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THE LONGITUDINAL STRUCTURE FUNCTION F_L: PERTURBATIVE QCD AND k_T-FACTORIZATION VERSUS EXPERIMENTAL DATA AT FIXED W

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We use the results for the structure function F_L for a gluon target with a nonzero transverse momentum squared at the order α_s, obtained in our previous paper, for comparison with recent H1 experimental data for F_L at fixed W values and with collinear GRV predictions in the leading-order and next-to-leading-order approximations.

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1. The longitudinal structure function $F_L(x, Q^2)$ is a very sensitive QCD characteristic and is directly related to the gluon content of the proton. It is equal to zero in the parton model with spin-1/2 partons and acquires nonzero values in the framework of perturbative QCD. The perturbative QCD, however, leads to quite controversial results. In the leading-order (LO) approximation, F_L amounts to about 10–20 % of the corresponding F_2 values at large Q^2 and, thus, acquires quite large contributions at low x. The next-to-leading-order (NLO) corrections to the longitudinal coefficient function are large and negative at small x [1–5] and can lead to negative F_L values at low x and low Q^2 values (see [5, 6]). Negative F_L values demonstrate limitations of the applicability of the perturbation theory and the necessity of a resummation procedure that leads to the coupling constant scale higher than Q^2 (see [5, 7–9]).

The experimental extraction of the F_L data requires a rather cumbersome procedure, especially at small values of x (e.g., see [10]). However, new precise preliminary H1 data [11] on the longitudinal structure function F_L presented recently have probed the small-x region $10^{-5} \leq x \leq 10^{-2}$.

In this paper, the standard perturbative QCD formulas and the so-called k_T-factorization approach [12] based on the Balitsky – Fadin – Kuraev – Lipatov (BFKL) dynamics [13] (also see recent review [14] and the references therein) are used for the analysis of the above data. The perturbative QCD approach is hereafter called the collinear approximation and is applied at the LO and NLO levels using Gluck – Reya – Vogt (GRV) parameterizations for parton densities (see [15]). The corresponding coefficient functions are taken from papers [1, 3].

In the framework of the k_T-factorization approach, which we primarily consider in this paper, the longitudinal structure function F_L was first studied in Ref. [16], where the small-x asymptotics of F_L was obtained analytically using the BFKL results for the Mellin transform of the unintegrated gluon distribution, and the longitudinal Wilson coefficient functions for the full perturbative series were calculated at asymptotically small x values. In this paper, we follow a more phenomenological approach in [17], where we analyzed the F_L data in a broader range at small x; we

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thus use parameterizations of the unintegrated gluon distribution function $\Phi_g(x, k_\perp^2)$ (see Ref. [14]).

A similar study has already been done in our paper [17] using previous H1 data [18][1]. The recent H1 preliminary experimental data [11] is essentially more precise, which stimulates the present additional study.

2. The unintegrated gluon distribution $\Phi_g(x, k_\perp^2)$ (where f_g is the integrated gluon distribution in the proton multiplied by x and $k_\perp$ is the transverse part of the gluon 4-momentum k^μ),

$$f_g(x, Q^2) = \int^{Q^2} dk_\perp^2\, \Phi_g(x, k_\perp^2) \qquad (1)$$

(hereafter, $k^2 = -k_\perp^2$), is the basic dynamical quantity in the k_T-factorization approach[2]. It satisfies the BFKL equation [13].

In the k_T-factorization approach, the structure functions $F_{2,L}(x, Q^2)$ are driven at small x primarily by gluons and are related to the unintegrated distribution $\Phi_g(x, k_\perp^2)$ as

$$F_{2,L}(x, Q^2) = \int_x^1 \frac{dz}{z} \int^{Q^2} dk_\perp^2 \times \\ \times \sum_{i=u,d,s,c} e_i^2 \hat{C}^g_{2,L}(x/z, Q^2, m_i^2, k_\perp^2)\; \Phi_g(z, k_\perp^2), \qquad (2)$$

where e_i are charges of active quarks.

The functions $\hat{C}^g_{2,L}(x, Q^2, m_i^2, k_\perp^2)$ can be regarded as structure functions of the off-shell gluons with virtuality $k_\perp^2$ (hereafter, we call them hard structure functions by analogy with similar relations between cross sections and hard cross sections). They are described by the sum of the quark box (and crossed box) diagram contribution to the photon–gluon interaction (e.g., see Fig. 1 in [17] and [21]).

3. We note that the $k_\perp^2$-integral in Eqs. (1) and (2) can be divergent at the lower limit, at least for some parameterizations of $\Phi_g(x, k_\perp^2)$. To overcome this problem, we change the low Q^2 asymptotics of the QCD coupling constant within hard structure functions. We here apply two models: the «freezing» procedure and the Shirkov – Solovtsov analytization.

The «freezing» of the strong coupling constant is a very popular phenomenological model for the infrared behavior of $\alpha_s(Q^2)$ (e.g., see [22]). The «freezing» can be done in the hard way and in the soft way.

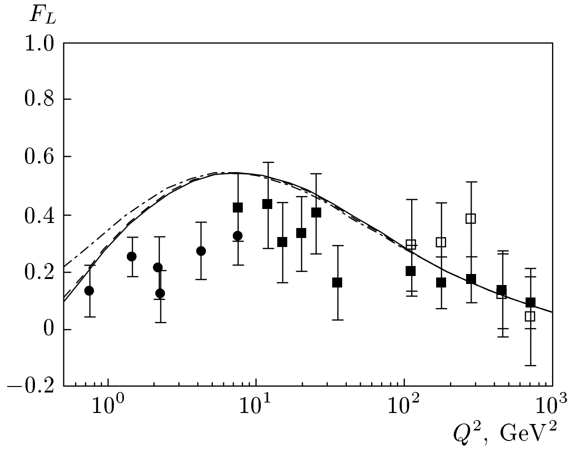

Fig. 1. Q^2 dependence of $F_L(x, Q^2)$ (at fixed $W = 276$ GeV). The H1 preliminary e^+p and e^-p experimental data are shown as the black points, black and white squares, respectively (see [11]). Theoretical curves are obtained in the k_T-factorization approach with the JB unintegrated gluon distribution: the solid curve corresponds to a «frozen» coupling constant, the dashed curve shows the case where the argument is «frozen» in both the coupling constant and in the unintegrated gluon distribution functions. The dash-dotted curve to the «frozen» argument of the unintegrated gluon distribution function

In the hard case (e.g., see [23]), the strong coupling constant is itself modified: it is taken to be constant at all Q^2 values less than some Q_0^2, i.e.,

$$\alpha_s(Q^2) = \alpha_s(Q_0^2) \quad \text{if} \quad Q^2 \le Q_0^2.$$

In the soft case (e.g., see [20]), the subject of the modification is the argument of the strong coupling constant. It contains the shift $Q^2 \to Q^2 + M^2$, where M is an additional scale, which strongly modifies the infrared α_s properties. For massless produced quarks, the ρ-meson mass m_ρ is usually taken as the M value, i.e., $M = m_\rho$. In the case of massive quarks with a mass m_i, the $M = 2m_i$ value is typically used. Below, we use the soft version of the «freezing» procedure.

Shirkov and Solovtsov proposed [24] a procedure of analytization of the strong coupling constant $\alpha_s(Q^2)$, which leads to a new strong analytical coupling constant $a_{an}(Q^2)$ having nonstandard infrared properties. Here, we do not discuss theoretical aspects of the procedure and use only the final formulas for the analytical

[1] We note that the F_L structure function has also been studied in the framework of the k_T-factorization in [19, 20].

[2] In our previous analysis [21], we have shown that the property $k^2 = -k_\perp^2$ leads to the equality of the Bjorken x value in the standard renormalization-group approach and in the Sudakov one.

coupling constant $a_{an}(Q^2)$. They are given by

$$\frac{a_{an}(Q^2)}{4\pi} = \frac{1}{\beta_0}\left[\frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right] \qquad (3)$$

in the LO approximation and

$$\frac{a_{an}(Q^2)}{4\pi} = \\ = \frac{1}{\beta_0}\left[\frac{1}{\ln(Q^2/\Lambda^2) + b_1\ln[1+\ln(Q^2/\Lambda^2)/b_1]} + \right. \\ \left. + \frac{1}{2}\frac{\Lambda^2}{\Lambda^2 - Q^2} - \frac{\Lambda^2}{Q^2}C_1\right], \qquad (4)$$

in the NLO approximation, where β_0 and β_1 are the first two terms in the α_s-expansion of the β-function and $b_1 = \beta_1/\beta_0^2$. The constant $C_1 = 0.0354$ is very small.

The first terms in the right-hand sides of Eqs. (3) and (4) are the standard LO and NLO representations for $\alpha_s(Q^2)$. The additional terms modify its infrared properties.

We note that numerically, both infrared transformations, the «freezing» procedure and the Shirkov–Solovtsov analytization, lead to very close results (see Fig. 1 and also Ref. [25] and the discussion therein).

4. As was already noted above, the purpose of this paper is to describe new preliminary H1 experimental data for the longitudinal structure function $F_L(x, Q^2)$ using our calculations of the hard structure function $\hat{C}^g_{2,L}(x, Q^2, m^2, k_\perp^2)$ given in our previous study [21] and infrared modifications of $\alpha_s(Q^2)$ explained above. For the unintegrated gluon distribution $\Phi(x, k_\perp^2, Q_0^2)$, we use the so-called Blumlein's parameterization (JB) [26]. We note that there are also several other popular parameterizations, which give quite similar results, with a possible exception of contributions from the small-$k_\perp^2$ range $k_\perp^2 \leq 1$ GeV^2 (see Ref. [14] and the references therein).

The JB form depends strongly on the Pomeron intercept value. In different models, the Pomeron intercept has different values (see [27]). In our calculations, we apply the H1 parameterization [28] based on the corresponding H1 data, which are in good agreement with perturbative QCD (see Refs. [28, 29]).

We calculate the structure function F_L as the sum of two types of contributions: that of the charm quark, F_L^c, and of the light quark, F_L^l:

$$F_L = F_L^l + F_L^c. \qquad (5)$$

For the F_L^l part, we use the massless limit of the hard structure function (see [17, 21]). We always use $f = 4$ in our fits, because our results depend on the exact f value very weakly (for similar results, see fits of experimental data in [30] and discussions therein). The weak dependence comes from two basic properties. First, the charm part of F_L, F_L^c, is quite small at the considered Q^2 values (see Ref. [17] for the F_L^c study). Second, the strong coupling constant depends on f very weakly because of the corresponding relations between Λ values at different f (see [31]).

In Fig. 1, we show the structure function F_L with «frozen» and analytical coupling constants, respectively, as functions of Q^2 for fixed W in comparison with the H1 experimental data sets (see [11]). The results mostly coincide with each other. They are presented as solid and dashed curves, which cannot be actually resolved in the figure.

The dash-dotted curve shows the results obtained with a «frozen» argument also added to the unintegrated gluon density. The difference between the solid and dash-dotted lines is not very big, which demonstrates the unimportance of the infrared modifications of the density argument. Below, we restrict ourselves to only the modification of the argument in the strong coupling constant entering the hard structure function.

Figure 2 contains the same solid curve as Fig. 1 and also shows the collinear results for F_L values. We use the popular GRV parameterizations [15] in the LO and NLO approximations. The k_T-factorization results lie between the collinear ones, which clearly demonstrates the particular resummation of high-order collinear contributions at small x values in the k_T-factorization approach.

We also see exellent agreement between the experimental data and the collinear approach with GRV parton densities in the NLO approximation. The NLO corrections are large and negative and reduce the F_L value by an approximate factor of 2 at $Q^2 < 10$ GeV^2.

In Figs. 1 and 2, our k_T-factorization results are in good agreement with the data for large and small parts of the Q^2 range. We have, however, some disagreement between the data and theoretical predictions at $Q^2 \approx 3$ GeV^2. The disagreement exists in both cases: for collinear QCD approach in the LO approximation and for the k_T-factorization approach.

Comparing these results with Fig. 4 in Lobodzinska's talk in Ref. [11], we conclude that the disagreement comes from the use of the LO approximation. Unfortunately, at the moment, only the LO terms are available in the k_T-factorization approach. The calculation of the NLO corrections is a very complicated problem (see [32] and the discussion therein).

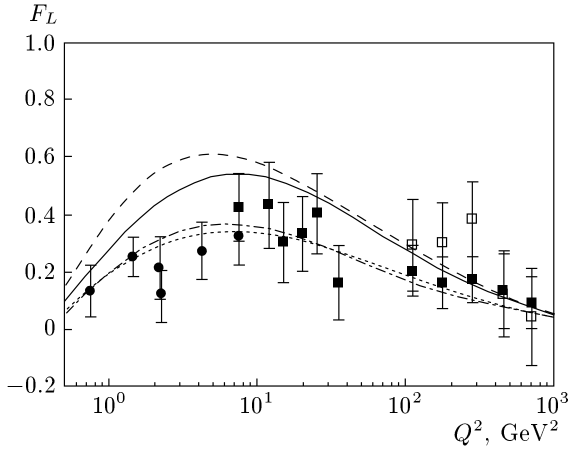

Fig. 2. The Q^2 dependence of $F_L(x,Q^2)$ (at fixed $W = 276$ GeV). The experimental points are as in Fig. 1. The solid curve is the result of the k_T-factorization approach with the JB unintegrated gluon distribution and «frozen» coupling constant, the dashed curve is the GRV LO calculations, the dash-dotted curve is the GRV NLO calculations, the dotted curve is the result of the GRV LO calculations with $\mu^2 = 127Q^2$

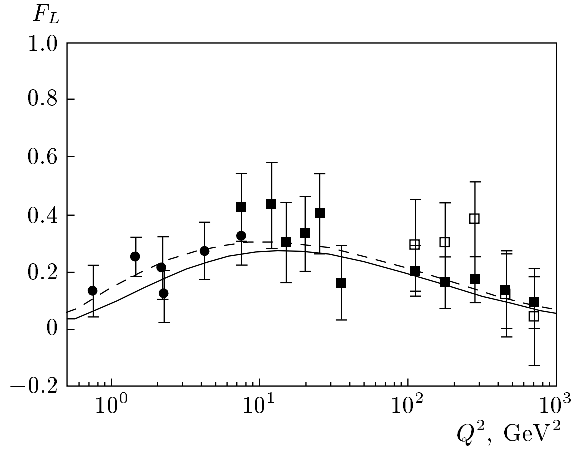

Fig. 3. The Q^2 dependence of $F_L(x,Q^2)$ (at fixed $W = 276$ GeV). The experimental points are as in Fig. 1. The solid curve is the result of the k_T-factorization approach with the JB unintegrated gluon distribution and $\mu^2 = M_Z^2$, the dashed curve is the GRV LO calculations at $\mu^2 = M_Z^2$

A rough estimate of the NLO corrections in the k_T-factorization approach can be done as follows. We first consider the BFKL approach. A popular resummation of the NLO corrections is done in [8] in some approximation. It is demonstrated in Ref. [8] that the basic effect of the NLO corrections is a strong rise of the α_s argument from Q^2 to $Q^2_{eff} = KQ^2$, where $K = 127$, i.e., $K \gg 1$, which is in agreement with [5, 7, 9].

The use of the effective argument Q^2_{eff} in the DGLAP approach in the LO approximation leads to results that are very close to the ones obtained in the NLO approximation, see the dot-dashed and dotted curves in Fig. 2. Thus, we hope that the effective argument represents the basic effect of the NLO corrections in the k_T-factorization framework, which in some sense lies between the DGLAP and BFKL approaches, as was already noted above.

The necessity of large effective arguments is also demonstrated in Fig. 3, where we show the k_T-factorization and collinear results for the nonrunning coupling constant. Its argument is fixed at $Q^2 = M_Z^2$, giving $\alpha_s \approx 0.118$ (see [33]), i.e., the considered argument is larger than most part of the Q^2-values of the considered experimental data[3].

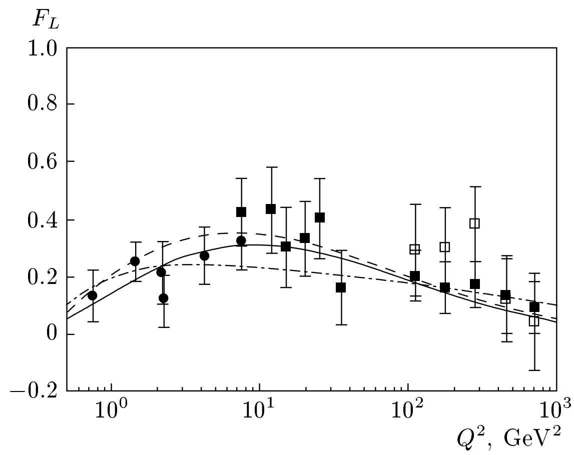

Fig. 4. The Q^2 dependence of $F_L(x,Q^2)$ (at fixed $W = 276$ GeV). The experimental points are as in Fig. 1. The solid curve is the result of the k_T-factorization approach with the JB unintegrated gluon distribution and at $\mu^2 = 127Q^2$, the dashed curve is the GRV LO calculations at $\mu^2 = 127Q^2$, the dash-dotted curve is from the R_{world}-parametrization

[3] The study is also initiated by a conversation with L. Lönnblad, whom we thank.

The results obtained in the k_T-factorization and collinear approaches based on Q^2_{eff} argument are presented in Fig. 4. In comparison with the ones shown in Fig. 1, they are close to each other because the effective argument is essentially larger than the Q^2 value. There is a very good agreement between the experimental data and both theoretical approaches.

Moreover, in Fig. 4, we also present the F_L results based on the R_{world}-parameterization for the $R = \sigma_L/\sigma_T$ ratio (see [34]) (because $F_L = F_2R/(1+R)$), improved in [35, 36] for low Q^2 values and the parameterization of F_2 data used in our previous paper [17]. The results are in good agreement with other theoretical predictions as well as with experimental data.

5. *Summary.* In the k_T-factorization framework, we have applied the results of the calculation of the perturbative parts for the structure functions F_L and F_L^c for a gluon target, having nonzero momentum squared, in the process of photon–gluon fusion [17, 21] to the analysis of recent H1 preliminary data. The perturbative QCD predictions are also presented in the LO and NLO approximations.

We have found a very good agreement between the experimental data and collinear results based on the GRV parameterization in the NLO approximation. The LO collinear and k_T-factorization results show disagreement with the data at some Q^2 values. We argued that the disagreement comes from the absence of NLO corrections in the k_T-factorization approach. Another reason is discussed in Ref. [36]. We modeled these NLO corrections by choosing a large effective argument of the strong coupling constant and argued for our choice. The effective corrections significantly improve the agreement with the H1 data under consideration.

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