exist, we consider the regions $x_+ \sim x'_+ \sim y_- \sim y'_-$ and $x_- \sim x'_- \sim y_+ \sim y'_+$ in (15). In these regions, the amplitude K tends to $V_0(x_+,\dots)\,V_0(x_-,\dots)$ where

$$V_0(x_\alpha,\dots) = \left[(x'_\alpha - x_\alpha - i\delta)\left(y_{-\alpha} - y'_{-\alpha} - i\delta\right)\right]^{-1}\times$$
$$\times\left[(x_\alpha - y_{-\alpha} + i\delta)\left(x'_\alpha - y'_{-\alpha} + i\delta\right)\right]^{1/2}\times$$
$$\times\left[\left(x_\alpha - y'_{-\alpha} + i\delta\right)\left(x'_\alpha - y'_{-\alpha} + i\delta\right)\right]^{-1/2}. \quad (21)$$

This means that this quantity can be represented as a product of the amplitudes each of which remains finite as the distance between them tends to infinity.

It is now convenient to introduce the intercomplex scattering amplitude V_{coll}. By derivation, it tends to zero with the intercomplex distance tending to ∞:

$$V_{coll}(x_+,\dots) = K(x_+,\dots) - V_{+4}(x_+,\dots)\times$$
$$\times\,V_{-4}(x'_+,\dots) - V_0(x_+,\dots)\,V_0(x_-,\dots). \quad (22)$$

The contribution to the ground-state wave function can be represented in terms of these amplitudes as

$$\int \frac{dx_+dx'_+dx_-dx'_-}{(2\pi i)^2}\,\frac{dy_+dy'_+dy_-dy'_-}{(2\pi i)^2}\times$$
$$\times\,\hat{a}^\dagger_{R,+}(x_+)\,\hat{b}^\dagger_{R,+}(x'_+)\,\hat{a}^\dagger_{R,-}(x_-)\times$$
$$\times\hat{b}^\dagger_{R,-}(x'_-)\,\hat{a}^\dagger_{L,+}(y_+)\,\hat{b}^\dagger_{L,+}(y'_+)\,\hat{a}^\dagger_{L,-}(y_-)\,\hat{b}^\dagger_{L,-}(y'_-)\times$$
$$\times\left(V_{+4}(x_+,\dots)\,V_{-4}(x'_+,\dots) + \right.$$
$$\left. + V_0(x_+,\dots)\,V_0(x_-,\dots) + V_{coll}(x_+,\dots)\right)|F\rangle. \quad (23)$$

The first term here describes noninteracting complexes with a nonzero chirality, the second describes 4-particle neutral complexes, and the third (the connected part) describes their collision. We are mainly interested in the first term because it is related to the chiral symmetry breakdown.

The chiral complex that we have obtained is already connected and cannot be split into simpler ones. This means that its wave function is a decreasing function of the interparticle distances. It is shown in the Appendix that the probability to find particles of the complex far from each other is negligibly small.

In addition, we should take into consideration that in the temperature interval $\Theta \ll \Theta_{chiral}$, where the last term in Eq. (12) need not be considered, there is a one-to-one correspondence between complexes with the chiralities $Q = 4$ and $Q = -4$. Hence, the total

chirality of the state is zero. The theorem of logarithmic connectedness states that the ground-state wave function can be represented as

$$
\begin{aligned}
|\Omega\rangle = P_{Q=0} \times \\
\times \exp \sum_{\alpha} \mathrm{Tr} \left[\frac{1}{(2\pi i)^2} V_{+4}\left(x_\alpha, x_{-\alpha}, y'_{-\alpha}, y'_\alpha\right) \times \right. \\
\times \left(\hat{a}^\dagger_{R,\alpha}\left(x_\alpha\right) \hat{a}^\dagger_{R,-\alpha}\left(x_{-\alpha}\right) \hat{b}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right) \times \right. \\
\times \hat{b}^\dagger_{L,\alpha}\left(y'_\alpha\right) + \hat{a}^\dagger_{L,\alpha}\left(y'_\alpha\right) \hat{a}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right) \times \\
\left. \times \hat{b}^\dagger_{R,-\alpha}\left(x_{-\alpha}\right) \hat{b}^\dagger_{R,\alpha}\left(x_\alpha\right) \right) + \\
+ \frac{1}{(2\pi i)^2} V_{+0}\left(x_\alpha, x_{-\alpha}, y'_{-\alpha}, y'_\alpha\right) \left(\hat{a}^\dagger_{R,\alpha}\left(x_\alpha\right) \times \right. \\
\times \hat{b}^\dagger_{R,\alpha}\left(x'_\alpha\right) \hat{a}^\dagger_{L,-\alpha}\left(y_{-\alpha}\right) \hat{b}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right) + \\
\left. + \hat{a}^\dagger_{R,-\alpha}\left(x_-\alpha\right) \hat{b}^\dagger_{R,-\alpha}\left(x'_{-\alpha}\right) \hat{a}^\dagger_{L,\alpha}\left(y_\alpha\right) \hat{b}^\dagger_{L,\alpha}\left(y'_\alpha\right) \right) + \\
+ \frac{1}{(2\pi i)^4} V_{coll}\left(x_\alpha, x_{-\alpha}, \dots\right) \hat{a}^\dagger_{R,\alpha}\left(x_\alpha\right) \hat{a}^\dagger_{R,-\alpha}\left(x_{-\alpha}\right) \times \\
\times \hat{b}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right) \hat{b}^\dagger_{L,\alpha}\left(y'_\alpha\right) \hat{a}^\dagger_{L,\alpha}\left(y_\alpha\right) \times \\
\left. \times \hat{a}^\dagger_{L,-\alpha}\left(y_{-\alpha}\right) \hat{b}^\dagger_{R,-\alpha}\left(x'_{-\alpha}\right) \hat{b}^\dagger_{R,\alpha}\left(x'_\alpha\right) + \dots \right], \quad (24)
\end{aligned}
$$

where $P_{Q=0}$ is the projector on the state with zero chirality. The symbol Tr includes the integrations over the particle coordinates. The terms omitted in Eq. (24) describe collision of three and more complexes, while all the elementary complexes are present here. We note that the complexes with a nonzero chirality have appeared in the theory. Nevertheless, the wave function of the ground state as a whole describes the state with $Q = 0$, i.e., the symmetry of the ground state is the same as that of the Hamiltonian. The states with a nonzero chirality have a higher energy (of the order of $2\pi v_F/L_\parallel$). Therefore, the spontaneous symmetry breaking may occur only in the region of higher temperatures $\Theta \gg \Theta_{chiral}$, where such an energy difference is not essential. In this temperature region, the term where the time arguments of the Green's functions differ by T must also be considered in Eq. (9) for the action $\mathcal{S}_0$. (In practice, it is more convenient to introduce the temperature by the replacement $T \to -i/\Theta$ in the final equations.) We then have the following nontrivial term in the evolution operator:

$$
\begin{aligned}
\mathrm{Tr} \frac{1}{(2\pi i)^4} \frac{\hat{a}^\dagger_{R,+}\left(x_+\right) \hat{a}_R\left(\tilde{x}_+\right)}{\tilde{x}_+ - x_+ + v_F T - i\delta} \frac{\hat{a}^\dagger_{R,-}\left(x_-\right) \hat{a}_R\left(\tilde{x}_-\right)}{\tilde{x}_- - x_- + v_F T - i\delta} \times \\
\times \frac{\hat{b}^\dagger_{L,+}\left(y'_+\right) \hat{b}_L\left(\tilde{y}'_+\right)}{\tilde{y}'_+ - y'_+ - v_F T + i\delta} \frac{\hat{b}^\dagger_{L,-}\left(y'_-\right) \hat{b}_L\left(\tilde{y}'_-\right)}{\tilde{y}'_- - y'_- - v_F T + i\delta} \times \\
\times \exp[\mathcal{S}^f_{eff}]\left(x_+, \dots\right) |F\rangle\langle F|. \quad (25)
\end{aligned}
$$

The action $\mathcal{S}^f_{eff}$ for this configuration is

$$
\frac{1}{2} \ln \frac{\prod_{\alpha,\alpha'\dots} \left(\tilde{y}'_\alpha - y'_\beta - v_F T + i\delta\right) \left(x_{\alpha'} - \tilde{x}_{\beta'} - v_F T + i\delta\right)}{\prod_{\alpha,\alpha'\dots} \left(x_\alpha - y'_\beta + i\delta\right) \left(\tilde{y}'_{\alpha'} - \tilde{x}_{\beta'} + i\delta\right)}. \quad (26)
$$

It is readily seen from the operator structure of this term that the amplitude V_4 appears here automatically (without extracting the amplitudes of neutral complexes and scattering channels), as it should. This is a consequence of the theorem of logarithmic connectedness. It guarantees the coincidence of the amplitude in this term with V_4. We verify this. We consider the region where the same variables with spin up and spin down are quite close to each other (for instance, $x_\alpha \sim x_{-\alpha}$). Besides, the coordinates in the creation and annihilation operators are apart at the distance of the order of $L_\parallel$ ($x_\alpha \sim \tilde{x}_\alpha \sim L_\parallel \gg 2\pi v_F/\Theta$). Then the C-factor in the integrand of (25) turns into the factored expression

$$
V_{+4}\left(x_+, \dots\right) V^*_{-4}\left(\tilde{x}_+, \dots\right).
$$

This proves that the chiral four-particle complexes were segregated properly from a more complicated expression (15). Such terms in the evolution operator result in the existence of the ground state with a nonzero chirality. But any state with fixed chiralities should be unstable under weak backscattering once states with different chiralities are degenerate. Therefore, we should consider a superposition of all such states. As a result, in the same way as in the theory of superconductivity, we have to introduce a condensate with a fixed phase instead of a state with a fixed chirality:

$$
\begin{aligned}
|\Omega\rangle_\theta = \exp \sum_{\alpha} \mathrm{Tr} \left[\frac{1}{(2\pi i)^2} V_{+4}\left(x_\alpha, x_{-\alpha}, y'_{-\alpha}, y'_\alpha\right) \times \right. \\
\times \left(\exp\left(i\theta\right) \hat{a}^\dagger_{R,\alpha}\left(x_\alpha\right) \hat{a}^\dagger_{R,-\alpha}\left(x_{-\alpha}\right) \times \right. \\
\left. \times \hat{b}^\dagger_{L,-\alpha}\left(y'_{-\alpha}\right) \hat{b}^\dagger_{L,\alpha}\left(y'_\alpha\right) + \exp\left(-i\theta\right) \hat{a}^\dagger_{L,\alpha}\left(y'_\alpha\right) \times \right.
\end{aligned}
$$

$$\times \hat{a}^{\dagger}_{L,-\alpha}(y'_{-\alpha})\,\hat{b}^{\dagger}_{R,-\alpha}(x_{-\alpha})\,\hat{b}^{\dagger}_{R,\alpha}(x_{\alpha})\Big) + \\ + \frac{1}{(2\pi i)^2}V_{+0}\left(x_{\alpha},x_{-\alpha},y'_{-\alpha},y'_{\alpha}\right)\Big(\hat{a}^{\dagger}_{R,\alpha}(x_{\alpha})\times \\ \times \hat{b}^{\dagger}_{R,\alpha}(x'_{\alpha})\,\hat{a}^{\dagger}_{L,-\alpha}(y_{-\alpha})\,\hat{b}^{\dagger}_{L,-\alpha}(y'_{-\alpha}) + \\ + \hat{a}^{\dagger}_{R,-\alpha}(x_{-}\alpha)\,\hat{b}^{\dagger}_{R,-\alpha}(x'_{-\alpha})\,\hat{a}^{\dagger}_{L,\alpha}(y_{\alpha})\,\hat{b}^{\dagger}_{L,\alpha}(y'_{\alpha})\Big) + \\ + \frac{1}{(2\pi i)^4}V_{coll}(x_{\alpha},x_{-\alpha},\dots)\,\hat{a}^{\dagger}_{R,\alpha}(x_{\alpha})\,\hat{a}^{\dagger}_{R,-\alpha}(x_{-\alpha})\times \\ \times \hat{b}^{\dagger}_{L,-\alpha}(y'_{-\alpha})\,\hat{b}^{\dagger}_{L,\alpha}(y'_{\alpha})\,\hat{a}^{\dagger}_{L,\alpha}(y_{\alpha})\times \\ \times \hat{a}^{\dagger}_{L,-\alpha}(y_{-\alpha})\,\hat{b}^{\dagger}_{R,-\alpha}(x'_{-\alpha})\,\hat{b}^{\dagger}_{R,\alpha}(x'_{\alpha})+\dots\Big]. \quad (27)$$

Equation (27) demonstrates that for a strong electron–electron interaction, a spontaneous symmetry breaking occurs. The first two terms in the equation for the ground-state wave function are not invariant under the chiral transformation

$$\Psi_{R,L}(x) \to \exp(\pm i\Lambda)\Psi_{R,L}, \quad (28)$$

while the Hamiltonian retains the invariance. Here, Λ is an arbitrary constant.

Such a form of the bound state permits introducing the order parameter. It is equal to zero in the high-symmetry phase ($\Theta \gg \Theta_c = \omega_{p_{min}}$) and is proportional to $L_{\|}$ in the low-symmetry phase (in the low-temperatures region). (These statements can be proved using ordinary symmetry considerations or by direct analytic calculation.) In our case, the following quantity can be considered an order parameter:

$$\int dx\ {}_{\theta}\langle\Omega|\hat{a}^{\dagger}_{R,\alpha}(x)\,\hat{a}^{\dagger}_{R,-\alpha}(x)\times \\ \times \hat{b}^{\dagger}_{L,\alpha}(x)\,\hat{b}^{\dagger}_{L,-\alpha}(x)\,|\Omega\rangle_{\theta}. \quad (29)$$

We can see from Eq. (29) why the second-order phase transition requires the chirality degeneracy of the ground state. For the order parameter to be nonvanishing, it is necessary to be able to add an extra four-particle complex to the ground state. This requires the condition $\Theta \gg \Theta_{chiral}$ because the zero-chirality state has the lowest energy and the energy difference between states with different chiralities is about Θ_{chiral}. There is also an upper bound for existence of the low-symmetry phase,

$$\Theta \ll \omega_{p_{min}}. \quad (30)$$

This limitation is because the long-range order is suppressed by thermal excitations in one-dimensional systems. In the temperature region given by (30), it can be neglected; this condition is independent of the number of fermion components and is discussed in detail in [5]. Hence, the temperature region

$$\Theta_{chiral} \ll \Theta \ll \Theta_c \quad (31)$$

is the region where the broken symmetry phase exists.

3. CARBON NANOTUBES

Conducting carbon nanotubes may give one more example of multicomponent electrons. The theory developed above can be generalized to this case. We consider one-dimensional conducting tubes and the e–e interaction such that it can be rewritten in the density–density form (cf. Eq. (1)). This means that the backward and inter-component (see below) scattering can be neglected. Following Ref. [19], we can visualize a nanotube as a cylinder constructed of a monoatomic layer of graphite. Graphite has a lattice of adjoining regular hexagons, with the angle between the neighboring basis vectors $\mathbf{a}$ and $\mathbf{b}$ being $2\pi/3$. Choosing the coordinates ξ_1 and ξ_2 such that the $0\xi_1$ axis is parallel to the vector $\mathbf{a}$ and the $0\xi_2$ axis is perpendicular to $\mathbf{a}$, we can represent these vectors as

$$\mathbf{a} = a(1,0), \quad \mathbf{b} = a\left(-1/2,\sqrt{3}/2\right). \quad (32)$$

Here, a is the lattice constant, equal to $d\sqrt{3}$, d =1.44 Å being the interatomic distance [20].

The circumferential vector $\mathbf{L}$ can be written as

$$\mathbf{L} = n_a\mathbf{a} + n_b\mathbf{b}, \quad (33)$$

where n_a and n_b are integers.

The electron effective Hamiltonian for a graphite sheet is

$$H = \begin{pmatrix} 0 & h^* \\ h & 0 \end{pmatrix}. \quad (34)$$

It can be expanded in the vicinity of the points

$$\mathbf{P} = \frac{4\pi}{3a}(-1,0), \quad \mathbf{P}' = \frac{4\pi}{3a}(1,0) \quad (35)$$

up to the first order in the small deviations $\mathbf{p}$ and $\mathbf{p}'$ from the values respectively given by the first and the second equation in (35),

$$h(\mathbf{P},\mathbf{p}) = \gamma e^{-i\theta}(p_{\perp} - ip_z), \\ h(\mathbf{P}',\mathbf{p}') = \gamma e^{i\theta}(-p'_{\perp} - ip'_z), \quad (36)$$

where $\gamma = (\sqrt{3}/2)\gamma_0 a$, $\gamma_0 \approx 3$ eV (see Eqs. (21) and (22)) is the transfer integral between the neighboring π orbitals, and θ is the angle between the vectors $\mathbf{L}$

and **a**. The subscripts z and $\perp$ refer to the components of $\mathbf{p}$ relative to the direction of $\mathbf{L}$, $p_z \perp \mathbf{L}$ and $p_\perp \| \mathbf{L}$, such that $p_z (p_\perp)$ is parallel (perpendicular) to the axis of the tube.

The spectrum near the point $\mathbf{P}$ is given by

$$E(\mathbf{P},\mathbf{p}) = \pm\gamma\sqrt{p_z^2 + p_\perp^2}, \tag{37}$$

where the upper (lower) sign corresponds to the conduction (valence) band in this equation. The spectrum near the $\mathbf{P}'$ point is obtained by the replacement $\mathbf{p} \to \mathbf{p}'$.

The electron wave function $\Psi(\mathbf{r})$ must satisfy the cyclic boundary condition

$$\Psi(\mathbf{r}) = \Psi(\mathbf{r} + \mathbf{L}), \tag{38}$$

and hence the discrete values of $p_\perp$ and $p'_\perp$ are given by (see Ref. [19])

$$p_\perp = \frac{2\pi}{|\mathbf{L}|}\left(n - \frac{\nu}{3}\right), \quad p'_\perp = \frac{2\pi}{|\mathbf{L}|}\left(n + \frac{\nu}{3}\right), \tag{39}$$

where $n = 0, \pm1, \pm2, \ldots$, $|\mathbf{L}| = a\sqrt{n_a^2 + n_b^2 - n_a n_b}$, and $\nu = 0$ or ±1 is determined by representing the sum $n_a + n_b$ as $3N + \nu$ (with N being an integer). The nanotubes are conductive (metallic) for the combination

$$n = \nu = 0 \tag{40}$$

and we consider this case in what follows. In other words, there are two conic bands in such tubes, i.e., the points $\alpha_P \mathbf{P}$ with $\alpha_P = \pm1$. The large phase corresponding to the quasimomentum $\alpha_P \mathbf{P}$ should be extracted in the same way as this was done in Eq. (2). In addition, we assume that due to the presence of gate electrodes, the Fermi level is well above (or below) the points $\alpha_P \mathbf{P}$ (cf. Ref. [8]). As a result, we have a theory with four-component fermions. There are two extra branches corresponding to two values of α_P. In each of them, there are analogues of R- and L-particles. To establish a correspondence between the present model and the Luttinger model, we should be able to neglect the transitions both between different branches (different values of α_P) and between R- and L-particles within the same branch. As indicated in Ref. [21], nanotubes have relatively large radii that encompass $N \gg 1$ atoms. Therefore, the only e–e scattering that is important in this limit is the forward scattering with a small quasimomentum transfer. The matrix element describing the backscattering within a band as well as the $\mathbf{P} \leftrightarrow \mathbf{P}'$ scattering acquire an extra small factor of the order of $1/N$. This is why these types of scattering can be neglected. This means that we can use the results obtained in Sec. 2. Repeating the arguments in that section for $n = 4$, we conclude that a condensate is formed in the ground state. It consists of the 8-plets of the form

$$\hat{a}^\dagger_{R,\alpha,\alpha_P}(x)\,\hat{a}^\dagger_{R,-\alpha,\alpha_P}(x)\,\hat{b}^\dagger_{L,\alpha,\alpha_P}(x)\,\hat{b}^\dagger_{L,-\alpha,\alpha_P}(x) \times \\ \times \hat{a}^\dagger_{R,\alpha,-\alpha_P}(x)\,\hat{a}^\dagger_{R,-\alpha,-\alpha_P}(x)\,\hat{b}^\dagger_{L,\alpha,-\alpha_P}(x) \times \\ \times \hat{b}^\dagger_{L,-\alpha,-\alpha_P}(x).$$

Their chirality is ±8.

4. CONCLUSION

We have demonstrated that the ground-state wave function of the system of $1D$ n-component electrons consists of point-like neutral complexes containing $2n$ particles and having the chirality $\pm2n$. It is essential to distinguish between three temperature regions:

1) The lowest temperature region $\Theta \ll \Theta_{chiral} = 2\pi v_F/L_\|$. Here, the numbers of the complexes with opposite chiralities are equal. Hence, the chirality of the state is zero and this is a state without symmetry breaking (see Eq. (24)). We can say that the size of the system is too small for the existence of a long-range order.

2) The intermediate region $2\pi v_F^c/L_\| \gg \Theta \gg 2\pi v_F/L_\|$. Here, the states with different chiralities are degenerate in energy ($v_F^c = v_F\sqrt{1 + nV_0/\pi v_F}$ is the renormalized excitation velocity, see Eq. (13)). This allows constructing a symmetry-breaking wave function, Eq. (27), although the Hamiltonian has no symmetry-breaking term. This means that adding a chiral complex to the condensate does not cost any energy. (An analogous property is imperative for any system with a second-order phase transition.) Therefore, the degeneracy is possible only if the size of the system is large enough and $L_\|^{min} \sim 2\pi v_F/\Theta$. A specimen can be considered infinite with regard to symmetry breaking if its size is bigger than $L_\|^{min}$.

3) In the high-temperature region $\Theta \gg \Theta_c = 2\pi v_F^c/L_\|$, the thermal excitations destroy the phase with a long-range order. The unique property of a $1D$ electron system is that the phase transition temperature tends to zero in an infinite sample. This is not the case for a $3D$ system. As it should, the upper temperature border of the chiral phase coincides with the temperature region where power-law correlators exist. To verify this statement, we recall that at a finite temperature, the correlators decrease exponentially at dis-

tances longer than v_F^c/Θ. In order not to reach the region within the specimen where the asymptotic behavior is exponential, the size of the specimen has to be smaller than v_F^c/Θ or, in other words, the temperature must be such that $\Theta \ll v_F^c/L_\parallel$.

For a real channel, the temperature Θ_c is not so small. For $v_F \sim 10^7$ cm/s and $L_\parallel \sim 10^{-4}$ cm, we have $\Theta_c \sim 1$ K (v_F^c/v_F).

In summary, in the system of interacting multicomponent fermions, the most correlated state consists of spinless point-like neutral complexes each containing $2n$ partticles with chiralities $\pm 2n$. In the limit of the infinitely strong interaction, the density of the complexes is finite. The phase with a broken symmetry exists in the temperature region $\Theta_{chiral} \ll \Theta \ll \Theta_c$.

We discuss an effect that could indicate the existence of the chiral phase. Because of the charge neutrality of the condensate, it is difficult to believe that an effect specific for the broken-symmetry system can be found in experiment associated with charge transfer. At the same time, the condensate cannot transfer heat but can transfer energy. This means that an effect similar to the thermomechanical effect in superfluid helium may be expected (decreasing the temperature of the sample with increasing the superfluid mass [22]). In this connection, we note that chiral complexes can be added to the condensate using an external electric field with a nonzero topological number (see Refs. [23, 24]).

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APPENDIX

Characteristic dimensions of correlated complexes

The simplest way to prove the existence of a bound chiral complex and to determine its characteristic dimensions is to consider the state with a single chiral complex:

$$|\Phi_c\rangle = \\ = \mathrm{Tr}\left[\frac{\hat a^\dagger_{R,+}(x_+)\hat a^\dagger_{R,-}(x_-)\hat b^\dagger_{L,-}(y'_-)\hat b^\dagger_{L,+}(y'_+)}{\sqrt{\prod_{\alpha,\alpha'=\pm}(x_\alpha - y'_{\alpha'} + i\delta)(x_\alpha - y'_{\alpha'} + i\delta)}}\right] \times \\ \times |F\rangle. \quad \text{(A.1)}$$

For instance, we can calculate the probability to find an electron with spin up at a distance $|z_+ - z_-| > d$ from the electron with spin down as

$$\langle\Phi_c|\hat a^\dagger_{R,+}(z_+)\, a_{R,+}(z_+)\, \hat a^\dagger_{R,-}(z_-)\, \hat a_{R,-}(z_-)\,|\Phi_c\rangle.$$

Moving all the creation operators to the right and all the annihilation operators to the left, we obtain $A^2/|z_+ - z_-|^2$, where A is a constant equal to

$$\int dx \left((1+x)^2 + \epsilon^2\right)^{-1/2} \left((1-x)^2 + \epsilon^2\right)^{-1/2},$$

where ϵ is an infinitesimal parameter. This means that the most probable is the particle configuration where $|z_+ - z_-| \sim d$ (to obtain a physical parameter, we should replace, as usual, δ by d). In other words, the right and left electrons with opposite spins are always near one another, forming a chiral complex. In this sense, the chiral 4-plets are point-like entities, as the $R\overline{L}$-pairs for one-component fermions. In the same way, we can estimate the distances between all the particles belonging to a four-particle complex.

The same calculation for a neutral four-particle complex is somewhat more cumbersome. The state with a single neutral pair is described by the wave function

$$|\Phi_0\rangle = \mathrm{Tr}\Big[\hat a^\dagger_{R,-}(x_-)\,\hat b^\dagger_{R,-}(x'_-)\,\hat a^\dagger_{L,+}(y_+)\,\hat b^\dagger_{L,+}(y'_+)\times \\ \times (x'_- - x_- - i\delta)^{-1}(y_+ - y'_+ - i\delta)^{-1}\times \\ \times \left[(x_- - y_+ + i\delta)(x'_- - y'_+ + i\delta)\right]^{1/2}\times \\ \times \left[(x_- - y'_+ + i\delta)(x'_- - y_+ + i\delta)\right]^{-1/2}|F\rangle. \quad \text{(A.2)}$$

To find the characteristic size of a neutral complex, we consider the matrix element

$$\langle\Phi_0|\hat a^\dagger_{R,-}(z_-)\, a_{R,-}(z_-)\, \hat a^\dagger_{L,+}(z_+)\, \hat a_{L+}(z_+)\,|\Phi_0\rangle.$$

It is equal to

$$\mathrm{Tr}'\Bigg[\left(\tilde{x}'_- - \overline{\tilde{x}'}_- - i\delta\right)^{-1}\left(\overline{\tilde{y}'}_+ - \tilde{y}'_+ - i\delta\right)^{-1}\times$$

$$\times\left(\overline{\tilde{x}'}_- + i\delta\right)^{-1}\left(-\overline{\tilde{y}'}_+ + i\delta\right)^{-1}\times$$

$$\times\left[\left(z_- - z_+ - i\delta\right)\left(z_- - z_+ + \overline{\tilde{x}'}_- - \overline{\tilde{y}'}_+ - i\delta\right)\right]^{1/2}\times$$

$$\times\left[\left(z_- - z_+ - \overline{\tilde{y}'}_+ - i\delta\right)\left(z_- - z_+ + \overline{\tilde{x}'}_- - i\delta\right)\right]^{-1/2}\times$$

$$\times\left(\tilde{x}'_- - i\delta\right)^{-1}\left(-\tilde{y}'_+ - i\delta\right)^{-1}\times$$

$$\times\left[\left(z_- - z_+ + i\delta\right)\left(z_- - z_+ + \tilde{x}'_- - \tilde{y}'_+ + i\delta\right)\right]^{1/2}\times$$

$$\times\left[\left(z_- - z_+ - \tilde{y}'_+ + i\delta\right)\times\right.$$

$$\left.\times\left(z_- - z_+ + \tilde{x}'_- + i\delta\right)\right]^{-1/2}\Bigg] \quad \text{(A.3)}$$

(Tr' means that we integrate over all variables except z_α). The exact expression for this matrix element for arbitrary values of $\Delta Z = z_+ - z_-$ is rather cumbersome and noninformative. It suffices to prove that the most probable is the particle configuration where $\Delta Z \sim d$.

For this, we note that for $\Delta Z \sim \delta$, all the integrals in (A.3) converge and are dominated by the regions of the order of δ. We show that for $\Delta Z \gg \delta$, matrix element (A.3) has an additional small factor $\delta/\Delta Z$. For instance, we consider the integration over $\tilde{x}'_-$. Only the factor

$$\frac{\sqrt{\left(z_- - z_+ + \tilde{x}'_- - \tilde{y}'_+ + i\delta\right)}}{\sqrt{\left(z_- - z_+ + \tilde{x}'_- + i\delta\right)}} \quad \text{(A.4)}$$

in the integrand has a singularity in the lower half-plane. The rest of the integrand is independent of ΔZ and has singularities only in the upper half-plane. In the leading approximation in $\delta/\Delta Z$, (A.4) tends to 1 ($\Delta Z \gg \delta$, while the region $\tilde{x}'_-, \tilde{y}'_+$ giving the leading contribution to the integral is of the order of δ). Hence, in this approximation, the integral of (A.3) vanishes. It is nonzero only in the next approximation due to the factors of the type $\sqrt{\left(\tilde{x}'_- + i\delta\right)/\left(z_- - z_+\right)} \ll 1$. Hence, the probability to find the electrons at a large distance is small. The most probable is a 4-plet where these particles are at a distances about d.

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