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CONTROLLING CHAOS IN A BOSE–EINSTEIN CONDENSATE LOADED INTO A MOVING OPTICAL LATTICE POTENTIAL

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The spatial structure of a Bose–Einstein condensate loaded into an optical lattice potential is investigated and spatially chaotic distributions of the condensates are revealed. Through changing the s-wave scattering length by using a Feshbach resonance, the chaotic behavior can be well controlled to enter into periodicity. Numerical simulation shows that there are different periodic orbits according to different s-wave scattering lengths only if the maximal Lyapunov exponent of the system is negative.

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1. INTRODUCTION

Eighty years after its prediction, the Bose–Einstein condensate (BEC) has been observed in trapped gases of rubidium, sodium, and lithium [1]. The mean-field theory (Gross–Pitaevskii (GP) equation) has been quite successful in quantitatively reproducing many experimental observations [2].

The realization of BEC in dilute alkali vapors has opened the field of a weakly interacting degenerate Bose gas. Subsequent experimental and theoretical progress has been made in studying the properties of this new state of matter. Several remarkable phenomena, which strongly resemble well-known effects in nonlinear optics, have been observed in BEC, such as four-wave mixing, vortices, dark and bright solitons, and chaos [3–12]. In realistic experimental setting, an external electromagnetic field is used to produce, trap, and manipulate the BEC. In early experiments, only the harmonic potential was used, but a wide variety of potentials can now be constructed experimentally. Among the most frequently studied both experimentally and theoretically are periodic optical lattice potentials. The optical lattice is created as a standing-wave interference pattern of mutually coherent laser beams. With each lattice site occupied by one mass of alkali atoms in its ground state, the BEC in optical lattices shows a number of potential applications, such as an atomic interferometer, registers for quantum computers, an atom laser, quantum information processing on the nanometer scale, and others. Optical lattices are therefore of particular interest from the perspective of both fundamental quantum physics and its relation to applications [8].

Numerous experimental studies have confirmed the general validity of the time-dependent nonlinear Schrödinger equation, also called the GP equation, used to calculate the ground state and excitations of various BECs of trapped alkali atoms. The dynamics of the system are described by a Schrödinger equation with a nonlinear term that represents many-body interactions in the mean-field approximation. This nonlinearity allows introducing chaos into a quantum system. The existence of BEC chaos has been proved and the chaos properties have also been extensively investigated in many previous works. Naturally, chaos, which plays a role in the regularity of the system, causes instability of the condensate wave function. The study of chaos in nonlinear deterministic systems has been underway for many years. Besides addressing the basic questions about the mechanisms and the predictions of chaos, however, the ability to control it to a regular state is

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also an important subject for the relevant studies.

For the purpose of applications, the control of chaos is anticipated in practical investigations. In [13], the Ott–Grebogi–Yorker scheme of controlling chaos in a BEC system was proposed. Chaos control has always been a widely attractive field since the pioneering work [13]. Controlling chaos can be separated into two categories: feedback control (active control) and non-feedback control (passive control). The general method for feedback control is to push a system state onto a stable manifold of a target orbit, that is, to stabilize the unstable target orbits embedded within a chaotic attractor. The main purpose of the present paper is to control the chaos into the stable states in the BEC through changing the s-wave scattering length by using the Feshbach resonance. We can force the system to the stable periodic orbit.

2. CONTROLLING THE CHAOS IN THE BEC SYSTEM

The BEC system considered here is created in a harmonically trapped potential and is then loaded into a moving optical lattice. The $3D$ combined potential is therefore given by

$$V(x,y,z,t_1) = V_1\cos^2(k\xi)+m(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)/2,$$

where the second term is the harmonic magnetic potential, with m being the atomic mass and ω_x, ω_y, ω_z the trap frequencies. The periodic potential is a moving optical lattice with the space–time variable $\xi = x+\delta t_1/2k$, where δ is the frequency difference between the two counterpropagating laser beams and k is the laser wave vector that determines the velocity of the traveling lattice as $V_L = \delta/2k$. When the BEC is formed in the region near the center of the magnetic trap, the magnetic potential is much weaker than the lattice one and can be neglected. We find that in the region

$$k\sqrt{x^2+y^2/2+z^2/4} \le 100\pi,$$

the harmonic potential is much smaller than the lattice potential. Therefore, the $1D$ optical potential plays the main role in the system and the quasi-$1D$ approximation is valid in this region. On the other hand, for a time-dependent lattice, the damping effect should be considered. The damping effect caused by the incoherent exchange of normal atoms and the finite-temperature effect [14–16] has been analyzed in detail for the two-junction linking of two BECs [14]. For the system considered here, it is similar to the case of the linear junction linking of many BECs. Thus, a damping effect caused by similar elements or other factors may also exist. With these considerations, the system is governed by the quasi-$1D$ GP equation [17]

$$i\hbar(1-i\lambda)\frac{\partial\psi}{\partial t_1} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + g_0|\psi|^2\psi + \nu_1\cos^2(k\xi)\psi, \quad (1)$$

where $g_0 = 4\pi\hbar^2 a/m$ denotes the interatomic interaction with a being the s-wave scattering length. The case $a > 0$ represents a repulsive interatomic interaction, and $a < 0$ implies the case of attraction. The parameter ψ is the macroscopic quantum wave function. The term proportional to γ represents the damping effect.

We focus our attention on only the traveling-wave solution of this equation and write Eq. (1) in the form

$$\psi = \varphi(\xi)\exp[i(\alpha x+\beta t_1)] \quad (2)$$

such that the matter wave is a Bloch-like wave. Here, α and β are two undetermined real constants. According to the definition of the space–time variable $\xi = x+V_Lt_1$, the traveling wave $\varphi(\xi)$ moves with the same velocity as the optical lattice. Inserting Eq. (2) in Eq. (1), we can easily turn partial differential equation (1) into the ordinary differential equation

$$\frac{\hbar^2}{2m}\frac{d^2\varphi}{d\xi^2} + i\left(\frac{\hbar^2\alpha}{m}+\hbar\nu_L - i\hbar\gamma\nu_L\right)\frac{d\varphi}{d\xi} - \left(\hbar\beta+\frac{\hbar^2\alpha^2}{2m}-i\hbar\beta\gamma\right)\varphi - g_0|\varphi|^2\varphi = \nu_1\cos^2(k\xi)\varphi. \quad (3)$$

For simplicity, using the dimensionless variables and parameters

$$\eta = k\xi, \quad \nu = \frac{2m\nu_L}{\hbar k},$$

$$\beta_1 = \frac{\hbar\beta}{E_r}, \quad \alpha_1 = \frac{\alpha}{k}, \quad I_0 = \frac{\nu_1}{E_r},$$

we set

$$\varphi = R(\eta)e^{i\theta(\eta)}, \quad \frac{d\theta}{d\eta} = -\frac{\beta_1}{\nu} = -\left(\frac{\nu}{2}+\alpha_1\right).$$

Then Eq. (3) becomes

$$\begin{cases} \dfrac{dy_1}{d\eta} = \dfrac{dR}{d\eta} = y_2, \\ \dfrac{dy_2}{d\eta} = \dfrac{d^2R}{d\eta^2} = \dfrac{1}{4}\nu^2y_1+gy_1^3+I_0y_1\cos^2\eta-\gamma\nu y_2, \end{cases} \quad (4)$$

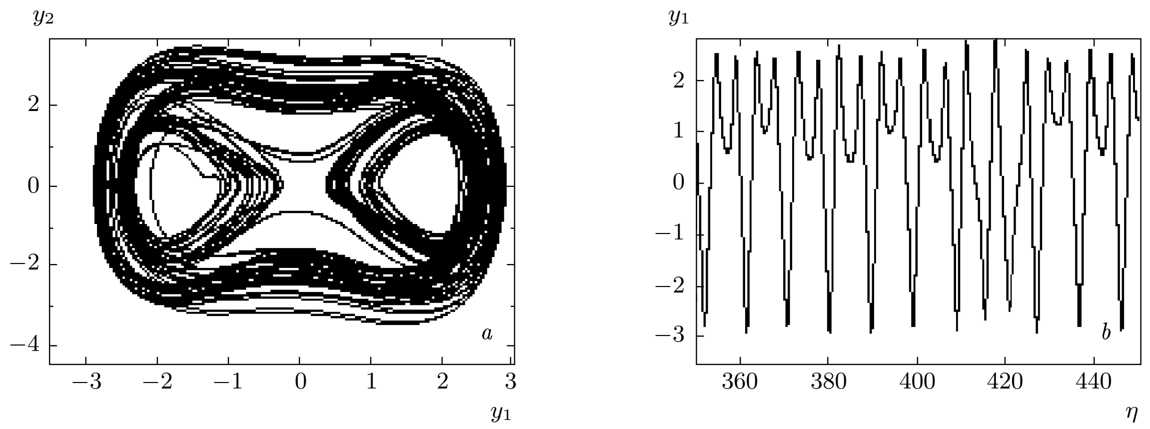

Fig. 1. The chaotic attractor projection on the y_1y_2 plane and the time series with $I_0 = 1.85$, $\gamma = 0.05$, $y = 2.03$, and $g = -0.75$

where I_0 is the optical intensity and $\nu = 2m\nu_L/\hbar k$. The square of the amplitude R is just the particle number density because $|R| = |\varphi| = |\psi|$, and θ is the phase of φ [5].

According to the general theory of the Duffing equation, Eq. (4) has a monoclinic solution only when the coefficients of the linear (R) and nonlinear (R^3) terms in the left-hand side of Eq. (4) have opposite signs. Therefore, to study the chaos for a negative R term, we must consider the case of attractive atom–atom interactions, i.e., $g < 0$; Eq. (4) is just the parametrically driven Duffing equation with a damping term. The square of the amplitude R is just the particle number density.

We solve Eq. (4) numerically using the fourth Runge–Kutta (RK) algorithm. To avoid transient chaos, y_1 and y_2 in the initial 10000 steps are eliminated. The initial conditions are $y_1 = 18.0$, $y_2 = 0.1$, and $\eta = 0$. The parameters in Eq. (4) are $I_0 = 1.85$, $\gamma = 0.05$, $\nu = 2.03$, and $g = -0.75$.

Figure 1*a* shows the strange attractor projected onto the y_1y_2 plane; however, we cannot tell whether this attractor is chaotic. We calculate the maximal Lyapunov exponent of the BEC system using the algorithms presented in [11, 18, 19]. The maximal Lyapunov exponent of the BEC system is $\lambda_{max} = 0.0792$. The system lies in a chaotic state because there exist one positive Lyapunov exponent. Figure 1*b* shows the time series of y_1, and we can find that the value seems to be random, but it is different from noise signals without rules and seems to change following some regularity.

3. NUMERICAL RESULTS

To control the chaos in a BEC loaded into a moving optical lattice potential, we adjust the two-body interaction by changing the s-wave scattering length, that is, changing the value of g. In this paper, we only consider the effect of the s-wave.

Figure 2 shows the maximal Lyapunov exponent as a function of the s-wave scattering length g. The middle point-drawing line stands for the value of zero. We find that in many ranges, for example $-0.651 < g < -0.6525$, $-0.575 < g < -0.578$, the maximal Lyapunov exponent is negative. If g takes a value in these ranges, the BEC is in a periodic state. The BEC is in a periodic state when g takes values as -0.661 and -0.561.

We solve Eq. (4) numerically by using the fourth RK algorithm. The values of y_1 and y_2 in the initial 30000 steps are eliminated. The last 10000 steps of y_1 and y_2 are retained. The initial conditions are $y_1 = 18.0$, $y_2 = 0.1$, and $\eta = 0$.

Figure 3 shows the attractor projected onto the y_1y_2 plane, and the time series of y_1. The parameters are the same as in Fig. 2, the other parameters being $g = -0.661$ and -0.561. We find that in Fig. 3*a*, C is in periods 1 and 3 respectively when $g = -0.661$ and -0.561. Figure 3*b,d* shows the respective time series. We can therefore transform the chaotic state into the periodic regular state by modulating the s-wave length g.

4. CONCLUSIONS

In summary, we have investigated the chaotic features in the spatial distributions of the BEC. We present a method of controlling chaos via changing the

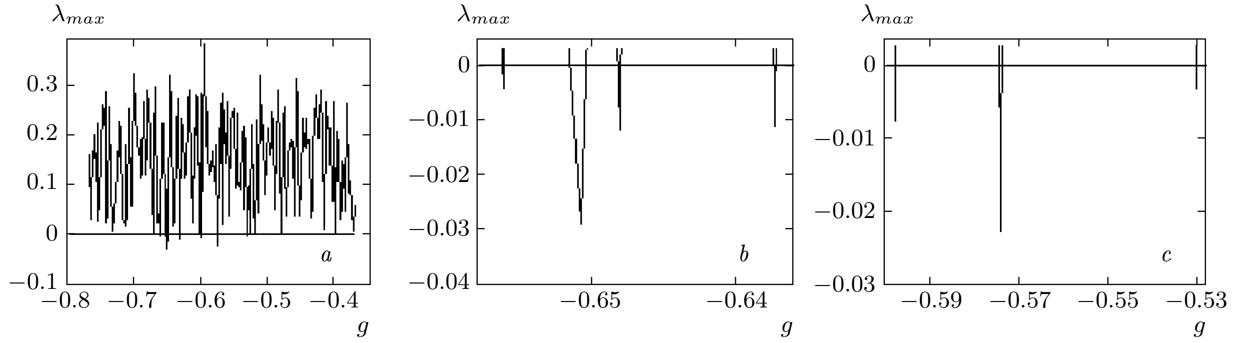

Fig. 2. The maximal Lyapunov exponent λ_{max} as a function of the s-wave scattering length g (a) with $\gamma = 0.05$, $\nu = 2.03$, and $I_0 = 1.85$

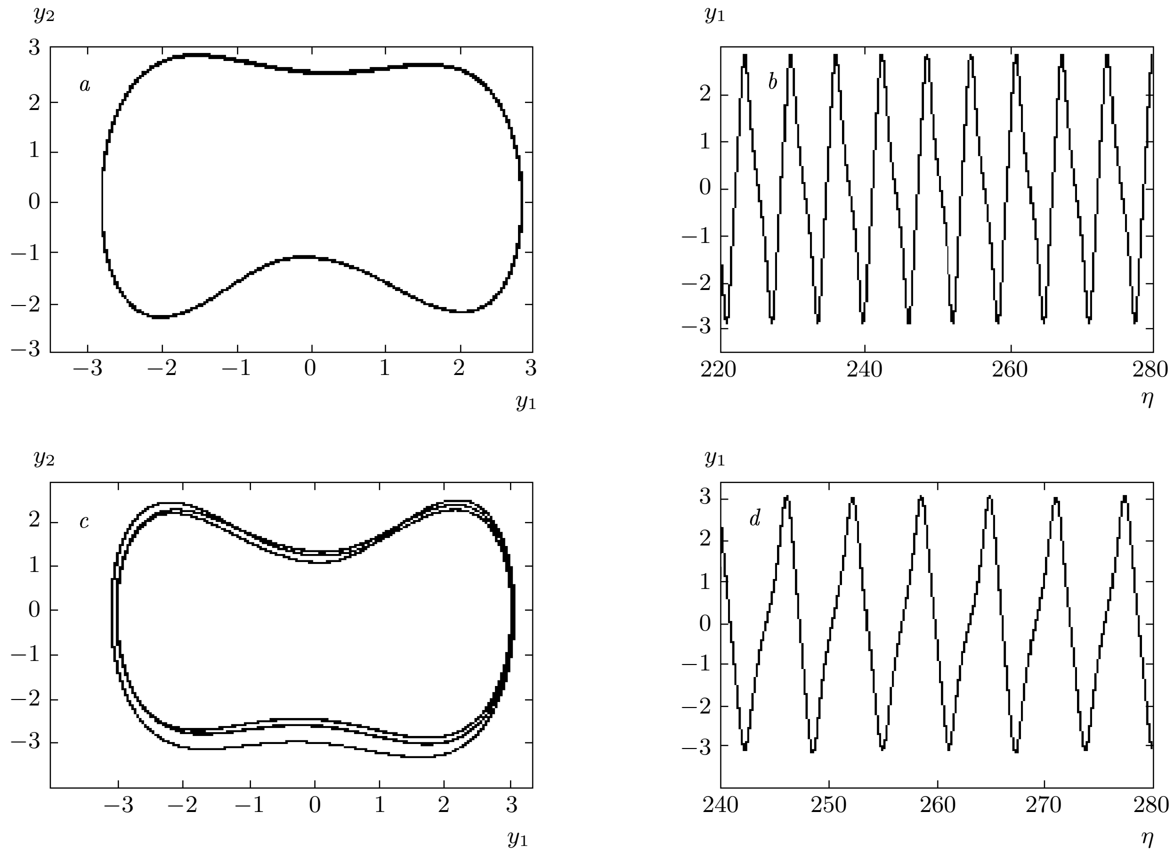

Fig. 3. The attractor projection on the $y_1 y_2$ plane and the time series of y_1 at different s-wave scattering lengths with $\gamma = 0.05$, $\nu = 2.03$, $I_0 = 1.85$, $g = -0.661$ (a,b), $g = -0.561$ (c,d)

s-wave scattering length. Numerical simulation shows that the period is different for different s-wave scattering lengths.

It is well known that the periodic lattice systems in BEC have many fantastic properties. For example, quantum computation with BEC atoms in a Mott insulating state is an interesting advancement in application of the BEC. On the other hand, chaos is associated with quantum entanglement and quantum error correcting, which are both fundamental subjects in quantum computations. Thus, it is important to apply or control the chaos in a system.

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