

# BILEPTON CONTRIBUTIONS TO THE NEUTRINOLESS DOUBLE BETA DECAY IN THE ECONOMICAL 3–3–1 MODEL

*D. V. Soa<sup>a\*</sup>, P. V. Dong<sup>b\*\*</sup>, T. T. Huong<sup>b</sup>, H. N. Long<sup>b</sup>*

<sup>a</sup>*Department of Physics, Hanoi University of Education  
10000, Hanoi, Vietnam*

<sup>b</sup>*Institute of Physics, VAST  
10000, Hanoi, Vietnam*

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A new bound of the mixing angle between charged gauge bosons (the standard-model  $W$  and the bilepton  $Y$ ) in the economical 3–3–1 model is given. Possible contributions of the charged bileptons to the neutrinoless double beta  $((\beta\beta)_{0\nu})$  decay are discussed. We show that the  $(\beta\beta)_{0\nu}$  decay in this model is due to both the Majorana  $\langle M_\nu \rangle_L$  and Dirac  $\langle M_\nu \rangle_D$  neutrino masses. If the mixing angle is in the range of the ratio of neutrino masses  $\langle M_\nu \rangle_L / \langle M_\nu \rangle_D$ , the Majorana and Dirac masses are comparable to each other and both may give the main contribution to the decay. As a result, constraints on the bilepton mass are given.

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## 1. INTRODUCTION

The  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  standard model (SM) of the strong and electroweak interactions, with the  $SU(2)_L \otimes U(1)_Y$  symmetry spontaneously broken down to the  $U(1)_Q$  of electromagnetism, is an excellent description of the interactions of elementary particles down to distances of the order of  $10^{-16}$  cm. But the SM also leaves many striking features of the physics of our world unexplained. Some of them are the generation number problem, the electric charge quantization, and the neutrino mass. Recent experimental results of SuperKamiokande Collaboration [1], KamLAND [2], and SNO [3] confirm that the neutrinos are massive and the flavor lepton number is not conserved; this implies that the SM must be extended.

A very common proposal to solve some of these problems consists in enlarging the gauge symmetry group, to the one that properly contains the SM group. For instance, the  $SU(5)$  grand unification model [4] can unify the interactions and predicts the electric charge quantization, and the  $E_6$  group can also unify the interactions and might explain the masses of the neutrinos [5, 6]. Nevertheless, such models can-

not explain the generation number problem. Among the extensions of the SM, the models based on the  $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$  (3–3–1) gauge group [7, 8] have some intriguing features. First, they can partly explain the number of generations. This is because the models are anomaly-free only if the number of generations  $N$  is a multiple of three. If the condition of the asymptotic freedom in QCD is also added, which is valid only if the number of generations of quarks is not less than 5, then it follows that the number of generations is equal to 3. Second, the third quark generation has to be different from the first two, which leads to a possible explanation of why the top quark is uncharacteristically heavy. Besides, the Peccei–Quinn symmetry naturally occurs in these models [9].

A few different versions of the 3–3–1 model have been proposed. In the minimal version [10], the three known left-handed lepton components for each generation are associated with three  $SU(3)_L$  triplets as  $(\nu_l, l, l^c)_L$ , where  $l^c_L$  is related to the right-handed isospin singlet of the charged lepton  $l$  in the SM. The scalar sector of this model is quite complicated (three triplets and one sextet). In the variant model, i.e., the model with right-handed neutrinos [11], three  $SU(3)_L$  lepton triplets are of the form  $(\nu, l, \nu^c)_L$ , where  $\nu^c_L$  is related to the right-handed component of the neutrino

\*E-mail: dvsoa@assoc.iop.vast.ac.vn

\*\*E-mail: pvdong@iop.vast.ac.vn

field  $\nu_L$ . The scalar sector of this model requires three Higgs triplets. It is interesting to note that in this model, two Higgs triplets have the same  $U(1)_X$  charge with two neutral components at their top and bottom. Allowing vacuum expectation values (VEVs) of these neutral components, we can reduce the number of Higgs triplets to two. A model of this kind was proposed recently [12, 13]. The scalar sector of this model is minimal with just two Higgs triplets, and hence it has been called the economical 3–3–1 model [14]. The phenomenology of this model is presented in detail in [15, 16].

Despite the recent experimental advances in neutrino physics, we do not yet know if the neutrinos are Dirac or Majorana particles. If the neutrinos are Majorana particles, then the mass terms violate the lepton number by two units, which may result in important consequences in particle physics and cosmology. A crucial process that will help in determining the neutrino nature is the  $(\beta\beta)_{0\nu}$  decay<sup>1)</sup>. It is also a typical process that requires lepton number violation, although it can say nothing about the value of the mass because, although right-handed currents and/or scalar bosons may affect the decay rate, it has been shown that whatever the mechanism of this decay is, it implies a nonvanishing neutrino mass [18]. In some models, the  $(\beta\beta)_{0\nu}$  decay can proceed with an arbitrarily small neutrino mass via a scalar boson exchange [19].

The mechanism involving a trilinear coupling of scalar bosons was proposed in Ref. [20] in the context of a model with the  $SU(2) \otimes U(1)$  symmetry with doublets and a triplet of scalar bosons. But because there is no large mass scale in these types of models [21], the contribution of the trilinear coupling is, in fact, negligible. In general, in models with that symmetry, a fine tuning is needed if we want the trilinear terms to give important contributions to the  $(\beta\beta)_{0\nu}$  decay [22]. It was shown in Ref. [23] that in 3–3–1 model, which has a rich Higgs boson sector, there are many new contributions to the  $(\beta\beta)_{0\nu}$  decay. In recent work [24], the authors showed that the implementation of spontaneous breaking of the lepton number in the 3–3–1 model with right-handed neutrinos gives rise to a fast neutrino decay with a Majoron emission and generates numerous new contributions to the  $(\beta\beta)_{0\nu}$  decay.

In our earlier work [25], we analyzed the neutrino masses in the economical 3–3–1 model. The masses of neutrinos are given by three different sources widely ranging over the mass scales including the GUT's and the small VEV  $u$  of spontaneous lepton number break-

ing. With a finite renormalization in mass, the spectrum of neutrino masses is neat and can fit the data. In this work, we discuss possible contributions of the bilepton to the  $(\beta\beta)_{0\nu}$  decay in the model under consideration. We show that in contradiction with the previous analysis, the  $(\beta\beta)_{0\nu}$  decay arises from two different sources, which require both Majorana and Dirac neutrino masses to be nonvanishing. If the mixing angle between the charged gauge bosons is in the range of the ratio of neutrino masses  $\langle M_\nu \rangle_L / \langle M_\nu \rangle_D$ , then the Majorana and Dirac masses are comparable to each other and may give the main contribution to the decay. The constraints on the bilepton mass are also given.

The rest of this paper is organized as follows. In Sec. 2, we briefly review the economical 3–3–1 model. Charged currents and a new bound on the mixing angle are given in Sec. 3. Section 4 is devoted to a detailed analysis of the possible contributions of the bilepton to the  $(\beta\beta)_{0\nu}$  decay. We summarize our results and make conclusions in Sec. 5.

## 2. A REVIEW OF THE MODEL

The particle content in this anomaly-free model is given by [13]

$$\begin{aligned} \psi_{aL} &= (\nu_{aL}, l_{aL}, (\nu_{aR})^c)^T \sim (3, -1/3), \\ l_{aR} &\sim (1, -1), \quad a = 1, 2, 3, \\ Q_{1L} &= (u_{1L}, d_{1L}, U_L)^T \sim (3, 1/3), \\ Q_{\alpha L} &= (d_{\alpha L}, -u_{\alpha L}, D_{\alpha L})^T \sim (3^*, 0), \quad \alpha = 2, 3, \\ u_{aR} &\sim (1, 2/3), \quad d_{aR} \sim (1, -1/3), \\ U_R &\sim (1, 2/3), \quad D_{\alpha R} \sim (1, -1/3), \end{aligned} \quad (1)$$

where the values in the parentheses denote quantum numbers based on the  $SU(3)_L \otimes U(1)_X$  symmetry. Unlike the usual 3–3–1 model with right-handed neutrinos, where the third family of quarks should be discriminating, the first family has to be different from the other two in the model under consideration [16]. The electric charge operator in this case takes the form

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \quad (2)$$

where the  $T_i$  ( $i = 1, 2, \dots, 8$ ) and  $X$  are respectively the  $SU(3)_L$  and  $U(1)_X$  charges. The electric charges of the exotic quarks  $U$  and  $D_\alpha$  are the same as for the usual quarks, i.e.,  $q_U = 2/3$  and  $q_{D_\alpha} = -1/3$ .

The spontaneous symmetry breaking in this model is obtained in two stages:

$$SU(3)_L \otimes U(1)_X \rightarrow SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q. \quad (3)$$

<sup>1)</sup> For experimental projects in preparation, see [17].

The first stage is achieved by a Higgs scalar triplet with the VEV given by

$$\begin{aligned} \chi &= (\chi_1^0, \chi_2^-, \chi_3^0)^T \sim (3, -1/3), \\ \langle \chi \rangle &= \frac{1}{\sqrt{2}} (u, 0, \omega)^T. \end{aligned} \tag{4}$$

The last stage is achieved by another Higgs scalar triplet needed with the VEV

$$\begin{aligned} \phi &= (\phi_1^+, \phi_2^0, \phi_3^+)^T \sim (3, 2/3), \\ \langle \phi \rangle &= \frac{1}{\sqrt{2}} (0, v, 0)^T. \end{aligned} \tag{5}$$

The VEV  $\omega$  gives mass to the exotic quarks  $U$  and  $D_\alpha$  and the new gauge bosons  $Z^2$ ,  $X$ , and  $Y$ , while the VEVs  $u$  and  $v$  give mass to all the ordinary fermions and gauge bosons [13, 16]. The VEV  $\omega$  is responsible for the first step of symmetry breaking; the second step is due to  $u$  and  $v$ . Therefore, the VEVs in this model have to satisfy the constraint  $u, v \ll \omega$ . It is interesting to note that the VEV  $v$  is close to the SM one,  $v \approx 246$  GeV; this is due to identification of the charged gauge boson  $W$  as the  $W$  in the SM. From the  $\rho$  parameter, we obtain the constraint on  $u$  as  $u \leq 2.46$  GeV [13], which implies that  $u$  is much smaller than  $v$ . Therefore, the VEVs in this model must satisfy the constraint

$$u \ll v \ll \omega. \tag{6}$$

The masses of the gauge bosons are

$$M_W^2 = \frac{g^2 v^2}{4}, \tag{7}$$

$$M_Y^2 = \frac{g^2}{4} (u^2 + v^2 + \omega^2), \tag{8}$$

$$M_X^2 = \frac{g^2}{4} (\omega^2 + u^2), \tag{9}$$

and

$$M_{Z^1}^2 \approx \frac{g^2}{4c_W^2} (v^2 - 3u^2), \tag{10}$$

$$M_{Z^2}^2 \approx \frac{g^2 c_W^2 \omega^2}{3 - 4s_W^2}. \tag{11}$$

It follows from (7), (8), and (9) that the splitting between the bilepton masses is governed by the law of Pythagoras

$$M_Y^2 = M_X^2 + M_W^2. \tag{12}$$

Hence, the charged bilepton  $Y$  is slightly heavier than the neutral bilepton  $X$ . We recall that a similar relation in the model with the right-handed neutrino is  $|M_Y^2 - M_X^2| \leq m_W^2$  [11].

### 3. CHARGED CURRENTS AND A NEW BOUND ON THE MIXING ANGLE

The consequence of  $u \neq 0$  in this model is a mixing of the SM gauge boson  $W'$  and bilepton  $Y'$ ,

$$\begin{aligned} \mathcal{L}_{mass}^{CG} &= \frac{g^2}{4} (W'^-, Y'^-) \begin{pmatrix} u^2 + v^2 & u\omega \\ u\omega & \omega^2 + v^2 \end{pmatrix} \times \\ &\times \begin{pmatrix} W'^+ \\ Y'^+ \end{pmatrix}. \end{aligned}$$

The physical charged gauge bosons are given by

$$\begin{aligned} W &= c_\theta W' + s_\theta Y', \\ Y &= -s_\theta W' + c_\theta Y', \end{aligned} \tag{13}$$

where the mixing angle is defined by

$$\text{tg } \theta = \frac{u}{\omega} \tag{14}$$

and we use the notation  $c_\theta = \cos \theta$  and  $s_\theta = \sin \theta$ .

As a consequence of this mixing, there exist lepton-number violating (LNV) terms in the charged currents proportional to  $s_\theta$ ,

$$H^{CC} = \frac{g}{\sqrt{2}} (J_W^{\mu+} W_\mu^- + J_Y^{\mu+} Y_\mu^- + \text{H.c.}), \tag{15}$$

with

$$\begin{aligned} J_W^{\mu+} &= c_\theta (\bar{l}_{\alpha L} \gamma^\mu \nu_{\alpha L} + \bar{d}_{\alpha L} \gamma^\mu u_{\alpha L}) - \\ &- s_\theta (\bar{l}_{\alpha L} \gamma^\mu \nu_{\alpha R}^c + \bar{d}_{1L} \gamma^\mu U_L + \bar{D}_{\alpha L} \gamma^\mu u_{\alpha L}), \end{aligned} \tag{16}$$

$$\begin{aligned} J_Y^{\mu+} &= c_\theta (\bar{l}_{\alpha L} \gamma^\mu \nu_{\alpha R}^c + \bar{d}_{1L} \gamma^\mu U_L + \bar{D}_{\alpha L} \gamma^\mu u_{\alpha L}) + \\ &+ s_\theta (\bar{l}_{\alpha L} \gamma^\mu \nu_{\alpha L} + \bar{d}_{\alpha L} \gamma^\mu u_{\alpha L}). \end{aligned} \tag{17}$$

As in Ref. [13], the constraint on the  $W$ - $Y$  mixing angle  $\theta$  from the  $W$  width is given by  $s_\theta \leq 0.08$ . But we show in what follows that a stricter bound can obtain from the invisible  $Z$  width through the unnormal neutral LNV current

$$\begin{aligned} \mathcal{L}_{unnormal}^{NC} &= -\frac{gt_{2\theta} g_{kV}(\nu)}{c_W} (\bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha R}^c + \bar{u}_{1L} \gamma^\mu U_L - \\ &- \bar{D}_{\alpha L} \gamma^\mu d_{\alpha L}) Z_\mu^k + \text{H.c.}, \end{aligned} \tag{18}$$

where the neutrino coupling constants ( $g_{kV}, k = 1, 2$ ) are given by

$$g_{1V}(\nu_L) \approx \frac{c_\varphi - s_\varphi \sqrt{4c_W^2 - 1}}{2}, \tag{19}$$

$$g_{2V}(\nu_L) \approx \frac{s_\varphi + c_\varphi \sqrt{4c_W^2 - 1}}{2}. \tag{20}$$

It is worth mentioning that the mixing angle  $\varphi$  between the  $Z$ - $Z'$  neutral bosons is very small. In the case where  $u \rightarrow 0$ , the analysis of the  $Z$  decay width [26] shows that the  $Z$ - $Z'$  mixing angle is constrained as  $-0.0015 \leq \varphi \leq 0.001$ . The neutrino couplings in (18) lead to additional invisible-decay modes to the  $Z$  boson. For each generation of leptons, the corresponding invisible-decay width can be approximately written as

$$\Gamma_{\nu_L N_L} \approx \frac{1}{2} t_{2\theta}^2 \left(1 + \mathcal{O}(s_\varphi^2)\right) \Gamma_{\nu\bar{\nu}}^{SM}, \quad (21)$$

where  $N_L = \nu_{aR}^c$  and  $\Gamma_{\nu\bar{\nu}}^{SM} = G_F M_Z^3 / 12\pi\sqrt{2}$  is the SM prediction for the decay rate of  $Z$  into a pair of neutrinos. The experimental data for the total invisible neutrino decay modes give [27]

$$\Gamma_{invisible}^{exp} = (2.994 \pm 0.012) \Gamma_{\nu\bar{\nu}}^{SM}. \quad (22)$$

From (21) and (22), we obtain the upper limit for the mixing angle

$$\text{tg } \theta \leq 0.03, \quad (23)$$

which is smaller than that given in Ref. [13].

#### 4. BILEPTON CONTRIBUTIONS TO THE NEUTRINOLESS DOUBLE BETA DECAY

The  $(\beta\beta)_{0\nu}$  decay is a typical process that requires the lepton number violation, and hence it can be useful in probing new physics beyond the SM [17, 18]. The interactions that lead to the  $(\beta\beta)_{0\nu}$  decay involve hadrons and leptons. For the standard contribution, its amplitude can be written as [24]

$$M_{(\beta\beta)_{0\nu}} = \frac{g^4}{4m_W^4} M_{\mu\nu}^h \bar{U} \gamma^\mu P_L \frac{\not{q} + m_\nu}{q^2 - m_\nu^2} \gamma^\nu P_R V, \quad (24)$$

where  $M_{\mu\nu}^h$  carries the hadronic information of the process,  $P_{R,L} = (1 \pm \gamma_5)/2$ , and  $U$  and  $V$  are Dirac spinors. In the presence of neutrino mixing, assuming that  $m_\nu^2 \ll q^2$ , we can write

$$M_{(\beta\beta)_{0\nu}} = A_{(\beta\beta)_{0\nu}} M_{\mu\nu}^h \bar{U} P_R \gamma^\mu \gamma^\nu V, \quad (25)$$

where

$$A_{(\beta\beta)_{0\nu}} = \frac{g^4 \langle M_\nu \rangle}{4m_W^4 \langle q^2 \rangle} \quad (26)$$

is the strength of the effective coupling of the standard contribution. In the case of three neutrino species,  $\langle M_\nu \rangle = \sum U_{ei}^2 m_{\nu i}$  is the effective neutrino mass and  $\langle q^2 \rangle$  is the average of the transferred squared four-momentum.

The contributions to the  $(\beta\beta)_{0\nu}$  decay in our model coming from the charged gauge bosons  $W^-$  and  $Y^-$  dominate the process. Because the  $(\beta\beta)_{0\nu}$  decay has not yet been experimentally detected, the aim of our analysis here is to obtain new contributions and to compare them with the standard one [18, 23]. Feynman diagrams for the contributions are depicted in Figs. 1, 2, and 3. Left-handed figures (a) are given by the nonvanishing Majorana mass, and the right-handed figures (b) by the Dirac mass.

For the standard contribution as depicted in Fig. 1a, its effective coupling takes the form

$$A_{(\beta\beta)_{0\nu}}(1a) = \frac{g^4 \langle M_\nu \rangle_L}{4m_W^4 \langle q^2 \rangle} c_\theta^4, \quad (27)$$

where  $M_L$  is the Majorana mass. The first new contribution involves only  $W^-$ , as in the standard contribution, but  $W^-$  now interacts with two charged currents  $J_\mu$  and  $J_\mu^c$  as depicted in Fig. 1b. We note that in this case, the Dirac mass gives the contribution to the effective coupling

$$A_{(\beta\beta)_{0\nu}}(1b) = \frac{g^4 \langle M_\nu \rangle_D}{4m_W^4 \langle q^2 \rangle} c_\theta^3 s_\theta, \quad (28)$$

where  $M_D$  is the Dirac mass.

We see from Eqs. (27) and (28) that the LNV in the  $(\beta\beta)_{0\nu}$  decay arises from two different sources respectively identified by the nonvanishing Majorana and Dirac mass terms. In Fig. 1a, the LNV is due to the Majorana mass, and the LNV in Fig. 1b is due to the coupling of the  $W$  boson to the charged current (the term is proportional to  $s_\theta$ ). Comparing these effective couplings, we obtain the ratio

$$\frac{A_{(\beta\beta)_{0\nu}}(1b)}{A_{(\beta\beta)_{0\nu}}(1a)} = \frac{\langle M_\nu \rangle_D}{\langle M_\nu \rangle_L} \text{tg } \theta. \quad (29)$$

We see from (29) that the relevance of this contribution depends on the angle  $\theta$  and on the ratio between  $\langle M_\nu \rangle_D$  and  $\langle M_\nu \rangle_L$ . It is worth noting that if  $\langle M_\nu \rangle_D \text{tg } \theta \sim \langle M_\nu \rangle_L$ , then the Majorana and Dirac masses are comparable to each other and both may give the main contribution to the decay.

Next, we consider contributions that involve both  $W^-$  and  $Y^-$ . They involve the two currents  $J_\mu$  and  $J_\mu^c$  interacting with  $W$  and  $Y$  as depicted in Fig. 2a for  $\langle M_\nu \rangle_L$  and Fig. 2b for  $\langle M_\nu \rangle_D$ . The effective couplings in this case are

$$A_{(\beta\beta)_{0\nu}}(2a) = \frac{g^4 \langle M_\nu \rangle_L c_\theta^2 s_\theta^2}{4m_W^2 m_Y^2 \langle q^2 \rangle} \quad (30)$$

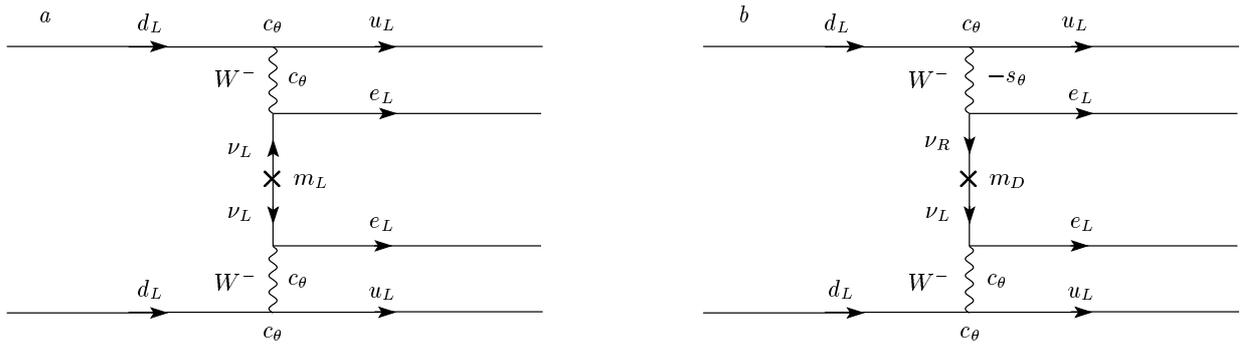


Fig. 1. Contribution of the SM bosons  $W$  to the  $(\beta\beta)_{0\nu}$  decay. Figure  $a$  is for the Majorana mass, figure  $b$  is for the Dirac mass

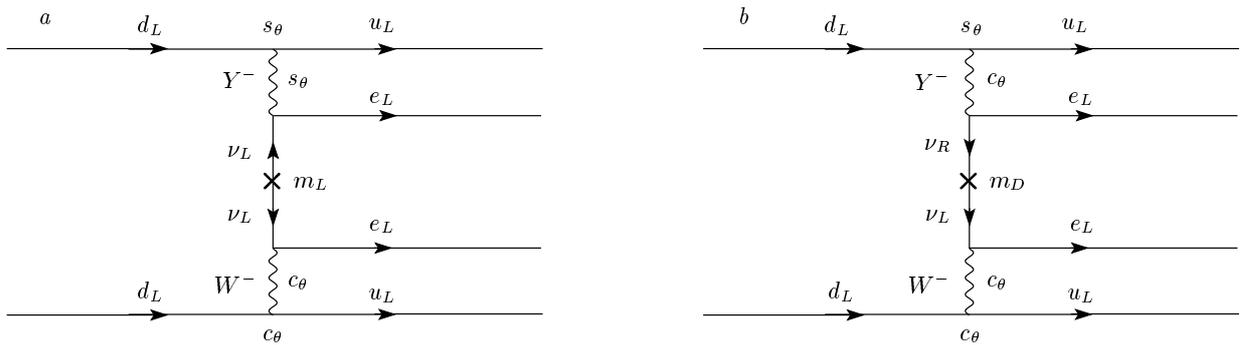


Fig. 2. Associated contribution of the  $W$  boson and the bilepton  $Y$  to the  $(\beta\beta)_{0\nu}$  decay

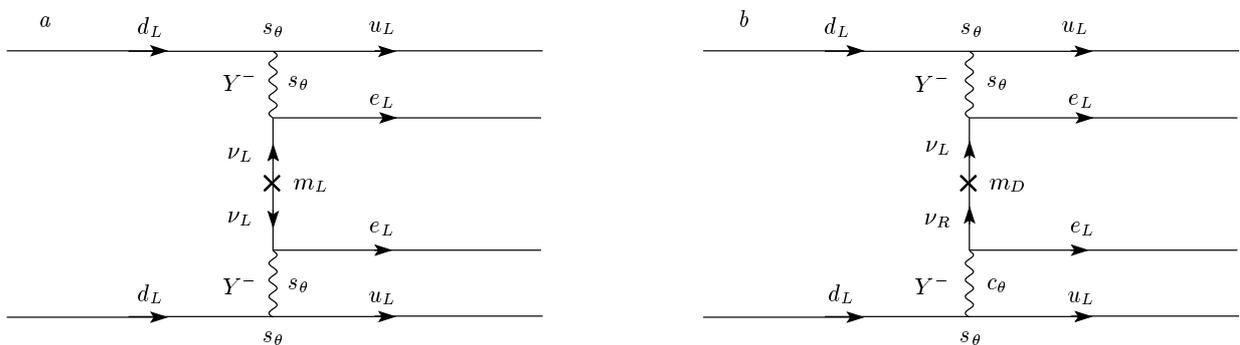


Fig. 3. Contribution of the bileptons  $Y$  to the  $(\beta\beta)_{0\nu}$  decay

and

$$A_{(\beta\beta)_{0\nu}}(2b) = \frac{g^4 \langle M_\nu \rangle_D c_\theta^3 s_\theta}{4m_W^2 m_Y^2 \langle q^2 \rangle}. \quad (31)$$

We see from (30) and (31) that the Majorana mass gives the contribution to the  $(\beta\beta)_{0\nu}$  decay much smaller than the Dirac mass. Comparing with the standard effective coupling, we obtain the ratios

$$\frac{A_{(\beta\beta)_{0\nu}}(2b)}{A_{(\beta\beta)_{0\nu}}(1a)} = \frac{m_W^2}{m_Y^2} \frac{\langle M_\nu \rangle_D}{\langle M_\nu \rangle_L} \text{tg} \theta \quad (32)$$

and

$$\frac{A_{(\beta\beta)_{0\nu}}(2a)}{A_{(\beta\beta)_{0\nu}}(1a)} = \frac{m_W^2}{m_Y^2} \text{tg}^2 \theta. \quad (33)$$

In contrast to the previous case, Eq. (32) shows that the relevance of these contributions depends on the angle  $\theta$ , the ratio  $\langle M_\nu \rangle_D / \langle M_\nu \rangle_L$ , and the bilepton mass. We suppose that the new contributions are smaller than the standard one; from Eq. (32), we then obtain a lower

**Table.** Low bounds on the bilepton mass in range of  $\langle M_\nu \rangle_D / \langle M_\nu \rangle_L$

| $\langle M_\nu \rangle_D / \langle M_\nu \rangle_L$ | 100   | 200   | 400   | 600   | 800   | 1000  |
|---|-------|-------|-------|-------|-------|-------|
| $m_Y, \text{ GeV}$                                  | 139.0 | 197.0 | 278.6 | 341.2 | 394.0 | 440.5 |

bound on the bilepton mass as

$$m_Y^2 > m_W^2 \frac{\langle M_\nu \rangle_D}{\langle M_\nu \rangle_L} \text{tg} \theta. \tag{34}$$

With  $m_W^2 = 80.425 \text{ GeV}$  and  $\text{tg} \theta = 0.03$ , the low bounds on the mass  $m_Y$  in range of  $\langle M_\nu \rangle_D / \langle M_\nu \rangle_L \sim 10^2 - 10^3$  [25] are given in the Table. It is interesting to note that “wrong” muon decay experiments imply a bound for the bilepton mass  $m_Y \geq 230 \text{ GeV}$  [13, 28], and a stronger mass bound has been derived from considering an experimental limit of lepton-number-violating charged lepton decays [29] of 440 GeV.

We see from Eq. (33) that the order of the contribution is much smaller than the standard contribution; this is because the LNV in the  $(\beta\beta)_{0\nu}$  decay arises from the Majorana mass term and the LNV coupling between the bilepton  $Y$  and the charged current  $J^\mu$  of ordinary quarks and leptons. Taking  $m_Y = 139 \text{ GeV}$ , we obtain

$$\frac{A_{(\beta\beta)_{0\nu}}(2a)}{A_{(\beta\beta)_{0\nu}}(1a)} \leq 3.0 \cdot 10^{-4}. \tag{35}$$

We now examine the next four contributions that involve only the bileptons  $Y$ . In Fig. 3a, we show an example of this kind of contribution where the current  $J_\mu^c$  appears in two vertices. The effective coupling is

$$A_{(\beta\beta)_{0\nu}}(3a) = \frac{g^4 \langle M_\nu \rangle_L s_\theta^4}{4m_Y^4 \langle q^2 \rangle}. \tag{36}$$

In another case, we also have

$$A_{(\beta\beta)_{0\nu}}(3b) = \frac{g^4 \langle M_\nu \rangle_D c_\theta s_\theta^3}{4m_Y^4 \langle q^2 \rangle}. \tag{37}$$

Comparing with the standard effective coupling, we obtain

$$\frac{A_{(\beta\beta)_{0\nu}}(3a)}{A_{(\beta\beta)_{0\nu}}(1a)} = \left( \frac{m_W}{m_Y} \right)^4 \text{tg}^4 \theta. \tag{38}$$

With the above data, the ratio upper limit is

$$\frac{A_{(\beta\beta)_{0\nu}}(3a)}{A_{(\beta\beta)_{0\nu}}(1a)} \leq 9.0 \cdot 10^{-8}, \tag{39}$$

which is very small. It is easy to verify that the remaining contributions are much smaller than those with the charged  $W$  bosons. This is because all the couplings of the bilepton with ordinary quarks and leptons in the diagrams in Fig. 3 are lepton number violating.

### 5. CONCLUSION

We have obtained a new bound on the mixing angle between charged gauge bosons in the economical 3–3–1 model from the invisible decay modes of the neutral gauge boson  $Z$ . We have also investigated the implications of spontaneous breaking of the lepton number in the  $(\beta\beta)_{0\nu}$  decay and systematically analyzed the couplings of all possible contributions of charged gauge bosons to the decay. The result shows that, in contradiction with previous analysis [23, 24], the  $(\beta\beta)_{0\nu}$  decay mechanism in the considered model requires both Majorana and Dirac nonvanishing masses. If the mixing angle between the charged gauge boson and the bilepton is in the range of the ratio of neutrino masses  $\langle M_\nu \rangle_L$  and  $\langle M_\nu \rangle_D$ , then the Majorana and Dirac masses are comparable to each other and both may give the main contribution to the decay. Based on the result, the constraints on the bilepton mass are given. It is interesting to note that the relevance of the new contributions is dictated by the mixing angle  $\theta$ , the effective neutrino mass, and the bilepton mass. By estimating the order of magnitude of the new contributions, we predicted that the most robust one is the contribution depicted in Fig. 2, whose order of magnitude is  $10^{-4}$  of the standard contribution.

Finally, we emphasize that in the considered model, the charged Higgs boson is a scalar bilepton (with the lepton number  $L = \pm 2$ ). Therefore, their Yukawa couplings to ordinary quarks and leptons violate the lepton number and are very weak (see Ref. [30] for the details). This means that their possible contributions to the  $(\beta\beta)_{0\nu}$  decay must be much smaller than the contributions of charged gauge bosons.

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