ЖЭТФ, 2004, том 126, вып. 1 (7), стр. 89–98

DYNAMICS OF JUMP-WISE TEMPERATURE PITCH VARIATIONS IN PLANAR CHOLESTERIC LAYERS FOR FINITE STRENGTH OF SURFACE ANCHORING

V. A. Belyakov [*a], ***I. W. Stewart*** [b], ***M. A. Osipov*** [b]

[a] *Landau Institute for Theoretical Physics*
117334, Moscow, Russia

[b] *Department of Mathematics, University of Strathclyde*
Glasgow G1 1XH, Great Britain

Submitted 15 September 2003

The dynamics of pitch jumps in cholesteric layers with finite surface anchoring strength at temperature variations is investigated theoretically. General expressions are presented that connect the dynamics of the pitch jumps with the parameters that determine the process, such as the viscosity, the specific form of the anchoring potential, and the dimensionless parameter $S_d = K_{22}/Wd$, where W is the depth of the anchoring potential, K_{22} is the twist elastic modulus, and d is the layer thickness. It is found that the shape of the anchoring potential essentially influences the temporal behavior of the cholesteric helix in the process of a pitch jump. To illustrate this revealed dependence of the pitch jump dynamics on the shape and strength of the anchoring potential, the problem was investigated for two different models of the surface anchoring potential for a jump mechanism connected with the slipping of the director at the surface over the barrier of the anchoring potential. Calculations for the unwinding (winding) of the helix in the process of the jump were performed to investigate the case of infinitely strong anchoring on one surface and finite anchoring on the other, which is important in applications. The results show that an experimental investigation of the dynamics of the pitch jumps will allow one to distinguish different shapes of the finite strength anchoring potential, and in particular, will provide a means for determining whether the well-known Rapini–Papoular anchoring potential is the best suited potential relevant to the dynamics of pitch jumps in cholesteric layers with finite surface anchoring strength. The optimal conditions for the experimental observation of the phenomena discussed here are briefly considered.

PACS: 61.30.-v, 68.15.+e

1. INTRODUCTION

Recent investigations of the temperature pitch variations in planar cholesteric layers and of the influence of finite surface anchoring and thermodynamical fluctuations on these variations [1, 2] have revealed some novel effects that are interesting in the general context of the physics of liquid crystals and in the practical applications of liquid crystals. Similar investigations of pitch variation under the influence of applied external fields have also been carried out [3, 4]. Some experimental and applied aspects of pitch variations in external fields have been considered in [5]. It has been known for quite some time that the temperature evolution of a cholesteric liquid crystal (CLC) structure [6, 7] in samples with a finite surface anchoring energy may be continuous in some ranges of the temperature with jump-wise changes at definite temperature points, with a strong hysteresis effect occurring when the direction of the temperature variations is changed [8–10]. This problem has been recently investigated in [1, 2]. But only thermodynamic equilibrium states of cholesteric layers were studied in these theoretical papers and the problem of the dynamics of jump-wise transitions was not considered. Nevertheless, the dynamics of liquid crystals in restricted geometries present general physical interest and is of especially great concern in liquid crystal applications [6]. For example, the switching time between two bistable states studied experimen-

*E-mail: bel@landau.ac.ru

tally in [11, 12], as was shown theoretically [13], is directly connected to the liquid crystal dynamics in the corresponding restricted geometry. In this article, the dynamics of the aforementioned jump-wise transitions is studied. We begin with the simplest cases that reveal jump-wise transitions, namely, theoretical investigations of the dynamics of the temperature-induced jump-wise variations of the cholesteric pitch and director distribution in a planar layer of CLC with infinitely strong anchoring at one of its boundary surfaces and anchoring of a finite strength at its other boundary surface. We note that the dynamics of such jump-wise transitions is directly dependent on both the viscosity properties of liquid crystals and the characteristics of the surface anchoring potential. This is the motivation for introducing and considering different models for the surface anchoring potential. In addition to the well-known Rapini–Papoular anchoring potential, another possible model for the anchoring potential, the B-potential, is used in the calculations presented below.

2. SOME RESULTS OF EQUILIBRIUM INVESTIGATIONS

We present some results on the temperature behavior of the cholesteric helix in a planar cholesteric layer of finite thickness having finite strength of anchoring at one of its surfaces and infinite anchoring at the other, as depicted in Fig. 1. We first restrict the analysis of the temperature variations of the director configuration in the layer by assuming that the pitch-jump mechanism is connected with overcoming the anchoring barrier by the director at the surface and, moreover, that any liquid crystal (LC) thermal fluctuations may be neglected.

Below, we follow the approach and investigations reported earlier [1, 2], and concentrate on the transitions between N and $N+1$ half-turns of the director in the layer, which proceed without strong local disturbances of the director configuration. Such a transition is unique if the finite anchoring at the second surface

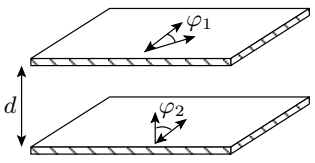

Fig. 1. The case of nonidentical anchoring at the surfaces of a cholesteric layer (for infinitely strong anchoring at the bottom surface $\varphi_2 = 0$)

is sufficiently weak, i.e., the dimensionless parameter $S_d = K_{22}/Wd > 1/2\pi$, where W is the depth of the anchoring potential, K_{22} is the twist elastic constant, and d is the layer thickness. The case of small $S_d < 1/2\pi$ has been studied numerically in [9].

We start by finding the temperature behavior of φ, the angle of deviation of the director from the alignment direction orientation at the surface of the cholesteric layer with finite anchoring, see Fig. 1. Following [6], we can write the free energy of the layer as

$$F(T) = W_s(\varphi) + \frac{1}{2}K_{22}d\left[\frac{2\pi}{p_d(T)} - \frac{2\pi}{p(T)}\right]^2, \qquad (1)$$

where $W_s(\varphi)$ is the surface anchoring potential, K_{22} is the elastic twist modulus, d is the layer thickness, $p(T)$ is the natural pitch value in an infinite sample of the cholesteric liquid crystal, and $p_d(T)$ is the corresponding pitch value in the layer.

Because the pitch value $p_d(T)$ in the layer is determined by the angle φ and the natural pitch $p(T)$ may be expressed via the angle $\varphi_0(T)$, the angle of the director deviation from the alignment direction at the surfaces with finite anchoring in the absence of any anchoring forces, expression (1) for the free energy is readily represented as a function of these angles, namely,

$$F(T) = W_s(\varphi) + \frac{1}{2}\frac{K_{22}}{d}\left[\varphi - \varphi_0(T)\right]^2. \qquad (2)$$

We note that Eq. (2) is obtained from Eq. (1) using a simple change of variables $\varphi = qd$, where $q = 2\pi/p$ and $z = 0$ at the surface with infinite anchoring. The angle φ can be found from the condition for a minimum of the free energy in Eq. (2). Consequently, φ must satisfy the equation

$$\frac{\partial W_s}{\partial \varphi} + \frac{K_{22}}{d}\left[\varphi - \varphi_0(T)\right] = 0. \qquad (3)$$

Analysis of Eq. (3) reveals that a smooth change of the director deviation angle φ is possible when φ is less than some critical angle φ_c. As φ achieves the critical value φ_c, a jump-like change of the pitch occurs. For $S_d > 1/2\pi$, the transition to the unique new configuration of the helix, differing by one in the number of half-turns N, occurs. In this case, it is possible to restrict the range of values of φ to the interval $[-\pi/2, \pi/2]$ using the formula $\varphi = N\pi + \varphi'$, where the integer $N = \mathrm{Int}[\varphi/\pi]$ is the number of half-turns within the layer thickness. In the case where $S_d > 1/2\pi$, all solutions for φ' fit into the interval $[-\pi/2, \pi/2]$. In the rest of the paper, we only use the variable φ', with the prime dropped for simplicity. The critical value of the

director deviation angle φ_c corresponds to the configuration with N director half-turns in the layer when it is at an instability point.

The pitch in the layer just before the jump and the corresponding natural pitch are expressed through φ_c as

$$p_d(T_c) = \frac{2d}{N + \varphi_c/\pi}, \quad p(T_c) = \frac{2d}{N + \varphi_0(T_c)/\pi}, \quad (4)$$

where T_c is the jump temperature. The angle $\varphi_0(T_c)$ (the natural one at the jump point temperature) is given by

$$\varphi_0(T_c) = \varphi_c + \frac{d}{K_{22}} \left[\frac{\partial W_s(\varphi)}{\partial \varphi}\right]_{\varphi=\varphi_c}. \quad (5)$$

The value of φ after the jump, denoted by φ_j, which is basically connected to the pitch $p_{dj}(T_c)$ in the layer after the jump, is determined by the solution of the equation

$$\frac{\partial W_s(\varphi)}{\partial \varphi} + \frac{K_{22}}{d}\left[\varphi - \varphi_0(T_c) + \pi\right] = 0, \quad (6)$$

where $\varphi_0(T_c)$ is determined by Eq. (5). As has been determined previously [1, 2], the variations of the pitch in the layer and, in particular, the hysteresis are determined by the dimensionless parameter $S_d = K_{22}/Wd$, where W is the depth of the anchoring potential, and are rather universal phenomena because they are not directly dependent on the sample thickness. This means that for every specific form of the anchoring potential, expressions (3)–(6) can be transformed to forms that include the parameters of the problem, namely, d, K_{22}, and W, which only appear in combinations reducing to the dimensionless parameter S_d.

3. MODEL ANCHORING POTENTIALS

To obtain some quantitative predictions, we must assume some specific form of the anchoring potential. We use the widely known Rapini–Papoular (RP) anchoring model potential [6, 14]

$$W_s(\varphi) = -\frac{W}{2}\cos^2\varphi. \quad (7)$$

For this potential, the critical angle $\varphi_c = \pi/4$ when the anchoring is identical at both surfaces of the layer. The analysis in [1, 2] demonstrated that the essential features of the director temperature variations are directly dependent on the particular shape of the anchoring potential. It is therefore quite natural to perform similar calculations for a potential that differs from the RP model anchoring potential. The second model potential to be investigated, which we call the B-potential, is given by the expression (see Fig. 2)

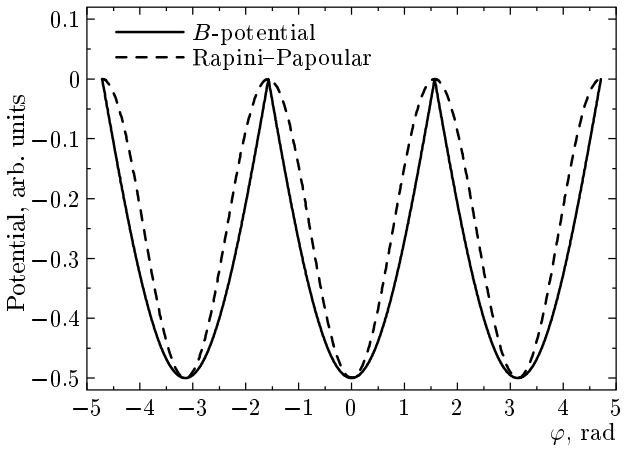

Fig. 2. Qualitative plots of RP potential (7) and B-potential (8)

$$W_s(\varphi) = -W\left(\cos^2\frac{\varphi}{2} - \frac{1}{2}\right), \quad -\frac{\pi}{2} < \varphi < \frac{\pi}{2}, \quad (8)$$

which is continued periodically for $|\varphi| > \pi/2$ in accordance with the relation $W_s(\varphi) = W_s(\varphi - \pi)$. The behavior of potential (8) is similar to the case of the RP potential for small φ. However, it differs essentially from the RP model when φ is close to $\pi/2$. In particular, for identical anchoring at both surfaces, the critical angle φ_c for potential (8) is independent of the strength of the anchoring (via the parameter S_d) and is equal to $\pi/2$. Because there are too many parameters in the general case for different anchoring at the two surfaces, we consider a specific case in detail, namely, the case where there is infinitely strong anchoring on one surface and finite anchoring on the other surface of a layer.

4. INFINITELY STRONG ANCHORING AT ONE SURFACE

We now apply the above general expressions to the specific case where infinitely strong anchoring is assumed at one of the layer surfaces and finite anchoring, described by the potential $W_s(\varphi)$, at the other, where $W_s(\varphi)$ is either of the two finite anchoring potentials mentioned above.

4.1. The Rapini–Papoular potential

For the RP potential (7), the free energy given by Eq. (2) becomes

$$\frac{F(T)}{W} = \frac{1}{2}\left[-\cos^2\varphi + S_d\left(\varphi - \varphi_0(T)\right)^2\right]. \qquad (9)$$

The angle of the director deviation at the surface, φ, is determined by

$$\sin(2\varphi) + 2S_d\left(\varphi - \varphi_0(T)\right) = 0, \qquad (10)$$

while the critical angle φ_c is determined by the relation $\cos(2\varphi_c) + S_d = 0$, that is,

$$\varphi_c = \frac{1}{2}\arccos(-S_d). \qquad (11)$$

Equation (11) shows that the critical angle φ_c depends on the parameter S_d, in contrast to the case where the anchoring is identical at both surfaces [1, 2]. It changes from $\pi/4$ at $S_d = 0$ to $\pi/2$ at $S_d = 1$.

The value of the pitch for a bulk sample, or the alternative description involving the free rotation angle $\varphi_0(T_c)$ corresponding to the jump point, is determined from Eq. (10) as

$$\varphi_0(T_c) = \varphi_c + \frac{1}{2S_d}\sin(2\varphi_c). \qquad (12)$$

The solution of Eq. (11) exists only for $0 < S_d < 1$. This means that for weak anchoring (or thin layers), jump-wise changes of the director configuration in the layer may be absent. But it should be mentioned that because Eq. (11) for the critical angle was obtained for the RP anchoring model potential, the previous statement is model-dependent. Therefore, experimental investigations of jump-wise changes of the director configuration in a layer may be used for determining the shape of the actual anchoring potential and its deviations from the RP anchoring model potential. The value of φ_j, or its equivalent in terms of the pitch $p_{d_j}(T_c)$ in a layer after the jump, is determined by the solution of

$$\sin(2\varphi_j) + \\ +2S_d\left\{\varphi_j - \frac{1}{2}\left[\arccos(-S_d)\right] - \frac{\sin(2\varphi_c)}{2S_d} + \pi\right\} = 0. \qquad (13)$$

4.2. The B-potential

When the finite anchoring potential is taken as the B-potential in Eq. (8), the expression for the free energy given by Eq. (2) becomes

$$\frac{F(T)}{W} = \frac{1}{2}\left[-2\cos^2\frac{\varphi}{2} + 1 + S_d\left(\varphi - \varphi_0(T)\right)^2\right]. \qquad (14)$$

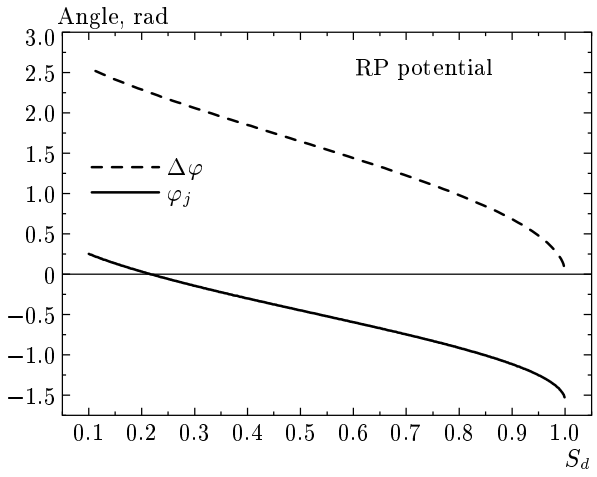

Fig. 3. The dependence of the post-jump angle φ_j and the angular width $\Delta\varphi$ of the jump on the dimensionless parameter S_d for the RP potential

The angle φ of the director deviation is determined from the relation

$$\sin\varphi + 2S_d\left(\varphi - \varphi_0(T)\right) = 0. \qquad (15)$$

The critical angle φ_c for the B-potential is given by $\pi/2$ for any value of S_d (see Fig. 2), as in the case for identical anchoring at both surfaces. The value of the pitch for a bulk sample, or its equivalent in terms of the free rotation angle $\varphi_0(T_c)$ corresponding to the jump point, is determined from Eq. (15) as

$$\varphi_0(T_c) = \frac{\pi}{2} + \frac{1}{2S_d}. \qquad (16)$$

The value of φ_j, equivalent to the knowledge of the pitch in a layer after the jump at $p_{d_j}(T_c)$, is determined by the solution of the equation

$$\sin\varphi_j + 2S_d\left(\varphi_j - \frac{1}{2S_d} + \frac{\pi}{2}\right) = 0. \qquad (17)$$

The values given above for the angles of the director deviation just before and after the jump, equivalent to knowing the values of the pitches in the layer and the corresponding value of the pitch in a bulk sample, completely determine the initial and final states of the dynamical problem to be solved. As an example, φ_j and the angular width of the jump, i.e., $\Delta\varphi = \varphi_j - \varphi_c$, calculated versus S_d and the layer thickness d for the RP and B-potentials are presented in Figs. 3, 4 and Figs. 5, 6, respectively.

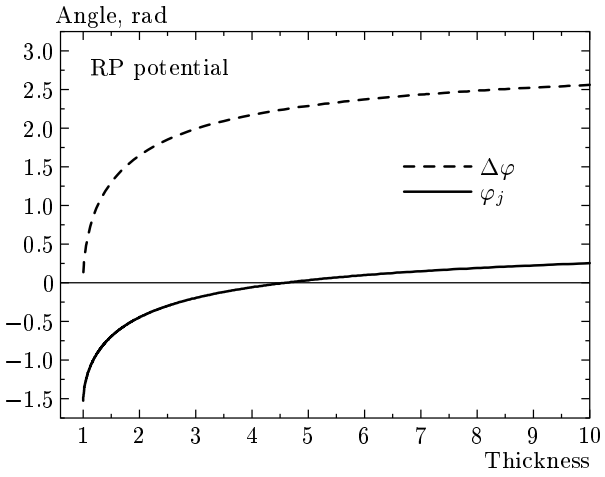

Fig. 4. The dependence of the post-jump angle φ_j and the angular width $\Delta\varphi$ of the jump on the layer thickness (in units of the penetration length K_{22}/W for fixed values of K_{22} and W) calculated for the RP potential

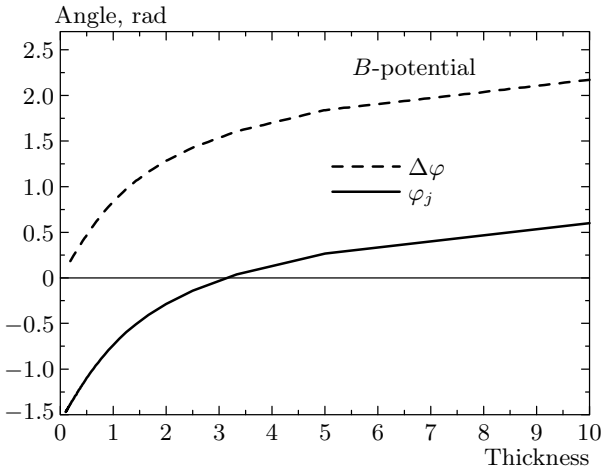

Fig. 6. The dependence of the post-jump angle φ_j and the angular width $\Delta\varphi$ of the jump on the layer thickness (in units of the penetration length K_{22}/W for fixed values of K_{22} and W) calculated for the B-potential

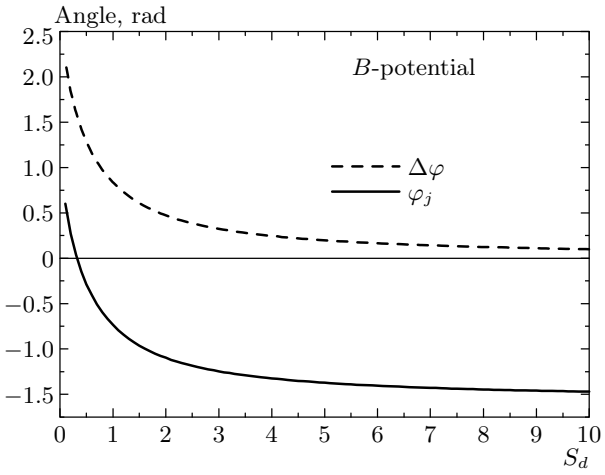

Fig. 5. The dependence of the post-jump angle φ_j and the angular width $\Delta\varphi$ of the jump on the dimensionless parameter S_d for the B-potential

5. JUMP DYNAMICS

The plane geometry of the problem under consideration, and symmetry arguments, allow us to suppose, in a first approximation to the problem, that the hydrodynamical flow during the pitch jump motion in a planar cholesteric layer is of minor significance. We therefore initiate a study of the above problem by neglecting the hydrodynamical flow. Moreover, we also neglect the fluid and director inertial terms; this approach is usually adopted in the theory of liquid crystals.

In our approach, the director configuration is specified by one time-dependent variable $\varphi(t)$, i.e., by the director orientation at the surface, which, unlike in the previous sections, is now time-dependent. The dynamics of φ is in general described by the Landau–Khalatnikov equation [15]

$$\gamma \frac{d\varphi}{dt} = -\frac{\partial F}{\partial \varphi}, \tag{18}$$

where $F = F(\varphi)$ is the total free energy and γ is the kinetic coefficient. This equation can also be derived using the general continuum theory of liquid crystals, which enables one to determine the parameter γ. In the absence of flow and general external forces, it is known from the general continuum theory of liquid crystals [16] that

$$\frac{d}{dt}F = -\int\limits_{\Omega} \mathcal{D}\, d\Omega, \tag{19}$$

where $\mathcal{D}$ is the Rayleigh dissipation function [17] and Ω is the volume of the sample.

5.1. Simplified dynamical solutions

For the further simplification of the problem, we assume that the director distribution in the layer is quasi-static. This means that the director orientation angle $\phi(t, z)$ in the bulk of the sample can be easily related to the above time-dependent orientation angle $\varphi(t)$ through the equation

$$\frac{\partial \phi}{\partial t} = \frac{z}{d}\frac{d\varphi}{dt}, \tag{20}$$

a form that is motivated by the well-known twist solution that occurs in the isothermal situation of cholesterics. This is a consequence of assuming that the director configuration inside the layer at any given time t is quasi-static, that is, the helical structure within the layer remains undistorted and corresponds to some value of the pitch, which is changing with time; infinitely strong anchoring at one boundary then justifies the assumption made in Eq. (20). It follows from Eqs. (18) and (19) that for a sample of unit dimensions in the x and y directions,

$$\frac{d}{dt}F\left(\varphi(t)\right) = -\int\limits_0^d \mathcal{D}\, dz, \tag{21}$$

With the above assumptions on flow, the dissipation function in this case is simply (cf. [18, pp. 11–13] and [19])

$$\mathcal{D} = \gamma_1 \left(\frac{\partial \phi}{\partial t}\right)^2, \tag{22}$$

where $\gamma_1 > 0$ is the twist viscosity. Because

$$\frac{dF}{dt} = \frac{d\varphi}{dt}\,\frac{dF}{d\varphi}\,, \tag{23}$$

we can use relations (20) and (23) and insert $\mathcal{D}$ defined by Eq. (22) in Eq. (21) to find that

$$\frac{d\varphi}{dt} = -\frac{1}{\gamma_1}\,\frac{3}{d}\,\frac{dF}{d\varphi}\,. \tag{24}$$

This expression for the dynamics in the layer allows us to derive the solution for φ implicitly by integration of (24) from time $t = 0$, where the director deviation angle at the surface is equal to the critical value, i.e., $\varphi(0) = \varphi_c$, up to time t, where $\varphi = \varphi(t)$. The resulting solution for φ is

$$t = -\gamma_1 \frac{d}{3} \int\limits_{\varphi_c}^{\varphi} \left(\frac{dF}{d\widehat{\varphi}}\right)^{-1} d\widehat{\varphi}. \tag{25}$$

The solution φ must of course lie within the range $\varphi_c \le \varphi < \varphi_j$, that is, between the initial and final states identified above in Sec. 4. The values of φ_c and φ_j were discussed in Sec. 4 and are clearly dependent on the specific form of the anchoring potential and may be dependent on the value of the parameter S_d. The complete duration τ of the pitch jump is found by replacing the upper limit in the integral in Eq. (25) by φ_j.

The solution in Eq. (25) also allows us to define the relaxation time for the jump process. For example, the relaxation time t_r may be defined as the derivative $dt/d\varphi$ evaluated at the middle-during-the-jump-value of $\varphi = (\varphi_c + \varphi_j)/2$. Explicitly, we have from (25) that

$$t_r = -\gamma_1 \frac{d}{3} \left(\frac{dF}{d\varphi}\right)^{-1}_{\varphi=(\varphi_c+\varphi_j)/2}. \tag{26}$$

We now discuss the solutions for the RP and B-potentials separately.

5.2. Rapini–Papoular potential

In the case of the RP potential, the pitch jump only occurs when $0 < S_d < 1$ (sufficiently strong anchoring). The explicit form of the integral appearing in solution (25) can be obtained via Eqs. (2) and (7), with the result that the solution can be written as

$$t = -S_d \frac{2\gamma_1 d^2}{3K_{22}} \int\limits_{\varphi_c}^{\varphi} \Bigg[\sin(2\widehat{\varphi}) + \\ + 2S_d \left(\widehat{\varphi} + \pi - \varphi_c - \frac{\sin(2\varphi_c)}{2S_d}\right)\Bigg]^{-1} d\widehat{\varphi}, \tag{27}$$

where the critical angle φ_c is given by Eq. (11). The maximum possible upper limit of integration is φ_j, which is determined from Eq. (13) for a given value of S_d.

In the simple special case where $S_d = 1/2$, $\varphi_c = \pi/3$ and the upper limit φ_j is determined from the equation

$$\sin(2\varphi_j) + \varphi_j + \frac{\pi}{3} = 0, \tag{28}$$

obtained from Eq. (13).

5.3. The B-potential

For the B-potential, the pitch jump occurs for $0 < S_d < \infty$ (i.e., at any strength of anchoring) and the explicit expression for the implicit solution is given via Eqs. (2), (8), and (25). The solution is given by

$$t = -S_d \frac{2\gamma_1 d^2}{3K_{22}} \times \\ \times \int\limits_{-\pi/2}^{\varphi} \left[\sin\widehat{\varphi} - 1 + 2S_d \left(\widehat{\varphi} + \frac{\pi}{2}\right)\right]^{-1} d\widehat{\varphi}, \tag{29}$$

where we recall that φ_c is always $-\pi/2$ for the B-potential. The upper limit of integration φ_j is determined from Eq. (17) for any given value of S_d. We note that in this result, the form of the integrand and integration limits are given for the director configuration

after the jump, that is, when N has changed by 1, and, consequently, $\varphi_0(T_c)$ is replaced by $\varphi_0(T_c) - \pi$.

As a simple example, we consider the case where $S_d = 1/\pi$. The implicit solution (29) for φ is then given by

$$t = -\frac{1}{\pi}\frac{2\gamma_1 d^2}{3K_{22}} \int\limits_{-\pi/2}^{\varphi} \left[\sin\widehat{\varphi} + 2\frac{\widehat{\varphi}}{\pi}\right]^{-1} d\widehat{\varphi}\,, \qquad (30)$$

where the upper integration limit in the right-hand side is formally zero, because, φ_j must be zero at $S_d = 1/\pi$ by relation (17). We note that the integral in (30) diverges logarithmically as φ approaches $\varphi_j = 0$, which indicates formally that the time taken to complete the jump is infinite. Nevertheless, there are physical mechanisms that ensure a cut-off to this limit such that this divergence does not occur in reality. For example, the cut-off may be due to thermal fluctuations within the cholesteric layer and a nonzero upper limit may then be determined by the temperature.

6. GENERAL RESULTS

We now use solutions (27) and (29) for the RP and B-potentials, respectively. To simplify the results and give qualitative plots for data, we introduce the timescale [6, p. 226]

$$\overline{t} = t\frac{\pi^2 K_{22}}{\gamma_1 d^2}\,, \qquad (31)$$

which is a typical kind of scaling that occurs in liquid crystal problems [16, 19].

6.1. Rapini–Papoular potential

Solution (27) for the RP potential is given by

$$\overline{t} = -S_d\,\pi^2\frac{2}{3}\int\limits_{\varphi_c}^{\varphi}\Bigg[\sin(2\widehat{\varphi}) + 2S_d\left(\widehat{\varphi} + \pi - \varphi_c - \frac{\sin(2\varphi_c)}{2S_d}\right)\Bigg]^{-1} d\widehat{\varphi}, \qquad (32)$$

where, by (11), $\varphi_c = \arccos(-S_d)/2$ and the relaxation time is given, via (26), by

$$\overline{t}_r = -S_d\,\pi^2\frac{2}{3}\Bigg[\sin(2\varphi) + 2S_d\left(\varphi + \pi - \varphi_c - \frac{\sin(2\varphi_c)}{2S_d}\right)\Bigg]^{-1}_{\varphi=\frac{\varphi_c-\pi+\varphi_j}{2}}, \qquad (33)$$

with φ_j obtained from Eq. (13). Figure 3 shows the dependence of φ_j on S_d. It is also possible to define the switching time. We define the switching time $\overline{t}_s$ as the time taken for the orientation angle φ to change by a half of the jump angle width, i.e., to change from φ_c to $(\varphi_c - \pi + \varphi_j)/2$. It is given by

$$\overline{t}_s = -S_d\,\pi^2\frac{2}{3}\int\limits_{\varphi_c}^{\frac{\varphi_c-\pi+\varphi_j}{2}}\Bigg[\sin(2\widehat{\varphi}) + 2S_d\left(\widehat{\varphi} + \pi - \varphi_c - \frac{\sin(2\varphi_c)}{2S_d}\right)\Bigg]^{-1} d\widehat{\varphi}. \qquad (34)$$

The solution φ, the relaxation time $\overline{t}_r$, and dependence of the switching time $\overline{t}_s$ on S_d for the RP potential are shown in Figs. 7, 8, and 9.

6.2. The B-potential

Solution (29) for the B-potential is given by

$$\overline{t} = -S_d\,\pi^2\frac{2}{3}\int\limits_{-\frac{\pi}{2}}^{\varphi}\left[\sin\widehat{\varphi} - 1 + 2S_d\left(\widehat{\varphi} + \frac{\pi}{2}\right)\right]^{-1} d\widehat{\varphi}, \qquad (35)$$

and the relaxation time is

$$\overline{t}_r = -S_d\pi^2\frac{2}{3}\left[\sin\varphi - 1 + 2S_d\left(\varphi + \frac{\pi}{2}\right)\right]^{-1}_{\varphi=\frac{\varphi_j-\pi/2}{2}}, \qquad (36)$$

where φ_j is calculated from Eq. (17). It is also possible to define a switching time analogous to that for the RP potential by the relation

$$\overline{t}_s = -S_d\pi^2\frac{2}{3}\times \int\limits_{-\frac{\pi}{2}}^{\frac{\varphi_j-\pi/2}{2}}\left[\sin\widehat{\varphi} - 1 + 2S_d\left(\widehat{\varphi} + \frac{\pi}{2}\right)\right]^{-1} d\widehat{\varphi}. \qquad (37)$$

The solution φ and the relaxation time $\overline{t}_r$ for the B-potential are shown in Figs. 10 and 11, while the dependence of the switching time $\overline{t}_s$ on S_d is shown in Fig. 12.

We note that at the initial stage of the jump, the surface viscosity (see, e.g., the introduction of the surface viscosity discussed in [13]) may restrict the velocity of director rotation at the layer surface. It may be taken into account by adding the surface viscosity term $\gamma_s(d\varphi/dt)^2$ to Eq. (21). The corresponding additional term may play a role in very thin layers and may in principle be detected by experiment.

7. CONCLUSION

The results obtained above reveal the qualitatively important physical properties of jump dynamics. For instance, such dynamics have a direct dependence on the strength and shape of the anchoring potential. Although the specific calculations of the jump dynamics were performed under the simplifying assumptions mentioned above, there is no doubt that the qualitative features of jump dynamics remain valid in general for the phenomenon as a whole. It is therefore interesting to discuss under which circumstances the solutions described above are quantitatively valid, and what modifications to these solutions would be required under other conditions in order to obtain a further quantitative description of pitch jump dynamics.

We consider the quasi-static approximation to the dynamics of the pitch jump. It may work quantitatively if the time of propagation of the disturbance between the surfaces of the layer is smaller than the characteristic time of the jump. Estimating the velocity of perturbation propagation in the cholesteric as $v_p = K_{22}/\gamma_1 p$, where p is the cholesteric pitch, we find that the perturbation propagation time is p/d, in the de Gennes units of time used in the calculations (cf. Eq. (31)), and is therefore small if p/d is small. Examination of Figs. 7–12 allows us to determine in which range of the parameter S_d and for which potential the accepted approximations are valid. In any case, it is clear that in general, the jump time (or the corresponding relaxation time) is shorter for the B-potential compared to the RP potential, and therefore, as the values of S_d decrease, the approximation becomes invalid for the B-potential at larger values of S_d than for the RP potential. If we assume that $p/d \leq 0.1$, then the calculations presented above show that the obtained results are of quantitative meaning for $S_d > 0.1$.

As a first step in overcoming the quasi-static approximation made here, we may regard solutions of the equations in the previous section with a time-dependent space scale and limit the integration in Eq. (19) to the time t by the distance relation $d(t) = v_p t$ if $d(t) < d$. Within this approach, a quantitative description of the pitch jump dynamics may be obtained for values of the parameter S_d not limited by the condition $S_d > 0.1$ above. However, one has to bear in mind that other mechanisms of the pitch jumps may be at work for smaller values of S_d. This is connected with the fact that for values of S_d smaller than its critical value $1/2\pi$ [1], along with the jumps corresponding to a change in the number N of half-turns at the layer thickness by one (which correspond to the transitions of the director configuration not currently in the ground state), jumps greater than the increment of one to the value of N are possible. The calculated results for $S_d < 1/2\pi$ therefore require a special discussion to determine the range of applicability to the pitch jump dynamics in the framework of the mechanism accepted here.

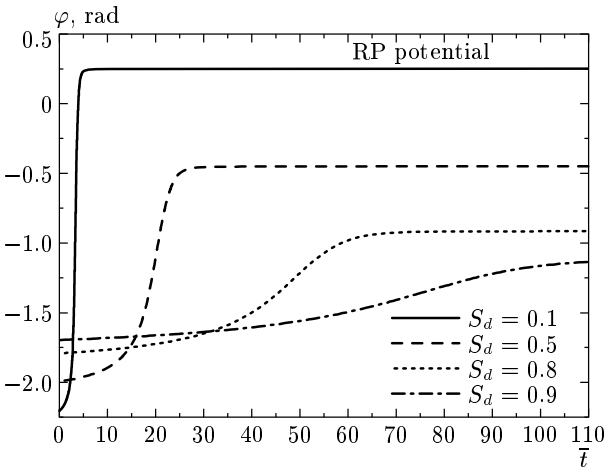

Fig. 7. Temporal behavior of the director orientation angle φ at the surface during a jump for the RP potential at the indicated values of S_d

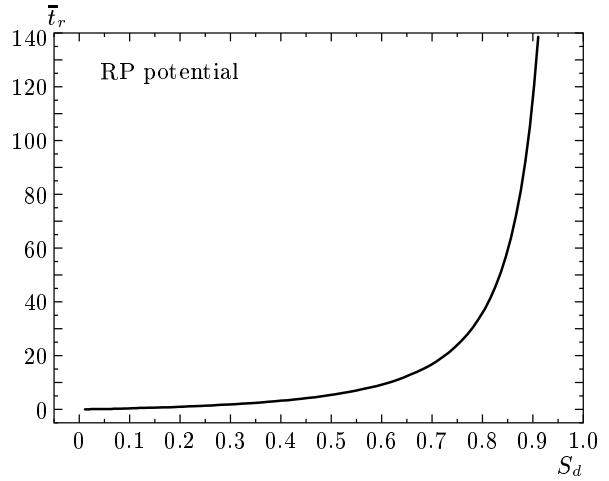

Fig. 8. The dependence of the relaxation time $\bar{t}_r$ defined by (33) on S_d for the RP potential

The comparison of Figs. 7–12 (see also Fig. 3 and Fig. 5) shows that experimental measurements allow one to obtain a qualitative conclusion about the applicability of the RP potential to describing jumps (at $S_d > 1$, the jumps are absent at all for RP potential). It is commonly accepted that for small angular deviations

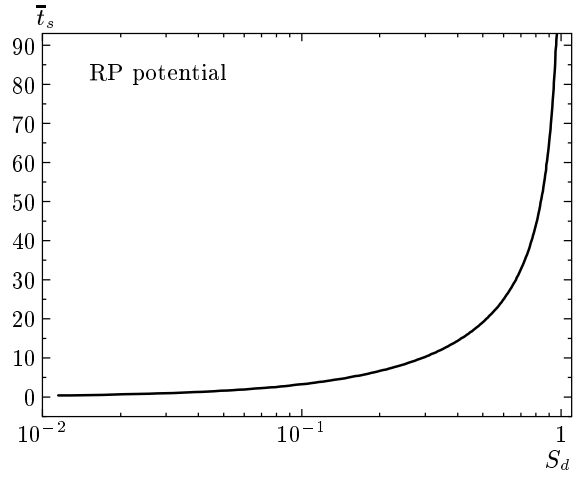

Fig. 9. The dependence of the switching time $\overline{t}_s$ defined by (34) on S_d calculated for the RP potential

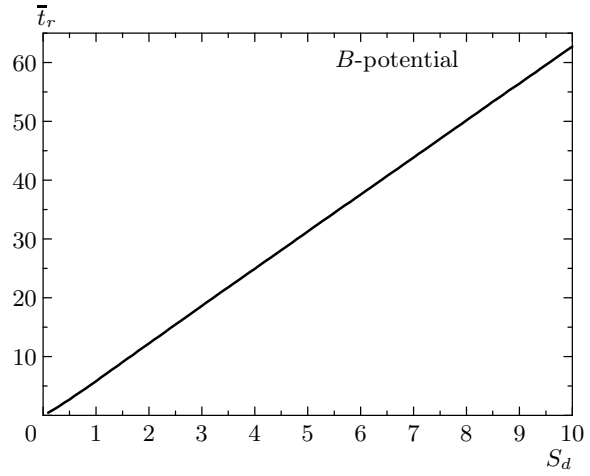

Fig. 11. The dependence of the relaxation time $\overline{t}_r$ defined by (33) on S_d for the B-potential

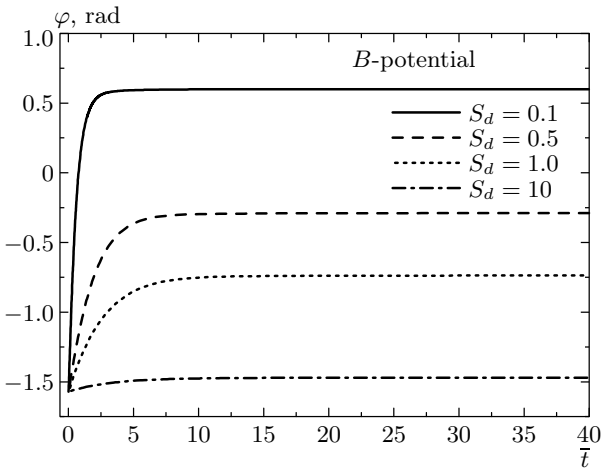

Fig. 10. Temporal behavior of the director orientation angle φ at the surface during a jump for the B-potential at the indicated values of S_d

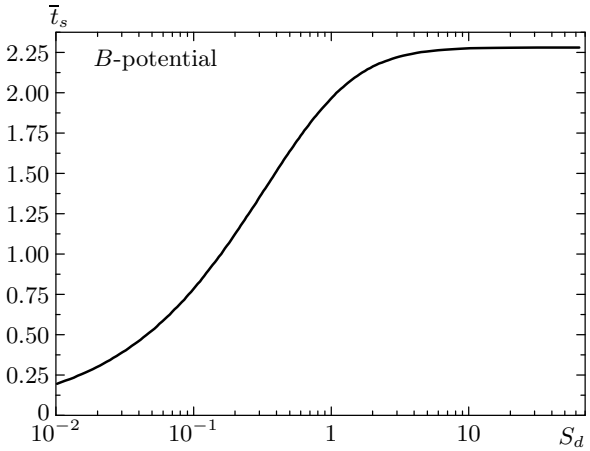

Fig. 12. The dependence of the switching time $\overline{t}_s$ defined by (37) on S_d calculated for the B-potential

of the director from the alignment direction, the surface anchoring potential is quadratic in the deviation angle, as is the case with the RP potential. But for large deviation angles (which are essential for the occurrence of pitch jumps), the question about the shape of the associated anchoring potential remains open. Therefore, the results presented above show that experimental investigations of the pitch jump dynamics give a unique opportunity to study the actual shape of the anchoring potential at large angular deviations of the director from the alignment direction. Other approaches employed so far in this area (see, e.g., [20, 21]) mainly enable one to determine the anchoring strength that characterizes the anchoring energy at small deviations from the alignment direction.

Concerning the experimental observation of the dynamics of pitch jumps, it should be kept in mind that without special precautions, it is quite improbable that the pitch jump in a cell occurs in the whole cell simultaneously, and that it most probably occurs in limited areas of its surface, whereas the formulas presented above assume that the jump process occurs throughout the whole cell simultaneously. To ensure that the process occurs in the whole cell, it is possible to perform an experiment stabilizing the director configuration in the layer by applying a rather weak external field and turning the field off just as the director at the surface achieves the critical angle. This approach seems to be

very similar to the one applied for studying the dynamics of the Frederiks transition [6, 22]. The same effect of a jump in a whole cell may be achieved by mechanical rotation of the layer surface by a small angle in the director configuration state of the layer close to the jump.

When the homogeneity of the jump over the surface of the cell is ensured, the dynamics of the jump may be followed by the conventional approach of measuring the time-dependent transmission [8], reflection spectra [23] or rotation of the plane of light polarization [10].

The authors are grateful for the UK EPSRC Grant GR/S34311/01 that enabled this work to be undertaken. Partial funding of this work by the RFBR (grant № 03-02-16173) is greatly appreciated by V. A. B.

REFERENCES

1. V. A. Belyakov and E. I. Kats, Zh. Eksp. Teor. Fiz. **118**, 560 (2000).
2. V. A. Belyakov, P. Oswald, and E. I. Kats, Zh. Eksp. Teor. Fiz. **123**, 1040 (2003).
3. V. A. Belyakov and E. I. Kats, Zh. Eksp. Teor. Fiz. **120**, 430 (2001).
4. V. A. Belyakov, Pis'ma v Zh. Eksp. Teor. Fiz. **76**, 99 (2002).
5. G. Chilaya, G. Hauck, H. D. Koswig, G. Petriashvili, and D. Sikharulidze, Crystal Res. Technol. **32**, 401 (1997).
6. P. G. de Gennes and J. Prost, *The Physics of Liquid Crystals*, Clarendon Press, Oxford (1993).
7. P. Oswald and P. Pieranski, *Les cristaux liquides: concepts et propriétés physiques illustrées par des expériences*, Gordon and Breach Science Publishers, Paris (2000).
8. H. Zink and V. A. Belyakov, Mol. Cryst. Liq. Cryst. **265**, 445 (1995); Pis'ma v Zh. Eksp. Teor. Fiz. **63**, 37 (1996).
9. S. P. Palto, Zh. Eksp. Teor. Fiz. **121**, 308 (2002).
10. W. Kuczynski (private communication).
11. R. Barberi and G. Durand, Appl. Phys. Lett. **58**, 2907 (1991).
12. R. Barberi, M. Giocondo, and G. Durand, Appl. Phys. Lett. **60**, 1085 (1992).
13. P. J. Kedney and F. M. Leslie, Liq. Cryst. **24**, 613 (1998).
14. L. M. Blinov, E. I. Kats, and A. A. Sonin, Uspekhi Fiz. Nauk **152**, 449 (1987).
15. L. D. Landau and L. P. Pitaevskii, *Physical Kinetics, Course of Theoretical Physics*, Vol. 10, Pergamon, London (1981).
16. G. Vertogen and W. H. de Jeu, *Thermotropic Liquid Crystals, Fundamentals*, Springer-Verlag, Berlin (1988).
17. L. D. Landau and E. M. Lifshitz, *Fluid Mechanics, Course of Theoretical Physics*, Vol. 6, Pergamon, London (1959).
18. F. M. Leslie, Adv. Liq. Cryst. **4**, 1 (1979).
19. I. W. Stewart, *The Static and Dynamic Continuum Theory of Liquid Crystals*, Taylor and Francis, London, in press.
20. B. Jerome, Rep. Progr. Phys. **54**, 391 (1991).
21. H. Yokoyama, Mol. Cryst. Liq. Cryst. **165**, 265 (1988).
22. F. Brochard, P. Pieranski, and E. Guyon, Phys. Rev. Lett. **28**, 1681 (1972).
23. H. F. Gleeson (private communication).