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ČERENKOV RADIATION OF A SPINNING PARTICLE

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We consider the Čerenkov radiation of a neutral particle with magnetic moment, and the spin-dependent contribution to the Čerenkov radiation of a charged spinning particle. The corresponding radiation intensity is obtained for an arbitrary value of spin and for an arbitrary spin orientation with respect to velocity.

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1. The problem of Čerenkov radiation of a neutral particle with magnetic moment, moving in a medium with the refractive index n with a velocity $v > c/n$, was considered previously in Refs. [1–6]. The magnetic dipole was modeled therein classically, either by a loop with a current or by a magnetic monopole–antimonopole pair. The results thus obtained are rather model dependent, and the conclusion made in Ref. [6] is that the situation with the problem of Čerenkov radiation by a magnetic moment is not entirely clear.

In the present work, the problem is addressed as follows. A spinning particle, charged or neutral, with magnetic moment is treated as a point-like particle, i.e., is described by a well-localized wave packet. As regards the spin s, it has an arbitrary half-integer or integer value, starting with $s = 1/2$. In particular, in the limit $s \gg 1$, we arrive at the classical internal angular momentum and classical magnetic moment. The result obtained below for a neutral particle with magnetic moment differs considerably from all the previous ones. As regards the spin-dependent contribution to the Čerenkov radiation of a charged particle, the author is not aware of any previous results for it.

Certainly, the effects analyzed here are tiny, too small perhaps to be observed experimentally. Hopefully, however, their investigation is of some theoretical interest.

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2. We start with the electric and magnetic fields created by a point-like neutral particle with the magnetic moment

$$\frac{e\mathbf{s}g}{2m} = \frac{esg}{2m}\boldsymbol{\sigma};$$

here and below, g is the g-factor, and $\boldsymbol{\sigma} = \mathbf{s}/s$. Of course, for $s = 1/2$, vector $\boldsymbol{\sigma}$ consists of the common spin σ-matrices, and in the classical limit $s \gg 1$, $\boldsymbol{\sigma}$ is just a unit vector directed along $\mathbf{s}$. In the particle rest frame, the four-dimensional current density is

$$j_\alpha^{(rf)} = (0, \mathbf{j}^{(rf)}) = \frac{esg}{2m}\left(0, \nabla \times \boldsymbol{\sigma}^{(rf)}\right)\delta(\mathbf{r}^{(rf)}). \quad (1)$$

In the laboratory frame, in which we are working, this Lorentz-transformed current is formally given by

$$j_\alpha = \left(\gamma v(\mathbf{n}\cdot\mathbf{j}^{(rf)}), \mathbf{j}^{(rf)} - \mathbf{n}(\mathbf{n}\cdot\mathbf{j}^{(rf)}) + \gamma\mathbf{n}(\mathbf{n}\cdot\mathbf{j}^{(rf)})\right);$$

$$\gamma = 1/\sqrt{1-v^2}, \quad \mathbf{n} = \mathbf{v}/v$$

(we set $c = 1$ throughout). Now, we have to pass in $\mathbf{j}^{(rf)}$ from the rest-frame coordinates $\mathbf{r}^{(rf)}$ to the laboratory ones:

$$\mathbf{r}^{(rf)} = (\gamma(x - vt), y, z).$$

Under this Lorentz transformation,

$$\delta(\mathbf{r}^{(rf)}) = \delta(\gamma(x - vt))\delta(y)\delta(z) = \frac{1}{\gamma}\delta(x - vt)\delta(y)\delta(z) = \frac{1}{\gamma}\delta(\mathbf{r} - \mathbf{v}t).$$

Besides the overall factor $1/\gamma$, the components of gradient transform obviously as follows:

$$\nabla_x^{(rf)}\delta(\mathbf{r}-\mathbf{v}t)=\frac{1}{\gamma}\nabla_x\delta(\mathbf{r}-\mathbf{v}t),$$

$$\nabla_{y,z}^{(rf)}\delta(\mathbf{r}-\mathbf{v}t)=\nabla_{y,z}\delta(\mathbf{r}-\mathbf{v}t).$$

The spin operators $\boldsymbol{\sigma}^{(rf)}$, also entering $\mathbf{j}^{(rf)}$, transform the same as $\mathbf{j}^{(rf)}$ itself:

$$\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)=\boldsymbol{\sigma}^{(rf)}-\mathbf{n}(\mathbf{n}\cdot\boldsymbol{\sigma}^{(rf)})+\gamma\mathbf{n}(\mathbf{n}\cdot\boldsymbol{\sigma}^{(rf)})=$$
$$=(\gamma\sigma_x^{(rf)},\sigma_y^{(rf)},\sigma_z^{(rf)}),$$

or

$$\sigma_x^{(rf)}=\frac{1}{\gamma}\sigma_x,\quad \sigma_{y,z}^{(rf)}=\sigma_{y,z}.$$

Therefore, in the laboratory frame, the four-dimensional current density created by the magnetic moment $(esg/2m)\boldsymbol{\sigma}$ is[1)]

$$j_\alpha^g(\mathbf{r},t)=\frac{esg}{2m}\left((\boldsymbol{\sigma}\mathbf{v}\nabla),(1-v^2)\nabla\times\boldsymbol{\sigma}+\mathbf{v}(\boldsymbol{\sigma}\mathbf{v}\nabla)\right)\times$$
$$\times\,\delta(\mathbf{r}-\mathbf{v}t).\quad(2)$$

We note that this 4-current density, as well as the initial rest-frame one (1), is orthogonal to the 4-velocity u_α: $u_\alpha j_\alpha=0$. This is an extra check of the above transformations. We note also that the current density (2) can be conveniently rewritten as the sum of two four-currents, each of them being conserved separately:

$$j_\alpha^{1g}(\mathbf{r},t)=\frac{esg}{2m}(\boldsymbol{\sigma}\mathbf{v}\nabla)\,(1,\mathbf{v})\,\delta(\mathbf{r}-\mathbf{v}t),\qquad(3)$$

$$j_\alpha^{2g}(\mathbf{r},t)=\frac{esg}{2m}(1-v^2)\,(0,\nabla\times\boldsymbol{\sigma})\,\delta(\mathbf{r}-\mathbf{v}t).\qquad(4)$$

We are interested in the back-reaction of the field created by current (2) on the spin of the particle. This interaction is

$$H_g=\int d\mathbf{r}j_\alpha^g(\mathbf{r}-\mathbf{v}t)A_\alpha(\mathbf{r})=$$
$$=\frac{esg}{2m}\boldsymbol{\sigma}\left[\mathbf{H}-\frac{\gamma}{\gamma+1}\,\mathbf{v}(\mathbf{v}\cdot\mathbf{H})-\mathbf{v}\times\mathbf{E}\right],\quad(5)$$

where both field strengths, $\mathbf{H}$ and $\mathbf{E}$, are taken at the point of spin location $\mathbf{r}=\mathbf{v}t$. This is the usual interaction of the magnetic moment of a relativistic neutral particle with an external electromagnetic field. In the final expression, we actually omitted a term proportional to the total derivative of the vector potential,

$$\frac{d\mathbf{A}}{dt}=\frac{\partial\mathbf{A}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{A},$$

because a total time derivative in interaction does not result in any observable effects. Moreover, in the present case, the vector potential $\mathbf{A}$, together with the current creating it, depends on the combination $\mathbf{r}-\mathbf{v}t$ only, and therefore this total derivative vanishes identically.

This line of reasoning is generalized easily to the case of a charged particle. For this, we supplement spin current (2) with the following, also conserved, contribution:

$$j_\alpha^{th}(\mathbf{r},t)=-\frac{es}{m}\frac{\gamma}{\gamma+1}(\boldsymbol{\sigma}\mathbf{v}\nabla)\,(1,\mathbf{v})\,\delta(\mathbf{r}-\mathbf{v}t).\qquad(6)$$

In its turn, this current generates one more contribution to the spin interaction with the electromagnetic field:

$$H_{th}=\int d\mathbf{r}j_\alpha^{th}(\mathbf{r}-\mathbf{v}t)A_\alpha(\mathbf{r})=\frac{es}{m}\boldsymbol{\sigma}\times$$
$$\times\left[\left(1-\frac{1}{\gamma}\right)\mathbf{H}-\frac{\gamma}{\gamma+1}\,\mathbf{v}(\mathbf{v}\cdot\mathbf{H})-\frac{\gamma}{\gamma+1}\mathbf{v}\times\mathbf{E}\right],\quad(7)$$

which describes the well-known Thomas precession. In this expression, by the same reasons as above, we also omitted a term proportional to the total derivative $d\mathbf{A}/dt$. Finally, from now on, we work with the total interaction

$$H=H_g+H_{th}=-\frac{es}{2m}\boldsymbol{\sigma}\left[\left(g-2+\frac{2}{\gamma}\right)\mathbf{H}-\right.$$
$$\left.-\,(g-2)\frac{\gamma}{\gamma+1}\,\mathbf{v}(\mathbf{v}\cdot\mathbf{H})-\left(g-\frac{2\gamma}{\gamma+1}\right)\mathbf{v}\times\mathbf{E}\right]\quad(8)$$

and the total spin current

$$j_\alpha(\mathbf{r},t)=j_\alpha^g(\mathbf{r},t)+j_\alpha^{th}(\mathbf{r},t)=\frac{es}{2m}\times$$
$$\times\left\{\left(g-\frac{2\gamma}{\gamma+1}\right)(\boldsymbol{\sigma}\mathbf{v}\nabla)\,(1,\mathbf{v})+\right.$$
$$\left.+\,g(1-v^2)\,(0,\nabla\times\boldsymbol{\sigma})\right\}\delta(\mathbf{r}-\mathbf{v}t).\quad(9)$$

Hamiltonian (8) not only generates the spin precession, including of course the Thomas effect. It also produces the relativistic Stern–Gerlach force

$$\mathbf{F}=-\nabla\mathbf{H}.\qquad(10)$$

[1)] Here and below, $(\boldsymbol{\sigma}\mathbf{v}\nabla)=\boldsymbol{\sigma}\cdot[\mathbf{v}\times\nabla]=[\boldsymbol{\sigma}\times\mathbf{v}]\cdot\nabla$, etc.

Obviously, this force results in the energy loss and therefore is antiparallel to the velocity $\mathbf{v}$ of the spinning particle. Thus, the energy loss per unit time, or the (positive) radiation intensity, is

$$I = -\mathbf{F}\cdot\mathbf{v} = (\mathbf{v}\cdot\nabla)H. \tag{11}$$

We note here that the field strengths $\mathbf{H}$ and $\mathbf{E}$, being created by the current density $j_\alpha(\mathbf{r},t)$, depend on the noncommuting operators $\boldsymbol{\sigma}$. Therefore, to guarantee that expression (11) is Hermitian, we should, strictly speaking, properly symmetrize the products of σ-operators therein. In fact, however, the final result (see (21) below) proves to be Hermitian automatically, without extra effort.

3. The derivation in this section, resulting in general expression (21) (see below) for the spectral intensity, essentially follows that used in Ref. [7] in the problem of the usual Čerenkov radiation.

We calculate the radiation intensity by passing to the Fourier transforms $\mathbf{H}_\mathbf{k}$ and $\mathbf{E}_\mathbf{k}$ of the field strengths, defined as follows:

$$\mathbf{H}(\mathbf{r}-\mathbf{v}t) = \int d^3k \exp(i\mathbf{k}\cdot(\mathbf{r}-\mathbf{v}t))\mathbf{H}_\mathbf{k},$$

$$\mathbf{E}(\mathbf{r}-\mathbf{v}t) = \int d^3k \exp(i\mathbf{k}\cdot(\mathbf{r}-\mathbf{v}t))\mathbf{E}_\mathbf{k}.$$

For our purpose, the wave vectors $\mathbf{k}$ are conveniently decomposed into the components parallel to the velocity $\mathbf{v}$ and orthogonal to it:

$$\mathbf{k} = \mathbf{q} + \mathbf{n}\omega/v, \quad \omega = \mathbf{kv}, \quad (\mathbf{q}\cdot\mathbf{v}) = 0.$$

At the position of the point-like source, we then have

$$(\mathbf{v}\nabla)\mathbf{H}(\mathbf{r}=\mathbf{v}t) = \int d^3k\, i\omega\mathbf{H}_\mathbf{k} = -\frac{1}{v}\int d^2q \int\limits_{-\infty}^{\infty} d\omega\,\omega\mathbf{k}\times\mathbf{A}_\mathbf{k}, \tag{12}$$

$$(\mathbf{v}\nabla)\mathbf{E}(\mathbf{r}=\mathbf{v}t) = \int d^3k i\omega\mathbf{E}_\mathbf{k} = -\frac{1}{v}\int d^2q \int\limits_{-\infty}^{\infty} d\omega\omega(\omega\mathbf{A}_\mathbf{k} - \mathbf{k}\phi_\mathbf{k}), \tag{13}$$

where $\phi_\mathbf{k}$ and $\mathbf{A}_\mathbf{k}$ are the Fourier transforms of the electromagnetic scalar and vector potentials.

In the generalized Lorenz gauge

$$\operatorname{div}\mathbf{A} + \frac{\partial\hat{\varepsilon}\phi}{\partial t} = 0,$$

the wave equations for potentials are

$$\hat{\varepsilon}\left(\Delta\phi - \hat{\varepsilon}\frac{\partial^2\phi}{\partial t^2}\right) = -4\pi j_0(\mathbf{r}-\mathbf{v}t) = -4\pi\frac{es}{2m}\left(g - \frac{2\gamma}{\gamma+1}\right)(\boldsymbol{\sigma}\mathbf{v}\nabla)\delta(\mathbf{r}-\mathbf{v}t), \tag{14}$$

$$\Delta\mathbf{A} - \hat{\varepsilon}\frac{\partial^2\mathbf{A}}{\partial t^2} = -4\pi\mathbf{j}(\mathbf{r}-\mathbf{v}t) = -4\pi\frac{es}{2m}\times \left\{\left(g - \frac{2\gamma}{\gamma+1}\right)(\boldsymbol{\sigma}\mathbf{v}\nabla)\mathbf{v} + g(1-v^2)\nabla\times\boldsymbol{\sigma}\right\}\times \delta(\mathbf{r}-\mathbf{v}t), \tag{15}$$

where the "dielectric constant" $\hat{\varepsilon}$ should be understood as an operator; below, we use its Fourier transform $\varepsilon(\omega)$. As regards the permeability $\mu(\omega)$, it can be put equal to unity for the frequencies of interest to us.

For the Fourier transforms of the potentials, we now obtain

$$\phi_\mathbf{k} = \frac{i}{2\pi^2}\frac{1}{\varepsilon(\omega)}\frac{es}{2m}\left(g - \frac{2\gamma}{\gamma+1}\right)\frac{(\mathbf{vk}\boldsymbol{\sigma})}{k^2 - \varepsilon(\omega)\omega^2} = \frac{i}{2\pi^2}\frac{1}{\varepsilon(\omega)}\frac{es}{2m}\left(g - \frac{2\gamma}{\gamma+1}\right)\times \frac{(\mathbf{vq}\boldsymbol{\sigma})}{q^2 - [\varepsilon(\omega) - 1/v^2]\omega^2}, \tag{16}$$

$$\mathbf{A}_\mathbf{k} = \frac{i}{2\pi^2}\frac{es}{2m}\frac{g(1-v^2)[\mathbf{k}\times\boldsymbol{\sigma}] + (g - 2\gamma/(\gamma+1))\,\mathbf{v}(\mathbf{vk}\boldsymbol{\sigma})}{k^2 - \varepsilon(\omega)\omega^2} = \frac{i}{2\pi^2}\frac{es}{2m}\frac{g(1-v^2)[(\mathbf{q}+\mathbf{n}\omega/v)\times\boldsymbol{\sigma}] + (g - 2\gamma/(\gamma+1))\,\mathbf{v}(\mathbf{vq}\boldsymbol{\sigma})}{q^2 - [\varepsilon(\omega) - 1/v^2]\omega^2}. \tag{17}$$

After substituting (16) and (17) in (12) and (13), we note that

$$\int d^2q \to \pi\int dq^2, \quad \int d^2q\, q_m \to 0,$$

$$\int d^2q\, q_m q_n = \frac{1}{2}\delta_{mn}\pi\int dq^2 q^2.$$

We also note that

$$\int\limits_{-\infty}^{\infty} \frac{d\omega\,\omega q^2}{q^2-[\varepsilon(\omega)-1/v^2]\omega^2} = \\ = \int\limits_{-\infty}^{\infty} d\omega\,\omega \left\{1+\frac{[\varepsilon(\omega)-1/v^2]\omega^2}{q^2-[\varepsilon(\omega)-1/v^2]\omega^2}\right\} = \\ = \int\limits_{-\infty}^{\infty} \frac{d\omega\omega^3[\varepsilon(\omega)-1/v^2]}{q^2-[\varepsilon(\omega)-1/v^2]\omega^2}.$$

Then the integral over q^2 is conveniently combined with the explicit dependence on ω into the following overall factor for the spectral intensity:

$$I(\omega) \sim f(\omega) = \\ = -i\sum \omega^3 \int \frac{dq^2}{q^2-[\varepsilon(\omega)-1/v^2]\omega^2}. \quad (18)$$

The symbol $\sum$ in this expression means that we should sum over the signs of the frequency: both $\omega = +|\omega|$ and $\omega = -|\omega|$ contribute to the intensity $I(\omega)$. All other dependence of the total result on ω is via $\varepsilon(\omega)$ only; in our problem of the Čerenkov radiation, we restrict ourself to the frequencies corresponding to the transparency region, i.e., to real $\varepsilon(\omega)$, which is an even function of ω.

We now analyze the expression

$$f(\omega) = -i\sum \omega^3 \int \frac{dq^2}{q^2-[\varepsilon(\omega)-1/v^2]\omega^2}$$

entering result (18). The poles of its integrand obviously correspond to the vanishing four-momentum squared of a photon in the medium. Here, however, we should retain a small imaginary part in $\varepsilon(\omega)$: $\operatorname{Im}\varepsilon(\omega) > 0$ for $\omega > 0$, and $\operatorname{Im}\varepsilon(\omega) < 0$ for $\omega < 0$. In other words, the poles of the integrand in $f(\omega)$ tend to the real axis from above for $\omega > 0$, and from below for $\omega < 0$. Therefore, their contributions to the integral are $i\pi$ and $-i\pi$, respectively. The real part of the integral is an even function of ω (together with $\operatorname{Re}\varepsilon(\omega)$), and therefore its contributions to the sum $f(\omega)$ cancel. Returning to the poles, their contributions to $f(\omega)$ are $i\pi\omega^3$ and $-i\pi(-\omega)^3 = i\pi\omega^3$, where ω is positive from now on. Thus,

$$f(\omega) = 2\pi\omega^3,$$

Quite straightforward (although rather tedious) transformations now result in the following expressions for $(\mathbf{v}\nabla)\mathbf{H}$ and $(\mathbf{v}\nabla)\mathbf{E}$:

$$(\mathbf{v}\nabla)\mathbf{H}(\mathbf{r}=\mathbf{v}t) = \frac{es}{2m}\frac{\omega^3 d\omega}{2v}\times \\ \times\left\{-\boldsymbol{\sigma}_\perp\left[\left(g-2+\frac{2}{\gamma}\right)\left(\varepsilon-\frac{1}{v^2}\right)+\frac{2g}{\gamma^2 v^2}\right]- \\ -\boldsymbol{\sigma}_\parallel \frac{g}{\gamma^2}\left(\varepsilon-\frac{1}{v^2}\right)\right\}, \quad (19)$$

$$(\mathbf{v}\nabla)\mathbf{E}(\mathbf{r}=\mathbf{v}t) = \frac{es}{2m}\frac{\omega^3 d\omega}{2v}\times \\ \times\left[\left(g-\frac{2\gamma}{\gamma+1}\right)\frac{1}{\varepsilon}\left(\varepsilon-\frac{1}{v^2}\right)+2g(1-v^2)\frac{1}{v^2}\right]\times \\ \times[\mathbf{v}\times\boldsymbol{\sigma}_\perp], \quad (20)$$

here and below, $\boldsymbol{\sigma}_\perp$ and $\boldsymbol{\sigma}_\parallel$ are the components of the vector $\boldsymbol{\sigma}$, orthogonal and parallel to the velocity $\mathbf{v}$.

Substituting these expressions in (8) and (10), we obtain at the final general result for the spectral intensity of Čerenkov radiation by a spinning particle:

$$I(\omega)d\omega = \left(\frac{es}{2m}\right)^2\frac{\omega^3 d\omega}{2v}\left\{\left[\left(g-2+\frac{2}{\gamma}\right)^2\times\right.\right. \\ \times\left(n^2(\omega)-\frac{1}{v^2}\right)-\left(g-2+\frac{2}{\gamma+1}\right)^2\left(v^2-\frac{1}{n^2(\omega)}\right)+ \\ \left.\left.+\frac{2g^2}{\gamma^4 v^2}\right]\boldsymbol{\sigma}_\perp^2+\frac{g^2}{\gamma^3}\left(n^2(\omega)-\frac{1}{v^2}\right)\boldsymbol{\sigma}_\parallel^2\right\}. \quad (21)$$

Few remarks on this result are in order.

First, the formal singularity of (21) in v should not worry us: anyway, Čerenkov radiation occurs for $v \ge 1/n$ only. Second, as distinct from the common Čerenkov radiation, the contribution to the energy loss due to $\boldsymbol{\sigma}_\perp$ does not vanish here at the threshold $v = 1/n$. Finally, it is not exactly clear at first glance whether the structure

$$\left(g-2+\frac{2}{\gamma}\right)^2\left(n^2-\frac{1}{v^2}\right)- \\ -\left(g-2+\frac{2}{\gamma+1}\right)^2\left(v^2-\frac{1}{n^2}\right)+\frac{2g^2}{\gamma^4 v^2} \quad (22)$$

at $\boldsymbol{\sigma}_\perp^2$ is positive definite (as it should be for arbitrary g and γ!). To prove that it is, we note that the discussed quadratic function of g is certainly positive definite as $g \to \infty$ for $v \ge 1/n$. On the other hand, the discriminant d of this quadratic form is negative definite:

$$d = -4\varepsilon\frac{v^2}{\gamma^2}\left(1-\frac{1}{n^2v^2}\right)^2.$$

Therefore, quadratic form (22) is indeed positive definite.

Of course, in the case of a charged spinning particle, the common Čerenkov radiation occurs as well (and is strongly dominating quantitatively). But don't we have then some combined effect, a Čerenkov-type radiation of the first order in spin? By symmetry reasons, it is practically obvious that such an effect should not exist, but we present somewhat more quantitative arguments. The effect could arise due to the Lorentz force

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{H}),$$

with $\mathbf{E}$ and $\mathbf{H}$ generated by spin current density (9). However, the magnetic contribution $e\mathbf{v} \times \mathbf{H}$ to the energy loss $-\mathbf{vF}$ vanishes trivially. As regards the corresponding electric contribution $-e\mathbf{v} \cdot \mathbf{E}(\mathbf{r} = \mathbf{v}t)$ to the energy loss, it can be demonstrated explicitly with formulas (16) and (17) that it also vanishes. Equally explicitly, it can be demonstrated that the contribution to the energy loss due to Stern–Gerlach force (11), but now with $\mathbf{H}$ and $\mathbf{E}$ generated by the common convection current $j_\mu(\mathbf{r}, t) = e(1, \mathbf{v})\delta(\mathbf{r} - \mathbf{v}t)$, vanishes as well.

4. In conclusion, we consider some particular cases of general result (21).

We start with a neutral particle with a finite magnetic moment μ. As $e \to 0$, $g \to \infty$, and $\mu = esg/2m \to$ $\to$ const, we obtain

$$I(\omega)d\omega = \frac{\mu^2\omega^3}{2v}d\omega\left[\left(n^2 - \frac{1}{v^2} - v^2 + \frac{1}{n^2} + \frac{2}{\gamma^4 v^2}\right)\boldsymbol{\sigma}_\perp^2 + \frac{1}{\gamma^3}\left(n^2 - \frac{1}{v^2}\right)\boldsymbol{\sigma}_\parallel^2\right]. \quad (23)$$

For $s = 1/2$ (i. e., for the Dirac neutrino with a mass and magnetic moment), $\boldsymbol{\sigma}_\perp^2 = \boldsymbol{\sigma}^2 - \sigma_z^2 = 2$ and $(\boldsymbol{\sigma} \cdot \mathbf{n})^2 = \sigma_z^2 = 1$. Therefore, it follows from (23) that

$$I(\omega)d\omega = \frac{\mu^2\omega^3}{v}d\omega\left[\left(n^2 - \frac{1}{v^2} - v^2 + \frac{1}{n^2}\right) + \frac{1}{2\gamma^3}\left(n^2 - \frac{1}{v^2}\right) + \frac{2}{\gamma^4 v^2}\right]. \quad (24)$$

In the classical limit $s \gg 1$, radiation intensity (23) becomes

$$I(\omega)d\omega = \frac{\mu^2\omega^3}{2v}d\omega\left[\left(n^2 - \frac{1}{v^2} - v^2 + \frac{1}{n^2} + \frac{2}{\gamma^4 v^2}\right)\sin^2\theta + \frac{1}{\gamma^3}\left(n^2 - \frac{1}{v^2}\right)\cos^2\theta\right], \quad (25)$$

where θ is the angle between the spin and velocity.

The opposite limit is that of a charged particle with the vanishing g-factor. The effect here is finite and is given by

$$I(\omega)d\omega = \left(\frac{es}{2m}\right)^2\frac{2\omega^3 d\omega}{v}\left[\left(\frac{\gamma-1}{\gamma}\right)^2\left(n^2 - \frac{1}{v^2}\right) - \left(\frac{\gamma}{\gamma+1}\right)^2\left(v^2 - \frac{1}{n^2}\right)\right]\boldsymbol{\sigma}_\perp^2. \quad (26)$$

We finally mention the case $g = 2$ (applicable for instance, to an electron if its small anomalous magnetic moment is neglected). Here,

$$I(\omega)d\omega = \left(\frac{es}{2m}\right)^2\frac{2\omega^3 d\omega}{v}\left\{\left[\frac{1}{\gamma^2}\left(n^2 - \frac{1}{v^2}\right) - \frac{1}{(\gamma+1)^2}\left(v^2 - \frac{1}{n^2}\right) + \frac{2}{\gamma^4 v^2}\right]\boldsymbol{\sigma}_\perp^2 + \frac{1}{\gamma^3}\left(n^2 - \frac{1}{v^2}\right)\boldsymbol{\sigma}_\parallel^2\right\}. \quad (27)$$

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