ЖЭТФ, 2004, том 126, вып. 3 (9), стр. 734–740



# COMPOSITE FERMIONS, TRIOS, AND QUARTETS IN THE FERMI–BOSE MIXTURE OF NEUTRAL PARTICLES

*M. Yu. Kagan*[*], *I. V. Brodsky*

*Kapitza Institute for Physical Problems*
*119334, Moscow, Russia*

*D. V. Efremov*

*Technische Universität Dresden Institut für Theoretische Physik*
*01062, Dresden, Germany*

*A. V. Klaptsov*

*Russian Research Centre «Kurchatov Institute»*
*123182, Moscow, Russia*

Submitted 17 March 2004

We consider the model of a Fermi–Bose mixture with strong hard-core repulsion between particles of the same sort and attraction between particles of different sorts. In this case, in addition to the standard anomalous averages of the type $\langle b\rangle$, $\langle bb\rangle$, and $\langle cc\rangle$, a pairing between fermions and bosons of the type $\langle bc\rangle$ is possible. This pairing corresponds to creation of composite fermions in the system. At low temperatures and equal densities of fermions and bosons, composite fermions are further paired into the quartets. At higher temperatures, trios consising of composite fermions and elementary bosons are also present in the system. Our investigations are important in connection with the recent observation of weakly bound dimers in magnetic and optical dipole traps at ultralow temperatures and with the observation of collapse of a Fermi gas in an attractive Fermi–Bose mixture of neutral particles.

PACS: 32.80.Pj, 05.30.Fk, 05.30.Jp, 03.75.Mn

## 1. INTRODUCTION

The Fermi–Bose mixture model is currently very popular in connection with different problems in condensed matter physics, such as high-$T_c$ superconductivity, superfluidity in $^3$He–$^4$He mixtures [1], and fermionic superfluidity in magnetic traps.

In high-$T_c$ superconductivity, this model was first proposed by Ranninger [2, 3] for simultaneous description of the high transition temperature and short coherence length of superconductive pairs on one hand and of the presence of a well-defined Fermi surface on the other.

In this paper, we show that the Fermi–Bose mixture with attractive interaction between fermions and bosons is unstable with respect to the creation of composite fermions $f = bc$. Moreover, for low temperatures and equal densities of fermions and bosons, the composite fermions are further paired into the quartets $\langle ff\rangle$. We note that the matrix element $\langle f\rangle = \langle bc\rangle$ is nonzero only for the transitions between states $|N_B; N_F\rangle$ and $\langle N_B - 1; N_F - 1|$, where $N_B$ and $N_F$ are particle numbers of elementary bosons and fermions, respectively. For the superconductive state, the matrix element $\langle ff\rangle$ is nonzero only for the transitions between states $|N_B; N_F\rangle$ and $\langle N_B - 2; N_F - 2|$. Our results are interesting not only for the physics of high-$T_c$ superconductors but also for Fermi–Bose mixtures in magnetic and optical dipole traps as well as in optical lattices, where we can easily tune the parameters of the system such as the particle density and the sign and strength of the interparticle interaction [4, 5].

[*] E-mail: kagan@kapitza.ras.ru

## 2. THEORETICAL MODEL

The model of the Fermi – Bose mixture has the following form on a lattice:

$$
\begin{aligned}
H =& H_F + H_B + H_{BF},\\
H_F =& -t_F \sum_{\langle i,j\rangle} c^+_{i\sigma} c_{j\sigma} + U_{FF}\sum_i n^F_{i\uparrow} n^F_{i\downarrow} - \\
&- \mu_F \sum_{i,\sigma} n^F_{i\sigma},\\
H_B =& -t_B \sum_{\langle i,j\rangle} b^+_i b_j + \frac{1}{2} U_{BB} \sum_i n^B_i n^B_i - \\
&- \mu_B \sum_i n^B_i,\\
H_{BF} =& -U_{BF} \sum_{i,\sigma} n^B_i n^F_{i\sigma}.
\end{aligned}
\tag{1}
$$

This is a lattice analogue of the standard Hamiltonian considered, for example, in Ref. [6]. Here, $t_F$ and $t_B$ are fermionic and bosonic hopping amplitudes, and $c^+_{i\sigma}$, $c_{i\sigma}$, $b^+_i$, and $b_i$ are fermionic and bosonic creation and annihilation operators; the Hubbard interactions $U_{FF}$ and $U_{BB}$ correspond to hard-core repulsion between particles of the same sort; the interaction $U_{BF}$ corresponds to attraction between fermions and bosons; $W_F = 8t_F$ and $W_B = 8t_B$ are the bandwidths in the two-dimensional case; and finally, $\mu_F$ and $\mu_B$ are fermionic and bosonic chemical potentials. For the square lattice, the spectra of fermions and bosons after the Fourier transformation are given by

$$\xi_{p\sigma} = -2t_F(\cos p_x d + \cos p_y d) - \mu_F$$

for fermions and

$$\eta_p = -2t_B(\cos p_x d + \cos p_y d) - \mu_B$$

for bosons, where $d$ is the lattice constant. In the intermediate coupling case

$$\frac{W_{BF}}{\ln(W_{BF}/T_{0BF})} < U_{BF} < W_{BF},$$

the energy of the bound state is given by

$$|E_b| = \frac{1}{2m_{BF}d^2}\,\frac{1}{\exp\left[2\pi/m_{BF}U_{BF}\right] - 1}, \tag{2}$$

where

$$m_{BF} = \frac{m_B m_F}{m_B + m_F}$$

is an effective mass and

$$W_{BF} = \frac{4}{m_{BF}d^2},$$

$$T_{0BF} = 2\pi n/m_{BF}.$$

For simplicity, we consider the case of equal densities $n_B = n_F = n$.

We note that in the intermediate coupling case, the binding energy $|E_b|$ between a fermion and a boson is larger than the bosonic and fermionic degeneracy temperatures

$$T_{0B} = \frac{2\pi n_B}{m_B}$$

and

$$T_{0F} = \frac{2\pi n_F}{m_F} \equiv \varepsilon_F,$$

but smaller than the bandwidths $W_B$ and $W_F$. In this case, pairing of fermions and bosons, $\langle bc\rangle \neq 0$, occurs earlier (at higher temperatures) than both Bose – Einstein condensation of bosons (or bibosons) ($\langle b\rangle \neq 0$ or $\langle bb\rangle \neq 0$) and superconductive pairing of fermions ($\langle cc\rangle \neq 0$). We note that in the case of a very strong attraction $U_{BF} > W_{BF}$, we have the natural result $|E_b| = U_{BF}$, and the effective mass

$$m^*_{BF} = \frac{m_{BF}U_{BF}}{W_{BF}} \gg m_{BF}$$

is additionally enhanced on the lattice [7]. We also note that the Hubbard interactions $U_{FF}$ and $U_{BB}$ satisfy the inequalities

$$
\begin{aligned}
U_{FF} &> \frac{W_F}{\ln(W_F/|E_b|)},\\
U_{BB} &> \frac{W_B}{\ln(W_B/|E_b|)}.
\end{aligned}
$$

We now consider the temperature evolution of the system. It is governed by the corresponding Bethe – Salpeter equation. After the analytic continuation $i\omega_n \to \omega + i0$ (see Ref. [8]), the solution of this equation becomes

$$
\Gamma(\mathbf{q},\,\omega) = \\
= \frac{-U_{BF}}{1 - U_{BF}\displaystyle\int \frac{d^2p}{(2\pi)^2}\,\frac{1 - n_F(\xi(\mathbf{p})) + n_B(\eta(\mathbf{q}-\mathbf{p}))}{\xi(\mathbf{p}) + \eta(\mathbf{q}-\mathbf{p}) - \omega - i0}}, \tag{3}
$$

where

$$\xi(\mathbf{p}) = \frac{p^2}{2m_F} - \mu_F,$$

$$\eta(\mathbf{p}) = \frac{p^2}{2m_B} - \mu_B$$

are spectra of fermions and bosons at low densities $n_F d^2 \ll 1$ and $n_B d^2 \ll 1$. We note that the temperature factor

$$1 - n_F(\xi(\mathbf{p})) + n_B(\eta(\mathbf{q}-\mathbf{p}))$$

enters the pole of the Bethe – Salpeter equation, in contrast with the factor

$$1 - n_F(\xi(\mathbf{p})) - n_F(\xi(\mathbf{q}-\mathbf{p}))$$

for two-fermion superconductive pairing and

$$1 + n_B(\eta(\mathbf{p})) + n_B(\eta(\mathbf{q}-\mathbf{p}))$$

for two-boson pairing. The pole of the Bethe–Salpeter equation corresponds to the spectrum of the composite fermions,

$$\omega \equiv \xi^*_{\mathbf{p}} = \frac{p^2}{2(m_B+m_F)} - \mu_{comp}, \tag{4}$$

where

$$\mu_{comp} = \mu_B + \mu_F + |E_b| \tag{5}$$

is the chemical potential of the composite fermions. Composite fermions are well-defined quasiparticles, because the damping of quasiparticles is equal to zero in the case of a bound state ($E_b < 0$), but becomes nonzero and is proportional to $E_b$ in the case of a virtual state ($E_b > 0$). The dynamical equilibrium (boson + fermion $\rightleftarrows$ composite fermion) is governed by the standard Saha formula [9]. In the two-dimensional case, it is

$$\frac{n_B n_F}{n_{comp}} = \frac{m_{BF}T}{2\pi}\exp\left\{-\frac{|E_b|}{T}\right\}. \tag{6}$$

The crossover temperature $T_*$ is determined, as usual, from the condition that the number of composite fermions is equal to the number of unbound fermions and bosons:

$$n_{comp} = n_B = n_F = n.$$

This conditions yields

$$T_* \approx \frac{|E_b|}{\ln(|E_b|/2T_{0BF})} \gg \{T_{0B}; T_{0F}\}. \tag{7}$$

We note that in the Boltzmann regime $|E_b| > \{T_{0B}; T_{0F}\}$, we actually deal with the pairing of two Boltzmann particles. Therefore, this pairing does not differ drastically from the pairing of two particles of the same type of statistics. Indeed, if we replace $\mu_B + \mu_F$ in (5) with $2\mu_B$ or $2\mu_F$, we obtain the familiar expressions for chemical potentials of composite bosons consisting of either two bosons [10, 11] or two fermions [12, 13]. The crossover temperature $T_*$ plays the role of a pseudogap temperature, and therefore the Green's functions of elementary fermions and bosons acquire a two-pole structure below $T_*$ in similarity with Ref. [13].

For lower temperatures $T_0 < T < T_*$, where

$$T_0 = \frac{2\pi n}{m_F + m_B}$$

is the degeneracy temperature of composite fermions, the numbers of elementary fermions and bosons are exponentially small. The chemical potential of composite fermions is given by

$$\mu_{comp} = -T\ln(T/T_0),$$

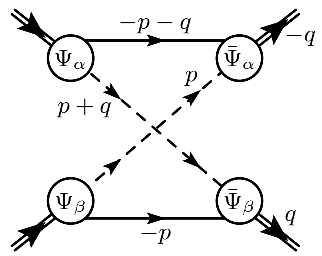


**Fig. 1.** The skeleton diagram for the coefficient $b$ at $\Psi^4$ in the effective action. The dashed lines correspond to bosons, the solid lines correspond to fermions

and hence

$$|\mu_{comp}| \ll |E_b| \quad \text{for } T \ll T_*.$$

By the Hubbard–Stratonovich transformation, the original partition function

$$Z = \int \mathcal{D}\bar b\,\mathcal{D}b\,\mathcal{D}\bar c\,\mathcal{D}c \exp\{-\beta F\}$$

can be written in terms of the composite fermions,

$$Z = \int \mathcal{D}\bar\Psi_\alpha \mathcal{D}\Psi_\alpha \exp\{-\beta F_{eff}\}.$$

This procedure gives the magnitude of the interaction between the composite fermions. The lowest order of the series expansion is given in Fig. 1. Analytically, this diagram is given by

$$-\frac{1}{2}\sum_n \int \frac{d^2p}{(2\pi)^2}\left\{G_F^2(\mathbf{p}; i\omega_{nF})G_B^2(-\mathbf{p}; -i\omega_{nB}) + \right. \\ \left. + G_F^2(-\mathbf{p}; -i\omega_{nF})G_B^2(\mathbf{p}; i\omega_{nB})\right\}, \tag{8}$$

where

$$G_F = \frac{1}{i\omega_{nF} - \xi(\mathbf{p})},$$
$$G_B = \frac{1}{i\omega_{nB} - \eta(\mathbf{p})}$$

are the fermion and boson Matsubara Green's functions, and

$$\omega_{nF} = (2n+1)\pi T, \quad \omega_{nB} = 2n\pi T$$

are the fermion and boson Matsubara frequencies. This integral actually determines the coefficient $b$ at $\Psi^4$ in the effective action. Evaluation of integral (8) yields

$$b \approx -\frac{N(0)}{|E_b|^2}, \tag{9}$$

where

$$N(0) = m_{BF}/2\pi.$$

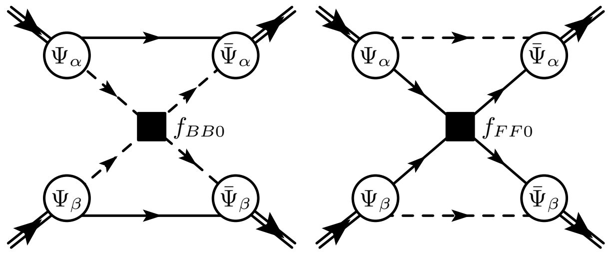


**Fig. 2.** The corrections to the coefficient $b$ containing boson–boson and fermion–fermion interactions

The corrections to the coefficient $b$ are presented in Fig. 2. They explicitly contain the $T$-matrices for the boson–boson and fermion–fermion interactions. In the intermediate coupling case, these diagrams are small in the small parameters

$$f_{BB0} \sim \frac{1}{\ln(W_B/|E_b|)}, \quad f_{FF0} \sim \frac{1}{\ln(W_F/|E_b|)}.$$

Therefore, the exchange diagram indeed gives the main contribution to the coefficient $b$.

The coefficient at the quadratic term $\Psi^2$ in the effective action, in agreement with general rules of diagram technique (see Ref. [8]), is given by

$$a + \frac{cq^2}{2(m_B + m_F)} = -\frac{1}{\Gamma(q;0)}, \tag{10}$$

where $\Gamma(q;0)$ is given by (3). The solution of (10) yields

$$c = \frac{N(0)}{|E_b|}, \quad a = N(0)\ln(T/T_*).$$

Therefore, although $T_*$ in reality corresponds to a smooth crossover and not to a real second-order phase transition, the effective action of composite fermions at temperatures $T \sim T_*$ formally resembles the Ginzburg–Landau functional for the Grassmann field $\Psi_\alpha$.

If we want to rewrite the effective action with gradient terms

$$\Delta F = a\bar{\Psi}_\alpha\Psi_\alpha + \frac{c}{2(m_F + m_B)}(\nabla\bar{\Psi}_\alpha)(\nabla\Psi_\alpha) + \\ + \frac{1}{2}b\bar{\Psi}_\alpha\bar{\Psi}_\beta\Psi_\beta\Psi_\alpha \tag{11}$$

in the form of the energy functional of a nonlinear Schrödinger equation for the composite particle with the mass $m_B + m_F$, we have to introduce the effective order parameter

$$\Delta_\alpha = \sqrt{c}\,\Psi_\alpha.$$

Accordingly, in terms of $\Delta_\alpha$, the new coefficients $\tilde{a}$ and $\tilde{b}$ at the quadratic and quartic terms become

$$\tilde{a} = \frac{a}{c}, \quad \tilde{b} = \frac{b}{c^2}.$$

We note that the Grassmann field $\Delta_\alpha$ corresponds to the composite fermions and is normalized by the condition

$$\Delta_\alpha^+\Delta_\alpha = n_{comp}.$$

Hence, the coefficient $\tilde{b}$ plays the role of the effective interaction between composite particles. From Eqs. (9) and (10),

$$\tilde{b} = -\frac{1}{N(0)}.$$

This result coincides by the absolute value with the result in [14], but has the opposite sign. In [14], the residual interaction between two composite bosons, each consisting of two elementary fermions, was calculated in the two-dimensional case. The sign difference between these two results is due to different statistics of elementary particles in the two cases. It is also important to calculate $b(q)$, where the momenta of the incoming composite fermions are equal to $(\mathbf{q}, -\mathbf{q})$. It is easy to find that

$$b(q) = -\frac{1}{2}\sum_n \int \frac{d^2p}{(2\pi)^2} \times \\ \times \{G_B(\mathbf{p}; i\omega_{nB})G_F(\mathbf{p}; -i\omega_{nF}) \times \\ \times G_B(\mathbf{p}+\mathbf{q}; i\omega_{nB})G_F(\mathbf{p}-\mathbf{q}; -i\omega_{nF}) + \\ + G_B(\mathbf{p}; -i\omega_{nB})G_F(\mathbf{p}; i\omega_{nF}) \times \\ \times G_B(\mathbf{p}-\mathbf{q}; -i\omega_{nB})G_F(\mathbf{p}+\mathbf{q}; i\omega_{nF})\}. \tag{12}$$

In the case of equal masses $m_B = m_F = m$, a straightforward calculation for small $q$ yields

$$b(q) = -\frac{m}{4\pi(|E_b| + q^2/4m)^2}. \tag{13}$$

Accordingly,

$$\tilde{b} = \frac{b}{c^2} \approx -\frac{4\pi}{m(1 + q^2/4m|E_b|)^2}, \tag{14}$$

where

$$|E_b| = \frac{1}{ma^2}.$$

A similar result in the three-dimensional case was obtained in [15]. Hence, the four-particle interaction has a Yukawa form in momentum space. Therefore,

$$U_4(r) \approx -\frac{1}{ma^2}\sqrt{\frac{a}{2r}}\exp\left(-\frac{2r}{a}\right)$$

corresponds to an attractive potential with the interaction radius equal to $a/2$. We can now calculate the binding energy $|E_4|$ of quartets. A straightforward calculation, absolutely similar to the calculation of $|E_b|$, yields

$$1 = \frac{|\tilde{b}|(m_B + m_F)}{2\pi} \int\limits_0^{2/a} \frac{q\,dq}{q^2 + (m_B + m_F)|E_4|}. \qquad (15)$$

Hence,

$$|E_4| = \\ = \frac{4}{a^2(m_B + m_F)\left[\exp\left(\dfrac{4\pi}{|\tilde{b}|(m_B + m_F)}\right) - 1\right]}. \qquad (16)$$

For equal masses $m_B = m_F$, the coupling constant

$$\frac{|\tilde{b}|(m_B + m_F)}{4\pi} = \frac{1}{2},$$

and therefore

$$|E_4| = \frac{2|E_b|}{(e^{1/2} - 1)} \approx 3|E_b|. \qquad (17)$$

The dynamical equilibrium (composite fermion + composite fermion $\rightleftarrows$ quartet) is again governed by the Saha formula

$$\frac{n_{comp}^2}{n_4} = \frac{m_4 T}{2\pi} \exp\left\{-\frac{|E_4|}{T}\right\}, \qquad (18)$$

where

$$m_4 = \frac{m_B + m_F}{2}.$$

The number of composite fermions is equal to half the number of quartets, $n_4 = n_2/2$, for the crossover temperature

$$T_{**}^{(4)} = \frac{|E_4|}{\ln(|E_4|/2T_0)}. \qquad (19)$$

Below this temperature, the quartets of the type $\langle f_{i\uparrow} b_i; f_{j\downarrow} b_j \rangle$ play the dominant role in the system. We note that $T_{**}^{(4)} > T_*$, and therefore quartets are dominant over pairs (composite fermions) in the entire temperature interval. We also note that the quartets are in the spin-singlet state. The creation of spin-triplet quartets is prohibited or at least strongly reduced by the Pauli principle. The triplet $p$-wave pairs of composite fermions are possibly created in the strong-coupling case $|E_b| > W$, where the corrections to the coefficient $b$ given by the diagrams in Fig. 2 are large and repulsive. However small parameters are absent in this case, and it is very difficult to control the diagram expansion.

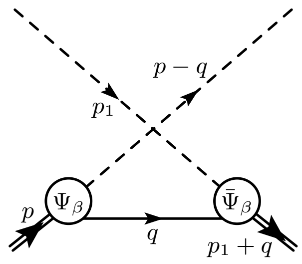


**Fig. 3.** The exchange diagram for the three-particle interaction

### 3. THREE-PARTICLE PROBLEM

If we consider the scattering process of an elementary fermion on a composite fermion, we obtain a repulsive sign of the interaction regardless of the relative spin orientation of the composite and elementary fermions. The same result in three dimensions for scattering of an elementary fermion on a dimer consisting of two fermions was obtained in [16]. However, for a scattering process of an elementary boson on a composite fermion, we obtain an attractive sign of the interaction. Moreover, in the two-dimensional case, the Fourier component of the three-particle interaction for $m_B = m_F = m$ is given by (see Fig. 3)

$$U_3(q) = \frac{1}{c} G_F(0, q) = -\frac{8\pi}{m(1 + q^2 a^2)}, \qquad (20)$$

where $G_F(0, q)$ is the Green's function of elementary fermions and $c = N(0)/|E_b|$. Hence,

$$U_3(r) \sim -\frac{1}{ma^2} K_0(r/a) \sim \\ \sim -\frac{1}{ma^2}\sqrt{\frac{a}{r}} \exp(-r/a), \qquad (21)$$

which again corresponds to an attracting potential of the Yukawa type, but now with the interaction range equal to $a$. Calculation of the three-particle bound-state energy yields

$$1 = \frac{|U_3(0)|}{2\pi} \times \\ \times \int\limits_0^{1/a} \frac{q\,dq}{q^2/2m_B + q^2/2(m_B + m_F) + |E_3|}. \qquad (22)$$

Hence, for $m_B = m_F = m$, we have

$$|E_3| = \frac{3}{4ma^2} \frac{1}{[\exp(3\pi/m|U_3|) - 1]} = \\ = \frac{3|E_b|}{4(e^{3/8} - 1)} \approx 1.65|E_b|. \qquad (23)$$

We note that we are studying trios and quartets in the zeroth-order exchange approximation. A more rigorous solution of the three- and four-particle problems requires analyzing the Skorniakov – Ter-Martirosian type of equations [17]. This investigation will be the subject of a separate publication. The dynamical equilibrium of the type composite fermion + boson $\rightleftarrows$ trio is governed by the Saha formula

$$\frac{n_B n_{comp}}{n_3} = \frac{m_3 T}{2\pi} \exp\left\{-\frac{|E_3|}{T}\right\}, \tag{24}$$

where

$$m_3 = \frac{m_B(m_B + m_F)}{2m_B + m_F}.$$

Accordingly, trios dominate over unbound bosons for temperatures $T < T_{**}^{(3)}$, where

$$T_{**}^{(3)} = \frac{|E_3|}{\ln(|E_3|/2T_0)}. \tag{25}$$

We note that $T_{**}^{(3)} < T_{**}^{(4)}$, and therefore trios are not so important as quartets.

As a result, there are mostly quartets in the system for $T < T_{**}^{(4)}$. The quartets are Bose-condensed at the critical temperature

$$T_c = \frac{T_0}{8 \ln\ln(4/na^2)}$$

in the case of equal masses. It is important to note that in the Feshbach resonance scheme [4, 5, 18], we are usually in the regime $T \sim T_0$, where quartets prevail over trios and pairs. In this scheme, the particles are first cooled to very low temperatures $T < T_0$ and only then the sign of the scattering length is changed by a magnetic field to support the formation of bound pairs. We emphasize that in the restricted geometry of magnetic or optical dipole traps, our theory is valid under the condition $T_c > \omega$, where $\omega$ is the level spacing in the trap. For a large number of particles $N \gg 1$ in the two-dimensional trap, $\omega \sim T_0/N^{1/2}$ ($\omega \sim T_0/N^{1/3}$ in three-dimensional traps), and this condition is therefore easily satisfied. We also note that octets are not formed in the system because two quartets repel each other due to the Pauli principle, in similarity with the results in [14, 19].

## 4. CONCLUSIONS

We have considered the appearance and pairing of composite fermions in a Fermi – Bose mixture with an attractive interaction between fermions and bosons.

At equal densities of elementary fermions and bosons, the system is described at low temperatures by a one-component attractive Fermi gas for composite fermions and is unstable with respect to the formation of quartets.

The problem that we considered is important for theoretical understanding of high-temperature superconductive materials and for the investigation of Fermi – Bose mixtures of neutral particles at low and ultralow temperatures. In high-$T_c$ superconductors, quartets play the role of singlet superconductive pairs. The radius of the quartets (the coherence length of the superconducting pair) is governed by the binding energy $|E_4|$ of the quartets. If $|E_4|$ is larger that $T_0$, the quartets are local: $p_F a < 1$. For

$$T_c = \frac{T_0}{8 \ln\ln(4/na^2)},$$

the local quartets are Bose-condensed and the system becomes superconductive. We note that at higher temperatures $T > T_0$, some amount of trios is also present in the system in addition to the quartets. The role of trios is usually neglected in the standard theories of high-$T_c$ superconductivity.

We also note that we consider the low-density limit $|E_b| \gg T_0$. In the opposite case of higher densities $T_0 \gg |E_b|$, Bose – Einstein condensation of bosons or bibosons (see Refs. [11, 20] and [21]) occurs earlier than the creation of composite fermions and quartets. Such a state can be distinguished from the ordinary BCS-superconductor by measuring the temperature dependence of the specific heat and the normal density.

For Fermi – Bose mixtures, our investigations enrich superfluid phase diagram in magnetic and optical dipole traps and are important in connection with recent experiments where weakly bound dimers $^6Li_2$ and $^{40}K_2$, consisting of two elementary fermions, were observed [22, 23]. We note that in an optical dipole trap, it is possible to obtain an attractive scattering length for fermion–boson interaction with the help of the Feshbach resonance [18]. We also note that even in the absence of the Feshbach resonance, it is experimentally possible now to create a Fermi – Bose mixture with attractive interaction between fermions and bosons. For example, in Refs. [24, 25], such a mixture of $^{87}Rb$ (bosons) and $^{40}K$ (fermions) was experimentally studied. Moreover, the authors of Refs. [24, 25] experimentally observed the collapse of the Fermi gas with a sudden disappearance of fermionic $^{40}K$ atoms when the system enters the degenerate regime. We cannot exclude in principle that it is just a manifestation of the creation of the $\langle bc; bc\rangle$ quartets in the system. We note that in the regime of strong attraction between fermions and bosons, phase separation with the creation of larger clusters or droplets is also possible. We also note that much slower collapse in the Bose subsystem of $^{87}$Rb atoms can possibly be explained by the fact that the number of Rb atoms in the trap is much larger than the number of K atoms, and therefore after the formation of composite fermions, many residual bosons are still present in the system. A more thorough comparison of our results with an experimental situation will be the subject of a separate publication. Here, we only mention that for the experiments performed in Refs. [24, 25], the three-dimensional case is more relevant. In the three-dimensional case, the attractive interaction between composite fermions acquires the form

$$\tilde{b}(q) = -\frac{\pi a_{eff}}{m_{BF}[1 + q^2/2(m_B + m_F)|E_b|]}, \qquad (26)$$

where

$$|E_b| = \frac{1}{2m_{BF}a^2}$$

is a shallow level of a fermion–boson bound state. We note that in the case of a repulsive interaction between two bosons (each of which consists of two fermions), $a_{eff} = 2a$ in the mean-field theory in [19], $a_{eff} = 0.75a$ in the calculations in [15], and $a_{eff} = 0.6a$ in the calculations in [16]. The shallow bound state of quartets exists in the three-dimensional case only if

$$a_{eff} > 2\pi a \left(\frac{m_{BF}}{m_B + m_F}\right)^{3/2}. \qquad (27)$$

For $m_B = m_F = m$, we have

$$a_{eff} > \pi a/4.$$

The authors acknowledge helpful discussions with A. Andreev, Yu. Kagan, L. Keldysh, B. Meierovich, P. Wölfle, G. Khaliullin, A. Chernyshev, I. Fomin, M. Mar'enko, A. Smirnov, P. Arseev, E. Maksimov, G. Shlyapnikov, M. Baranov, and A. Sudbø. This work was supported by the Russian Foundation for Basic Research (grant № 02-02-17520), Russian President Program for Science Support (grant № 00-15-96-9694), and by the Grant of Russian Academy of Sciences for Young Scientists.

## REFERENCES

1. J. Bardeen, G. Baym, and D. Pines, Phys. Rev. **156**, 207 (1967).
2. J. Ranninger and S. Robaszkiewicz, Physica B **135**, 468 (1985).
3. B. K. Chakraverty, J. Ranninger, and D. Feinberg, Phys. Rev. Lett. **81**, 433 (1998).
4. W. Hofstetter, J. I. Cirac, P. Zoller, E. Demler, and M. D. Lukin, E-print archives cond-mat/0204237.
5. J. N. Milstein, S. J. J. M. F. Kokkelmans, and M. J. Holland, E-print archives cond-mat/0204334.
6. D. V. Efremov and L. Viverit, Phys. Rev. B **65**, 134519 (2002).
7. P. Nozieres and S. Schmittt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
8. A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinkski, *Methods of Quantum Field Theory in Statisitical Physics*, Dover, New York (1963).
9. L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Butterworth–Heinemann (1999).
10. P. Nozieres and D. Saint James, J. de Phys. **4**, 1133 (1982).
11. M. Yu. Kagan and D. V. Efremov, Phys. Rev. B **65**, 195103 (2002).
12. M. Yu. Kagan, R. Fresard, M. Capezzali, and H. Beck, Phys. Rev. B **57**, 5995 (1998).
13. M. Yu. Kagan, R. Fresard, M. Capezzali, and H. Beck, Physica B **284**–**288**, 447 (2000).
14. M. Drechsler and W. Zwerger, Ann. Phys. **1**, 15 (1992).
15. P. Pieri and G. C. Strinati, Phys. Rev. B **61**, 15370 (2000).
16. D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, E-print archives cond-mat/0309010.
17. G. V. Skorniakov and K. A. Ter-Martirosian, Zh. Eksp. Teor. Fiz. **31**, 775 (1956).
18. E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, Phys. Rep. **315**, 199 (1999).
19. R. Haussmann, Z. Phys. B **91**, 291 (1993).
20. P. A. Lee and N. Nagaosa, Phys. Rev. B **46**, 5621 (1992).
21. P. A. Lee, N. Nagaosa, T. K. Ng, and X. G. Wen, Phys. Rev. B **57**, 6003 (1998).
22. C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Nature **424**, 47 (2003).
23. B. G. Levi, Phys. Today **56**, 18 (2003).
24. G. Roati, F. Riboli, G. Modugno, and M. Inguscio, Phys. Rev. Lett. **89**, 150403 (2002).
25. G. Mondugno, G. Roati, F. Ferlaino, R. J. Brecha, and M. Inguscio, Science **297**, 2240 (2002).