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TWO-DIMENSIONAL S–N–S JUNCTION WITH THE RASHBA SPIN-ORBIT INTERACTION

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The effect of Rashba spin-orbit interaction on the supercurrent in S – 2DEG – S proximity junctions is investigated in the clean limit. A generalization of Beenakker's formula for Andreev levels to the case of spin-orbit scattering is presented. Spin-orbit-induced splitting of Andreev bound states is predicted for an infinite-width junction with nonvanishing normal backscattering at S–N interfaces. However, a semiclassical average of the Josephson current is insensitive to the Rashba coupling as long as the electron-electron interaction in 2DEG is neglected.

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1. INTRODUCTION

Josephson junctions of two superconductors via two-dimensional electron gas (most usually implemented in the Nb/InAs/Nb structures) were actively studied both experimentally and theoretically, see, e.g., [1–6]. A generic feature of all these devices is strong reduction of the experimentally measured product $I_c R_N$ with respect to the theoretical predictions. In particular, this discrepancy is known for short junctions with high-quality S–N interfaces, demonstrated by measurement of a nonsinusoidal current–phase relation [6]. At the temperatures much below T_c, the parameter $I_c R_N \approx 0.22$ mV was measured in Ref. [6], to be compared with the Niobium superconductive gap $\Delta \approx 1.5$ meV. It therefore seems natural to look for some effects that were not taken into account in the existing theory, see, e.g., [7, 8], but could be responsible for such a drastic suppression of the critical current.

An obvious candidate to be explored is the Rashba spin-orbit interaction [9] $H_R = \alpha[\sigma \times \mathbf{p}] \cdot \mathbf{n}$, known to exist in the 2DEG structures due to the up–down asymmetry of the quantum well (here, $\mathbf{n}$ is the unit vector normal to the plane of the 2DEG). In the InAs heterostructures, this term is especially large (see Ref. [10]), leading to the band splitting $\Delta_R = 2\alpha p_F \approx 5$ meV, i.e., is considerably larger than the Niobium superconductive gap. Therefore, it seems natural that taking the Rashba term into account might be important for the analysis of the Josephson current in these devices. In this relation, we also note paper [11], where it was shown that persistent currents in mesoscopic metal rings should be strongly modified by spin-orbit coupling, which seems to indicate the existence of a similar effect on the Josephson current.

However, it is frequently assumed that the spin-orbit interaction cannot influence the proximity effect in superconductive structures, because it respects the time-reversal invariance. But this argument is not valid when the critical Josephson current is considered, because the presence of a current already violates the time-reversal symmetry. More detailed arguments seem to come from recent papers [12, 13], where the effect of both the Rashba coupling and the Zeeman magnetic field on the critical current of S–N–S junctions was considered. In both these papers, it was found that in the absence of the Zeeman term, the Rashba interaction (if treated within the simplest model of equal Fermi velocities on both chiral branches) totally cancels out from the equations for Andreev levels. We show, however, that this cancellation is not generic; rather, it is due to different simplifications used in the papers mentioned: a model of completely transparent S–N interfaces was employed in Ref. [12], and a purely one-dimensional model was used in Ref. [13].

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It is shown below that in the general case where some normal backscattering at an arbitrary incidence angle occurs at the S–N boundaries, the spin-orbit coupling does affect the energies of the Andreev levels and the supercurrent they carry on. We show that the spin-orbit interaction effect can be understood as being due to modification of the transmission channels defined by the scattering matrix $\check{S}$ that describes the junction properties in the normal state. For a model junction with an infinite length (or periodic boundary conditions) in the direction transverse to the supercurrent, a spin-orbit splitting of transmission eigenvalues is found, which results in splitting of each Andreev level into a pair of spin-polarized levels, with a phase-dependent energy difference $\delta E(\chi)$. We note that $\delta E(0) = 0$, in agreement with the time-reversal invariance restored in the absence of the phase bias. The idea that the Andreev levels can be spin-splitted due to the spin-orbit coupling was proposed in Ref. [14] for a narrow (few-channel) junction. The spin-orbit effect we discuss in this paper is different from the one presented in Ref. [14].

In this paper, we consider the simplest two-dimensional model of a ballistic S – 2DEG – S junction (see, e.g., Ref. [8]) of an infinite width in the lateral direction transverse to the current flow, see Fig. 1. We neglect possible potential barriers at the S–N interfaces, assuming that the normal backscattering is due to a Fermi-velocity mismatch only, and consider ballistic electron propagation along the 2D structure between superconductive terminals. In Sec. 2, we show that in the short-junction limit (junction length $L \ll \xi_0 = \hbar v_F/\Delta$, where v_F is the Fermi velocity of 2DEG), the positions of the Andreev levels can be expressed via the transmission eigenvalues $\mathcal{T}$ of the full scattering matrix $\check{S}$ in precisely the same way as was found by Beenakker [16] for junctions with spin-independent scattering. In Sec. 3, we then present calculations of the scattering matrix $\check{S}$ for the simplest two-dimensional model of a ballistic S – 2DEG – S junction (see, e.g., Ref. [8]) of infinite width in the lateral direction transverse to the current flow, see Fig. 1. We explicitly demonstrate the spin-splitting of the transmission probabilities $\mathcal{T}_\pm(p_y)$ for the transmission channels characterized by the momentum component p_y. We then show that the distribution function for the transmission probabilities $\mathcal{P}(\mathcal{T})$ coincides with the one discussed by Melsen and Beenakker [18] in the absence of spin-orbit coupling. In Sec. 4, we derive an expression for the Josephson current of a short junction and demonstrate that the average current is insensitive to the Rashba coupling. In Sec. 5, we go beyond the short-junction limit: we derive an equation for spin-splitted Andreev levels for the junction with an arbitrary L/ξ_0 ratio and demonstrate that their contribution to the average (semiclassical) supercurrent is insensitive to the spin-orbit coupling. Section 6 is devoted to the discussion of the applicability of our results to S – 2DEG – S junctions of finite width and possible ways to detect spin-splitted Andreev levels. Finally, in Sec. 7, we present our conclusions and discuss open problems; in particular, it is proposed that the account of the electron-electron interaction together with the Rashba coupling might be able to explain the Josephson current.

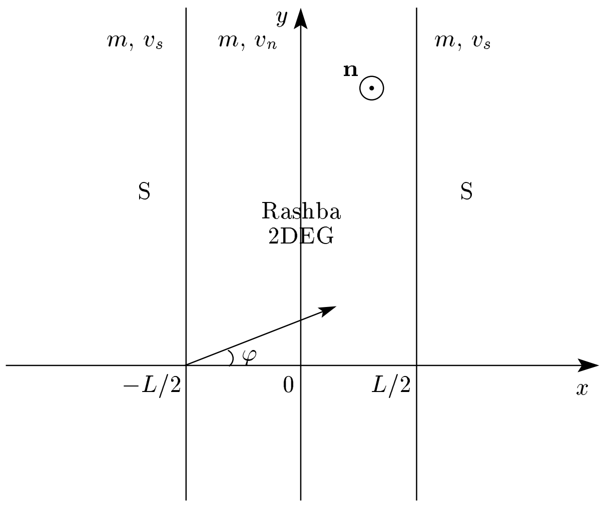

Fig. 1. Two-dimensional model of a superconductor/Rashba 2DEG/superconductor Josephson junction infinite in the direction perpendicular to the current (along the y axis). The Rashba 2DEG region has a thickness L; m/m_n is the effective mass and v_s/v_n is the Fermi velocity in the S/2DEG; φ is the angle between the velocity direction of a quasiparticle and the x axis in the 2DEG region; $\mathbf{n}$ is a unit vector normal to the plane of the 2DEG

2. SPECTRUM OF ANDREEV LEVELS

The excitation spectrum consists of the positive eigenvalues of the Bogolyubov–de Gennes (BdG) equation,

$$\begin{aligned} \epsilon_\nu u^\alpha &= [\xi + U]^\alpha{}_\beta u^\beta + \Delta v^\alpha, \\ \epsilon_\nu v^\alpha &= -[\xi^* + U^*]^\alpha{}_\beta v^\beta + \Delta^* u^\alpha, \end{aligned} \tag{1}$$

where $(U^*)^\alpha{}_\beta = \hat{g}^{\nu\alpha}(U(\nu,\mu))^* \hat{g}_{\mu\beta}$, α and β are spinor indices, $\hat{g} = i\hat{\sigma}_y$ is a metric tensor in spin space, $\xi = \mathbf{p}^2/2m - E_F$ is the kinetic energy of a quasiparticle (energies are measured relative to the Fermi energy), and

$$u^\alpha(\mathbf{r}) = \begin{pmatrix} u(\mathbf{r}\uparrow) \\ u(\mathbf{r}\downarrow) \end{pmatrix}, \quad u_\alpha = \hat{g}_{\alpha\beta} u^\beta, \quad u^\alpha = u_\beta \hat{g}^{\beta\alpha}, \quad U(\sigma,\mu) = U^\sigma{}_\mu. \tag{2}$$

In our model, in the normal region (mapped as «Rashba 2DEG» in Fig. 1), the operator $U = \alpha[\sigma \times \mathbf{p}] \cdot \mathbf{n}$ is the spin-orbit interaction, which preserves the time-reversal invariance. In superconductors, the Rashba term is absent, $U = 0$. The superconducting gap Δ is assumed to be a step-like function: it is equal to zero in the normal region and its modulus $|\Delta|$ is constant and has the same value in both superconductors.

The equation that relates the excitation spectrum of the Josephson junction to the scattering matrix in the normal state $\check{S}$ was derived in Ref. [16]:

$$\det[1 - r_{he}\check{S}_e(\epsilon) r_{eh} \check{S}_h(\epsilon)] = 0, \tag{3}$$

where

$$r_{he} = \gamma r_A, \quad r_{eh} = \gamma r_A^*, \quad r_A = \begin{pmatrix} e^{i\chi/2} & 0 \\ 0 & e^{-i\chi/2} \end{pmatrix}, \quad \gamma = e^{-i\arccos(\epsilon/\Delta)}, \tag{4}$$

r_{he} is the Andreev reflection matrix for the $e \to h$ scattering in the space of channels incident (reflected) on the left and right N–S boundaries, $\pm\chi/2$ are the phases of the left (right) superconductor, and $\check{S}_{e(h)}$ is the electron (hole) scattering matrix of the normal state.

When no spin-dependent scattering is present, the normal scattering matrix $\check{S}_e$ is trivial in spin space, i.e., proportional to the unit matrix $\hat{\sigma}_0$. Furthermore, in the short-junction limit $L \ll \xi_0$, the scattering matrices $\check{S}_{e,h}$ are independent of energy, and, moreover, $\check{S}_h = \check{S}_e^*$. Therefore, Eq. (3) can be transformed to an explicit solution [16] for spin-degenerate Andreev levels, $\epsilon_j = \pm\sqrt{1 - \mathcal{T}_j \sin^2(\chi/2)}$, where $\mathcal{T}_j$ is the jth eigenvalue of the transmission probability matrix $\hat{T}^\dagger \hat{T}$ (eigenvectors of this matrix define scattering channels). Below, we show that a solution of the same kind can be obtained when the spin-orbit scattering is present.

In the presence of the spin-orbit interaction, the scattering matrix in (3) becomes spin-dependent but still obeys the time-reversal invariance. This allows generalizing Beenakker's derivation for the Andreev levels in a short junction [20] using the following set of relations for the $\check{S}$-matrix:

$$\check{S}\check{S}^\dagger = 1, \quad \check{S}^T(-p_y) = \hat{g}^T \check{S}(p_y)\hat{g}, \quad \check{S}_h(\epsilon, p_y) = \hat{g}^T \check{S}_e^*(-\epsilon, -p_y)\hat{g}, \tag{5}$$

where the superscript «T» denotes the full matrix transposition. The first relation in (5) is just the unitarity condition, the second follows from the time-reversal invariance (we here used the time-reversal transformation of the wavefunctions $\psi^{t-r}(p_y) = \hat{g}\psi^*(-p_y)$). Finally, the third relation in (5) is due to a special symmetry of the BdG equations: $\psi_h(\epsilon, p_y) = \hat{g}^T \psi_e^*(-\epsilon, -p_y)$. It is important to note the sign change of the parameter p_y in the second and the third relations above: when all scattering states are characterized by a conserved momentum (p_y), the time-conjugation operation involves complex conjugation and the $p_y \to -p_y$ inversion, because time reversal of the scattering matrix should change the sign of p_x only, while keeping p_y intact. In other words, the additional $p_y \to -p_y$ operation is needed due to the use of scattering channels characterized by complex eigenfunctions proportional to $e^{ip_y y}$. In calculations of this kind, a real basis of transmission channels is typically used, in which case such an additional operation is absent.

Using relations (5), we can transform Eq. (3) to the form

$$\det\left[\frac{1}{\gamma}\hat{g}^T \check{S}_e^*(\epsilon, p_y)\hat{g} r_A^* - \gamma r_A^* \hat{g}^T \check{S}_e^*(-\epsilon, p_y)\hat{g}\right] = 0. \tag{6}$$

For a short contact $L \ll \xi_0$, we neglect the energy dependence of the scattering matrix in Eq. (6) and obtain a second-order equation for ϵ^2, which results in the solution

$$\epsilon_{s,\eta}(p_y) = \eta\Delta\sqrt{1 - \mathcal{T}_s(p_y)\sin^2\frac{\chi}{2}}, \tag{7}$$

where $\eta = \pm$ and $\mathcal{T}_s(p_y)$ are transmission probabilities — the eigenvalues of the matrix $\hat{T}^\dagger\hat{T}$, depending on the spin index $s = \pm$ and the conserved momentum p_y. In general, $\mathcal{T}_+(p_y) \neq \mathcal{T}_-(p_y)$, and therefore four non-degenerate Andreev levels correspond to each p_y value, as shown in Fig. 3 below. We note, however, that the full family of Andreev levels still contains pairwise degeneracy within our model. Namely, degeneracy exists between states with $p_y = \pm|p_y|$. Below, we consider a specific example of the scattering problem relevant to S – 2DEG – S structures, and calculate the $\mathcal{T}_s(p_y)$ eigenvalues.

3. S-MATRIX AND TRANSMISSION EIGENVALUES

We are interested in specific spin-orbit effects and hence consider the simplest model of S–N boundaries, assuming that normal electron reflection is due to the Fermi velocity mismatch only, $v_s \neq v_n$ (where v_s and v_n are the respective Fermi velocities in the superconductive metal and in the 2DEG). An additional source of reflection due to an effective potential barrier at the interface (see, e.g., Ref. [8]) can be present, but does not affect our results qualitatively. Because the effective mass m_n in the 2DEG differs strongly from the effective mass m in the metallic superconductor (typically, $m_n/m \approx 0.03$ for 2D structures with InAs), the difference of these masses should be taken into account explicitly. Our first goal is now to find the reflection/transmission amplitudes on single S–N interfaces (for the normal state of the superconductive metal S). We follow Ref. [15] and use the continuity equations that follow from the Schrödinger equation with a space-dependent mass $m(x)$ and spin-orbit parameter $\alpha(x)$,

$$\left[\frac{\hat{p}_x}{m(x)} - \alpha(x)\right] \Psi \,|_N^S = 0\,, \quad \Psi\,|_N^S = 0, \qquad (8)$$

where $F\,|_N^S$ denotes $F(x=-L/2+0)-F(x=-L/2-0)$ for the left interface (cf. Fig. 1) and similarly for the right interface located at $x = L/2$. We let $P_F = mv_s$ and $p_F = m_n v_n$ denote the respective Fermi momenta in the S metal and in the 2DEG; usually, $p_F \ll P_F$, whereas v_s and v_n are of the same order of magnitude. Below, we assume that the parameter $\alpha/v_n \ll 1$ measuring the relative strength of the Rashba interaction is small in comparison with the Fermi-velocity mismatch, i.e., $\alpha \ll |v_s - v_n|$. Under this condition, the reflection amplitudes at each of the S–N boundaries are determined by the ratio v_n/v_s only. Then the reflection and transmission amplitudes are trivial in the spin space, e.g., $\overrightarrow{r}_1^{\alpha\beta} = \delta^{\alpha\beta}\overrightarrow{r}_1$. For an incident wave incoming from $x=-\infty$, the reflection and transmission amplitudes on the left (1) interface are

$$\overrightarrow{r}_1 = \frac{w-1}{1+w}, \quad \overrightarrow{t}_1 = \frac{2}{1+w}, \qquad (9)$$

where $w = v_{nx}/v_{sx}$ is the ratio of the x-components of the electron velocities, with $v_{nx} = v_n\cos\varphi$ and $v_{sx} = \left[v_s^2 - (m_n/m)^2 v_n^2 \sin^2\varphi\right]^{1/2} \approx v_s$. Here, φ is the angle between the velocity direction and the x axis in the 2DEG; we note that v_{sx} is very close to v_s for any angle φ because $(m_n/m)^2 \ll 1$. The other reflection/transmission amplitudes are determined as follows:

$$\begin{aligned} &\overleftarrow{r}_2 = \overrightarrow{r}_1, \quad \overleftarrow{t}_2 = \overrightarrow{t}_1, \\ &\overleftarrow{r}_1 = \overrightarrow{r}_2 = -\overrightarrow{r}_1, \quad \overleftarrow{t}_1 = \overrightarrow{t}_2 = \frac{2w}{1+w}. \end{aligned} \qquad (10)$$

The total scattering matrix $\check{S}$ of the S – Rashba 2DEG – S junction in the normal state, formed out of the «single boundary» amplitudes, Eq. (9), is given by (similar equations can be written for $\hat{T}_2$ and $\hat{R}_2$)

$$\begin{aligned} &\hat{T}_1 = \overrightarrow{t}_2 \hat{S}^r \left[1 - \overleftarrow{r}_1 (\hat{S}^l)^{-1} \overrightarrow{r}_2 \hat{S}^r\right]^{-1} \overrightarrow{t}_1, \\ &\hat{R}_1 = \overleftarrow{t}_1 (\hat{S}^l)^{-1} \overrightarrow{r}_2 \hat{S}^r \times \\ &\qquad \times \left[1 - \overleftarrow{r}_1 (\hat{S}^l)^{-1} \overrightarrow{r}_2 \hat{S}^r\right]^{-1} \overrightarrow{t}_1 + \overrightarrow{r}_1, \end{aligned} \qquad (11)$$

where $\hat{R}$ and $\hat{T}$ are the reflection and transmission blocks of the scattering matrix,

$$\check{S} = \begin{pmatrix} \hat{R}_1 & \hat{T}_2 \\ \hat{T}_1 & \hat{R}_2 \end{pmatrix}, \qquad (12)$$

and the subscript «1» in the amplitudes $\hat{R}$ and $\hat{T}$ in Eq. (11) indicates that the equations are written for the case of an electron propagating from left to right. The matrices in the spin space $\hat{S}^{r(l)}$ describe spin rotation during the electron propagation across the 2DEG region with the Rashba coupling between the two S–N boundaries. The explicit form of these matrices can be obtained by transformation of the plane-wave eigenmodes with a definite chirality to the spin basis with a definite S_y projection,

$$\begin{aligned} &\hat{S}^r = e^{i\xi}\left[\cos A - i\sin A\sin\varphi\hat{\sigma}_x + i\sin A\cos\varphi\hat{\sigma}_z\right], \\ &(\hat{S}^l)^{-1} = e^{i\xi}\left[\cos A - \right. \\ &\qquad \left. - i\sin A\sin\varphi\hat{\sigma}_x - i\sin A\cos\varphi\hat{\sigma}_z\right], \end{aligned} \qquad (13)$$

where $\xi(\epsilon) = k(\epsilon)L$, with $k(\epsilon) = k + m\epsilon/k$ and $k = p_F\cos\varphi$, is the main semiclassical phase and $A = m_n\alpha L/\cos\varphi$ is the additional phase due to the spin rotation by the Rashba coupling. Within our approximation $\alpha/v_n \ll 1$, the whole effect of the Rashba coupling is contained in the phase A, which is not small if the length L of the junction is comparable with or larger than the spin-rotation length $L_0 = \hbar/m_n\alpha$.

For further convenience, we define a new parameter $x = \lg[(1+w)/(1-w)]$, where w is defined after Eq. (9). Then, combining Eqs. (11) and (13), we obtain the transmission matrix as

$$\begin{aligned} &\hat{T}_1 = T_0 + T_1\hat{\sigma}_x + T_3\hat{\sigma}_z, \\ &\hat{T}_2 = T_0 + T_1\hat{\sigma}_x - T_3\hat{\sigma}_z \end{aligned} \qquad (14)$$

with

$$\begin{aligned} &T_0 = t\,\mathrm{sh}(x - i\xi)\cos A, \\ &T_1 = -it\,\mathrm{ch}(x - i\xi)\sin A\sin\varphi, \\ &T_3 = it\,\mathrm{sh}(x - i\xi)\sin A\,\cos\varphi, \end{aligned} \qquad (15)$$

where we set

$$t = \frac{\operatorname{sh} x}{\operatorname{sh}^2(x - i\xi) + \sin^2 A \sin^2 \varphi}. \qquad (16)$$

.

The reflection matrix $\hat{R}$ is given by

$$\hat{R}_1 = R_0 + R_1\hat{\sigma}_x + R_2\hat{\sigma}_y, \quad \hat{R}_2 = R_0 + R_1\hat{\sigma}_x - R_2\hat{\sigma}_y \qquad (17)$$

with

$$R_0 = t\left[\operatorname{ch} x\, \sin^2 A\, \sin^2\varphi - i\frac{\sin\xi}{\operatorname{sh} x}\operatorname{sh}(x - i\xi)\right], \quad R_1 = \frac{i}{2}\, t \sin 2A \sin\varphi, \quad R_2 = \frac{i}{2}\, t\sin^2 A \sin 2\varphi. \qquad (18)$$

We now use Eqs. (14) and (15) to obtain the transmission probabilities as the eigenvalues of the matrix $\hat{\mathcal{T}} = \hat{T}^\dagger\hat{T}$,

$$\mathcal{T}_\pm(\xi, x(\varphi)) = \frac{\operatorname{sh}^2 x}{\operatorname{sh}^2 x + \sin^2(\xi \pm \beta/2)}, \qquad (19)$$

where the phase β defined by

$$\cos\beta = 1 - 2\sin^2\varphi \sin^2 A \qquad (20)$$

is due to the Rashba interaction. Equation (19) explicitly demonstrates spin-splitting of the transmission eigenvalues $\mathcal{T}_\pm$. We note that $\beta = 0$ and the splitting is absent for trajectories with $\varphi = 0$, which is the case of a purely 1-dimensional (single-channel) contact [13]. In the absence of normal reflection, i.e., at $x \to \infty$, all transmission eigenvalues are equal to unity and the spin-orbit effects disappear as well [12].

The spin-orbit effect on $\mathcal{T}_\pm$ reduces, according to Eq. (19), to the shift $\xi \to \xi \pm \beta/2$ of the main semiclassical phase, in agreement with the result in Eq. (1) in Ref. [11]. An example of the dependence of the transmission eigenvalues $\mathcal{T}_\pm$ on the incidence angle is shown in Fig. 2. An important point to mention is that this dependence is even with respect to the $\phi \to -\phi$ reflection, cf. Eq. (20). This symmetry is a «trace» of the Kramers degeneracy, which is known to exist for the transmission eigenvalues defined within the real basis of scattering states (we note that the proof of the Kramers degeneracy of transmission eigenvalues is by far more complicated [17] than of the original Kramers theorem for the degeneracy of energy levels). We characterize scattering states by complex traveling waves $e^{ip_y y}$, which leads to violation of time invariance. Therefore,

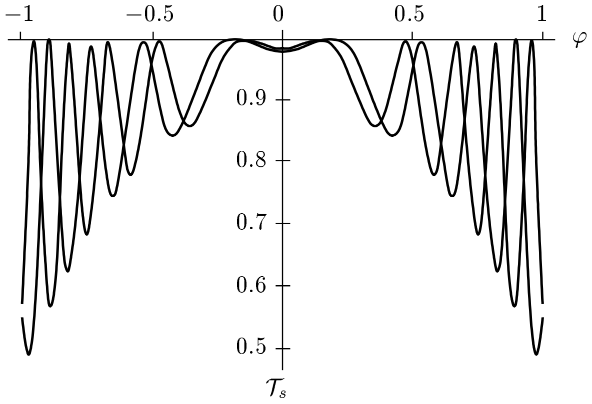

Fig. 2. The spin-splitted transmission eigenvalues $\mathcal{T}_s$, $s = \pm 1$, as functions of the angle of propagation φ of the quasiparticles inside the 2DEG, plotted for a realistic S–2DEG–S junction with the parameters $v_s = 7 \cdot 10^7$ cm/s, $v_n = 5 \cdot 10^7$ cm/s, $m = m_e$, $m_n = 0.035 m_e$, $m_n\alpha/\hbar = 5 \cdot 10^4$ cm^{-1}, and $L = 190$ nm. For these parameters and for the value of the superconducting gap $\Delta = 1.4$ meV: (1) the length of the contact L is shorter than the coherence length, $\xi_0 = \hbar v_s/\Delta = 330$ nm; (2) the Rashba velocity is much smaller than the Fermi velocity in the 2DEG, $\alpha/v_n \approx 0.03$; (3) the system is in the semiclassical limit, $p_F L/\hbar = m_n v_n L/\hbar \approx 30$; (4) the spin-orbit splitting $2\alpha p_F \approx 3.3$ meV is larger than the superconducting gap Δ; (5) the S–N interfaces are almost transparent ($v_s/v_n \approx 1.4$), which allows a large experimental value of the critical current

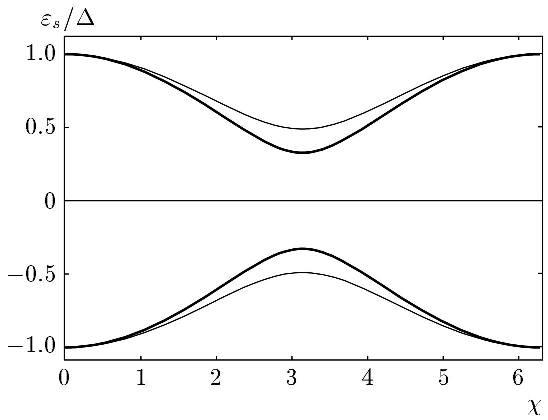

Fig. 3. The four spin-splitted Andreev levels $\pm\epsilon_s$, $s = \pm 1$, as functions of the superconducting phase difference χ, plotted for a value of the angle of propagation $\varphi = \pi/5$, and for realistic S–2DEG–S junctions with the parameters $v_s = 7 \cdot 10^7$ cm/s, $v_n = 5 \cdot 10^7$ cm/s, $\alpha \approx 0.2 \cdot 10^7$ cm/s, $m = m_e$, $m_n = 0.035 m_e$, and $L = 190$ nm

the Kramers theorem is not applicable to our model and spin splitting demonstrated in Eq. (19) may occur.

It follows from Eq. (19) that a semiclassical average of any physical quantity that can be expressed as a sum of terms containing individual $\mathcal{T}$ variables (i.e., does not contain cross-terms like $\mathcal{T}_+\mathcal{T}_-$), is independent of the spin-orbit coupling. Indeed, calculation of any average quantity in our model involves integration over the momentum component p_y parallel to the interfaces (or over the propagation angle φ, defined as $p_y = p_F \sin\varphi$). The integrand, as a function of φ, contains fast oscillations with the characteristic scale $1/p_F L$ and relatively slow dependence on $\cos\varphi$. It is convenient first to average over fast oscillations by going to the probability distribution of transmission eigenvalues defined as

$$\mathcal{P}_\varphi(\mathcal{T}_\pm) = \int \delta(\mathcal{T} - \mathcal{T}_\pm(\xi, x(\varphi))\, d\xi. \tag{21}$$

Clearly, the presence of the $\pm\beta$ phase shift does not alter the form of the probability distribution, which is of the same form as considered, e.g., in Ref. [18] and independent of the spin-orbit coupling:

$$\mathcal{P}_\varphi(\mathcal{T}) = \frac{\operatorname{th} x}{2\mathcal{T}\sqrt{1-\mathcal{T}}\sqrt{\mathcal{T} - \operatorname{th}^2 x}}. \tag{22}$$

As the simplest example, we next consider calculation of the average conductivity of a junction in the normal state. It can be written as

$$G = G_Q \int\limits_{-\pi/2}^{\pi/2} \cos\varphi \frac{d\varphi}{\pi} \int \mathcal{T}\mathcal{P}_\varphi(\mathcal{T})\, d\mathcal{T}. \tag{23}$$

The universality of the distribution function $\mathcal{P}_\varphi(\mathcal{T})$ leads to the independence of the average conductance G, as well as of other quantities that can be expressed through this distribution functions, from the spin-orbit phase A (we recall that we neglected weak effects of the order of $\alpha/v_n \ll 1$). We note once again that the above simple considerations cannot be applied to calculation of any quantity that is not additive as a function of different transmission channels, i.e., which contains products of different transmission eigenvalues.

4. JOSEPHSON CURRENT

Equation (7) for the Andreev levels together with Eq. (19) for transmission eigenvalues constitute the central result in this paper. We can now calculate the Josephson current [16] as

$$I(\chi) = \frac{e\Delta^2}{2\hbar}\sin\chi \times \\ \times \int \frac{L_y dp_y}{2\pi\hbar} \sum_{s=\pm 1} \frac{\mathcal{T}_s(p_y)}{\epsilon_{s,+}(\chi)} \operatorname{th}\frac{\epsilon_{s,+}(\chi)}{2T}. \tag{24}$$

Equation (24) is applicable to the temperature-dependent Josepshson current in the short-junction limit. In the semiclassical limit $p_F L \to \infty$, the average Josephson current can be calculated with the use of the distribution function $\mathcal{P}_\varphi(\mathcal{T})$ given by Eq. (22) as follows:

$$I(\chi) = \frac{e\Delta}{2\hbar} \int\limits_{-\pi/2}^{+\pi/2} \frac{d\varphi\cos\varphi}{\pi} \times \\ \times \int \mathcal{P}_\varphi(\mathcal{T})\, d\mathcal{T} \frac{\mathcal{T}\sin\chi}{\sqrt{1-\mathcal{T}\sin^2\dfrac{\chi}{2}}} \times \\ \times \operatorname{th}\frac{\Delta\sqrt{1-\mathcal{T}\sin^2\dfrac{\chi}{2}}}{2T}. \tag{25}$$

Equation (25) demonstrates the independence of the average Josephson current from the spin-orbit coupling. Such an average current is a meaningful characteristic of a junction with both lateral sizes much longer than the Fermi wavelength, $L, L_y \gg \hbar/p_F$.

Oscillations of I_c as a function of the electron density were discussed theoretically in Ref. [8] within the model very similar to the present one (but without the Rashba coupling). It was argued that oscillations should appear due to the presence of normal resonances in a double-barrier structure, like the ones described in our Eq. (19) as a function of $\xi = p_F L\cos\varphi$. A strong spin-rotation effect expected at $L \geq L_0$ produces an additional phase $\beta \sim 1$, which leads to splitting of the resonances as a function of $p_F L$. As a result, at $L \geq L_0$, the oscillations discussed in Ref. [8] have a twice shorter period and reduced amplitude.

5. SPECTRUM EQUATION AND CURRENT FOR THE JUNCTION OF AN ARBITRARY LENGTH

In this section, we find equations determining the Andreev levels for an arbitrary length of the contact, with the main purpose to demonstrate that the semiclassical average of the Josephson current is independent of the spin-orbit coupling for any L/ξ_0 ratio. Here, we use an alternative method of calculation: instead of expressing the Andreev levels via the transmission eigenvalues, we use direct matching of the wavefunctions obeying the BdG equations in the 2DEG and both superconductive regions. To simplify the calculations in this section, we consider the model with equal effective masses, $m_n = m$.

Eigenfunctions of the BdG equation for the S – 2DEG – S junction can be represented as 8-dimensional vectors, because they contain three two-dimensional blocks: i) electron and hole components, ii) two spin projections, and iii) two direction of momentum along the x axis, $p_x = \pm p_F|\cos\varphi|$. Matching conditions for the wavefunctions on both S–N interfaces consist of 16 scalar equations that relate 8 wavefunction components in the 2DEG region to 4 components in each of the superconductive terminals (in the case of subgap Andreev levels, which decay exponentially into the bulk of superconductors). The next step is to reduce this system of 16 equations to 8 equations that couple $4+4=8$ amplitudes of wavefunctions in superconductors. The solvability condition for this system of 8 linear equations is equivalent to the condition of vanishing of the corresponding determinant, $g(\epsilon,\chi)=0$, which is equivalent to the one defined in Eq. (6). After some tedious calculations, the equation $g(\epsilon,\chi)=0$ can be transformed to the form

$$g(\epsilon,\chi) \equiv g_+(\epsilon,\chi)g_-(\epsilon,\chi) = 0,$$

where

$$g_\pm(\epsilon,\chi) = \cos 2\xi - Q\cos\beta \pm \sqrt{1-Q^2}\sin\beta, \qquad (26)$$

where the parameter β is defined in Eq. (20),

$$\begin{aligned} Q &= \cos\Psi + \frac{4k^2K^2\Delta^2(\cos\Psi+\cos\chi)}{(K^2-k^2)^2(\Delta^2-\epsilon^2)}, \\ \Psi &= 2\,\mathrm{arctg}\,\frac{2kK\epsilon}{(K^2+k^2)\sqrt{\Delta^2-\epsilon^2}} + \mathcal{E}, \end{aligned} \qquad (27)$$

with $\mathcal{E} = 2m\epsilon L/k$ being the energy-dependent part of the phase $\xi(\epsilon)$. Equations (26) demonstrate that in the presence of the Rashba interaction, the Andreev levels are generically spin-splitted for the contact of an arbitrary length.

In the limit of vanishing spin-orbit interaction $\alpha = 0$, as well as for electron trajectories with $p_y = \varphi = 0$, spectrum equation (26) reduces to the standard equation $\cos 2\xi = Q$ with a two-fold degenerate (due to the spin) solutions. In the special case of ideally transparent boundaries $p_F = P_F$, general spectrum equation (26) also reduces to the standard equation $\cos 2\xi = Q$, which then simplifies to

$$\cos\left(-\mathcal{E} + 2\arccos\frac{\epsilon}{\Delta}\right) = \cos\chi. \qquad (28)$$

For a relatively short contact with $0 < \mathcal{E} \ll 1$, we expand Andreev spectrum equation (26) in powers of the small parameter $\kappa = m\Delta L/k$ and find the first correction to the result (7) obtained in Sec. 2 in the limit as $\kappa \to 0$,

$$\epsilon_\pm = \Delta\sqrt{1-\mathcal{T}_\pm\sin^2\frac{\chi}{2}}\left(1-\kappa\mathcal{T}_\pm^{3/2}\left|\sin\frac{\chi}{2}\right|\mathrm{cth}\,x\right), \qquad (29)$$

where $\mathcal{T}_\pm$ are defined in Eq. (19).

In the general case of an arbitrary length of the contact, spectral equation (26) is too complicated to be solved explicitly for the energies of the Andreev levels. Moreover, one should remember that for a junction with an arbitrary L/ξ ratio, the continuous part of the spectrum (scattering states) contributes to the Josephson current as well as the localized levels we have considered. However, the total Josephson current (carried by both the localized Andreev levels and the continuous part of the spectrum) can be found following the method in Ref. [21], in terms of the spectral function $g(\epsilon,\chi)$ itself.

We use Eqs. (I.9), (A.48), and (A.49) in Ref. [21], modified in our case due to the presence of the spin splitting and the continuous scattering channels characterized by the transverse momentum p_y. Therefore, the total current contains an integral over all p_y,

$$I_{total}(\chi) = \\ = L_y\frac{4e}{\hbar}T\sum_{s=\pm}\int\frac{dp_y}{2\pi\hbar}\sum_{\omega_n>0}\partial_\chi \ln g_s(i\omega,\chi), \qquad (30)$$

where the summation ranges over positive Matsubara frequencies $\omega = 2\pi T(n+1/2)$, $n = 0,1,\ldots$

In the semiclassical limit ($Lp_F \gg 1$), the calculation of the integral over p_y in Eq. (30) can be simplified by the same method that was used in the last part of Sec. 3. Namely, we first average over the period of fast oscillations of $\cos\xi \equiv \cos(kL)$ at a fixed angle φ and then do the integration over φ. The integration $\int_0^\pi d\xi\ldots$ in Eq. (30) leads to the result that does not contain the spin-orbit phase β:

$$I_{total}(\chi) = \\ = -L_y p_F\frac{4e}{\hbar^2}T\sum_{\omega_n>0}\int_{-\pi/2}^{\pi/2}\frac{d\varphi}{\pi}\cos\varphi\frac{\partial_\chi Q}{\sqrt{|1-Q^2|}}, \qquad (31)$$

where $Q \equiv Q(\epsilon = i\omega_n, \chi)$ in accordance with Eq. (27), and we took into account that $dp_y = p_F \cos\varphi \, d\varphi$. Equation (31) demonstrates that the semiclassical average of the Josephson current through the S – Rashba 2DEG – S contact is independent of the Rashba coupling constant, and this result is valid for an arbitrary Fermi velocity mismatch and arbitrary length of the contact. We note, however, that this result is valid as long as the electron-electron interaction in the 2DEG region is neglected.

6. DISCUSSION

The above results were obtained for a model of an infinitely long junction in the direction perpendicular to the current, in the case where the motion along the y axis was completely determined by the wavevector p_y of the corresponding plane wave due to the translational invariance. An obvious generalization of such a model system would be a junction with periodic boundary conditions in the y direction. In this case, all our results would stay intact, up to replacement of continuous p_y by the discrete set of wavevectors $p_n = 2\pi n/L_y$. Although somewhat exotic for SNS junctions, such a geometry does not seem to be impossible if we take the recent advances in fabrication of complicated InAs structures into account, cf., e.g., [22]. Usually, however, the S – 2DEG – S structure is of a finite length L_y in the y direction with closed boundary conditions, and therefore the channel eigenstates are characterized by standing waves — the mixtures of plane waves $e^{ip_y L_y}$ and $e^{-ip_y L_y}$. In the presence of the Rashba term, the direction of the electron momentum is coupled to the direction of its spin, and hence determination of the correct standing-wave eigenstates is nontrivial. The major effect of a finite $L_y \gg L_0$ is the existence of a discrete set of transmission channels, $N_{ch} = 2L_y/\lambda_F$, where λ_F is the Fermi wavelength of the 2DEG. However, some qualitative effects of closed boundary conditions then occur: because a real basis of scattering states is then used, the Kramers theorem for the transmission eigenvalues [17] becomes applicable. This means that for closed boundary conditions and in the short-junction limit $L/\xi_0 \to 0$, no spin splitting of the Andreev levels may occur. In other terms, in a closed (in the y direction) system, the Rashba coupling modifies transmission eigenvalues but does not split them. How can we reconcile this with a natural idea that for very long L_y, the type of boundary conditions should not be important? The point is that the total Andreev spectrum of the system is doubly degenerate for the periodic boundary conditions as well as for the closed ones. In the first case, the degeneracy is due to the symmetry of $\mathcal{T}_\pm(p_y)$ under the $p_y \to -p_y$ reflection, whereas in the second case, it is due to the Kramers theorem. In order to obtain a global Andreev spectrum without degeneracy, the time-reversal symmetry should be broken. In particular, this happens if a nonzero L/ξ_0 ratio is taken into account, as reported in Ref. [14]. Another possibility might be related to an open sample geometry like the one used in Ref. [5], where an additional current can be passed in the direction transverse to the supercurrent.

The individual Andreev levels could possibly be observed experimentally by microwave spectroscopy or by measurement of the tunnelling conductance into the 2DEG region from an additional point-like junction. One version of the former type of experiment was proposed theoretically, for a single-channel junction, in Ref. [19]. In this case, the resonant frequency is very high, of the order of $\Delta/\hbar$, because the only possible transitions are between the positive and negative Andreev levels. This frequency is about 0.4 THz for Nb terminals (considerably lower frequencies can be found in the case of a very small reflection probability, $1 - \mathcal{T} \ll 1$ and a phase difference $\chi \approx \pi$). In many-channel junctions, the energy spacing between neighboring Andreev levels is reduced as $\delta\epsilon \sim \Delta/N_{ch}$, but it is usually (without the spin-orbit coupling) impossible to observe microwave-induced transitions between levels belonging to different conduction channels. The reason is the momentum conservation: different transmission channels are characterized by different wavevectors $p_y/\hbar$, which are spaced by π/L_y, whereas the photon wavelength $\lambda_{ph} = hc/\delta\epsilon$ is much longer than L_y, their ratio is of the order of $(E_F^{2DEG}/\Delta)(c/v_n) \sim 10^4$. It seems possible that this selection rule will not be effective in the considered situation with the Rashba coupling, which modifies conduction channels considerably at $L_y \geq L_0$. The point is that conduction channels will then be defined in the entangled space of orbital and spin variables, and therefore there seems to be no reason for the vanishing of the inter-channel photon matrix element. However, this question certainly needs further investigation.

7. CONCLUSIONS

We investigated the dependence of the Josephson currents in clean S – Rashba 2DEG – S proximity junctions on the Rashba spin-orbit interaction. We have generalized the Beenakker formula for the Andreev levels to the case of spin-orbit scattering and found that for an infinitely wide junction (in the direction transverse to the current), the Andreev levels are spin-splitted. This result is in agreement with papers [12, 13], where the effect of the Rashba spin-orbit interaction on the supercurrent was studied either in the case of the absence of normal backscattering at the interfaces ($p_F = P_F$) [12] or in the one-dimensional case [13]. We have shown that the semiclassical average of the Josephson current is insensitive to the Rashba coupling as long as the electron-electron interaction in 2DEG is neglected.

Our results therefore show that the account of the Rashba spin-orbit interaction for the usual model of a SNS junction without the electron-electron interaction in the normal region is not sufficient to explain the experimentally observed strong suppression of the $I_c R_N$ parameter with respect to its theoretical value. We believe that to explain this suppression, the electron-electron interaction should be taken into account together with the spin-orbit effects. We note that electron-electron interactions in both density-density and spin-spin channels are not weak in 2DEG structures.

A related open problem is to find the average spin polarization $\langle S_y \rangle$ in the 2DEG region, which is expected to exist at a nonzero supercurrent in the S–2DEG–S structure due to symmetry considerations, see, e.g., Ref. [14]. We note that in the presence of the electron-electron interaction, a supercurrent-induced average spin polarization will induce an effective Zeeman field that may strongly modify the Andreev levels as well as the Josephson current.

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