UPDATE OF THE $e^+e^- \to \pi^+\pi^-$ CROSS SECTION MEASURED BY THE SPHERICAL NEUTRAL DETECTOR IN THE ENERGY REGION $400 < \sqrt{s} < 1000$ MeV

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The corrected cross section of the $e^+e^- \to \pi^+\pi^-$ process measured in the spherical neutral detector experiment at the VEPP-2M e^+e^- collider is presented. The update is necessary due to a flaw in the $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \mu^+\mu^-$ Monte Carlo event generators used previously in data analysis.

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The spherical neutral detector (SND) [1] operated from 1995 to 2000 at the VEPP-2M e^+e^- collider [2]. One of the recent SND results was the measurement of the $e^+e^- \to \pi^+\pi^-$ process cross section in the energy region $\sqrt{s} < 1000$ MeV [3]. The systematic error of the cross section determination was estimated to be 1.3 %. Studies of the $e^+e^- \to \pi^+\pi^-$ reaction allow determining the ρ and ω meson parameters and provide information on the G-parity violation mechanism in the $\omega \to \pi^+\pi^-$ decay.

Much attention was recently attracted by the $e^+e^- \to \pi^+\pi^-$ cross section because of the interest in high-precision results of the muon anomalous magnetic moment measurement [4, 5]. The comparison of the $e^+e^- \to \pi^+\pi^-$ process cross section with the spectral function in the $\tau^\pm \to \pi^\pm\pi^0\nu_\tau$ decay [6–8] is used to test the vector current conservation (VCC).

Theoretical calculations of cross sections of the $e^+e^- \to e^+e^-, \pi^+\pi^-, \mu^+\mu^-$ reactions play an important role in measurements of the $e^+e^- \to \pi^+\pi^-$
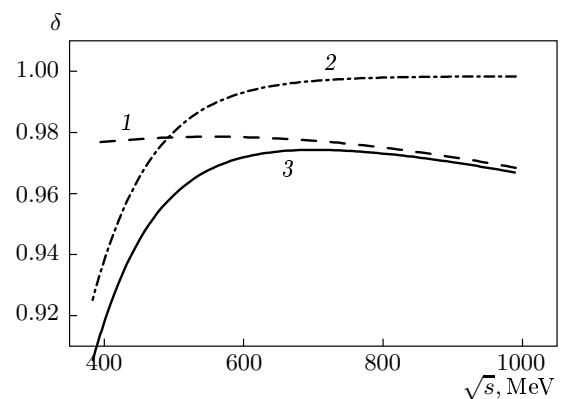

Fig. 1. Corrections $\delta = \delta_\pi \delta_\mu$ to the $e^+e^- \to \pi^+\pi^-$ cross section [3], which account for the mistakes of the $e^+e^- \to \pi^+\pi^-$ (1) and $e^+e^- \to \mu^+\mu^-$ (2) cross section calculations and the total correction δ (3)

process. Such calculations are necessary for luminosity measurements ($e^+e^- \to e^+e^-$ events), the $e^+e^- \to \mu^+\mu^-$ background subtraction, and determi-

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Table 1. The results of the $e^+e^- \to \pi^+\pi^-$ cross section measurements. $\sigma_{\pi\pi}$ and $|F_\pi|^2$ are the cross section and the form factor of the $e^+e^- \to \pi^+\pi^-$ process, $\sigma_{\pi\pi}^{pol}$ is the undressed $e^+e^- \to \pi^+\pi^-$ cross section without vacuum polarization but with the final state radiation. Only uncorrelated errors are shown. The correlated systematic error σ_{sys} is 1.3 % for $\sqrt{s} \geq 420$ MeV and 3.2 % for $\sqrt{s} < 420$ MeV

$\sqrt{s}$, MeV	$\sigma_{\pi\pi}$, nb	$|F_\pi|^2$	$\sigma_{\pi\pi}^{pol}$, nb
970.0	76.68 ± 1.79	3.78 ± 0.09	75.06 ± 1.75
958.0	91.33 ± 1.96	4.41 ± 0.09	89.22 ± 1.91
950.0	101.52 ± 1.93	4.83 ± 0.09	99.07 ± 1.88
940.0	115.14 ± 1.57	5.38 ± 0.07	112.25 ± 1.53
920.0	147.78 ± 5.15	6.66 ± 0.23	143.57 ± 5.00
880.0	246.38 ± 2.80	10.30 ± 0.12	237.93 ± 2.70
840.0	450.70 ± 4.19	17.46 ± 0.16	433.39 ± 4.03
820.0	622.54 ± 5.54	23.19 ± 0.21	597.24 ± 5.31
810.0	715.94 ± 6.21	26.15 ± 0.23	685.26 ± 5.94
800.0	822.66 ± 7.05	29.46 ± 0.25	785.42 ± 6.73
794.0	859.35 ± 7.19	30.41 ± 0.25	815.84 ± 6.83
790.0	855.55 ± 16.98	30.04 ± 0.60	806.96 ± 16.02
786.0	874.23 ± 7.42	30.45 ± 0.26	820.44 ± 6.96
785.0	887.68 ± 8.81	30.86 ± 0.31	835.25 ± 8.29
784.0	940.42 ± 19.12	32.62 ± 0.66	890.94 ± 18.11
783.0	1022.45 ± 10.99	35.40 ± 0.38	979.52 ± 10.53
782.0	1106.69 ± 26.44	38.24 ± 0.91	1073.85 ± 25.66
781.0	1161.62 ± 10.84	40.06 ± 0.37	1138.88 ± 10.63
780.0	1233.58 ± 10.17	42.45 ± 0.35	1220.00 ± 10.06
778.0	1314.33 ± 9.78	45.05 ± 0.34	1309.00 ± 9.74
774.0	1331.59 ± 9.87	45.28 ± 0.34	1326.86 ± 9.83
770.0	1302.16 ± 9.67	43.92 ± 0.33	1296.23 ± 9.63
764.0	1304.40 ± 9.80	43.47 ± 0.33	1297.32 ± 9.75
760.0	1308.40 ± 10.08	43.26 ± 0.33	1301.28 ± 10.03
750.0	1291.96 ± 22.80	41.86 ± 0.74	1288.31 ± 22.74
720.0	1060.14 ± 7.11	32.31 ± 0.22	1064.32 ± 7.14

Table 1. Continued

$\sqrt{s}$, MeV	$\sigma_{\pi\pi}$, nb	$|F_\pi|^2$	$\sigma_{\pi\pi}^{pol}$, nb
690.0	764.53 ± 8.31	21.92 ± 0.24	769.56 ± 8.36
660.0	543.75 ± 6.24	14.66 ± 0.17	546.05 ± 6.27
630.0	398.61 ± 8.73	10.11 ± 0.22	399.49 ± 8.75
600.0	296.06 ± 10.92	7.08 ± 0.26	296.17 ± 10.92
580.0	261.49 ± 14.78	6.01 ± 0.34	261.11 ± 14.76
560.0	230.91 ± 12.69	5.12 ± 0.28	230.54 ± 12.67
550.0	221.00 ± 17.83	4.81 ± 0.39	220.33 ± 17.78
540.0	215.61 ± 13.79	4.62 ± 0.30	214.99 ± 13.75
530.0	202.32 ± 23.04	4.26 ± 0.49	201.77 ± 22.98
520.0	179.55 ± 10.42	3.72 ± 0.22	179.10 ± 10.39
510.0	175.37 ± 16.81	3.58 ± 0.34	174.62 ± 16.74
500.0	176.32 ± 10.93	3.55 ± 0.22	175.60 ± 10.89
480.0	165.60 ± 9.72	3.26 ± 0.19	165.01 ± 9.69
470.0	143.61 ± 13.28	2.81 ± 0.26	143.14 ± 13.24
450.0	140.47 ± 14.24	2.71 ± 0.28	139.82 ± 14.17
440.0	114.75 ± 15.51	2.22 ± 0.30	114.26 ± 15.44
430.0	109.25 ± 12.54	2.11 ± 0.24	108.83 ± 12.49
410.0	125.06 ± 18.92	2.46 ± 0.37	124.70 ± 18.87
390.0	116.37 ± 21.78	2.39 ± 0.45	116.17 ± 21.74

It was found recently that the $e^+e^- \to \pi^+\pi^-$ and $\mu^+\mu^-$ Monte Carlo event generators used in the SND data analysis were not quite correct and understated the $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \mu^+\mu^-$ cross sections by about 2.5 % and 1.5 % respectively.

In this paper, in order to correct the error, the measured cross section σ_0 is multiplied by correction factors,

$$\sigma = \sigma_0 \cdot \delta_\pi \cdot \delta_\mu, \tag{1}$$

where δ_π and δ_μ are the respective corrections due to mistakes in calculations of the $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \mu^+\mu^-$ cross sections. The δ_π and δ_μ coefficients were determined using the MCGPJ $e^+e^- \to \mu^+\mu^-$, $\pi^+\pi^-$ event generator [11], which is based on the same approach as in [9, 10]. The applied corrections have not altered the systematic error value of the $e^+e^- \to \pi^+\pi^-$ cross section measurement, which is 1.3 % for the energy region $\sqrt{s} \geq 420$ MeV and 3.2 % for $\sqrt{s} < 420$ MeV.

The energy dependence of δ_π and δ_μ is shown in Fig. 1. The error in the $e^+e^- \to \mu^+\mu^-$ cross section calculation is significant only in the energy re-

nation of radiative corrections and the $e^+e^- \to \pi^+\pi^-$ detection efficiency. The $e^+e^- \to \pi^+\pi^-, \mu^+\mu^-$ cross sections were calculated in accordance with the formulas of Refs. [9, 10], which take the photons radiation by the initial and final state particles into account and have the accuracy about 0.2 %.

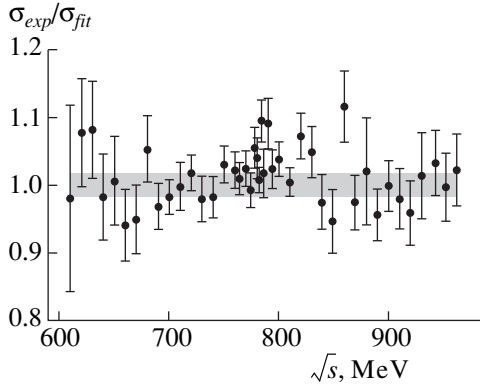**Fig. 2.** The ratio $\sigma_{exp}/\sigma_{fit}$ of the $e^+e^- \to \pi^+\pi^-$ cross section measured by CMD-2 [12] to the SND fit curve. The shaded area shows the systematic error
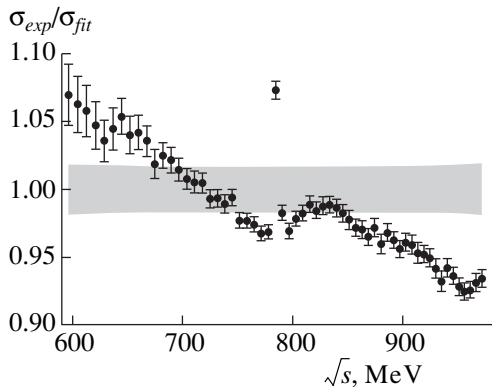

Fig. 3. The ratio $\sigma_{exp}/\sigma_{fit}$ of the $e^+e^- \to \pi^+\pi^-$ cross section measured by KLOE [13] to the SND fit curve. The shaded area shows the systematic error

Fig. 4. The ratio σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay spectral function measured by CLEOII [7] to the isovector part of the $e^+e^- \to \pi^+\pi^-$ cross section corrected in this work. The shaded area shows the systematic error

Fig. 5. The ratio σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay spectral function measured by ALEPH [8] to the isovector part of the $e^+e^- \to \pi^+\pi^-$ cross section corrected in this work. The shaded area shows the systematic error

gion $\sqrt{s} < 500$ MeV. The corrected values of the $e^+e^- \to \pi^+\pi^-$ cross section $\sigma_{\pi\pi}(s)$, of the form factor

$$|F_\pi(s)|^2 = \frac{3s}{\pi\alpha^2\beta^3}\sigma_{\pi\pi}(s), \quad \beta = \sqrt{1 - 4m_\pi^2/s}, \quad (2)$$

and of the bare cross section (the cross section without the vacuum polarization contribution but with the final state radiation taken into account) are listed in Table 1. The cross section decreased by two systematic errors on average. The results in Table 1 supersede the results quoted in Table 1 of the original work [3].

The comparison of the obtained cross section with CMD-2 [12] and KLOE [13] measurements is shown in Figs. 2 and 3. The CMD-2 result exceeds the SND data by 1.4 ± 1.5 % on average. Here, the error includes both systematic and statistical uncertainties. The uncorrected SND cross section exceeds the CMD-2 one
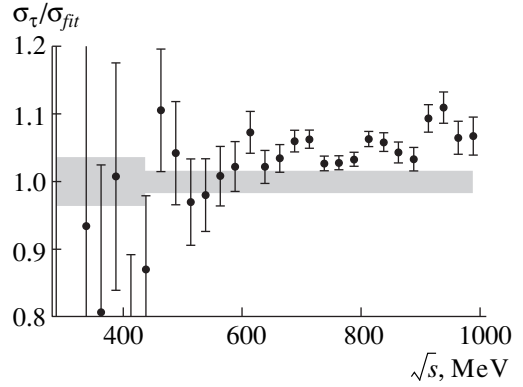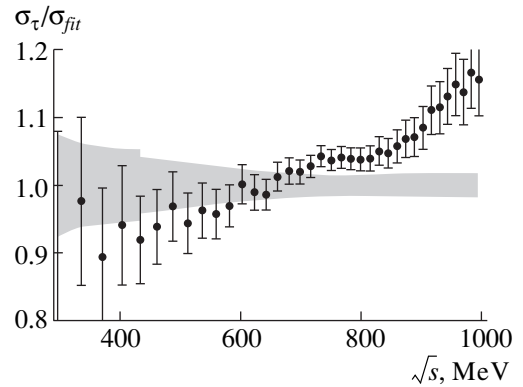
by the same value. In the KLOE experiment at the DAΦNE ϕ-factory, the form factor $|F_\pi(s)|^2$ was measured using the "radiative return" method with the systematic error 1.3 % [13]. In Ref. [13], the bare form factor values are listed. Therefore, in order to compare the KLOE result with the SND one, we appropriately dress the form factor. The results of this comparison are shown in Fig. 3. The difference between the SND and KLOE data is energy-dependent. The point that jumped out is situated in the region of a sharp rise of the cross section due to the $\rho - \omega$ interference. The KLOE measurement is in conflict with the SND result

Table 2. Parameters of the ρ and ω mesons measured in this work. In the third column, the parameter deviations from the original work results [3] in units of measurement errors are listed.

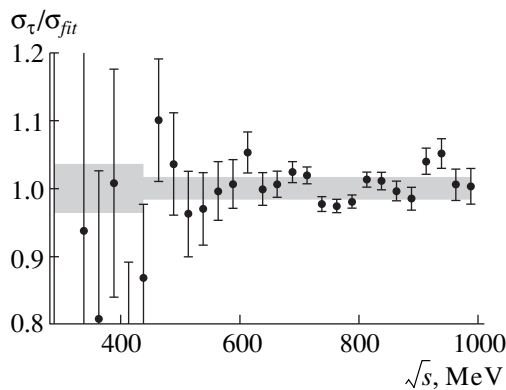| Parameters	Values	Deviations
m_ρ, MeV	$774.6 \pm 0.4 \pm 0.5$	0.5
Γ_ρ, MeV	$146.1 \pm 0.8 \pm 1.5$	0.2
$\sigma(\rho \to \pi^+\pi^-)$, nb	$1193 \pm 7 \pm 16$	1.6
$B(\rho \to e^+e^-) \times B(\rho \to \pi^+\pi^-)$	$(4.876 \pm 0.023 \pm 0.064) \cdot 10^{-5}$	1.6
$\Gamma(\rho \to e^+e^-)$, keV	$7.12 \pm 0.02 \pm 0.11$	1.7
$\sigma(\omega \to \pi^+\pi^-)$, nb	$29.3 \pm 1.4 \pm 1.0$	0.3
$B(\omega \to e^+e^-) \times B(\omega \to \pi^+\pi^-)$	$(1.225 \pm 0.058 \pm 0.041) \cdot 10^{-6}$	0.3
$\phi_{\rho\omega}$, degrees	$113.7 \pm 1.3 \pm 2.0$	0.1

Fig. 6. The ratio σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay spectral function measured by CLEOII [7] to the isovector part of the $e^+e^- \to \pi^+\pi^-$ cross section corrected in this work when the vacuum polarization contribution is not extracted from the SND data. The shaded area shows the systematic error
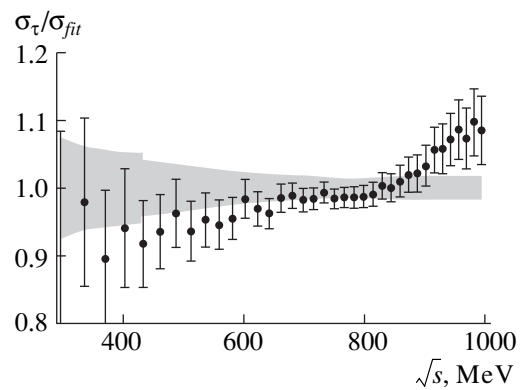

Fig. 7. The ratio σ_τ/σ_{fit} of the $e^+e^- \to \pi^+\pi^-$ cross section calculated from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decays spectral function measured by ALEPH [8] to the isovector part of the $e^+e^- \to \pi^+\pi^-$ cross section corrected in this work when the vacuum polarization contribution is not extracted from the SND data. The shaded area shows the systematic error

as well as with the CMD-2 one.

The cross section was fitted as described in the original work [3]. The fit results together with their deviation from the previous results [3] (in units of measurement errors) are listed in Table 2. These values supersede the results in [3]. All parameters except $\sigma(\rho \to \pi^+\pi^-)$, $B(\rho \to e^+e^-) \times B(\rho \to \pi^+\pi^-)$ and $\Gamma(\rho \to e^+e^-)$ changed by less than 0.5 error values, while $\sigma(\rho \to \pi^+\pi^-)$, $B(\rho \to e^+e^-) \times B(\rho \to \pi^+\pi^-)$ and $\Gamma(\rho \to e^+e^-)$ changed by less than two errors. The discussion of the parameters and conclusions made in the original work [3] are still valid.

The comparison of the $e^+e^- \to \pi^+\pi^-$ cross section obtained under the VCC hypothesis from the τ-lepton

spectral function from the $\tau^- \to \pi^-\pi^0\nu_\tau$ decay [7, 8] with the isovector part of the cross section measured by the SND is shown in Figs. 4 and 5. To compare with the τ spectral function, the radiative correction $S_{EW} = 1.0198 \pm 0.0006$ [8, 7, 14] was applied. The $e^+e^- \to \pi^+\pi^-$ cross section was undressed from the vacuum polarization, the contribution from the $\omega \to \pi^+\pi^-$ decay was excluded, and the correction for the $\pi^\pm$ and π^0 mass difference was applied. This yields the picture well-known from Ref. [8, 15]. It is interesting that the difference between the e^+e^- and τ data is approximately equal to the value of the accepted vacuum polarization contribution to the e^+e^- annihilation. The comparison of the τ data with

the dressed $e^+e^- \to \pi^+\pi^-$ cross section is shown in Figs. 6 and 7.

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