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# THE DOUBLING OF THE ANOMALOUS MAGNETIC MOMENT OF ELECTRON IN A VERY STRONG CONSTANT HOMOGENIOUS ELECTRIC FIELD

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Calculated earlier by the author [8], the anomalous magnetic moment (AMM) of the electron in an intense constant electric field changes nonmonotonically as the field increases, passing through a minimum and tending to the doubled Schwinger value for very strong fields. In the present paper, it is supposed that the AMM is related by the Lande factor to the angular momentum of a virtual electron accompanied by a virtual photon. This factor changes its effective value because of the influence of the external field on the motion of the virtual electron and its self-action. With the increase of the electric field, the virtual electron can successively occupy the excited states $l = 1$, $j = 1/2$ and $l = 1$, $j = 3/2$ in addition to the original state with the orbital angular momentum $l = 0$ and the total angular momentum $j = 1/2$. The first of these excited states decreases the AMM and the second increases and doubles it if only this state is occupied for a very strong field. The latter condition is equivalent to the alignment of the spin and the orbital angular momentum of the electron along the field, while the total angular momentum of the entire system of the virtual electron and the virtual photon remains equal to 1/2.

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## 1. INTRODUCTION

The purpose of this paper is to draw attention to an interesting dependence of the anomalous magnetic moment of electron on the intensity of the external constant homogeneous electric field. The dependence of the AMM on the constant magnetic or crossed field was considered by Demeur [1], Newton [2], Ternov et al. [3], Ritus [4], Jancovici [5], Tsai Wu-yang and Yildiz [6], and Baier et al. [7]. It was shown that the AMM tends to zero for a very strong magnetic or crossed field. The dependence of the AMM on the constant electric field cannot be obtained from its dependence on the magnetic field by the analytic continuation $\mathbf{H}^2 \to -\mathbf{E}^2$ because of the nonanalyticity at zero field.

In my paper [8], the eigenvalue of the mass operator of electron in a constant homogeneous electromagnetic field of an arbitrary intensity was found. In particular, this eigenvalue, or more precisely, the corresponding elastic scattering amplitude involves the dependence of the AMM $\Delta\mu$ on the electric and magnetic fields. If one keeps only the dependence on the electric field $\varepsilon$ and confines oneself to the state with $\mathbf{p}_\perp = 0$, the ratio of $\Delta\mu$ to the Bohr magneton $\mu_0$ is given by

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{\pi} J(\beta), \quad \beta = \frac{\hbar|e\varepsilon|}{m^2c^3}, \tag{1}$$

where $\beta$ is the electric field $\varepsilon$ in characteristic QED units and

$$\begin{aligned} &J(\beta) = 1 - I(\beta), \\ &I(\beta) = \frac{1}{\beta}\int_0^\infty dy\, \sin(y/\beta)\, \phi(y), \\ &\phi(y) = \int_y^\infty \frac{dx\, u^2}{x(1+u)^2}, \\ &u = x[\operatorname{cth}(x-y) - \operatorname{cth} x]. \end{aligned} \tag{2}$$

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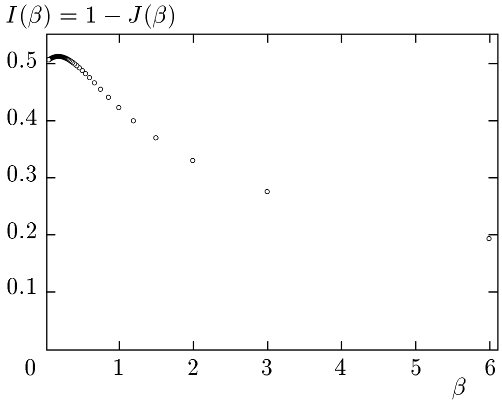


**Fig. 1.**

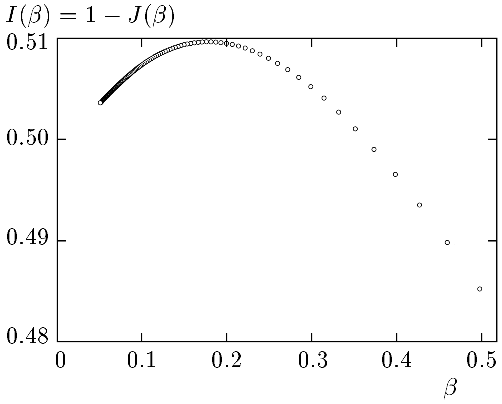


**Fig. 2.**

It was shown that in a weak field ($\beta \ll 1$),

$$J(\beta) = \frac{1}{2} - \frac{4}{3}\beta^2\left(\ln\frac{\gamma}{2\beta} - \frac{23}{12}\right) - \\ - \frac{128}{3}\beta^4\left(\ln\frac{\gamma}{2\beta} - \frac{81}{70}\zeta(3) - \frac{209}{1120}\right) + \ldots \quad (3)$$

and in a strong field ($\beta \gg 1$),

$$J(\beta) = 1 - \frac{\pi}{4\beta}\left(\ln\frac{2\beta}{\gamma} - 1\right) + \ldots, \quad (4)$$

where $\gamma = 1,781\ldots$ and $\zeta(3) = 1.202\ldots$

Thus, as the field increases, the AMM first decreases from the Schwinger value $\alpha/2\pi$, reaches a minimum, and then increases and approaches the doubled Schwinger value $\alpha/\pi$.

This intriguing dependence is also confirmed by the numerical calculation of the integral $J(\beta)$, see Fig. 1 and Fig. 2. The minimum of $J(\beta)$ is located at $\beta \approx 0.179$ and is equal to $J(0.179) \approx 0.49040$.

The most striking property of $J(\beta)$ is the doubling of this function and of the AMM in the strong-field limit compared to its value in the weak field limit,

$$\frac{J(\infty)}{J(0)} = 2. \quad (5)$$

In all other fields (magnetic, crossed) the AMM is equal to zero in the strong-field limit.

Another interesting property of $J(\beta)$ is its nonmonotonic dependence with one minimum.

The formula for the AMM is nothing else than the Fourier sine transform of the function $\phi(y)$. This function has a maximum at zero, which is equal to $1/2$, and monotonically decreases to zero as $y \to \infty$,

$$\begin{aligned} \phi(y) &= \frac{1}{2} - y^2\left(\frac{2}{3}\ln\frac{1}{2y} - \frac{5}{18}\right) - \\ &- y^3\left(\frac{1}{3} + \frac{8\pi^2}{45}\right) + \ldots, \quad y \ll 1, \\ \phi(y) &= \frac{1}{2y}(\ln 2y - 1) - \\ &- \frac{1}{8y^2}\left(\ln^2 2y - 8\ln 2y + \frac{\pi^2}{3} + 4\right) + \ldots, \quad y \gg 1. \end{aligned} \quad (6)$$

Qualitatively, a similar nonmonotonic behaviour for $J(\beta)$ with the minimum at $\beta \sim 1$ and the doubling at infinity, $J(\infty) = 2J(0)$, would be given by the Gaussian

$$\phi(y) = \frac{1}{2}e^{-y^2}$$

and the Lorentzian

$$\phi(y) = \frac{1}{2}(1 + y^2)^{-1}$$

functions of $y$.

We consider the physical meaning of the function $\phi(y)$ and its argument.

## 2. THE MASS OPERATOR OF ELECTRON IN A CONSTANT ELECTROMAGNETIC FIELD

The law of motion of a relativistic classical charge in the homogeneous constant electromagnetic field $F_{\alpha\beta}$ can be written as

$$x_\alpha(s) - x_\alpha(0) = \left(\frac{\exp(2eFs) - 1}{eF}\right)_{\alpha\beta}\pi_\beta(0), \quad (7)$$

where $x_\alpha(s)$ is the charge 4-coordinate depending on the proper time $s$ and $x_\alpha(0)$ and $\pi_\alpha(0)$ are the initial 4-coordinate and the kinetic 4-momentum respectively. In a Lorentz system where the electric and magnetic fields are parallel, the electron moves along a helical line with alternating pitch whose rate of change is defined by the electric field and the period of revolution is defined by the magnetic field.

Quantum motion of the electron in an external field, with the radiative corrections taken into account, is described by the Dirac wave equation with the mass operator. Roughly speaking, the mass operator is defined in the $e^2$-approximation by the product of the causal propagation functions $S^c(x,x')$ and $D^c(x-x')$ of the electron in an external field and of the photon in the vacuum:

$$M(x,x') = ie^2\gamma_\mu S^c(x,x')\gamma_\mu D^c(x-x'). \tag{8}$$

Here and below, we use the same notation as in [8]. In the proper time representation, we have

$$\begin{aligned} S^c(x,x') &= -\frac{ie^{i\varphi}}{16\pi^2}\int\limits_0^\infty \frac{ds\, e^2\eta\,\varepsilon}{\sin(e\eta s)\,\mathrm{sh}(e\varepsilon s)} \times \\ &\times \left(m-\frac{i}{2}\gamma Bz\right)\exp\left(-im^2 s+\frac{iz\beta z}{4}+\frac{ie\sigma Fs}{2}\right), \\ D^c(z) &= -\frac{i}{16\pi^2}\int\limits_0^\infty \frac{dt}{t^2}\exp\left(\frac{iz^2}{4t}\right), \quad z = x-x', \\ B_{\alpha\beta} &= \beta_{\alpha\beta}+eF_{\alpha\beta}, \quad \beta_{\alpha\beta} = (eF\,\mathrm{cth}\,(eFs))_{\alpha\beta}, \\ \varphi &= e\int\limits_{x'}^{x} dy_\alpha A_\alpha(y), \end{aligned} \tag{9}$$

and $\varepsilon$ and $\eta$ are the strengths of the electric and the magnetic field in the frame where they are parallel. The $\varphi$ integration goes along the straight line.

After going over to the $E_p(x)$-representation, the mass operator becomes diagonal and its renormalized eigenvalue is given by the $\gamma$-matrix,

$$\begin{aligned} M_R(\bar p,F) &= \frac{\alpha}{2\pi}\int\limits_0^\infty\int\limits_0^\infty \frac{ds\,dt}{t^2}\left\{\frac{\sin(e\eta w_1)\,\mathrm{sh}(e\varepsilon w_2)}{\sin(e\eta s)\,\mathrm{sh}(e\varepsilon s)}\times\right. \\ &\times \exp\left(-im^2 s - i\bar p w\bar p - \frac{ie\sigma Fw}{2}\right)\times \\ &\times \left[2m(S+i\gamma_5 P)+i\exp\left(-\frac{ie\sigma Fs}{2}\right)\times\right. \\ &\times \left.\gamma\exp(eF(w+s))\frac{\mathrm{sh}\,(eFw)}{\mathrm{sh}\,(eFs)}\,\bar p\right] - \frac{\omega^2}{s^2}\times \\ &\times \left.\exp(-im^2 s - ip^2\omega)\left(2m+i\gamma\bar p\frac{\omega}{s}\right)\right\} + M_R^0(\bar p). \end{aligned} \tag{10}$$

This is Eq. (52) in [8], where one can find all the details about the quantities involved here and the notation.

It is now important to discuss the transformation of the term $\gamma Bz$ entering the electron propagator. This term, being linear in the field (for a weak field) and linear in the coordinate difference $z = x - x'$, gives a contribution to the AMM and contains information about the motion of the virtual electron between the points $x'$ and $x$ when it is accompanied by a virtual photon and its motion is distorted by the external field. Because

$$\begin{aligned} \gamma_\mu\left(m-\frac{i}{2}\gamma Bz\right)\exp\left(\frac{ie\sigma Fs}{2}\right)\gamma_\mu &= \\ = 4m(S+i\gamma_5 P)+i\exp\left(-\frac{ie\sigma Fs}{2}\right)\gamma Bz, \end{aligned} \tag{11}$$

the term $\gamma Bz$ appears in $M_R(\bar p, F)$ in the second term in the square brackets as

$$\begin{aligned} \gamma Bz_{eff} &= 2\gamma\exp\left(eF(w+s)\right)\frac{\mathrm{sh}\,(eFw)}{\mathrm{sh}\,(eFs)}\,\bar p = \\ &= \gamma\,\frac{\exp(eFs)(\exp(2eFw)-1)}{\mathrm{sh}\,(eFs)}\,\bar p = \\ &= \gamma B\frac{\exp(2eFw)-1}{eF}\,\bar p. \end{aligned} \tag{12}$$

Therefore, after the integration over $x$ and $x'$ performed in passing from $M(x,x')$ to $M_R(\bar p, F)$, we obtain instead of $z \equiv x - x'$ the quantity

$$z_{\alpha\ eff} = \left(\frac{\exp(2eFw)-1}{eF}\right)_{\alpha\beta}\bar p_\beta, \tag{13}$$

which is the «mean» or the «effective» coordinate difference. Here, $\bar p_\beta$ is the constant momentum four-vector that characterizes the quantum motion of the electron in the external field and $w_{\alpha\beta}$ is the $4\times 4$ matrix

$$w = \frac{1}{eF}\,\mathrm{Ar\,cth}\left(\mathrm{cth}\,(eFs)+\frac{1}{eFt}\right) \tag{14}$$

with two doubly degenerate eigenvalues

$$\begin{aligned} w_1(s,t,\eta) &= \frac{1}{e\eta}\,\mathrm{Arc\,ctg}\left(\mathrm{ctg}\,(e\eta s)+\frac{1}{e\eta t}\right), \\ w_2(s,t,\varepsilon) &= \frac{1}{e\varepsilon}\,\mathrm{Ar\,cth}\left(\mathrm{cth}(e\varepsilon s)+\frac{1}{e\varepsilon t}\right) \end{aligned} \tag{15}$$

playing the roles of the magnetic and the electric proper times. Thus, the virtual electron moves between the points $x'$ and $x$ of the emission and the absorption of the virtual photon «as a classical charge» with two proper times.

Because the virtual electron is accompanied by a virtual photon with the proper time $t$ (or with the squared momentum $k^2 \sim t^{-1}$), the proper times $w_{1,2}$ are always less than $s$,

$$0 \le w_{1,2} \le s. \tag{16}$$

The symbol Arc ctg also indicates that $w_1$ is always in the same period with $s$,

$$n\pi \le e\eta w_1 \le e\eta s \le (n+1)\pi, \quad n = 0, 1, 2, \ldots$$

The argument $y$ of $\phi(y)$ is equal to

$$y = e\varepsilon(s - w_2) \equiv e\varepsilon s - \mathrm{Ar\,cth}\left(\mathrm{cth}\,(e\varepsilon s) + \frac{1}{e\varepsilon t}\right), \tag{17}$$

i.e., it is proportional to the delay of the electric proper time of the virtual electron compared to the proper time of the real electron in the field.

### 3. PHYSICAL INTERPRETATION OF THE AMM DOUBLING FOR A VERY STRONG ELECTRIC FIELD

The AMM $\Delta\mu$ explicitly appears in the elastic scattering amplitude [8]

$$T(\bar{p}, \bar{s}, F) = -\mathrm{Tr}\,\left(M_R(\bar{p}, F)\, u_{\bar{p}\zeta}\bar{u}_{\bar{p}\zeta}\right) \tag{18}$$

as the real part of the coefficient at the first of the two spin-dependent invariants

$$\begin{aligned} -\frac{\bar{s}F^*\bar{p}}{m}\,\mathrm{Tr}\,(u\bar{u}) &= \bar{u}_{\bar{p}\zeta}\,\frac{1}{2}\sigma F\, u_{\bar{p}\zeta}, \\ \frac{\bar{s}F\bar{p}}{m}\,\mathrm{Tr}\,(u\bar{u}) &= \bar{u}_{\bar{p}\zeta}\,\frac{1}{2}\sigma F^*\, u_{\bar{p}\zeta}. \end{aligned} \tag{19}$$

Here, $\bar{s}$ is the polarization 4-vector, $F^*_{\alpha\beta}$ is the field tensor dual to $F_{\alpha\beta}$, and

$$\begin{aligned} u_{\bar{p}\zeta}\bar{u}_{\bar{p}\zeta} &= \frac{\mathrm{Tr}\,(u\bar{u})}{4m}\,(m - i\gamma\bar{p})(1 + i\gamma_5\gamma\bar{s}), \\ \bar{p}^2 &= -m^2, \quad \bar{s}^2 = 1, \quad \bar{s}\bar{p} = 0, \end{aligned} \tag{20}$$

is the polarization density matrix. For the state with $\mathbf{p}_\perp = 0$ and the magnetic field $\eta \to 0$, $\Delta\mu$ is given by (1) and (2).

The main contribution to the integral $I(\beta)$ comes from $y \sim \beta$. The doubling of the AMM in a strong field is then related to a large delay $y \sim \beta \gg 1$ and the explicit expression for $y$ shows that

$$s \sim m^{-2}, \quad t \sim \frac{1}{e\varepsilon} \ll m^{-2} \quad \text{or} \quad k^2 \sim \frac{1}{t} \sim e\varepsilon \gg m^2. \tag{21}$$

In other words, in a strong field the virtual electron is accompanied by a «heavy» virtual photon with the squared momentum

$$k^2 \sim e\varepsilon \gg m^2.$$

We note that

$$s \sim t \sim m^{-2}$$

in a weak field.

The phenomenological and very speculative explanation of the doubling of the magnetic moment in the system of a virtual electron and a photon with the increase of the virtual photon momentum squared may be as follows.

There is the known relation between the magnetic moment $\mu$ and the angular momentum $j$ of an electrodynamical system,

$$\mu = \mu_0\, gj, \quad \mu_0 = \frac{\hbar e}{2mc}, \tag{22}$$

where $g$ is the gyromagnetic ratio. Writing this relation for the AMM and comparing it with the definition of the function $J(\beta)$,

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}\, gj, \quad \frac{\Delta\mu}{\mu_0} = \frac{\alpha}{\pi}\, J(\beta), \quad J(\beta) = \frac{1}{2}\, gj, \tag{23}$$

shows that $J(\beta)$ can be considered as half the product of the gyromagnetic ratio and the angular momentum of the virtual electron.

In a weak field, $gj/2 = 1/2$ because the virtual electron has the quantum numbers $s = 1/2$, $l = 0$, and $j = 1/2$ and the Lande formula

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \tag{24}$$

gives $g = 2$. In a strong field, the virtual electron can go to the state with $s = 1/2$, $l = 1$, and $j = 3/2$, for which $g = 4/3$. Then $gj/2 = 1$ and the AMM doubles.

For moderate field intensities, the virtual electron can be in a superposition of the states $s = 1/2$, $l = 0$, $j = 1/2$ and $s = 1/2$, $l = 1$, $j = 1/2$. Because $g = 2/3$ and $gj/2 = 1/6$ for the latter state, the decrease of the AMM with the increase of $\beta$ becomes clear untill $\beta$ is sufficiently small and the state with $s = 1/2$, $l = 1$, and $j = 3/2$ is not perceptibly excited.

Thus, the following physical picture can occur.

The electron interacting with itself via a virtual photon possesses the total angular momentum $J = 1/2$, which can be considered as the vector sum of the virtual electron angular momentum $j = 1/2$ and the proper moment (spin) $j_\gamma = 1$ of the virtual photon. The external electric field changes the motion of the virtual electron such that the electron can acquire the orbital angular momentum $l = 1$ and its total angular momentum $j$ can remain equal to $1/2$ or become equal to $3/2$. Besides, the vector sum

$$\mathbf{J} = \mathbf{j} + \mathbf{j}_\gamma$$

of the virtual electron and the virtual photon angular momenta remains equal to $J = 1/2$ and their projections on the electric field direction satisfy the conservation law

$$m_J = m_s + m_l + m_\gamma. \tag{25}$$

If the orbital angular momentum and the spin of the virtual electron prefer to be parallel as the electric field increases, such that

$$j = l + 1/2$$

and

$$m_j = m_s + m_l = \pm(l + 1/2),$$

then the appearance of the states with $l > 1$ becomes impossible.

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