ЖЭТФ, 2003, том 124, вып. 1 (7), стр. 5–13

MULTIDIMENSIONAL GLOBAL MONOPOLE AND NONSINGULAR COSMOLOGY

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Submitted 29 January 2003

We consider a spherically symmetric global monopole in general relativity in $(D = d + 2)$-dimensional space–time. For $\gamma < d - 1$, where γ is a parameter characterizing the gravitational field strength, the monopole is shown to be asymptotically flat up to a solid angle defect. In the range $d - 1 < \gamma < 2d(d + 1)/(d + 2)$, the monopole space–time contains a cosmological horizon. Outside the horizon, the metric corresponds to a cosmological model of the Kantowski–Sachs type, where spatial sections have the topology $\mathbb{R} \times \mathbb{S}^d$. In the important case where the horizon is far from the monopole core, the temporal evolution of the Kantowski–Sachs metric is described analytically. The Kantowski–Sachs space–time contains a subspace with a $(d + 1)$-dimensional Friedmann–Robertson–Walker metric, whose possible cosmological application is discussed. Some estimates in the $d = 3$ case show that this class of nonsingular cosmologies can be viable. In particular, the symmetry-breaking potential at late times can give rise to both dark matter and dark energy. Other results, generalizing those known in the 4-dimensional space–time, are derived, in particular, the existence of a large class of singular solutions with multiple zeros of the Higgs field magnitude.

PACS: 04.90.+e

1. INTRODUCTION

In our recent paper with Podolyak [1], we considered the general properties of global monopole solutions in general relativity and developed some earlier results (see [2, 3] and references therein). It was confirmed, in particular, that the properties of these objects are governed by a single parameter γ, squared energy of spontaneous symmetry breaking in Planck units. For $0 < \gamma < 1$, solutions with the entirely positive (or entirely negative) Higgs field are globally regular and asymptotically flat up to a solid angle deficit. In the range $1 < \gamma < 3$, the space–time of the solutions remains globally regular but contains a cosmological horizon at a finite distance from the center. Outside the horizon, the geometry corresponds to homogeneous anisotropic cosmological models of the Kantowski–Sachs type, whose spatial sections have the topology $\mathbb{R} \times \mathbb{S}^2$. The nonzero symmetry-breaking potential can be interpreted as a time-dependent cosmological constant, a kind of hidden vacuum matter. The potential tends to zero at late times, and the «hidden vacuum matter» disappears. This solution with a nonsingular static core and a cosmological metric outside the horizon drastically differs from the standard Big Bang models and conforms to the ideas advocated by Gliner and Dymnikova [4] that the standard Big Bang singularity could be replaced by a regular vacuum bounce.

The lack of isotropization at late times did not allow us to directly apply the toy model of a global monopole

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to the early phase of our Universe. But this circumstance does not seem to be a fatal shortcoming of the model because the anisotropy of the very early Universe could be damped by particle creation later, and the further stages with low energy densities might conform to the standard isotropic Friedmann cosmology. Another idea is to add a comparatively small positive quantity Λ to the symmetry-breaking potential (to «slightly raise the Mexican hat»). It can change nothing but the late-time asymptotic regime, which then becomes de Sitter, corresponding to the added cosmological constant Λ. These ideas deserve a further study.

In this paper, we study the gravitational properties of global monopoles in multidimensional general relativity. This analysis can be of interest in view of numerous attempts to construct a unified theory using the ideas of supersymmetry in higher dimensions. Objects like multidimensional monopoles, strings, and other topological defects might form due to phase transitions in the early Universe at possible stages when the present three spatial dimensions were not yet separated from others, and a greater number of dimensions were equally important.

More specifically, we consider a self-gravitating hedgehog-type configuration of a multiplet of scalar fields with the Mexican-hat potential

$$V = (\lambda/4)(\phi^2 - \eta^2)^2$$

in a D-dimensional space–time with the structure $\mathbb{R}_t \times \mathbb{R}_\rho \times \mathbb{S}^d$ ($d = D - 2$), where $\mathbb{R}_\rho$ is the range of the radial coordinate ρ and $\mathbb{R}_t$ is the time axis. The properties of such objects generalize the results obtained in Ref. [1] and earlier papers (e.g., [2, 3]) in a natural way. Thus, for small values of the parameter $\gamma = \kappa^2\eta^2$ characterizing the gravitational field strength, the solutions are asymptotically flat up to a solid angle deficit. Within a certain range $d - 1 < \gamma < \overline{\gamma}(d)$, the solutions are nonsingular but contain a Killing horizon and a cosmological metric of the Kantowski–Sachs type outside it. In the important case where the horizon is far from the monopole core, the temporal evolution of the Kantowski–Sachs metric is described analytically. The upper bound $\overline{\gamma}(d)$, beyond which there are no static solutions with a regular center, is also found analytically.

The above description applies to solutions with an entirely positive (or entirely negative) scalar field magnitude ϕ. As in [1], we here also find a class of solutions with any number n of zeros of $\phi(r)$, existing for $\gamma < \gamma_n(d)$, where the upper bounds γ_n are found analytically. All solutions with $n > 0$ describe space–times with a regular center, a horizon, and a singularity beyond this horizon.

We also discuss a possible cosmological application of multidimensional global monopoles, which can be of particular interest for a 5-dimensional space–time with 3-dimensional spheres $\mathbb{S}^d$. In this case, the Kantowski–Sachs type model has the spatial topology $\mathbb{R} \times \mathbb{S}^3$ outside the horizon. It is anisotropic in 4-dimensions, but the 3-dimensional spheres $\mathbb{S}^3$ are isotropic. The anisotropy is thus related only to the fourth coordinate t, which is spatial outside the horizon and is a cyclic variable from the dynamical viewpoint. If we identify $\mathbb{S}^3$ with the observed space, ignoring the extra coordinate, we obtain a closed cosmological model, with the Friedmann–Robertson–Walker line element in the ordinary $3+1$-dimensional space–time.

A natural question arises: why is the fourth spatial dimension unobservable today? The answer cannot be found within our macroscopic theory without specifying the physical nature of the vacuum. The conventional Kaluza–Klein compactification of the extra dimension on a small circle is not satisfactory in our case because it leads to a singularity at the horizon (as demonstrated in Sec. 3). We therefore leave this question open and note that the global monopole model has a chance to describe only the earliest phase of the cosmological evolution. Its later stages should involve creation of matter and a sequence of phase transitions possibly resulting in localization of particles across the t direction. We then obtain a model with a large but unobservable extra dimension, similar in spirit to the widely discussed brane world models, see reviews [5–7] and references therein.

The solutions of interest appear when the symmetry breaking scale η is sufficiently large, and one can suspect that quantum gravity effects are already important at this energy scale. We show in Sec. 2.3 that this is not the case if the monopole core radius is much greater than the Planck length: the curvature and energy scales in the whole space are then much smaller than their Planckian values.

The existence of nonsingular models of the early Universe on the basis of classical gravity supports the opinion that our Universe had never undergone a stage described by full quantum gravity. In addition to those discussed here, such models are rather numerous now ([1, 4, 8–10], see also references therein). All of them are evidently free of the long-standing problems of the standard Big Bang cosmology related to the existence of multiple causally disconnected regions [11, 12].

This paper is organized as follows. In Sec. 2, we analyze the properties of a global monopole in $D = d + 2$ dimensions (one time coordinate and $d + 1$ spatial coordinates). It is a generalization of our previous results [1]. In Sec. 3, the particular case where $d = 3$ is studied in more detail along with its possible cosmological application. Unless otherwise indicated, we use the natural units $\hbar = c = 1$.

2. MULTIDIMENSIONAL GLOBAL MONOPOLE

2.1. General characteristics

The most general form of a static, spherically symmetric metric in $D = d + 2$ dimensions is

$$ds^2 = e^{2F_0} dt^2 - e^{2F_1} d\rho^2 - e^{2F_\Omega} d\Omega^2, \tag{1}$$

where $d\Omega^2 = d\Omega_d^2$ is a linear element on a d-dimensional unit sphere parameterized by the angles $\varphi_1, \ldots, \varphi_d$,

$$d\Omega_d^2 = d\varphi_d^2 + \sin^2\varphi_d \left(d\varphi_{d-1}^2 + \sin^2\varphi_{d-1} \times \right.$$
$$\left. \times \left(d\varphi_{d-2}^2 + \ldots + \sin^2\varphi_3 \left(d\varphi_2^2 + \sin^2\varphi_2 d\varphi_1^2\right) \ldots\right)\right),$$

and F_0, F_1, and F_Ω are functions of the radial coordinate ρ that are not yet specified. The nonzero components of the Ricci tensor are (the prime denotes $d/d\rho$)

$$\begin{aligned} R_0^0 &= e^{-2F_1} \left[F_0'' + F_0' \left(F_0' + dF_\Omega' - F_1'\right)\right], \\ R_\rho^\rho &= e^{-2F_1} \left[dF_\Omega'' + F_0'' + dF_\Omega'^2 + F_0'^2 - \right. \\ &\quad \left. - F_1' \left(F_0' + dF_\Omega'\right)\right], \\ R_2^2 &= \ldots = R_{d+1}^{d+1} = -(d-1)\, e^{-2F_\Omega} + \\ &\quad + e^{-2F_1} \left[F_\Omega'' + F_\Omega' \left(F_0' + dF_\Omega' - F_1'\right)\right]. \end{aligned} \tag{2}$$

A global monopole with a nonzero topological charge can be constructed with a multiplet of real scalar fields ϕ^a $(a = 1, 2, \ldots, d+1)$ comprising a hedgehog configuration in $d+1$ spatial dimensions[1)],

$$\phi^a = \phi(\rho)\, n^a (\varphi_1, \ldots, \varphi_d),$$

where $n^a(\varphi_1, \ldots, \varphi_d)$ is a unit vector $(n^a n^a = 1)$ in the $d+1$-dimensional Euclidean target space, with the components

$$\begin{aligned} n^{d+1} &= \cos\varphi_d, \\ n^d &= \sin\varphi_d \cos\varphi_{d-1}, \\ n^{d-1} &= \sin\varphi_d \sin\varphi_{d-1} \cos\varphi_{d-2}, \\ &\ldots \\ n^{d-k} &= \sin\varphi_d \sin\varphi_{d-1} \ldots \sin\varphi_{d-k} \cos\varphi_{d-k-1}, \\ &\ldots \\ n^2 &= \sin\varphi_d \ldots \sin\varphi_2 \cos\varphi_1, \\ n^1 &= \sin\varphi_d \ldots \sin\varphi_2 \sin\varphi_1. \end{aligned}$$

The Lagrangian of a multidimensional global monopole in general relativity is given by

$$L = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - V(\phi) + \frac{R}{2\kappa^2},$$

where R is the scalar curvature, $\kappa = \kappa_D$ is the D-dimensional gravitational constant, and $V(\phi)$ is a symmetry-breaking potential depending on $\phi = \pm\sqrt{\phi^a \phi^a}$; it is natural to choose V as the Mexican-hat potential,

$$V = \frac{\lambda}{4}(\phi^2 - \eta^2)^2 = \frac{\lambda\eta^4}{4}(f^2 - 1)^2. \tag{3}$$

We have introduced the normalized field magnitude $f = \phi(\rho)/\eta$ playing the role of the order parameter. The model has a global $SO(d+1)$ symmetry, which can be spontaneously broken to $SO(d)$; $\eta^{2/d}$ is the energy of symmetry breaking.

The Einstein equations can be written as

$$R_\mu^\nu = -\kappa^2 \widetilde{T}_\mu^\nu = -\kappa^2 \left(T_\mu^\nu - \frac{1}{d} T \delta_\mu^\nu\right), \tag{4}$$

where T_μ^ν is the energy-momentum tensor. The nonzero components of $\widetilde{T}_\mu^\nu$ are

$$\begin{aligned} \widetilde{T}_0^0 &= -\frac{2}{d} V, \\ \widetilde{T}_\rho^\rho &= -e^{-2F_1} f'^2 - \frac{2}{d} V, \\ \widetilde{T}_2^2 &= \ldots = \widetilde{T}_{d+1}^{d+1} = -e^{-2F_\Omega} f^2 - \frac{2}{d} V. \end{aligned}$$

We now use the quasiglobal coordinate ρ specified by the condition

$$F_0 + F_1 = 0,$$

which is a convenient gauge for spherically symmetric systems with Killing horizons. Introducing the functions

$$A(\rho) = e^{2F_0} = e^{-2F_1}, \quad r(\rho) = e^{F_\Omega},$$

we reduce the metric to the form

$$ds^2 = A(\rho) dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho) d\Omega^2, \tag{5}$$

[1)] A $7D$ universe with a global monopole with a hedgehog configuration of scalar fields only in three extra dimensions was recently considered in [13]. Our approach is different. We consider a hedgehog configuration in all $D-1$ space dimensions of the D-dimensional space–time.

and obtain the equations

$$\left(Ar^d\phi'\right)' - dr^{d-2}\phi = r^d\frac{\partial V}{\partial\phi}, \tag{6}$$

$$r'' = -\frac{\kappa^2}{d}r\phi'^2, \tag{7}$$

$$\left(r^dA'\right)' = -\frac{4\kappa^2}{d}r^dV, \tag{8}$$

$$A\left(r^2\right)'' - r^2A'' - (d-2)\,r^3r'\left(\frac{A}{r^2}\right)' = \\ = 2(d-1-\kappa^2\phi^2) \tag{9}$$

for the unknown functions $\phi(\rho)$, $A(\rho)$, and $r(\rho)$. Only three of these four equations are independent: scalar field equation (6) follows from Einstein equations (7)–(9) because of the Bianchi identities.

Equations (6)–(8) have the same structure as Eqs. (13)–(15) in [1]. General properties of Eqs. (6)–(8) with an arbitrary value of d are the same as for $d = 2$, and the classification of their solutions is also the same. In particular, if $V(\phi) > 0$, the system with a regular center can have either no horizon or one simple horizon; in the latter case, its global structure is the same as that of the de Sitter space–time. Below, we focus our attention on solutions belonging to class (a1) according to [1], i.e., those with $r(\rho)$ monotonically growing from zero to infinity as $\rho \to \infty$ and $A(\rho)$ changing from $A = 1$ at the regular center to $A_\infty < 0$ as $\rho \to \infty$, and with a cosmological horizon (where $A = 0$) at some $\rho = \rho_h$.

Equation (9) is a second-order linear inhomogeneous differential equation for A. The corresponding homogeneous equation has the evident special solution

$$A(\rho) = \mathrm{const}\cdot r^2(\rho).$$

This allows expressing $A(\rho)$ in terms of $r(\rho)$ and $\phi(\rho)$ in an integral form,

$$A = C_1r^2 - C_2r^2\int\limits_\rho^\infty\frac{d\rho_1}{r^{d+2}(\rho_1)} + 2r^2\int\limits_\rho^\infty\frac{d\rho_1}{r^{d+2}(\rho_1)}\times \\ \times\int\limits_0^{\rho_1}d\rho_2r^{d-2}(\rho_2)\left[d-1-\kappa^2\phi^2(\rho_2)\right]. \tag{10}$$

We consider solutions with a large-r asymptotic behavior such that $r(\rho) \to \infty$ and $r'(\rho) \to \mathrm{const} > 0$ as $\rho \to \infty$. Equation (7) gives r' as $\int[r\phi'^2]d\rho$, and its convergence as $\rho \to \infty$ implies a sufficiently rapid decay of ϕ' at large ρ, and therefore $\phi \to \phi_\infty = \mathrm{const}$ as $\rho \to \infty$. The potential V then tends to a constant equal to $V(\phi_\infty)$. Furthermore, Eq. (8) shows that at large r, $A(\rho)$ can grow at most as r^2, and finally, substitution of the asymptotic form of $\phi(\rho)$, $A(\rho)$, and $r(\rho)$ in Eq. (6) leads to $dV/d\phi \to 0$ as $\rho \to \infty$. In application to field equations, the condition that there exists a large-r asymptotic regime implies that the scalar field then tends either to an extremum of the potential $V(\phi)$ or to an inflection point with zero derivative. For the Mexican-hat potential, it can be either the maximum at $\phi = 0$ (the trivial unstable solution for ϕ and the de Sitter metric with the cosmological constant $(1/4)\kappa^2\lambda\eta^4$) or a minimum of V where $f = 1$ and $V = 0$. For a «slightly raised Mexican hat» (potential (3) plus a small constant V_+), we have a de Sitter asymptotic behavior with $f = 1$ and $V = V_+$.

A regular center requires that $A = A_c + O(r^2)$ and $Ar'^2 \to 1$ as $\rho \to \rho_c$ such that $r(\rho_c) = 0$. Without loss of generality, we set $\rho_c = 0$ and $A_c = 1$.

For potential (3), regularity at $\rho = 0$ and the asymptotic condition at $\rho \to \infty$ lead to $C_1 = C_2 = 0$, and Eq. (10) then implies that

$$A(\rho) = 2r^2(\rho)\int\limits_\rho^\infty\frac{d\rho_1}{r^{d+2}(\rho_1)}\int\limits_0^{\rho_1}d\rho_2r^{d-2}(\rho_2)\times \\ \times\left[d-1-\kappa^2\phi^2(\rho_2)\right]. \tag{11}$$

Equation (8) provides another representation for $A(\rho)$ satisfying the regular center conditions,

$$A(\rho) = 1 - \frac{4\kappa^2}{d}\int\limits_0^\rho\frac{d\rho_1}{r^d(\rho_1)}\int\limits_0^{\rho_1}d\rho_2r^d(\rho_2)\,V(\rho_2). \tag{12}$$

From (11), we find the limiting value of A at $\rho \to \infty$,

$$A(\infty) = \frac{d-1-\gamma}{\alpha^2(d-1)}, \quad \gamma = \kappa^2\eta^2, \tag{13}$$

where $\alpha = dr/d\rho$ at $\rho \to \infty$,

$$\alpha = 1 - \frac{\kappa^2}{d}\int\limits_0^\infty r(\rho)\phi'^2(\rho)d\rho.$$

Equation (13) shows that $\gamma = d-1$ is a critical value of γ: the large-r asymptotic behavior can be static only if $\gamma \le d-1$; for $\gamma < d-1$, it is flat up to a solid angle deficit, in full similarity to the conventional case $d = 2$ [1, 2]. If $\gamma > d-1$, then $A(\infty) < 0$, and there is a horizon at some $\rho = \rho_h$ where $A = 0$. From (12),

$$\frac{4\kappa^2}{d}\int\limits_0^{\rho_h}\frac{d\rho_1}{r^d(\rho_1)}\int\limits_0^{\rho_1}d\rho_2r^d(\rho_2)\,V(\rho_2) = 1,$$

and we therefore have

$$A(\rho) = -\frac{4\kappa^2}{d} \int\limits_{\rho_h}^{\rho} \frac{d\rho_1}{r^d(\rho_1)} \int\limits_0^{\rho_1} d\rho_2 r^d(\rho_2) V(\rho_2). \quad (14)$$

The γ dependence of ρ_h, with $\gamma = \kappa^2\eta^2$, can be found from the relation

$$\frac{4\kappa^2}{d} \int\limits_{\rho_h}^{\infty} \frac{d\rho_1}{r^d(\rho_1)} \int\limits_0^{\rho_1} d\rho_2 r^d(\rho_2) V(\rho_2) = -\frac{d-1-\gamma}{\alpha^2(d-1)}. \quad (15)$$

2.2. Large-r asymptotic behavior

From (6), we can find the asymptotic behavior of the field $f(\rho)$ and the potential $V(\rho)$ as $r \to \infty$. At large ρ, we have $A \to A(\infty)$, see (13), and field equation (6) reduces to

$$\frac{1}{r^d}\frac{d}{dr}\left(r^d \frac{df}{dr}\right) - \frac{d-1}{\gamma - d + 1}\left[\lambda\eta^2\left(1 - f^2\right) - \frac{d}{r^2}\right] f = 0, \quad r \to \infty.$$

A regular solution of this equation must tend to unity as $r \to \infty$, and for $\psi = 1 - f$, we have the linear equation

$$\psi_{,rr} + \frac{d}{r}\psi_{,r} + \frac{2\lambda\eta^2(d-1)}{\gamma-d+1}\left(\psi - \frac{d}{2\lambda\eta^2 r^2}\right) = 0, \quad r \to \infty. \quad (16)$$

The general solution of the corresponding homogeneous equation

$$\psi_{0,rr} + \frac{d}{r}\psi_{0,r} + \frac{2\lambda\eta^2(d-1)}{\gamma - d + 1}\psi_0 = 0$$

can be expressed in terms of Bessel functions,

$$\psi_0(r) = r^{-(d-1)/2} \times \left[C_1 J_{-(d-1)/2}\left(\frac{r}{r_0}\right) + C_2 Y_{-(d-1)/2}\left(\frac{r}{r_0}\right)\right], \quad r_0^2 = \frac{\gamma - d + 1}{2\lambda\eta^2(d-1)}.$$

A special solution of inhomogeneous equation (16) at $r \to \infty$ is

$$\psi = \frac{d}{2\lambda\eta^2 r^2} + O\left(\frac{1}{r^4}\right).$$

The general solution of Eq. (16) gives the asymptotic behavior for the Higgs field magnitude f as $r \to \infty$,

$$f(r) = 1 - \frac{d}{2\lambda\eta^2 r^2} - \frac{C}{(\lambda\eta^2 r^2)^{d/4}} \times \sin\left(\frac{r}{r_0} + \frac{\pi d}{4} + \varphi\right), \quad r \to \infty. \quad (17)$$

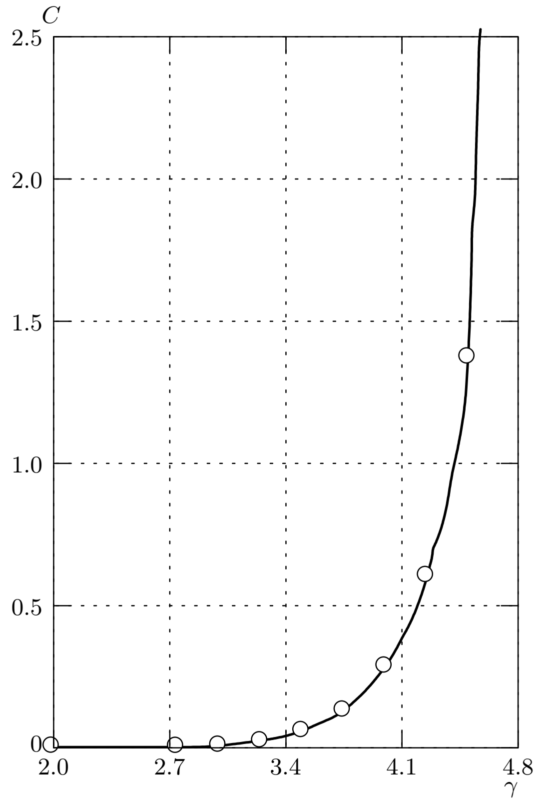

Fig. 1. The function $C(\gamma)$ found numerically for $d = 3$

Because of the boundary conditions imposed, the integration constants C and φ are functions of d and γ that can be found numerically. The function $C(\gamma)$ for $d = 3$ is presented in Fig. 1. From (17), we find the asymptotic behavior of V,

$$V(r) = \frac{\lambda\eta^4}{4}\left[\frac{d}{\lambda\eta^2 r^2} + \frac{2C}{(\lambda\eta^2 r^2)^{d/4}} \times \sin\left(\frac{r}{r_0} + \frac{\pi d}{4} + \varphi\right)\right]^2, \quad r \to \infty. \quad (18)$$

2.3. Bounds of the classical regime and the monopole core

Of certain interest are solutions with the cosmological large-r behavior, i.e., those with $\gamma > d - 1$. The latter condition means that the scalar field, approaching η at large r, actually takes near- or trans-Planckian values. Indeed, in D dimensions, the Planck length l_D and mass m_D are expressed in terms of the gravitational constant $\kappa = \kappa_D$ as

$$l_D = \kappa^{2/d}, \quad m_D = \kappa^{-2/d}, \quad d = D - 2.$$

Therefore,

$$\eta^2 = \frac{\gamma}{\kappa^2} = \gamma m_D^d,$$

and in the case of interest where $\gamma \sim d$, we have

$$\eta \sim (m_D)^{d/2}\sqrt{d}. \tag{19}$$

We can, however, remain at sub-Planckian curvature values, thus avoiding the necessity to invoke quantum gravity, if we require sub-Planckian values of the potential V in the entire space, i.e.,

$$\kappa^2 V = \frac{1}{4}\kappa^2\lambda\eta^4 \ll m_D^2.$$

For η given by (19), this implies that

$$\lambda \ll \frac{4}{d^2} m_D^{2-d}. \tag{20}$$

We can thus preserve the classical regime even with large η by choosing sufficiently small values of λ. In terms of lengths, this condition is equivalent to the requirement that the monopole core radius

$$r_{core} = \frac{1}{\sqrt{\lambda}\eta}$$

is much greater than the Planck length,

$$\frac{1}{\sqrt{\lambda}\eta} \gg l_D. \tag{21}$$

One may note that this condition is external with respect to the theory because general relativity does not contain an internal restriction on the gravitational field strength. Moreover, in ordinary units, our dimensionless gravitational field strength parameter, expressed as $\gamma = \kappa^2 c^{-4}\eta^2$, does not contain $\hbar$. We obtain restriction (20) or (21) only when we compare the characteristic length r_{core} existing in our theory with the Planck length $l_D = (\hbar\kappa^2/c^3)^{1/d}$.

We now discuss the solutions for γ slightly exceeding the critical value $d-1$. In the case where $\gamma - (d-1) \ll 1$, the horizon radius r_h is much greater than r_{core}, and the constant C turns out to be negligibly small (this is confirmed numerically, see Fig. 1). At large ρ_2, the integrand in the inner integrals in (12), (14), and (15) is then given by

$$d\rho_2 r^d(\rho_2)V(\rho_2) \approx \frac{d^2}{4\alpha\lambda}\frac{dr}{r^{4-d}}.$$

The main contribution to the above inner integrals comes from the monopole core if $d < 3$ and from the upper limit if $d > 3$. For $d = 3$, it is a logarithmic integral. As a result, we have different behaviors of $\rho_h(\gamma)$ at $\gamma - (d-1) \ll 1$ for $d = 2$ and $d \geq 3$[2)].

For $d = 2$ (4-dimensional general relativity),

$$\int_0^{\rho_1} d\rho_2 r^d(\rho_2)V(\rho_2) \approx \int_0^{\infty} d\rho_2 r^2(\rho_2)V(\rho_2) = \text{const},$$

and it follows from (15), in agreement with [1], that the horizon radius r_h is inversely proportional to $\gamma - 1$,

$$r_h = \frac{\text{const}}{\gamma - 1}, \quad \gamma - 1 \ll 1, \quad d = 2.$$

For $d > 3$, we find that at $\gamma - (d-1) \ll 1$, the horizon radius r_h is inversely proportional to the square root of $\gamma - (d-1) \ll 1$,

$$\begin{gathered} r_h = \sqrt{\frac{\gamma d(d-1)}{2(d-3)(\gamma - d + 1)}\frac{1}{\lambda\eta^2}}, \\ r_h^2 \gg \frac{1}{\lambda\eta^2}, \quad d > 3. \end{gathered} \tag{22}$$

It is thus confirmed that for $\gamma - (d-1) \ll 1$, the horizon is located far from the monopole core,

$$r_h^2 \gg \frac{1}{\lambda\eta^2}.$$

The function $A(r)$ at $r > r_h$ can then be found analytically. In this case, $r(\rho)$ is a linear function at $r > r_h$ and $dr = \alpha\, d\rho$. From (14) at $r > r_h$, we find

$$\begin{aligned} A(r) = &-\frac{\gamma + 1 - d}{\alpha^2(d-1)}\left(1 - \frac{r_h^{d-1}}{r^{d-1}}\right) + \\ &+ \frac{\gamma d}{2\alpha^2(d-3)\lambda\eta^2 r^2}\left[1 - \left(\frac{r_h}{r}\right)^{d-3}\right]. \end{aligned} \tag{23}$$

The condition of the applicability of (23) is $l_D \ll r_h$. In view of $r_{core} \ll r_h$, it is less restrictive than condition (21).

2.4. Solutions with $f(\phi)$ changing its sign

As in Ref. [1], numerical integration of the field equations shows that in addition to solutions with totally positive (or totally negative) $f(u)$, there also exist solutions with a regular center such that $f(u)$ changes its sign $n \geq 1$ times. All these solutions exist for $\gamma < \gamma_n(d)$, where $\gamma_n(d)$ are some critical values of the parameter γ. For $n > 0$, all of them have a horizon,

[2)] This is the only important qualitative difference between the general case $d \geq 3$ and the particular case $d = 2$ considered in [1].

and the absolute value of f at the horizon $|f_{h,n}(\rho_h)|$ is a decreasing function of γ, vanishing as $\gamma \to \gamma_n - 0$. Moreover, as $\gamma \to \gamma_n(d)$, the function $f(u)$ vanishes in the whole range $\rho \le \rho_h$ and is small inside the horizon for γ close to $\gamma_n(d)$. This allows us to find the critical values $\gamma_n(d)$ analytically: Eq. (6) reduces to a linear equation for f in a given (de Sitter) background, and combined with the boundary conditions $f(0) = 0$ and $f(\rho_h) < \infty$, leads to a linear eigenvalue problem. Its solution (see [1] for the details) in the d-dimensional case gives the upper limits $\gamma_n(d)$ and the corresponding minimal horizon radii $r_h = r_{hn}$ for solutions with the Higgs field magnitude f changing its sign n times,

$$r_{hn} = \sqrt{(2n+1)(2n+d+2)/\lambda\eta^2}, \tag{24}$$

$$\gamma_n = \frac{2d(d+1)}{(2n+1)(2n+d+2)}. \tag{25}$$

For $d = 2$, Eqs. (24) and (25) reduce to Eq. (52) in Ref. [1]. Under condition (21), these solutions remain in the classical gravity regime.

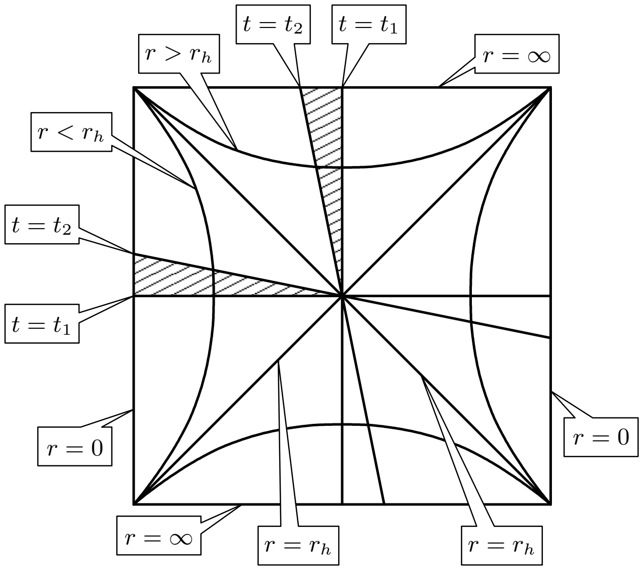

Fig. 2. The Carter–Penrose diagram of a global monopole with a cosmological horizon. The diagonals of the square represent the horizons. After identification of t_1 and t_2, only the dashed regions survive

3. 5-DIMENSIONAL MODELS AND NONSINGULAR COSMOLOGY

3.1. The extra dimension

At present, there is no evidence for the existence of more than three spatial dimensions up to the achievable energies about several hundred GeV. But this energy is quite small on the Planck scale (of the order 10^{19} GeV). Our solutions of a possible cosmological interest correspond to $\gamma > d - 1$, i.e., the Planck energy scale. Even under condition (21), there remains an enormous range of scales in the early Universe in which the number of equally important spatial dimensions can be greater than 3.

If we try to consider our $d = 3$ solutions in the cosmological context, the extra coordinate is t in (1) and (5). The coordinate t is time inside the horizon and becomes a fourth spatial coordinate outside it, where $A(\rho) < 0$. Metric (5) takes the form

$$ds^2 = \frac{d\rho^2}{|A(\rho)|} - |A(\rho)|dt^2 - r^2(\rho)d\Omega_3^2.$$

Introducing the proper time τ of a comoving observer outside the horizon,

$$\tau = \int\limits_{\rho_h}^{\rho} \frac{d\rho}{\sqrt{|A(\rho)|}}, \tag{26}$$

we obtain a 5-dimensional Kantowski–Sachs cosmology with a closed Friedmann–Robertson–Walker metric in the 3+1-dimensional space–time section of a constant t,

$$ds_4^2 = d\tau^2 - a^2(\tau)d\Omega_3^2 - |A(\rho(\tau))|dt^2. \tag{27}$$

The 4-dimensional spherical radius $r(\rho)$ now plays the role of the scale factor, $a(\tau) = r(\rho(\tau))$.

It is tempting to explain the unobservability of the extra dimension parameterized by the coordinate t by compactifying t with a certain «period» T in the spirit of Kaluza–Klein models. Such a compactification would lead to a singularity at $r = r_h$, however, as is clear from Fig. 2. If $t \in \mathbb{R}$, the static region (the left quadrant in the diagram) is connected with the future cosmological region (the upper quadrant) by the horizon, crossed by photons and massive particles without problems. But if the t axis is made compact by identifying, e.g., the points t_1 and t_2 on the t axis, the static and cosmological regions in the diagram take the form of the dashed sectors, actually tubes of a variable thickness, connected at one point only, the ends (tips) of the tubes. The curvature invariants do not change due to this identification and remain finite, and the emerging singularity in the ρt plane resembles the conical singularity.

Compactification is not the only possibility of explaining why the t coordinate is invisible. It can also be assumed that at some instant of the proper cosmological time τ of 5-dimensional model (27), a phase transition occurs at a certain energy scale $1/T$, leading to localization of matter on the 3-spheres in the spirit of brane world models. Anyway, within our macroscopic theory without specifying the structure of the physical vacuum, it is impossible to explain why the extra dimension is not seen now. It is nevertheless interesting to describe some cosmological characteristics of the $d = 3$ global monopole.

3.2. Some cosmological estimates

For $d = 3$, the inner integrals in (14) and (15) have a logarithmic character, and instead of (22) and (23), we obtain

$$\gamma - 2 = \frac{3}{\lambda\eta^2 r_h^2}\left[B + \ln\left(\lambda\eta^2 r_h^2\right)\right], \quad r_h^2 \gg \frac{1}{\lambda\eta^2}, \quad d = 3 \tag{28}$$

and

$$A(a) = -\frac{\gamma - 2}{2\alpha^2}\left(1 - \frac{r_h^2}{a^2}\right) + \frac{3\gamma}{2\alpha^2\lambda\eta^2}\frac{\ln(a/r_h)}{a^2}, \quad a > r_h, \quad d = 3. \tag{29}$$

The dependence $a(\tau)$ can be found from Eq. (26). In (28), B is a constant close to unity; our numerical estimate gives $B \approx 0.75$. The dimensionless radius of the horizon $\sqrt{\lambda}\eta r_h$ is presented in Fig. 3 as a function of γ for $d = 3$ (solid line). The dashed line is asymptotic dependence (28) valid for $\gamma - 2 \ll 1$. The function $A(\tau) \equiv A(a(\tau))$ is shown in Fig. 4 for $d = 3$ and $\gamma = 3, 3.5$, and 4. The numerical and analytic results are shown by solid and dashed lines, respectively. It is remarkable that only for $\gamma = 4$, the approximate analytic dependence (29), which is strictly speaking valid for $\gamma - 2 \ll 1$, is slightly different from the more precise dependence found numerically.

Far outside the horizon, $A(a)$ tends to a constant value,

$$A(a) \to -\frac{\gamma - 2}{2\alpha^2}, \quad a \gg r_h,$$

and metric (27) describes a uniformly expanding world with a linear dependence $a(\tau)$ at late times,

$$a(\tau) = \alpha\sqrt{|A(\infty)|}\tau = \sqrt{\frac{\gamma - 2}{2}}\tau, \quad \tau \to \infty. \tag{30}$$

The Hubble parameter $H = \dot{a}/a$, where the dot denotes $d/d\tau$, is found analytically from expression (29) for $A(a)$ ($d = 3$, $a > r_h \gg 1/\sqrt{\lambda}\eta$):

$$H(a) = \frac{1}{a}\sqrt{\frac{\gamma - 2}{2}\left(1 - \frac{r_h^2}{a^2}\right) - \frac{3\gamma}{2}\frac{\ln(a/r_h)}{\lambda\eta^2 a^2}}. \tag{31}$$

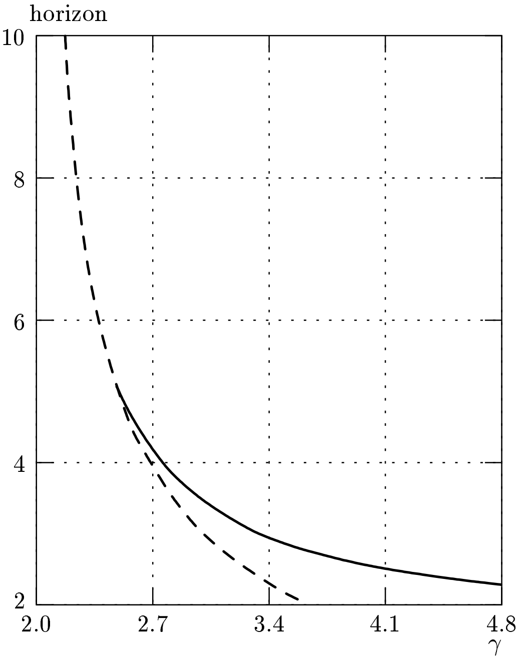

Fig. 3. The dimensionless horizon radius $\sqrt{\lambda}\eta r_h$ vs. γ for $d = 3$ (solid line). The dashed line is asymptotic dependence (28) valid for $\gamma - 2 \ll 1$

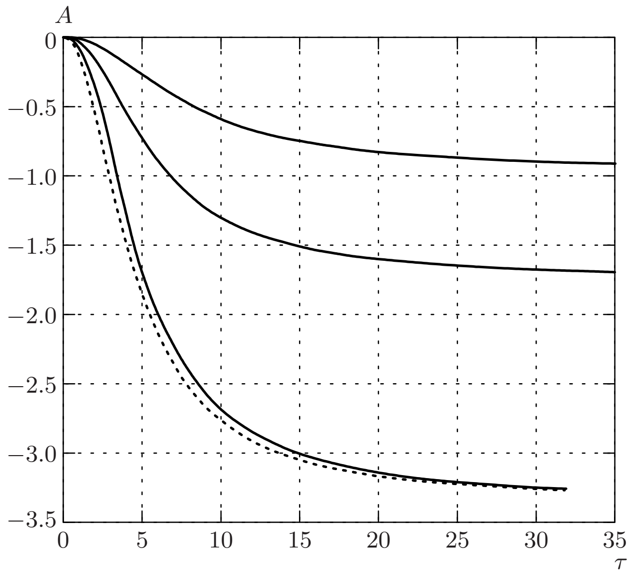

Fig. 4. The function $A(\tau) \equiv A(a(\tau))$ for $d = 3$ and $\gamma = 3$, 3.5, and 4 (from top down). Solid lines show numerical results and dashed lines show analytic dependence (29)

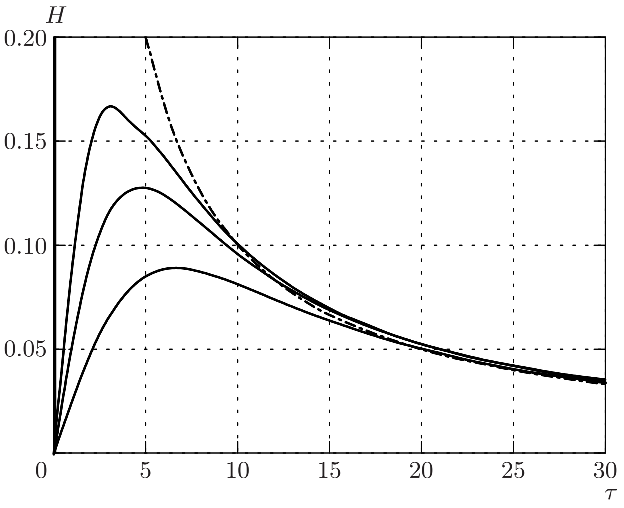

Fig. 5. The Hubble parameter $H(\tau)$ for $\gamma = 3$, 3.5, and 4 (from bottom up). At late times, $H(\tau) = 1/\tau$ (dashed curve)

The temporal evolution of the Hubble parameter $H(\tau)$ is shown in Fig. 5 for $\gamma = 3$, 3.5, and 4. The expansion starts from the horizon at $\tau = 0$ and rather quickly approaches the late-time behavior $H(\tau) = \tau^{-1}$. We actually have the asymptotic regime almost immediately after the beginning.

If we try to extrapolate this late-time regime to the present epoch, we can use the estimate given in Ref. [11] (Box 27.4), $\dot{a} \approx 0.66$; Eqs. (30) and (28) then lead to

$$\gamma = 2 + 2\dot{a}^2 = 2.87, \quad \sqrt{\lambda}\eta r_h \approx 3.65. \tag{32}$$

These estimates conform to the monopole parameter values leading to a nonsingular cosmology.

The symmetry-breaking potential (18), averaged over the oscillations, $V(\tau) \equiv \overline{V(a(\tau))}$, is a decreasing function of τ,

$$V(\tau) = \frac{9}{(\gamma-2)^2\lambda\tau^4} + \frac{\lambda\eta^4 C^2}{2[(\gamma/2-1)\lambda\eta^2\tau^2]^{3/2}}, \quad \tau \to \infty. \tag{33}$$

In cosmology, scalar field potentials are often interpreted as a time-dependent effective cosmological constant. The reason is that V enters the energy-momentum tensor as a Λ-term. In our case, as can be seen from (33), this term behaves as a mixture of two components, one decaying with the cosmological expansion as radiation ($\propto \tau^{-4} \propto a^{-4}$) and the other as matter without pressure ($\propto \tau^{-3} \propto a^{-3}$) in 4 dimensions. The four-dimensional energy density corresponding to V is proportional to $V\sqrt{|A|}$. But at late times, the extra-dimension scale factor $\sqrt{|A|}$ tends to a constant, and therefore the five- and four-dimensional behaviors of the energy density actually coincide at large τ. We can say that the potential $V(\phi)$ in the global monopole model gives rise to both dark radiation and dark matter. We recall that in accordance with modern views, both must necessarily be present in the Universe from the observational standpoint [12].

These estimates can only show that the 5-dimensional global monopole model is in principle able to give plausible cosmological parameters. Quantitative estimates certainly require a more complete model including further phase transitions, one of which should explain the unobservability of the fifth dimension.

The authors are grateful to A. F. Andreev, A. A. Starobinsky, V. A. Marchenko, and M. Yu. Kagan for useful discussions.

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