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# STOCHASTIC PROCESSES OF DEMARKOVIZATION AND MARKOVIZATION IN CHAOTIC SIGNALS OF THE HUMAN BRAIN ELECTRIC ACTIVITY FROM EEGs AT EPILEPSY

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We study the stochastic processes of markovization and demarkovization in chaotic signals of human electroencephalograms (EEGs) at epilepsy using various measures of demarkovization and markovization, namely, the statistical spectrum of a non-Markovity parameter, power spectra of the time correlation function and memory functions of junior orders, and local relaxation and kinetic parameters. The results demonstrate the superiority of the new measures in comparison to the traditional nonlinear measures. We conclude that the applied measures are more appropriate for the quantification of markovization and demarkovization in EEG data and the prediction of epilepsy seizure.

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## 1. INTRODUCTION

We develop a new approach that could provide us with a powerful means of discrete time series analysis and processing. The subject of our study is human electroencephalogram (EEG) records, because we address our work to those who are interested in signal processing in live complex systems. In studying the natural complex systems, very little is usually known about their internal structure and the relationship between their components. The time series describing the dynamics of one or several parameters are typically used for obtaining diagnostic information. The received information is not adequate for the description of all the degrees of freedom of this system. Quantitative and qualitative methods proposed recently allow constructing the framework for the description of natural complex systems. It allows diagnosing diseases without going into the detail of the internal structure underlying the natural complex systems. A similar approach can be used to describe and investigate the diversified complex systems as they are related only to the concepts of this framework. Here, we present the results of applying the new framework involving ideas of the discrete non-Markovian stochastic processes to the analysis of electric potentials of brain. It turns out that discussing the results in terms of demarkovization and markovization is the best way to uncover seizure dynamics features.

The brain cells communicate by producing tiny electrical pulses. In an EEG, electrodes are placed on the scalp over multiple areas of the brain to detect and record the electrical pulses within the brain. The EEG is used to help diagnose the presence and type of seizure disorders, confusion, head injuries, brain tumors, infections, degenerative diseases, and metabolic disturbances that affect the brain.

It is well known that epilepsy is one of the most serious diseases of human brain [1, 2]. The dynamics of the electric signals accompanying it belongs to a class of nonlinear, nonstationary and nonergodic processes of complex systems of a live nature [3, 4]. The discrete and non-Markovian properties of time variation of the signals and the sudden alternation of the behavior regimes must be taken into account in analyzing the electric activity of brain potentials. Together with the fast change

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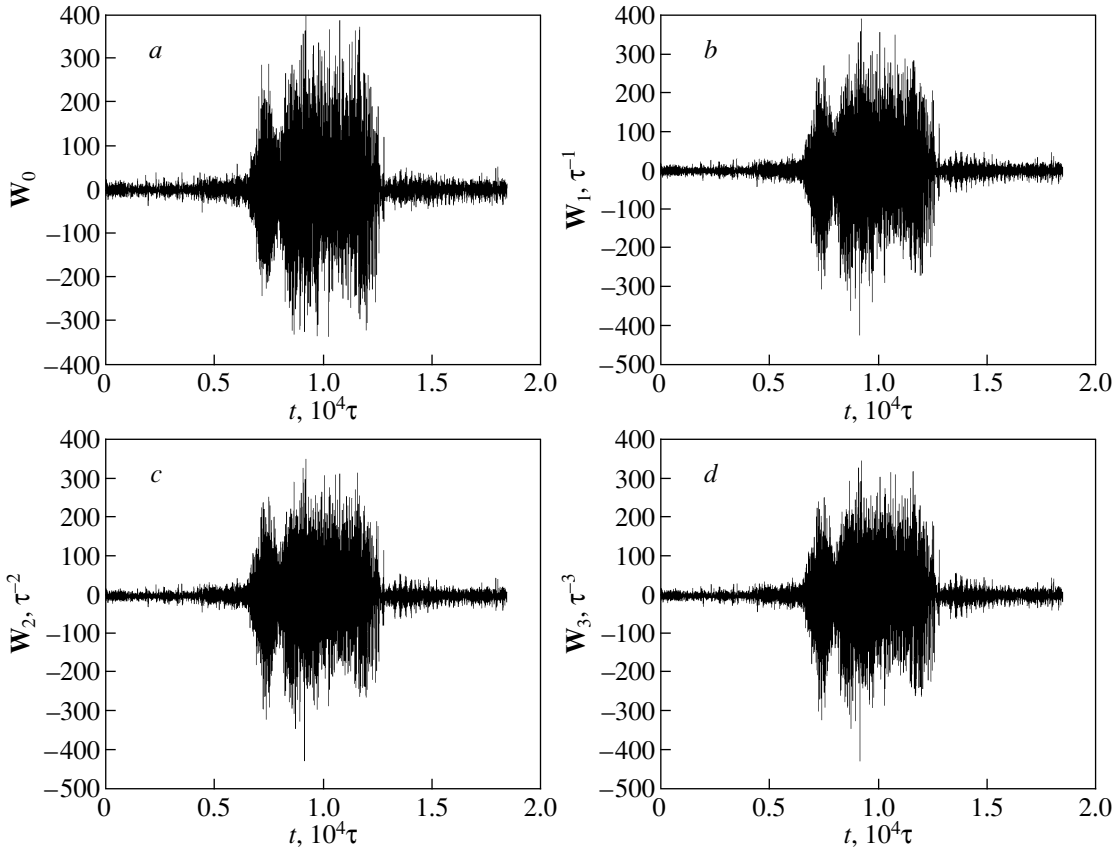



**Fig. 1.** The time record of the first four orthogonal variables $W_0$ ($a$), $W_1$ ($b$), $W_2$ ($c$), and $W_3$ ($d$) of the sampling of electric activity at the tonic-clonic seizure under study. The difference in the dimensions of the four variables must be taken into account in analyzing the scales. The general form of all signals has definite similarity. Simultaneously, some differences in time behavior $W_i(t)$ are made evident especially for the states before and after the seizure. We emphasize that the whimsical entanglement of regular and chaotic components is omnipresent in the window time recording of all the signals. We also note that the difference between the raw EEG data before, during, and after the seizure is sufficiently dramatic. But simple registration of this fact does not allow us to reveal such subtle features of EEG spectra as the presence or absence of the chaotic or regular components in the signal

of chaotic and regular modes in the behavior of the system, this creates serious problems for the diagnosis and treatment of patients with epilepsy seizure. This is why traditional methods of nonlinear dynamics such as the Lyapunov exponent, the Kolmogorov–Sinai entropy, and correlation and fractal dimensions are not sufficiently sensitive for the purpose of distinction between different chaotic regimes in epilepsy.

## 2. BASIC THEORY

Our approach is based on the recent theory for stationary [5] and nonstationary cases [6] of discrete stochastic processes in complex systems. We analyze the stochastic process on the basis of the chain of the coupled non-Markovian discrete equations for the initial discrete time correlation function (TCF) $a(t)$ $(t = m\tau)$,

$$\begin{aligned}
\frac{\Delta a(t)}{\Delta t} &= \lambda_1 a(t) - \tau\Lambda_1 \sum_{j=0}^{m-1} M_1(j\tau)a(t-j\tau),\\
\frac{\Delta M_1(t)}{\Delta t} &= \lambda_2 M_1(t) - \tau\Lambda_2 \sum_{j=0}^{m-1} M_2(j\tau)M_1(t-j\tau), \quad (1)\\
\frac{\Delta M_2(t)}{\Delta t} &= \lambda_3 M_2(t) - \tau\Lambda_3 \sum_{j=0}^{m-1} M_3(j\tau)M_2(t-j\tau),
\end{aligned}$$

where $\lambda_n$ is the eigenvalue spectrum of the Liouville operator $i\hat{L}$ and $\Lambda_n$ are the general relaxation parame-

ters,

$$\lambda_n = i\frac{\langle \mathbf{W}_n^* L \mathbf{W}_n\rangle}{\langle|\mathbf{W}_n|^2\rangle}, \quad \Lambda_n = -\frac{\langle \mathbf{W}_{n-1}(i\hat{L}-\lambda_{n+1})\mathbf{W}_n\rangle}{\langle|\mathbf{W}_{n-1}|^2\rangle}.$$

The kinetic nonlinear finite-difference equations (1) are analogous to the well-known chain of kinetic equations of the Zwanzig–Mori (ZM) type. These ZM equations play a fundamental role in the modern statistical mechanics of nonequilibrium phenomena with continuous time. Kinetic equations (1) can be considered as a discrete-difference analogy of hydrodynamic equations for physical phenomena with discrete time. By analogy with [5–7], we define the generalized nonlinear non-Markovity parameter in the frequency-dependent case as

$$\epsilon_i(\omega) = \left\{\frac{\mu_{i-1}(\omega)}{\mu_i(\omega)}\right\}^{1/2}, \qquad (2)$$

where $i = 1, 2, \ldots$ and $\mu_i(\omega)$ is the power spectrum of the $i$-th memory function. It is convenient to use this parameter for quantitative description of long-range memory effects in the system considered together with memory functions defined above. The values of $\varepsilon_i(\omega)$ allow us to obtain a quantitative estimate of non-Markovity effects and the statistical collective memory in the chaotic changes of the experimentally measured EEG data. The parameters $\varepsilon_i(\omega)$ allow classifying all the observed processes into three important types [5]. A Markov process corresponds to the situation where the non-Markovity parameter takes indefinitely large value $\varepsilon_i(\omega) \to \infty$, and the quasi-Markov processes correspond to the case where $\varepsilon_i(\omega) > 1$. The limit case $\varepsilon_i(\omega) \approx 1$ describes non-Markov processes. In this case, the time scale of memory processes and the correlation dynamics (or the nearest junior and senior memory function) coincide with each other.

## 3. EXPERIMENTAL DATA

We quantitatively demonstrate the stochastic description of the time-frequency peculiarities of epilepsy. We use experimental data [8] on human EEGs. These files show tonic-clonic seizures of two subjects recorded with a scalp right central (C4) electrode (linked earlobes reference). It contains a total of 3 minutes with about 1 min pre-seizure, the seizure, and some post-seizure activity. The sampling rate is 102.4 Hz (see the papers cited in [8] for more details).

## 4. NUMERICAL CALCULATIONS

We consider a discrete time series of the electric activity as a one-point stochastic process

$$X = \{x(T), x(T+\tau), x(T+2\tau), \ldots, x(T+k\tau), \ldots, x(T+\tau N-\tau)\}. \qquad (3)$$

It is convenient to introduce the normalized time correlation function for the quantitative description of time series,

$$a(t) = \frac{1}{(N-m)\sigma^2} \times \times \sum_{j=0}^{N-1-m} \delta x(T+j\tau)\delta x(T+(j+m\tau)), \qquad (4)$$

where $\sigma^2$ is the variance, $N$ is the number of measurements, and $\tau$ is a finite discretization time. The key element of the theory consists in transition from continuum values, variables, and equations to discrete ones. We then obtain a Liouville-like equation of motion for multidimensional state vectors. We can use the method of projection operators in a finite-dimensional vector space. This allows splitting the Liouville-like discrete equation of motion into two mutually orthogonal subspaces, one of which is relevant and the other is irrelevant to discrete time correlations. We have also developed the method for obtaining the set of dynamic orthogonal variables by the Gram–Schmidt orthogonalization procedure.

Dynamical orthogonal variables were calculated from initial time series (3) by the formulas (see [5, 6])

$$\hat{\mathbf{W}}_0 = \mathbf{A}_k^0, \quad \hat{\mathbf{W}}_1 = \left(\frac{\Delta}{\Delta t} - \lambda_1\right)\mathbf{A}_k^0,$$

$$\hat{\mathbf{W}}_2 = \left(\frac{\Delta}{\Delta t} - \lambda_2\right)\mathbf{W}_1 + \Lambda_1\mathbf{A}_k^0 = \left[\left(\frac{\Delta}{\Delta t}\right)^2 - \frac{\Delta}{\Delta t}(\lambda_1+\lambda_2) + \lambda_1\lambda_2 + \Lambda_1\right]\mathbf{A}_k^0, \qquad (5)$$

$$\hat{\mathbf{W}}_3 = \left(\frac{\Delta}{\Delta t} - \lambda_3\right)\mathbf{W}_2 + \Lambda_2\left(\frac{\Delta}{\Delta t} - \lambda_1\right)\mathbf{A}_k^0,$$

where the parameters $\lambda_i$ and $\Lambda_i$ were calculated using (2). Simple, but cumbersome calculations show that the first short-memory function $m_n(t)$ represents a normalized TCF of the first dynamic variable $W_n$,

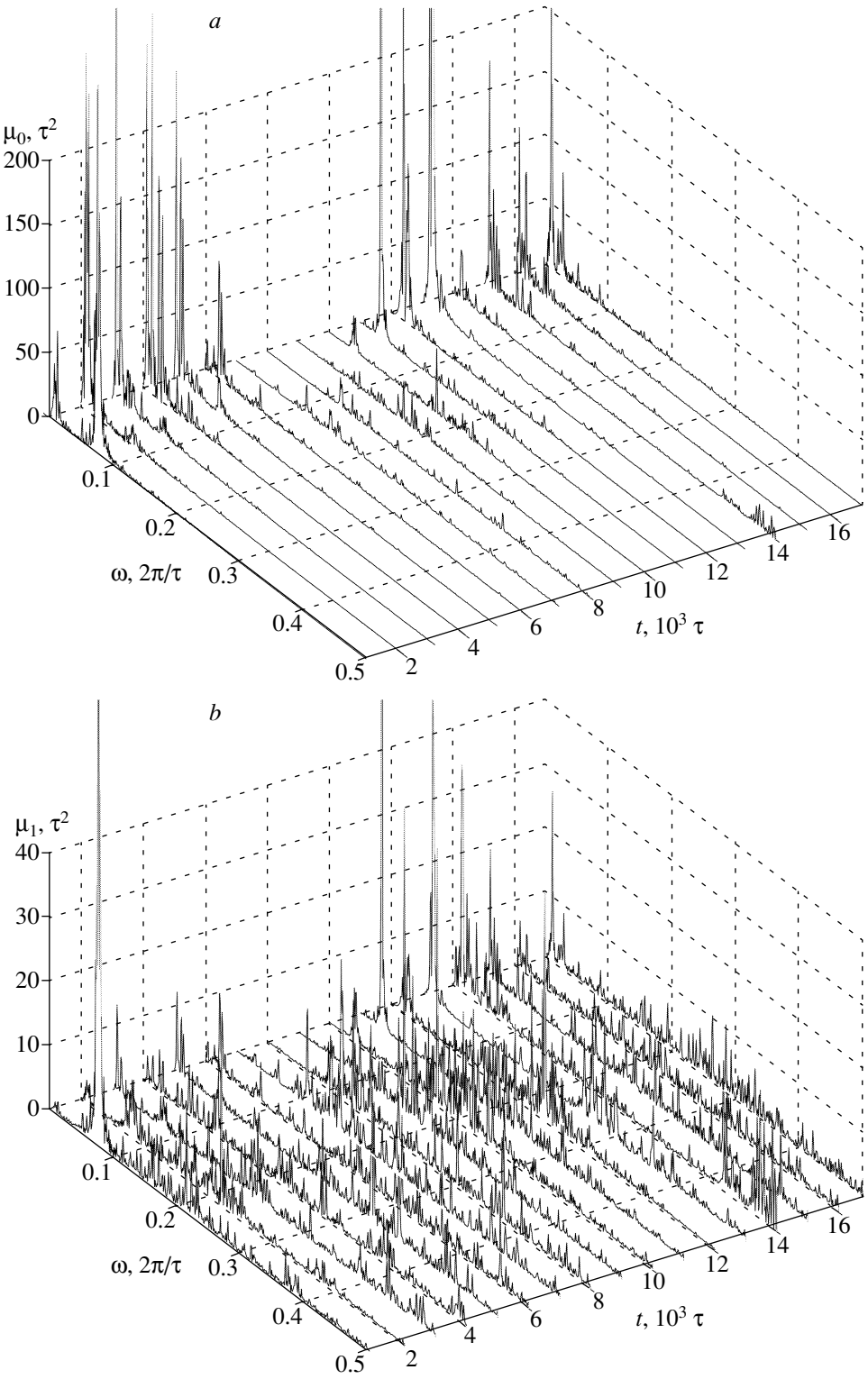
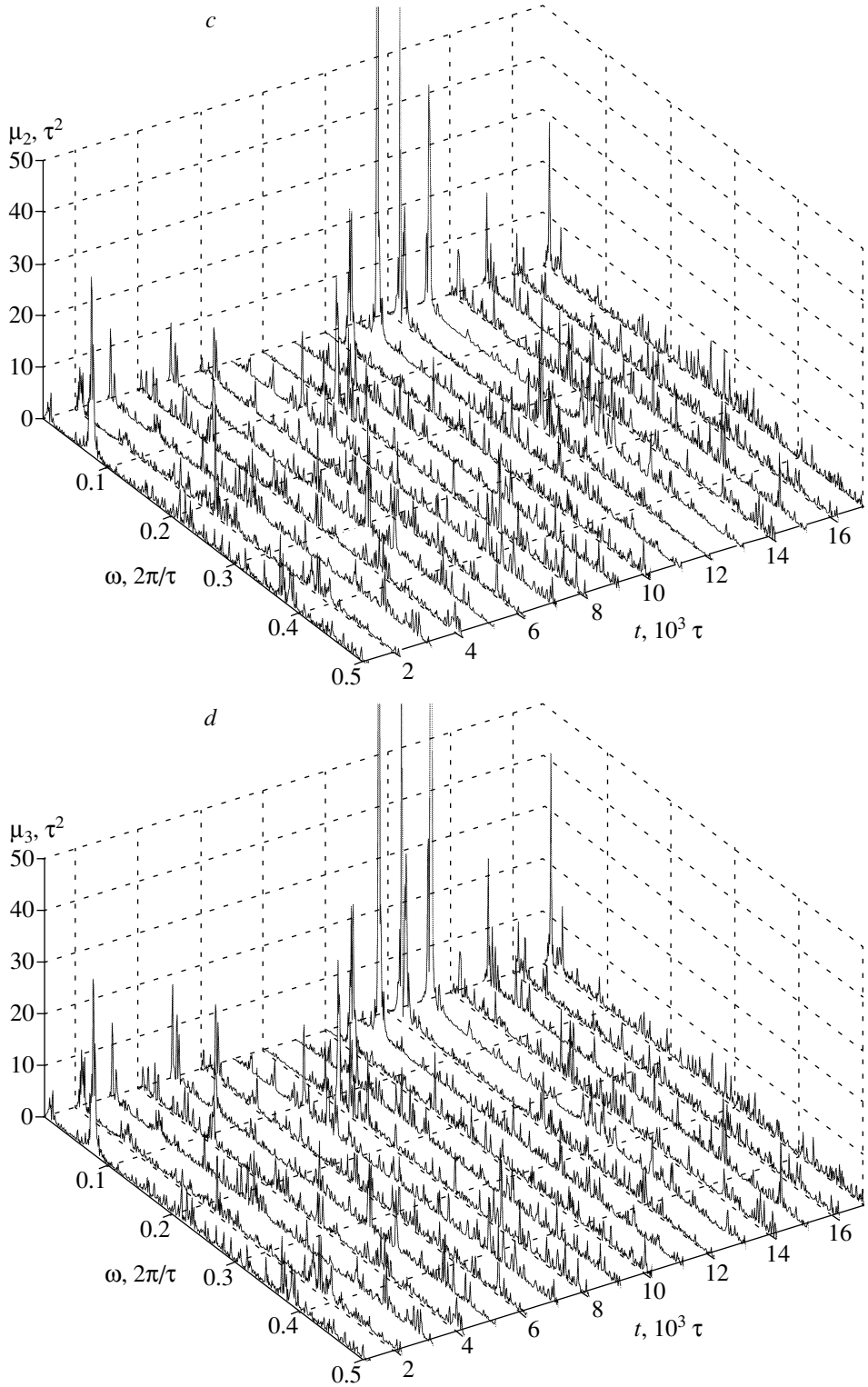
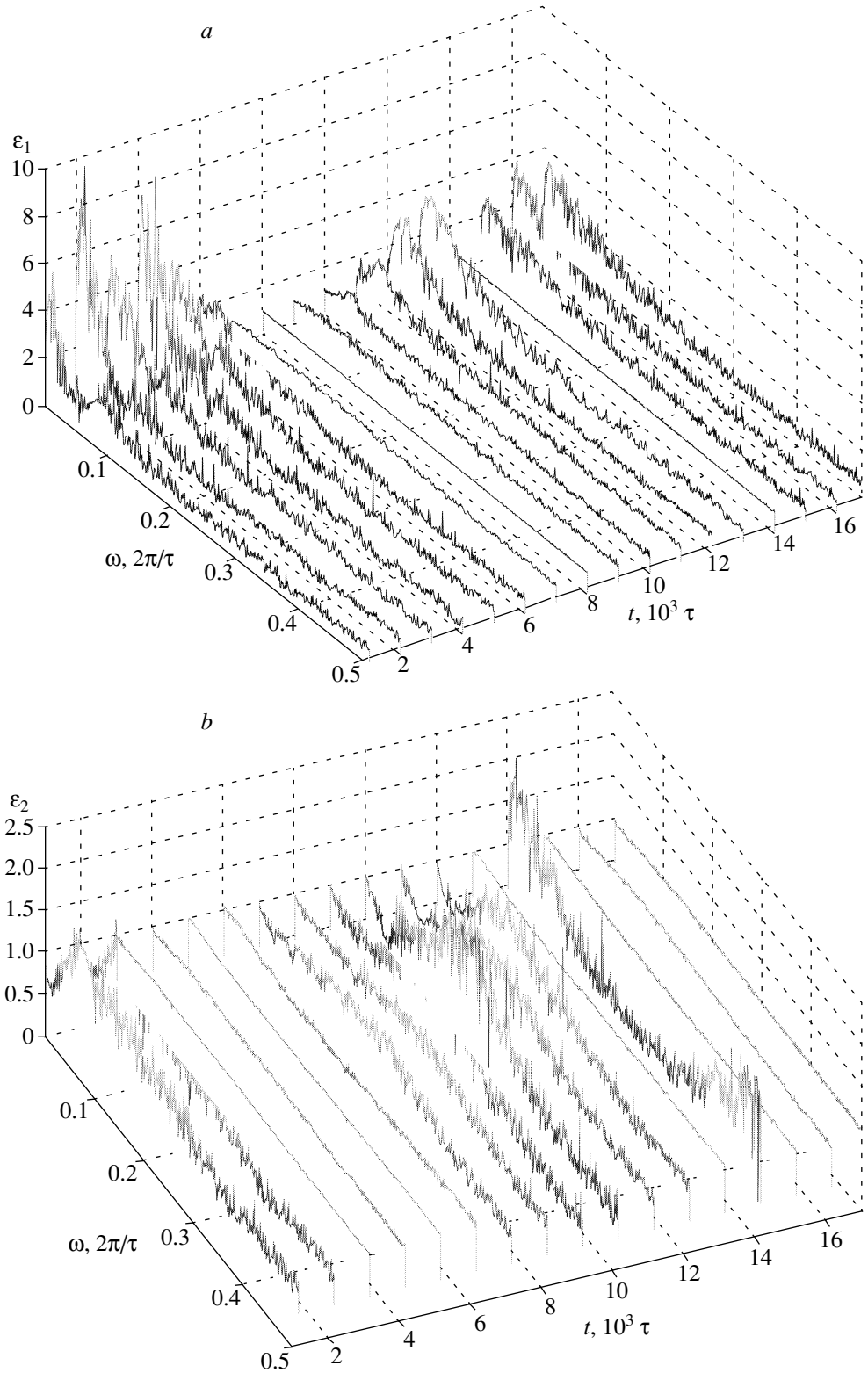



**Fig. 2.** The window-time behavior of the power spectra $\mu_i(\omega)$, $i = 0$ $(a)$, 1 $(b)$, 2 $(c)$, 3 $(d)$, for the considered sampling with the tonic-clonic seizure from the short-time window dynamics of the human brain electric activity. The sharp reduction (by almost one order) of intensity of the low-frequency components of the spectra (in the region of $\delta$ and $\vartheta$ rhythms) attracted our attention at the transition from $\mu_0$ to $\mu_1, \mu_2$, and $\mu_3$. The spectra $\mu_i(\omega)$, $i = 1, 2, 3$, contain rather strong




noises distributed at regular intervals in the entire frequency region. The intensity in the region of $\delta$ and $\vartheta$ rhythms sharply decreases in the first half of the seizure (the 7th, 8th, 9th and, in part, the 10th windows) in all $\mu_i$, $i = 0, 1, 2, 3$. The sharp increase of the intensity in the low-frequency region of the spectrum by almost 100 times (in the regions of $\delta$ and $\vartheta$ rhythms) is observed in the second half of the seizure (the 11th, 12th, and 13th windows)


a
ε1
10
8
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0.1
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ω, 2π/τ
0.3
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2
4
6
8
10
12
14
16
t, 10^3 τ
b
ε2
2.5
2.0
1.5
1.0
0.5
0
0.1
0.2
ω, 2π/τ
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Fig. 3.

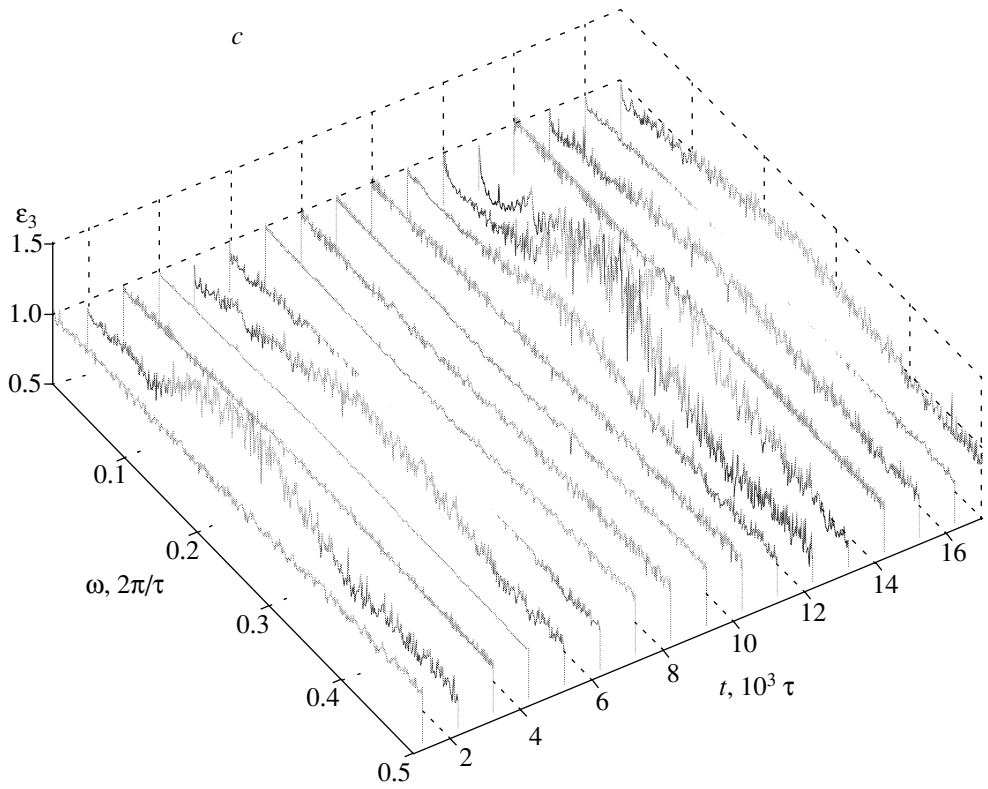


**Fig. 3.** The window-time behavior of the first three points of the non-Markovity parameter $\varepsilon_i(\omega)$, $i = 1$ ($a$), 2 ($b$), 3 ($c$), for the long sampling including the tonic-clonic seizure at epilepsy. For the state before the seizure, the quasi-Markovian behavior ($\varepsilon_1 \sim 10$) of the first point $\varepsilon_1(\omega)$ in the low-frequency region (with $\delta$ and $\alpha$ rhythms) is obvious. The beginning of the seizure (the 7th, 8th, 9th, and 10th windows) exhibits a strong non-Markovity ($\varepsilon_1 \sim 1$) on all frequencies of the full spectrum. A weak non-Markovity in the region of $\delta$ and $\vartheta$ rhythms ($\varepsilon \to 4$) is found during the seizure. A strong non-Markovity on all frequencies is established immediately after the termination of the seizure (the 14th window). Frequency behavior of $\varepsilon_3(\omega)$ is characterized by steady non-Markovity ($\varepsilon_3 \to 1$) in all the windows and in the entire frequency region. A weak quasi-Markov noise (in the region of $\alpha$ and $\beta$ rhythms) appears before the seizure (the 2nd and the 5th windows) and at the end of the seizure (in the 12th, 13th, and 14th windows). The behavior of the parameter $\varepsilon_2(\omega)$ is rather peculiar. A strong non-Markovity ($\varepsilon_2 \sim 1$) appears long before the seizure (in the 3rd, 4th, 5th, and 6th windows). Further development of the seizure is accompanied by a slight noise in $\varepsilon_2(\omega)$ in the region of $\alpha$ and $\beta$ rhythms. The termination of the seizure results in a strong non-Markovity ($\varepsilon_2 \sim 1$) in the 13th window. Noisiness in the entire frequency range of the 14th window then occurs. The steady non-Markovity ($\varepsilon_2 \sim 1$) is appreciable in the 15th, 16th, and 17th windows, appearing after the termination of the seizure. The low-frequency (in the region of $\delta$ rhythms with $\varepsilon_2 \sim 1$) and high-frequency (in the top border of the $\gamma$-spectrum with $\varepsilon_2 \sim 3$) sites of the spectrum are intensively noisy

$$m_n(t) = \frac{\langle \mathbf{W}_n(0)\mathbf{W}_n(t)\rangle}{\langle \mathbf{W}_n(0)\rangle^2}. \qquad (6)$$

We then obtain a chain of finite-difference discrete non-Markov kinetic equations for the initial time correlation function and memory functions of various orders. We note that all the involved kinetic and relaxation parameters, the time correlation function, and the memory functions can easily be found and calculated directly from the experimental time series. The spectra of memory functions were calculated using fast Fourier transform.

## 5. NON-MARKOV PROPERTIES OF EEGs

We have analyzed the time and frequency evolution of the signals during tonic-clonic seizure by means of the time-window technique. We find that the memory function spectra and the statistical spectrum of the non-Markovity parameter are valuable for the quantitative and qualitative analysis of epileptic seizures. Numerical parameters based on the theory of discrete non-Markov processes provide quantitative information about the state of brain before, during, and after the seizure.

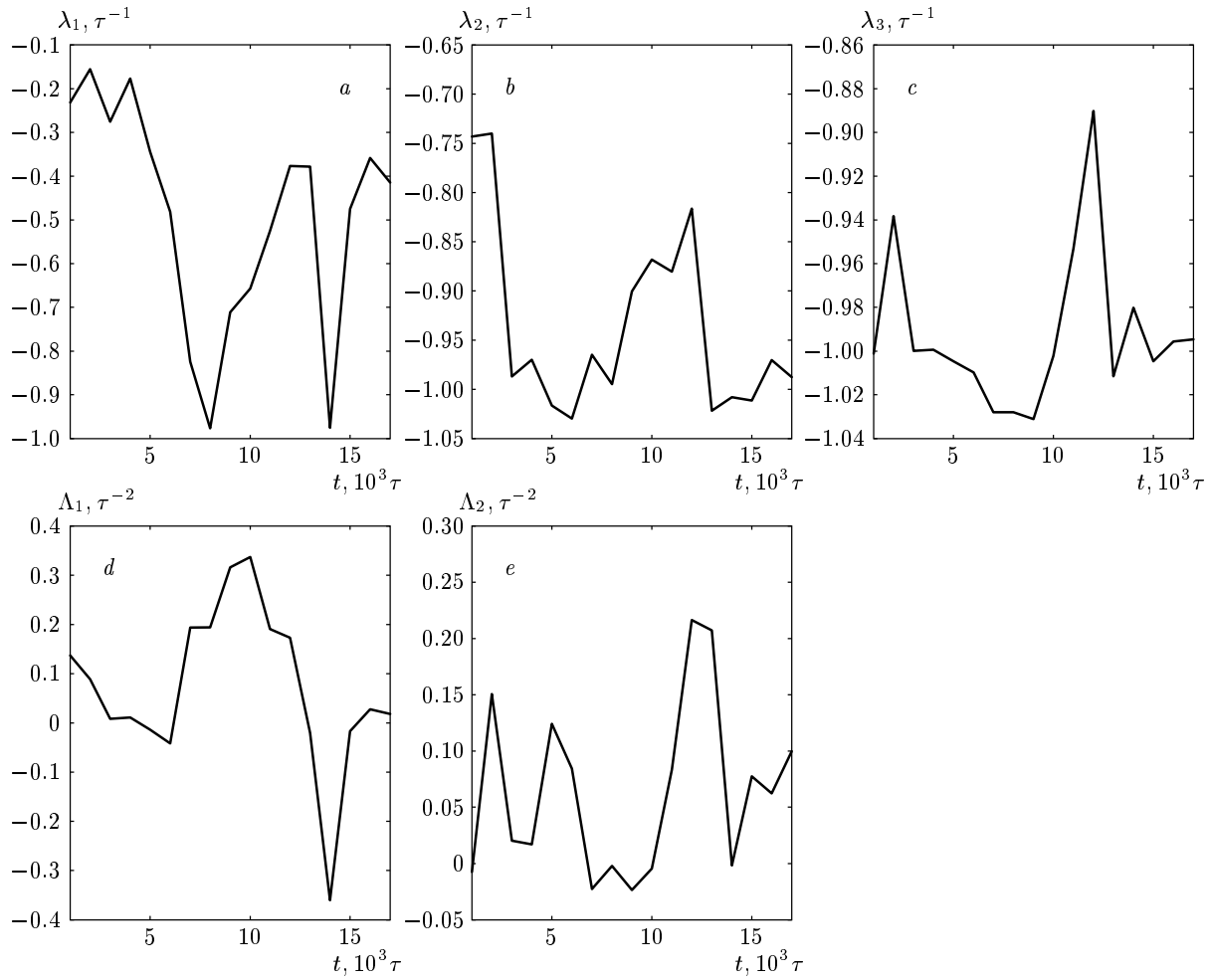


**Fig. 4.** The window-time behavior of the kinetic ($\lambda_1$ ($a$), $\lambda_2$ ($b$) and $\lambda_3$ ($c$)) and relaxation ($\Lambda_1$ ($d$), $\Lambda_2$ ($e$)) parameters for the time sampling at epilepsy with the tonic-clonic seizure. The kinetic parameters $\lambda_1$, $\lambda_2$, and $\lambda_3$ are always negative and increase with seizure. The relaxation parameters $\Lambda_1$ and $\Lambda_2$ change sharply with the sign change with the beginning of the seizure. The most dramatic changes in the behavior of $\Lambda_1$ and $\Lambda_2$ occur during the seizure in the opposite directions

Non-Markov properties are known to play an essential role in the time dynamics of complex systems. On the basis of our theory [5, 6], we can calculate memory functions $M_i(t)$, $i = 0, 1, 2, 3$, directly from experimental data by Eqs. (2.41)–(2.46) in [6]. We analyze the properties of memory functions by calculating their power frequency spectra. For a quantitative estimation of the non-Markovity degree, we use the frequency-dependent generalized non-Markovity parameter $\varepsilon_i(\omega)$ introduced by us previously [5]. From the theory in [5, 6] we can also calculate the quantitative values of the kinetic and relaxation parameters $\lambda_1$, $\lambda_2$, $\lambda_3$, $\Lambda_1$, and $\Lambda_2$ that give additional information about the properties of the complex system under study.

For the observed EEG spectra, we divide the entire time evolution data into nonoverlapping epochs of 1024 data points each. The dynamics of the first four dynamical orthogonal variables $W_0$, $W_1$, $W_2$, and $W_3$ of the entire data set is presented in Fig. 1. For each epoch, we have calculated the power spectra of the first four memory functions $\mu_0(\omega), \mu_1(\omega), \mu_2(\omega), \mu_3(\omega)$ and the three first points of statistical spectra of the non-Markovity parameter $\varepsilon_1(\omega)$, $\varepsilon_2(\omega)$, $\varepsilon_3(\omega)$ [5]. The time evolution of the spectra is shown in three-dimensional diagrams (Figs. 2 and 3). The time evolution of the numerical parameters $\lambda_1$, $\lambda_2$, $\lambda_3$, $\Lambda_1$, $\Lambda_2$ is presented in Fig. 4.

We emphasize that strong demarkovization of the stochastic changes of brain electric potentials with decreasing numerical values $\varepsilon_i$ to the point of a unit is exhibited at the tonic-clonic seizure. The chaotic regime of the system is then replaced by the steady

non-Markov state regime.

It can be seen from Figs. 1*a–d* that the time evolution of the dynamic orthogonal variables $W_i$, $i = 0, 1, 2, 3$, can be smoothed. Therefore, the scales of these variables before and during the seizure are practically idential. The beginning of the seizure (see Figs. 2*a–d*) is characterized by a sharp recession of low-frequency peaks in the spectrum $\mu_0(\omega)$ (7–10th windows); these peaks in $\mu_0(\omega)$ rise sharply at the end of the seizure and immediately after the seizure. The spectra of $\mu_j(\omega)$, $j = 1, 2, 3$, differ by white noise and low-frequency bursts on the tail of the seizure. These bursts are most appreciable in the behavior of the spectra $\mu_2(\omega)$ and $\mu_3(\omega)$.

The behavior of the first three points in the statistical spectrum of the non-Markovity parameter $\varepsilon_i(\omega)$, $i = 1, 2, 3$ (see Figs. 3), turn out to be most indicative and demonstrative. The state before the seizure can be considered a quasi-Markov one in the 1–6th windows for the first level in the low-frequency region (here, $\varepsilon_1(\omega)$ reaches the value 10) and in the 1st and 2nd windows for the second level ($\varepsilon_2(\omega) \sim 1.5$). The beginning of the seizure (the 7th and 8th windows) is accompanied by the strong non-Markovity of the first level ($\varepsilon_i(\omega) \approx 1$). The increase of the low-frequency non-Markovity on the first ($\varepsilon_1 \sim 3.8$), second ($\varepsilon_2 \sim 1.5$), and the third ($\varepsilon_3 \sim 1.5$) relaxation levels is visible at the end of the seizure (10–13th windows). The behavior on the third level with the value $\varepsilon_3 \approx 1$ can be considered a non-Markov one.

Non-Markov relaxation behavior on the second level is noteworthy (see Fig. 3*b*). The strong non-Markovity ($\varepsilon_2 \approx 1$) in the entire frequency region appears long before the seizure in the range from the 3rd to the 6th windows. The weak noise on the mean frequencies is appreciable during the seizure (10–12th windows). The ending of the seizure coincides with non-Markov 13th and quasi-Markov 14th windows. The appearance of a strongly pronounced non-Markov state on the second level with the value $\varepsilon_2 \approx 1$ is therefore a clear precursor of the seizure. It is significant that a similar precursor is absent in other non-Markov markers.

The relaxation ($\lambda_1$, $\lambda_2$, and $\lambda_3$) and kinetic ($\Lambda_1$ and $\Lambda_2$) parameters calculated with the formulas of the theory (see Fig. 4) are very sensitive to approaching the seizure. All the parameters $\lambda_i$, $i = 1, 2, 3$, always remain negative and change within wide limits: ($-0.97 \le \lambda_1 \le -0.15$, $-1.03 \le \lambda_2 \le -0.74$, and $-1.03 \le \lambda_3 \le -0.89$) in the units of $\tau^{-1}$. The parameters $\Lambda_1$ and $\Lambda_2$ change the sign at the time of the seizure. This corresponds to the alternation of the type of solution of the discrete nonlinear kinetic stochastic equation (see Eqs. (2.56)–(2.58) in [6]). All the above parameters are sensitive to approaching the seizure. A sharp decrease of the values and the sign alternation of $\lambda_i$ and $\Lambda_i$ can also be considered as a quantitative precursor of the seizure.

Therefore, the increase of the parameters $\varepsilon_i(\omega)$ can be considered as a markovization of the stochastic process. It may signify the increase of the chaotic components of EEG signals. Simultaneously, the decrease of $\varepsilon_i(\omega)$ up to a unit is related to demarkovization of the process under study and to the increase of the regular components of the signals. It is obvious from Figs. 1–4 that the specific alternations, fast and sudden changes of chaotic and regular regimes, are inherent features of the stochastic variation of electric potentials at epileptic seizure.

## 6. CONCLUSIONS

We have clearly demonstrated that the set of kinetic, relaxation, dynamic, and spectral parameters and characteristics of a discrete stochastic process are valuable for quantification of stochastic processes of markovization and demarkovization in EEG data and for prediction and precursor of the epileptic seizure. Because a similar situation is typical of the majority of the phenomena in live systems, our findings are most relevant for life sciences.

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