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SIZE RESONANCES IN PENETRATION OF INTENSE FEMTOSECOND LASER PULSES THROUGH ULTRA-THIN FOILS

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An analytic expression for the transmission coefficient as a function of the foil thickness d describing penetration of intense femtosecond laser pulses through ultra-thin foils with the thickness of the order of 30–100 nm is derived using the Vlasov–Boltzmann equation. It is found that the transmission of laser radiation stops at the skin depth c/ω_p, but sharp and narrow resonances occur for the foil thickness $d > c/\omega_p$ with the transmission coefficient $T = 1$.

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1. INTRODUCTION

When an intense femtosecond laser pulse impinges on a thin foil, the crystal lattice is destroyed and a dense plasma with a sharp boundary is produced. In a recent experiment in [1], under the irradiation of $10\,\mu$m thick Al foils by the laser pulse with the duration 300 fs and the peak intensity $5\cdot10^{19}$ W/cm^2, most of free electrons inside the foil had the kinetic energy about 100 eV. Of course, a small amount of electrons are heated up to 100–300 keV and more due to the inverse induced bremsstrahlung on the critical surface and other mechanisms [2] (these are the so-called hot electrons). But their number density is very small, of the order of 10^{19} cm^{-3}. These estimates of the energy of cold electrons are confirmed by the numerical derivations in [3]. The ratio of the energies of cold and hot electrons is typically about 0.01. We note that atomic ions inside the foil do not move significantly during femtosecond time intervals, and hence the foil surface remains immovable. The energy of bulk electrons in a foil of the order of 150 eV was also observed in Ref. [4] at the peak laser intensity 10^{18} W/cm^2 and the pulse duration 60 fs.

At the normal incidence of a linearly polarized laser beam, the tunneling and barrier suppression ionization of atoms or of atomic ions inside the foil produce electrons ejected along the laser polarization (i.e., parallel to the foil surface, the x axis) with the energies estimated as [5]

$$E_x \approx \frac{3F_{in}^3}{2\omega^2\,(2E_i)^{3/2}}.$$

Here, atomic units are used, $e = m_e = \hbar = 1$, F_{in} is the typical electric field strength inside the foil (it differs strongly from the external laser field strength F), ω is the laser frequency, and E_i is the ionization potential of an atom or of an atomic ion inside the foil. The typical energy in other directions (including the z axis normal to the foil surface) is much less, and its estimate is

$$E_z \approx \frac{F_{in}}{2\sqrt{2E_i}}.$$

Indeed, $E_z/E_x \approx \gamma^2$, where

$$\gamma = \frac{\omega\sqrt{2E_i}}{F_{in}} \ll 1$$

is so called Keldysh parameter [5]. In the tunneling and barrier suppression regime, the Keldysh parameter is very small. In addition, in dense plasma, the plasma frequency $\omega_p = \sqrt{4\pi n_e}$ is very large in comparison to the laser frequency ω. Here, $n_e \sim 10^{23}$ cm^{-3} is the typical number density of free electrons inside the foil plasma. These free electrons are added to the free electrons with relatively small kinetic energies that were in the metal foil before laser irradiation.

Cold electrons collide with each other and with atomic ions during the femtosecond laser pulse. Indeed,

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the time between two subsequent electron–electron collisions can be estimated according to Spitzer formula (in atomic units) as

$$\tau_{ee} \approx \frac{3(2E_x/3)^{3/2}}{4\sqrt{2\pi}n_e \ln \Lambda},$$

where $\ln \Lambda \approx 5$ is the Coulomb logarithm. In particular, we obtain $\tau_{ee} \approx 0.4$ fs at the electron number density $n_e = 10^{23}$ cm^{-3} for an overdense foil plasma and the averaged electron kinetic energy $E_x = 100$ eV. Thus, the Maxwell one-temperature distribution is established for the electron cloud during the femtosecond laser pulse. But for the laser intensity more than 10^{21} W/cm^2, the energy of cold electrons increases to 10 keV and more, and they do not collide with each other and with atomic ions during the femtosecond laser pulse. The energy of hot electrons is of the order of several MeV in this case, which is confirmed by PIC simulations [6]. We note that multiple ionization of atoms occurs at such ultrahigh laser intensities. In particular, in the experiments in [7], the laser intensity was $4 \cdot 10^{20}$ W/cm^2 at the pulse duration of 700 fs. Atomic ions were produced with high charged multiplicity in a Cu foil. Analogously, in the experiments in [8], the gold ions had the charge multiplicity up to 11+ at the irradiation of Au foils by laser with the peak intensity more than 10^{20} W/cm^2 and the pulse duration 100 fs.

The energy of bulk electrons in a foil depends significantly on the duration of the laser pulse. Using 3D particle-in-cell simulations, the author of Ref. [9] studied ion acceleration from a foil with the thickness 12 μm irradiated by a laser pulse at the intensity 10^{19} W/cm^2 with the pulse duration 700 fs. At the front side, the laser ponderomotive force pushes electrons inwards. The energy of bulk electrons in the foil was 50 keV, and the energy of hot electrons was 2.6 MeV. Then the two-temperature electron energy spectrum is realized without collisions. We consider just this case here.

2. THEORETICAL APPROACH

We assume that the electron beam is monochromatic in all directions and

$$f_0 = n_e \delta\left(v_x - \bar{v}_x\right) \delta\left(v_z - \bar{v}_z\right)$$

(where n_e is the electron number density) is the distribution function at $F = 0$. The integro-differential nonrelativistic Vlasov equation for the electric field strength $F_{in} = F(z)$ inside the foil is of the form (see [10])

$$F''(z) = -\frac{4\pi i\omega}{c^2} \int v_x \delta f dv_x dv_z. \qquad (1)$$

We replace $\bar{v}_x \to v_x$, $\bar{v}_z \to v_z$ in what follows. The plasma layer occupies the region $0 < z < d$, and x is the direction of polarization of the linearly polarized laser field. There is no dependence on the coordinate y. The small field correction to the electron distribution function δf must satisfy the kinetic equation

$$i\omega\delta f - v_z \frac{d}{dz}\delta f = F(z)\frac{\partial f_0}{\partial v_x} - \\ - \frac{i}{\omega}F'(z)\left[v_x \frac{\partial f_0}{\partial v_z} - v_z \frac{\partial f_0}{\partial v_x}\right]. \qquad (2)$$

We neglect collisions between electrons and atomic ions. We obtain δf from Eq. (2) by solving the first-order inhomogeneous differential equation with respect to z. The assumption of mirror reflection of electrons by foil boundaries is used:

$$\delta f(z = 0, v_z) = \delta f(z = 0, -v_z),$$
$$\delta f(z = d, v_z) = \delta f(z = d, -v_z).$$

Substituting δf from Eq. (2) in the right-hand side of Eq. (1) and assuming that all electrons have the same velocity v_z in the normal direction to the foil surface and the same velocity v_x along the laser field polarization, we then obtain the integro-differential equation for $F(z)$

$$F''(z) - \frac{\omega_p^2 \Delta}{c^2}F(z) = \frac{\omega \cdot \omega_p^2\left(1 - \Delta\right)}{c^2 v_z} \times \\ \times \left\{ \int_0^z dz' F(z') \sin\left[\frac{\omega}{v_z}\left(z - z'\right)\right] + I(z) \right\}, \qquad (3)$$

where

$$I(z) = \frac{\cos\left(\omega z/v_z\right)}{\sin\left(\omega d/v_z\right)} \int_0^d dz' F(z') \cos\left[\frac{\omega}{v_z}\left(d - z'\right)\right],$$

v_z is the electron velocity along the normal direction z, and

$$\Delta = 1 - \frac{v_x^2}{v_z^2} = 1 - \frac{E_x}{E_z}.$$

We introduce the notation

$$x \equiv \frac{\omega}{v_z}z, \quad D \equiv \frac{\omega}{v_z}d.$$

Then Eq. (3) can be rewritten as

$$F''(x) - \frac{\omega_p^2 \Delta v_z^2}{c^2\omega^2}F(x) = \\ = \frac{\omega_p^2 v_z^2\left(1-\Delta\right)}{c^2\omega^2}\left\{ \int_0^x dx' F(x') \sin\left(x-x'\right) + J(x) \right\}, \qquad (4)$$

where

$$J(x) = \frac{\cos x}{\sin D} \int_0^D dx' F(x') \cos(d - x') . \quad (5)$$

To obtain an ordinary differential equation, we introduce the new function $\tilde{F}(x)$ instead of the electric field strength $F(x)$:

$$\begin{aligned} \tilde{F}(x) &= \int_0^x dx' F(x') \sin(x - x') , \\ \tilde{F}'(x) &= \int_0^x dx' F(x') \cos(x - x') , \end{aligned} \quad (6)$$

$$\begin{aligned} \tilde{F}''(x) &= F(x) - \tilde{F}(x), \\ F''(x) &= \tilde{F}^{\mathrm{IV}}(x) + \tilde{F}''(x). \end{aligned} \quad (7)$$

Then Eq. (4) can be rewritten in the dimensionless form

$$\tilde{F}^{\mathrm{IV}}(x) + 2A\tilde{F}''(x) - B\tilde{F}(x) = C\cos x, \quad (8)$$

where we set

$$\begin{aligned} 2A &= 1 - \frac{v_z^2\omega_p^2\Delta}{\omega^2 c^2} > 0, \\ B &= \frac{v_z^2\omega_p^2}{\omega^2 c^2}, \\ C &= \frac{v_z^2\omega_p^2(1-\Delta)}{\omega^2 c^2} \frac{F'(D)}{\sin D}. \end{aligned} \quad (9)$$

The general solution of the homogeneous equation (8) is

$$F(x) \propto \exp(Kx),$$

where the quantity K (the complex wave number) is satisfied the equation

$$K^4 + 2AK^2 - B = 0. \quad (10)$$

The four solutions of this equation are

$$K = \pm\sqrt{-A \pm \sqrt{A^2 + B}}. \quad (11)$$

We introduce the notation

$$\begin{aligned} k &= \sqrt{A + \sqrt{A^2 + B}}, \\ \varkappa &= \sqrt{-A + \sqrt{A^2 + B}}. \end{aligned} \quad (12)$$

The particular solution of the inhomogeneous equation (8) is

$$\tilde{F}_p(x) = \frac{C}{1 - 2A - B} \cos x = -\frac{\tilde{F}'(D)}{\sin D} \cos x. \quad (13)$$

Hence, the general solution of the inhomogeneous equation (8) can be written as

$$\begin{aligned} &\tilde{F}(x) = C_1 \operatorname{sh} \varkappa x + C_2 \operatorname{ch} \varkappa x + \\ &\quad + C_3 \sin kx + C_4 \cos kx - \frac{\tilde{F}'(D)}{\sin D} \cos x, \\ &F(x) = C_1 \left(1+\varkappa^2\right) \operatorname{sh} \varkappa x + C_2 \left(1+\varkappa^2\right) \operatorname{ch} \varkappa x + \\ &\quad + C_3 \left(1 - k^2\right) \sin kx + C_4 \left(1 - k^2\right) \cos kx, \\ &F'(x) = C_1 \varkappa \left(1+\varkappa^2\right) \operatorname{ch} \varkappa x + C_2 \varkappa \left(1+\varkappa^2\right) \times \\ &\quad \times \operatorname{sh} \varkappa x + C_3 k \left(1 - k^2\right) \cos kx - \\ &\qquad - C_4 k \left(1 - k^2\right) \sin kx, \end{aligned} \quad (14)$$

where C_1, C_2, C_3, and C_4 are integration constants.

It follows from Eqs. (6) that $\tilde{F}(0) = \tilde{F}'(0) = 0$. Using Eq. (14), we then obtain the relations between these constants:

$$C_2 + C_4 = \frac{\tilde{F}'(D)}{\sin D}$$

and

$$\varkappa C_1 = -kC_3. \quad (15)$$

We differentiate Eq. (14) with respect to x and then set $x = D$. We find the second-relation for the constants,

$$\begin{aligned} C_1 \varkappa \operatorname{ch} \varkappa D + C_2 \varkappa \operatorname{sh} \varkappa D + \\ + C_3 k \cos kD - C_4 k \sin kD = 0. \end{aligned} \quad (16)$$

We next consider the boundary condition for the electric field strength at $z = 0$:

$$F(z = 0) - i\frac{c}{\omega}F'(z = 0) = 2 \quad (17)$$

(we set the electric field strength of the incident electromagnetic wave equal to 1, having in mind that a part of the incident laser energy is absorbed by foil electrons). This equation can be rewritten in terms of the dimensionless variable x:

$$F(x = 0) - i\frac{c}{v_z}F'(x = 0) = 2. \quad (18)$$

Substituting Eq. (14) at $x = 0$ in Eq. (17), we find the third relation between the constants:

$$\begin{aligned} C_2 \left(1 + \varkappa^2\right) + C_4 \left(1 - k^2\right) - 2 = \\ = i\frac{c}{v_z}\left[C_1 \varkappa \left(1 + \varkappa^2\right) + C_3 k \left(1 - k^2\right)\right]. \end{aligned} \quad (19)$$

The final equation is the boundary condition for the electric field strength at $z = d$:

$$F(z = d) + i\frac{c}{\omega}F'(z = d) = 0,$$

or

$$iF(x=D)=\frac{c}{v_z}F'(x=D). \qquad (20)$$

Substituting Eq. (14) at $x=D$ in Eq. (20), we obtain the fourth relation between the four constants:

$$\begin{aligned}&iC_1\left(1+\varkappa^2\right)\operatorname{sh}\varkappa D+iC_2\left(1+\varkappa^2\right)\operatorname{ch}\varkappa D+\\&\quad+iC_3\left(1-k^2\right)\sin kD+iC_4\left(1-k^2\right)\cos kD=\\&=\frac{c}{v_z}\left[C_1\varkappa\left(1+\varkappa^2\right)\operatorname{ch}\varkappa D+C_2\varkappa\left(1+\varkappa^2\right)\operatorname{sh}\varkappa D\right]+\\&+\frac{c}{v_z}\left[C_3k\left(1-k^2\right)\cos kD-C_4k\left(1-k^2\right)\sin kD\right]. \qquad (21)\end{aligned}$$

Therefore, we must solve the nonhomogeneous system of four equations, Eqs. (15), (16), (19), and (21) for the four constants C_1, C_2, C_3, C_4.

According to Eq. (14), the electric field strength is

$$F(0)=C_2\left(1+\varkappa^2\right)+C_4\left(1-k^2\right). \qquad (22)$$

According to the boundary condition, the electric field strength of the reflected electromagnetic wave is

$$F_r=1-F(0)=1-C_2\left(1+\varkappa^2\right)-C_4\left(1-k^2\right). \qquad (23)$$

The reflection coefficient is

$$R=|F_r|^2=1-T, \qquad (24)$$

where T is the transmission coefficient of the electromagnetic radiation through the foil. These equations were obtained and solved numerically in our previous work [11].

In this work, we obtain the analytic expression for the transmission coefficient T for the first time. Omitting the details of cumbersome derivations, we find

$$\begin{aligned}&T=\frac{1}{1+x^2}<1,\\&x=\frac{a_1\operatorname{sh}\varkappa D\sin kD+a_2\left(\operatorname{ch}\varkappa D\cos kD-1\right)}{a_3\operatorname{sh}\varkappa D+a_4\sin kD},\\&a_1=k\varkappa\left(k^2+\varkappa^2\right)^2+\\&\quad+\beta^2\left[k\left(1+\varkappa^2\right)^2/\varkappa-\varkappa\left(1-k^2\right)^2/k\right], \qquad (25)\\&a_2=2\beta^2\left(1+\varkappa^2\right)\left(1-k^2\right),\\&a_3=2\beta\varkappa\left(k^2+\varkappa^2\right)\left(1-k^2\right),\\&a_4=2\beta k\left(k^2+\varkappa^2\right)\left(1+\varkappa^2\right),\\&\beta=v_z/c.\end{aligned}$$

3. LIMIT CASES

In the case $\Delta=1$ ($E_x=0$), the transmission coefficient in Eq. (25) takes the simple form

$$T=\left\{1+\left(\frac{\beta^2+\varkappa^2}{2\beta\varkappa}\operatorname{sh}\varkappa D\right)^2\right\}^{-1}. \qquad (26)$$

In this case, $\varkappa=v_z\omega_p/c\omega$. It corresponds to the well-known decrease in the electric field in the skin layer. The skin depth is equal to c/ω_p.

In the limit of a thick foil, when $\operatorname{sh}\varkappa D\gg1$ and $\operatorname{ch}\varkappa D\gg1$, it follows from Eq. (25) that

$$T=\frac{1}{1+\left(a_1\sin kD+a_2\cos kD\right)^2/a_3^2}. \qquad (27)$$

Hence, resonances also occur for sufficiently thick foils, where $T=1$. The distance between neighboring resonances is determined by the condition $k\Delta D=\pi$, or $\Delta d\approx\pi v_z/\omega$. Of course, in practice, resonances disappears with increasing the foil thickness d because of the absorption of laser radiation. The resonance absorption in a foil on the critical surface can be very efficient [12].

4. ABSORPTION OF RADIATION

In each collision of an electron with an atomic ion with the charge multiplicity Z, the electron energy increases on average by the ponderomotive energy $F_{in}^2/4\omega^2$. This is the so-called inverse induced bremsstrahlung. The rate of electron–ion collisions is

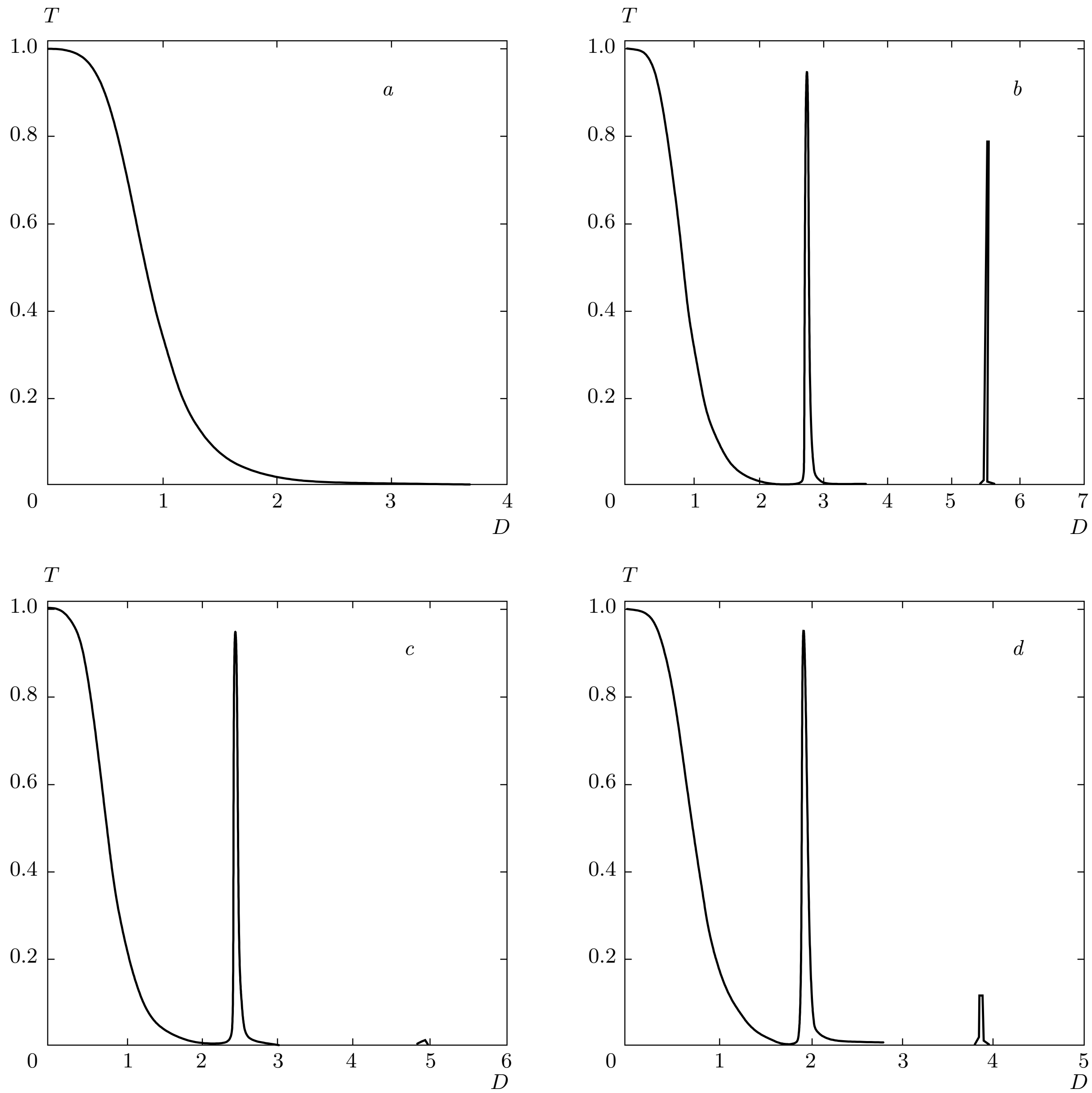
$$\tau_{ei}\approx\frac{3(2E_{in}/3)^{3/2}}{4\sqrt{2\pi}Z^2n_e\ln\Lambda}.$$

In addition, an electron can elastically collide with the foil surface. In each collision, its energy also increases by $F_{in}^2/4\omega^2$. The rate of electron–surface collisions is $\tau_{es}\approx d/v_z$. In the tau-approximation, kinetic equation (2) for the distribution function δf becomes

$$\begin{aligned}i\omega\delta f-v_z\frac{d}{dz}\delta f&=F(z)\frac{\partial f_0}{\partial v_x}-\\&-\frac{i}{\omega}F'(z)\left[v_x\frac{\partial f_0}{\partial v_z}-v_z\frac{\partial f_0}{\partial v_x}\right]-\frac{\delta f}{\tau},\end{aligned}$$

where τ is the collision time, i.e., the minimum value among τ_{ei} and τ_{es}. Hence, the above solution must be modified by the substitution $\omega\to\omega-i/\tau$ under the condition $\omega\tau\gg1$. In the nonrelativistic case $\beta^2\ll1$, Eq. (27) can be simplified to

$$T=\frac{1}{1+(a_1^2/a_3^2)\sin^2(k\omega d/v_z)}.$$



Dependence of the transmission coefficient on the dimensionless foil thickness $D = d\omega/v_z$ in the case where $E_x = 0$ (a), E_z (b), $2E_z$ (c), and $6E_z$ (d)

The transmission coefficient $T = 1$ when $kD = k\omega d/v_z = N\pi$ ($N = 1, 2, \dots$). After the substitution $\omega \to \omega - i/\tau$, we obtain

$$T = \frac{1}{1 + (a_1^2/a_3^2)\,\mathrm{sh}^2(kD/\omega\tau)} \qquad (28)$$

in the vicinity of these resonances. Hence, resonances in the transmission coefficient T disappear with increasing their number N because of the absorption of radiation.

We can generalize Eq. (25) by taking into account that Eq. (28) is valid in the vicinity of resonances, while outside the resonances, the transmission coefficient becomes very small as D increases:

$$T = \frac{1}{1 + x^2\,\mathrm{sh}^2(kD/\omega\tau)}. \qquad (29)$$

In the numerical example, we consider the typical case where $E_z = 1.6$ keV, i.e., $\beta = v_z/c = 0.08$. The photon energy of a Ti:Sapphire laser is $\hbar\omega = 1.5$ eV. In the Figure, the transmission coefficient T derived in accordance with Eq. (29) is presented as a function of the dimensionless foil thickness $D = \omega d/v_z$ at various values of the parameter $\Delta = 1 - E_x/E_z$ and with $\omega\tau = 2$ at $\tau = 1$ fs. In Fig. *a* ($\Delta = 1$, $E_x = 0$), the curve is described by Eq. (26), as it should be. The transmission of laser radiation is depleted at the skin depth c/ω_p. In other curves, where E_x is nonzero, the transmission of laser radiation is also depleted at the skin depth c/ω_p, but sharp and narrow resonances occur where the transmission coefficient T is sufficiently high at $d > c/\omega_p$.

In conclusion, we predict a nonmonotonic behavior of the transmission coefficient, with maxima and minima, as a function of the ultra-thin foil thickness. In contrast to the results in Ref. [10], this behavior can also be observed for the isotropic velocity distribution of heated electrons in foil plasma ($\Delta = 0$, $E_x = E_z$, Fig. *b*). Our results can be used, for example, for exact determination of the thickness of ultra-thin foils based on the measurements of resonant transmission coefficient at the penetration of femtosecond laser pulses, or for determination of the velocity distribution of heated electrons inside the foil plasma.

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