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SHORT OPTICAL PULSE POLARIZATION DYNAMICS IN A NONLINEAR BIREFRINGENT DOPED FIBER

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Numerical solutions are obtained of the full self-consistent system of equations for the counter-rotating polarization components of the field of a short optical pulse propagating in a birefringent nonlinear fiber and in the ensemble of the energy-level degenerate doped resonance atoms implanted in the fiber material. In every cross-section of the fiber, the ellipticity of the polarized wave experiences a complex evolution in time accompanied by rapid changes of the azimuthal angle due to the interplay of the dispersion and the Kerr nonlinear self- and cross-phase modulation. The reciprocal effect of the impurities on the traveling pulse causes oscillations of the pulse envelope that can completely distort the shape of the input signal, while the resonance absorption can drive the birefringence process from the nonlinear regime back to the linear one.

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1. INTRODUCTION

Much is known about the propagation of short optical pulses in nonlinear fibers [1–3]. In a nonlinear optical fiber, the propagation of distortionless pulses can be realized under conditions where the amplitude self-modulation effects compensate for the linear dispersion. In particular, for intensities at which the dielectric polarizability has a cubic field response (the Kerr nonlinearity), the envelopes of quasimonochromatic pulses are approximated by optical solitons. In the axisymmetric optical fiber, the fundamental mode consists of two copropagating and perpendicularly polarized linear fields. Non-axisymmetric imperfections to the fiber destroy this polarization degeneracy and introduce the linear birefringence — a difference in the propagation characteristic between the two polarizations. Furthermore, for a nonlinear fiber, the amplitude coupling causes an additional self-induced birefringence via the cross-phase modulation. Activating the fiber by resonance impurities, e.g., rare-earth ions, has given rise to an entire industry of fiber lasers and amplifiers whose physics is extensively discussed in the literature (see [4] and references therein).

In this paper, our approach is to consider the physical system in two associated parts. The first is a short optical pulse propagating in a nonlinear, dispersive and birefringent fiber. The second is an ensemble of two-level resonance atoms immersed in the fiber host material. The first part is modeled by the full fiber equations, broadly in the form of two coupled nonlinear Schrödinger (NLS) equations. The second is governed by a system of Bloch equations coupled to the fiber part by the resonance polarization.

Disregarding the linear birefringence, the difference of the group velocities of the polarized modes (i.e., the walk-off effect), and the polarization induced in the resonant subsystem, the nonlinear equations for the field components are an example of a completely integrable system [5, 6]. Under certain conditions, a short optical pulse in a resonant medium can in turn evolve into a steady-state solitary wave (a 2π-pulse) [7]. This means that in a model of this type, one could ideally observe the coexistence of the self-induced transparency (SIT) and NLS solitons [8–10]. But this can hardly occur for a moderately intense pulse in realistic doped fibers because the disparity in the spatial scales and the pulse energy for the SIT solitons and optical solitons are very substantial in a nonlinear fiber: one 2π SIT pulse corresponds to hundreds of NLS solitons by power.

The general problem then apparently reduces to two characteristic cases: (i) the Kerr nonlinearity and bire-

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fringence effect on the coherent propagation of short pulses in a short doped fiber when the dispersion of group velocities and the walk-off effect are insignificant, (ii) the weak effect of the resonant absorption and refraction on the coupled soliton-like pulse propagation in a nonlinear birefringent fiber. In this paper, we concentrate on the latter case. Basically, both the fiber effects and the two-level medium can affect the polarization state of the propagating light wave. In vector nonlinear wave equations, all the cross terms (linear birefringence, power-dependent cross-phase modulation, and the group velocity mismatch) contribute to variations of the polarization states [11]. At the same time, level-degenerate atoms possess their own polarization properties that have been discussed in numerous papers devoted, e.g., to polarization features of the photon–echo effect [2]. The eccentricity and polarization ellipse orientation can alter across the width of a 2π-pulse in degenerate self-induced transparency [12, 13].

In this paper, we consider a short optical pulse propagation in a nonlinear birefringent doped fiber by numerically solving the self-consistent system of equations for the optical field and the degenerate two-level medium. In the course of discussion, we introduce in succession the conventional fiber attributes (birefringence, dispersion, Kerr nonlinearity, and walk-off effect) followed by the resonance interaction of the light pulse with the impurity atoms in order to observe both the separate and the combined influence of these effects on the dynamics of polarization states and the waveforms of polarization modes.

2. POLARIZED WAVES IN A CUBIC MEDIUM WITH RESONANT IMPURITIES

We consider the electromagnetic wave propagation in an optical birefringent fiber with the cubic (Kerr) nonlinearity. We let this fiber contain doped two-level atoms with the transition energy in resonance with the frequency of the carrier. The description of the solitary wave propagation is conventionally based on the reduced Maxwell equations [9, 14–16] complemented with the Bloch equations [5] determining the evolution of the resonance subsystem. Hereafter, we follow the works by Boardman and Cooper [9, 14], where the propagation of polarized pulses in the Kerr medium was thoroughly observed. The resonance contribution is considered in the same way as in [5].

We write the electric field vector of the optical wave as $\mathbf{E} = E_x\mathbf{e}_x + E_y\mathbf{e}_y$, where $\mathbf{e}_x$ and $\mathbf{e}_y$ are orthogonal vectors in the x and y directions. The wave propagates in the z direction. Using the slowly varying (complex) envelope approximation (SVEA), we can write

$$E_x = \mathcal{E}_x(z,t)\Psi(x,y)\exp\left[i(\beta_x z - \omega_0 t)\right],$$

$$E_y = \mathcal{E}_y(z,t)\Psi(x,y)\exp\left[i(\beta_y z - \omega_0 t)\right],$$

where ω_0 is the carrier frequency and $\beta_x(\beta_y)$ is the linear propagation constant of the slow (fast) principal axis of the birefringent fiber [9, 14–16].

The radial distribution of the electric field in the fiber is described by the mode function $\Psi(x,y)$. We assume that the propagation constants slightly vary from some average value β such that $\beta_x = \beta + \Delta\beta$ and $\beta_y = \beta - \Delta\beta$. The complex envelopes are expressed in terms of real amplitudes and phases slowly varying in space and time, $\mathcal{E}_x = R_x\exp[i\varphi_x]$ and $\mathcal{E}_y = R_y\exp[i\varphi_y]$. The phases are given by $\varphi_x = \tilde{\varphi} + \phi$ and $\varphi_y = \tilde{\varphi} - \phi$, where $\tilde{\varphi}$ is the average value. Finally, the electric field components are given by

$$E_x = A_x(z,t)\Psi(x,y)\exp\left[i\beta z - i\omega_0 t\right],$$

$$E_y = A_y(z,t)\Psi(x,y)\exp\left[i\beta z - i\omega_0 t\right],$$

where

$$A_x = R_x\exp\left[i\left(\tilde{\varphi} + \phi + \Delta\beta z\right)\right],$$

$$A_y = R_y\exp\left[i\left(\tilde{\varphi} - \phi - \Delta\beta z\right)\right].$$

The presentation of the phase terms in the above form is attributable to the following effects: the intrinsic birefringence (i.e., the birefringence that would exist in the linear limit) is represented by $\pm\Delta\beta z$ and the self-phase modulation is described by $\pm\phi(t,z)$. In a weakly nonlinear and weakly birefringent medium, such as the typical glasses of optical fibres, the nonlinearity is assumed to be instantaneous. Generally speaking, the validity of this assumption depends on the pulse rise time. If the optical pulse becomes narrower, the assumption is no longer valid. We ignore this effect in this paper. We consider a Kerr-type nonlinear medium assuming that (i) the dielectric medium is isotropic, (ii) the third harmonic generation can be neglected, and (iii) the second-order nonlinear susceptibility is identically zero. Therefore, the slowly varying envelope of the nonresonance cubic polarization $\boldsymbol{\mathcal{P}}^{Kerr}(z,t)$ is

$$\boldsymbol{\mathcal{P}}^{Kerr} = 2\chi^{(3)}_{1122}(\boldsymbol{\mathcal{E}}\cdot\boldsymbol{\mathcal{E}}^*) + \chi^{(3)}_{1221}(\boldsymbol{\mathcal{E}}\cdot\boldsymbol{\mathcal{E}})\boldsymbol{\mathcal{E}}^*,$$

where the spatial and temporal dispersion is assumed to be absent. The above equality written in projections becomes

$$P_x^{Kerr} = \left[(a+b)|A_x|^2 + \right.$$
$$\left. + \left\{a + b\exp\left[-4i(\phi + \Delta\beta z)\right]\right\}|A_y|^2\right]A_x, \quad (1)$$

$$P_y^{Kerr} = \left[(a+b)|A_y|^2 + \right.$$
$$\left. + \left\{a + b\exp\left[4i(\phi+\Delta\beta z)\right]\right\}|A_x|^2\right] A_y, \quad (2)$$

where we use the notation $2\chi^{(3)}_{1122} = a$ and $\chi^{(3)}_{1221} = b$.

The Maxwell equations with nonlinear terms (1) and (2) and the resonance polarization included provide a set of coupled evolutionary equations for the amplitudes A_x and A_y, with the second-order group velocity dispersion taken into account:

$$i\frac{\partial A_x}{\partial z} + iv_x^{-1}\frac{\partial A_x}{\partial t} - \sigma_x\frac{\partial^2 A_x}{\partial t^2} + \Delta\beta A_x + $$
$$+ m_x\left(a_{11}|A_x|^2 + a_{12}|A_y|^2\right)A_x + qP_x = 0, \quad (3)$$

$$i\frac{\partial A_y}{\partial z} + iv_y^{-1}\frac{\partial A_y}{\partial t} - \sigma_y\frac{\partial^2 A_y}{\partial t^2} - \Delta\beta A_y + $$
$$+ m_y\left(a_{21}|A_x|^2 + a_{22}|A_y|^2\right)A_y + qP_y = 0. \quad (4)$$

In Eqs. (3) and (4), the effect of the resonance impurities is referred to by the slowly varying polarization envelopes P_x and P_y. The coefficient is defined in Eq. (15) in what follows. Nonresonance losses are ignored in (3) and (4) while the terms proportional to P_x and P_y represent the resonance absorption and refraction effects. In Eqs. (3) and (4), the following coefficients are introduced:

$$v_{x,y}^{-1} = \frac{d\beta_{x,y}}{d\omega}, \quad \sigma_{x,y} = \frac{1}{2}\frac{d^2\beta_{x,y}}{d\omega^2}, \quad m_{x,y} = \frac{\omega_0^2}{2c^2\beta_{x,y}}.$$

The self-modulation effect is represented by a_{11} and a_{22}. The factors a_{12} and a_{21} are responsible for cross-modulation. The effective nonlinear interaction parameter χ_{eff} is taken as a factor exclusively depending on the ratio of the material susceptibility tensor elements, i.e., on $\chi^{(3)}_{1221}/\chi^{(3)}_{1122}$. Thus, we have

$$a_{11} = a_{22} = \chi_{eff}, \quad a_{12} = \frac{a + b\exp\left[-4i(\phi+\Delta\beta z)\right]}{a+b}\chi_{eff},$$

$$a_{21} = \frac{a + b\exp\left[4i(\phi+\Delta\beta z)\right]}{a+b}\chi_{eff},$$

where the effective nonlinear interaction parameter χ_{eff} is defined as

$$\chi_{eff} = \frac{\int \chi^{(3)}_{1122}(\boldsymbol{\rho})|\Psi(\boldsymbol{\rho})|^4 d\boldsymbol{\rho}}{\int |\Psi(\boldsymbol{\rho})|^2 d\boldsymbol{\rho}}.$$

For the silica optical fiber, the third-order susceptibility mainly occurs because of a nonlinear electronic response and because $a = 2b$, and therefore,

$$a_{12} = \chi_{eff}\left(\frac{2}{3} + \frac{1}{3}\exp\left[-4i(\phi+\Delta\beta z)\right]\right),$$

$$a_{21} = \chi_{eff}\left(\frac{2}{3} + \frac{1}{3}\exp\left[4i(\phi+\Delta\beta z)\right]\right).$$

If we use the relations

$$\exp\left[4i(\phi+\Delta\beta z)\right] = \frac{E_xE_y^*}{E_x^*E_y} = \frac{A_xA_y^*}{A_x^*A_y},$$

$$\exp\left[-4i(\phi+\Delta\beta z)\right] = \frac{E_yE_x^*}{E_y^*E_x} = \frac{A_yA_x^*}{A_y^*A_x},$$

the nonlinear terms in Eqs. (3) and (4) become

$$\left(a_{11}|A_x|^2 + a_{12}|A_y|^2\right)A_x =$$
$$= \chi_{eff}\left(|A_x|^2A_x + \frac{2}{3}|A_y|^2A_x + \frac{1}{3}A_x^*A_y^2\right),$$

$$\left(a_{21}|A_x|^2 + a_{22}|A_y|^2\right)A_y =$$
$$= \chi_{eff}\left(|A_y|^2A_y + \frac{2}{3}|A_x|^2A_x + \frac{1}{3}A_y^*A_x^2\right).$$

The system of equations (3), (4) can now be rewritten in the final form

$$i\frac{\partial A_x}{\partial z} + iv_x^{-1}\frac{\partial A_x}{\partial t} - \sigma_x\frac{\partial^2 A_x}{\partial t^2} + \Delta\beta A_x +$$
$$+ m_x\chi_{eff}\left(|A_x|^2A_x + \frac{2}{3}|A_y|^2A_x + \frac{1}{3}A_x^*A_y^2\right) +$$
$$+ qP_x = 0, \quad (5)$$

$$i\frac{\partial A_y}{\partial z} + iv_y^{-1}\frac{\partial A_y}{\partial t} - \sigma_y\frac{\partial^2 A_y}{\partial t^2} - \Delta\beta A_y +$$
$$+ m_y\chi_{eff}\left(|A_y|^2A_y + \frac{2}{3}|A_x|^2A_x + \frac{1}{3}A_y^*A_x^2\right) +$$
$$+ qP_y = 0. \quad (6)$$

Equations (5) and (6) describe the propagation of a polarized radiation pulse in the birefringent fiber doped by resonance impurities. The electric field of the pulse is expressed by the Cartesian components. In order to emphasize the circular nature of birefringence, it seems reasonable to express the evolution equations in terms of the right- and left-hand circularly polarized fields

$$E_1 = E_x + iE_y, \quad E_2 = E_x - iE_y.$$

The corresponding complex envelopes can be written as

$$A_1 = A_x + iA_y, \quad A_2 = A_x - iA_y.$$

It is worth noting that

$$A_x^2 + A_y^2 = A_1A_2, \quad |A_x|^2 + |A_y|^2 = \frac{|A_1|^2 + |A_2|^2}{2}.$$

Hereafter, we set $\sigma_x = \sigma_y = \sigma$ and $m_x = m_y = m$. For the sake of generality, we also assume that the group velocities of the different polarization components are different. To proceed to numerical simulations of the nonlinear propagation of polarised pulses, it is convenient to introduce the dimensionless quantities

$$A_{1,2} = A_0 e_{1,2}, \quad z = \zeta L, \quad \tau = \left(t - \frac{z}{v}\right) t_0^{-1},$$

where t_0 is the characteristic time scale (which can be equal to the initial pulse duration t_{p0}), L and A_0 are the normalizing length and amplitude respectively, and the velocity of the time frame v is the velocity of the «center of gravity» of the optical pulse, $v^{-1} = (v_1^{-1} + v_2^{-1})/2$.

In terms of the new variables, the system of equations (4) becomes

$$i\frac{\partial e_1}{\partial \zeta} + i\frac{1}{\ell_g}\frac{\partial e_2}{\partial \tau} - \frac{s}{\ell_d}\frac{\partial^2 e_1}{\partial \tau^2} + \frac{1}{\ell_c}e_2 + \frac{1}{3\ell_k}\left(|e_1|^2 + 2|e_2|^2\right)e_1 + \left(\frac{Lq}{A_0}\right)P_1 = 0, \quad (7)$$

$$i\frac{\partial e_2}{\partial \zeta} + i\frac{1}{\ell_g}\frac{\partial e_1}{\partial \tau} - \frac{s}{\ell_d}\frac{\partial^2 e_2}{\partial \tau^2} + \frac{1}{\ell_c}e_1 + \frac{1}{3\ell_k}\left(|e_2|^2 + 2|e_1|^2\right)e_2 + \left(\frac{Lq}{A_0}\right)P_2 = 0, \quad (8)$$

where $P_1 = P_x + iP_y$ and $P_2 = P_x - iP_y$.

In Eqs. (7) and (8), the effect of resonance impurities is represented by the slowly varying polarization envelopes P_1 and P_2. The parameters ℓ_g, ℓ_c, ℓ_k, and ℓ_d are

$$\begin{aligned} \ell_g^{-1} &= \frac{L}{L_g} = \frac{L}{2t_0}\left(\frac{1}{v_1} - \frac{1}{v_2}\right), \\ \ell_c^{-1} &= \frac{L}{L_c} = \Delta\beta L, \\ \ell_k^{-1} &= \frac{L}{L_k} = L\chi_{eff}A_0^2\frac{\omega_0^2}{2c^2\beta}, \\ s = \operatorname{sign}\sigma, &\quad \ell_d^{-1} = \frac{L}{L_d} = \frac{L}{t_0^2}|\sigma|, \end{aligned} \quad (9)$$

where

$$\begin{aligned} L_d &= \frac{t_0^2}{|\sigma|}, \quad L_c = \frac{1}{\Delta\beta}, \\ L_k &= \frac{2\beta c^2}{\omega_0^2\chi_{eff}A_0^2}, \quad L_g = \frac{2v_1v_2t_0}{v_2 - v_1}. \end{aligned} \quad (10)$$

The length L_d characterizes the dispersion of the group velocities in each polarization mode. The quantity L_c stands for the coupling length. The corresponding terms in Eqs. (7) and (8) couple the right and left circular components of the electromagnetic wave, thereby implying the linear birefringence effect. The self- and cross-modulation effects reveal at the length L_k. The difference between the group velocities v_1 and v_2 of the counter-rotating polarized light waves causes a spatial divergence of the differently polarized components of the optical pulse (the walk-off effect) over the characteristic length L_g. A simple estimate gives the ratio

$$\frac{\ell_c}{\ell_g} \approx \frac{\lambda}{4\pi(ct_{p0})}.$$

It follows that in the picosecond pulse range, the terms related to the group velocities are small compared to the linear coupling terms. However, the walk-off effect can be important when we pass to the femto-second pulse duration domain.

Equations (7) and (8) must be supplemented by equations describing the temporal behavior of the density matrix elements for the ensemble of two-level atoms immersed in the fiber host material whose levels are degenerate with respect to the projections of the angular momenta j_a and j_b. For definiteness, we consider the case where $j_a = 1 \to j_b = 0$. The resonance transition is characterised by the dipole moment element $d_{13} = d_{23} = d_{31}^* = d_{32}^* = d$. The effective matrix element of the dipole transition is given by

$$d_{eff} = \frac{\int d(\boldsymbol{\rho})|\Psi(\boldsymbol{\rho})|^2 d\boldsymbol{\rho}}{\int |\Psi(\boldsymbol{\rho})|^2 d\boldsymbol{\rho}}.$$

The vector $\boldsymbol{\rho}$ lies in the plane normal to the optical fiber axis.

The temporal behavior of the resonant impurities is governed by a system of the generalized Bloch equations [13]. For slowly varying elements of the density matrix $\hat{\rho}$ describing the transition between the states $|a, m\rangle = |j_a = 1,\ m = \pm 1\rangle$, and $|b\rangle = |j_b = 0,\ m = 0\rangle$, we introduce the notation

$$\begin{aligned} &\rho_{12} = \langle a, -1|\hat{\rho}|a, +1\rangle, \quad \rho_{13} = \langle a, -1|\hat{\rho}|b\rangle, \\ &\rho_{23} = \langle a, +1|\hat{\rho}|b\rangle, \quad \rho_{11} = \langle a, -1|\hat{\rho}|a, -1\rangle, \\ &\rho_{22} = \langle a, +1|\hat{\rho}|a, +1\rangle, \quad \rho_{33} = \langle b|\hat{\rho}|b\rangle, \\ &\rho_{kl} = \rho_{lk}^*, \quad l, k = 1, 2, 3. \end{aligned} \quad (11)$$

The initial conditions are given by

$$\rho_{33}(0) = 1, \quad \rho_{22}(0) = \rho_{11}(0) = 0,$$

$$\rho_{12}(0) = \rho_{13}(0) = \rho_{23}(0) = 0.$$

The change of variables $\rho_{12} = m_{21}$, $\rho_{21} = m_{12}$, $\rho_{11} = m_{11}$, $\rho_{22} = m_{22}$, $\rho_{33} = n$, $p_1 = -\rho_{13}$, $p_2 = -\rho_{23}$

allows writing the generalized system of Bloch equations in the compact form

$$\frac{\partial p_\alpha}{\partial \tau} = i\nu p_\alpha - if\left(\sum_{\alpha'} e_{\alpha'} m_{\alpha'\alpha} - e_\alpha n\right), \qquad (12)$$

$$\frac{\partial m_{\alpha\alpha'}}{\partial \tau} = -if(e^*_\alpha p_{\alpha'} - e_{\alpha'} p^*_\alpha), \qquad (13)$$

$$\frac{\partial n}{\partial \tau} = -if\sum_\alpha (e_\alpha p^*_\alpha - e^*_\alpha p_\alpha), \quad \alpha, \alpha' = 1, 2. \qquad (14)$$

The initial conditions are given by $n(0) = 1$ for the ground level population, $p_\alpha(0) = 0$ for the polarization, and $m_{\alpha\alpha'}(0) = 0$. We also assume that the pulse duration is much shorter than all the relaxation times in the resonance subsystem, which allows us to omit the relaxation terms in Eqs. (12)–(14). The dimensionless variables p_α entering Eqs. (12)–(14) are related to the polarization terms in Eqs. (7) and (8) by

$$\left(\frac{Lq}{A_0}\right) P_\alpha = \frac{L}{L_r} P_\alpha = \frac{L}{L_r}\langle p_\alpha\rangle = \frac{1}{\ell_r}\langle p_\alpha\rangle, \qquad (15)$$

where

$$q = \frac{2\pi\omega_0 n_a d_{eff}}{cn(\omega_0)},$$

$L_r = fL_r^{(2\pi)}$, n_a is the concentration of the impurity atoms, and $\langle\ \rangle$ denotes the summation over all atoms with the frequency detunings $\nu = \Delta\omega t_0$ from the center of the inhomogeneously broadened line. In (15), the characteristic length of the resonance interaction is

$$L_r^{(2\pi)} = \frac{cn\hbar}{\pi d^2_{eff}\omega_0 n_a t_0}. \qquad (16)$$

In the system of equations (12)–(13) and in expression (15),

$$f = \frac{d_{eff} A_0 t_0}{2\hbar} = \frac{A_0}{A_{2\pi}}$$

is the normalized effective oscillation frequency of the material variables of the resonance subsystem affected by the field of the amplitude A_0 and $A_{2\pi}$ is the amplitude of the SIT 2π-pulse.

The coupled system of Maxwell-Bloch equations (7)–(8) and (12)–(14) provides the mathematical basis for numerically simulating the propagation of short pulses of circularly polarized light in a nonlinear waveguide doped by resonance impurities. The solution of field equations (7)–(8) was obtained using one of the popular [17] finite difference implicit–explicit Crank–Nicolson numerical schemes, where the desired accuracy 0.001 was reached by iterations. Bloch equations (12)–(14) coupled to field equations (7)–(8) by the resonance polarization terms were solved by the predictor–corrector procedure. The predictor–corrector was run at every iteration in the Crank–Nicolson algorithm until the accuracy about 0.001 was achieved for the polarization components p_α in Eqs. (12)–(14). Although the code could produce the integration over the inhomogeneously broadened line of the resonance absorption, we restricted it to the homogeneous case and the exact resonance at this stage of numerical simulation, i.e., to $\nu = 0$ in (12). The results of calculations were the absolute value of the complex amplitudes $e_{1,2}(\zeta, \tau)$ of the counter-rotating right- and left-handed oppositely polarized fields. Following Winful [18], we examined the polarization state of the field in the optical pulse in terms of the azimuthal angle

$$\theta(\zeta, \tau) = \frac{\arg\xi}{2}$$

and the ellipticity

$$\varepsilon(\zeta, \tau) = \frac{|\xi| - 1}{|\xi| + 1},$$

where $\xi = e_1 e_2^{-1}$ is a complex quantity. The characteristic values of ε are given by $\varepsilon = 0$ for the linearly polarized light, $\varepsilon = +1$ for the purely right-hand circularly polarized light, and $\varepsilon = -1$ for the purely left-hand circularly polarized light. The parameter θ is the angle between the axis of the polarization ellipse and the slow principal axis of the birefringent fiber. It can vary within the interval $(-\pi/4, \pi/4)$.

The launched pulses are assumed to have the sech form,

$$e_{1,2}(0, \tau) = e_{m1,2}\,\mathrm{sech}\left(\frac{\tau - \tau_0}{\delta}\right), \qquad (17)$$

where $\delta = t_{p0} t_0^{-1}$ and τ_0 is the temporal coordinate of the input pulse center.

3. NUMERICAL ESTIMATES

We let the group velocity dispersion $D = 4\pi c\sigma\lambda_0^{-2}$ of the silica-based monomode fiber host material be typically $D = 15$ ps·nm^{-1}·km^{-1} at $\lambda_0 = 1.55\,\mu$m and the nonlinear index $n_2 \approx 10^{-13}$ esu. It then follows that

$$\sigma = \frac{1}{2}\left|\frac{d^2\beta}{d\omega^2}\right| \approx 10^{-28}\ \mathrm{s}^2\cdot\mathrm{cm}^{-1}.$$

The effective nonlinear interaction parameter is

$$\chi_{eff} \approx \frac{n_2 n}{2\pi} \approx 2.3\cdot 10^{-14}\ \text{esu}.$$

We adopt the value $d \approx 5 \cdot 10^{-21}$ esu (the transition $^4I_{5/2} \rightarrow {}^4I_{5/2}$ in Er^{3+} ions) and the impurity concentration $n_a \approx 10^{18}$ cm^{-3} that corresponds to realistic samples [19]. With the input pulse duration $t_{p0} = t_0 = 0.1$ ps, the dispersion length is $L_d = t_0^2\sigma^{-1} \approx 10^2$ cm. The polarization mode coupling effect occurs over the distance

$$L_c = \frac{1}{\Delta\beta} \approx \frac{\lambda_0}{2\pi\Delta n} \approx 25 \text{ cm}.$$

Here, we set $\Delta n \sim 10^{-6}$ [20]. The effect of the group velocity mismatch becomes noticeable at the characteristic distance

$$L_g = \frac{2v_1v_2}{v_2 - v_1}t_0 \approx \frac{2ct_0}{\Delta n} \approx 6 \cdot 10^4 \text{ cm}.$$

The spatial scale of the Kerr self- and cross-modulation process L_k depends on the field amplitude A_0 as

$$L_k \approx \frac{n\lambda_0}{\pi\chi_{eff}A_0^2}.$$

The balance between the fiber group velocity dispersion and the nonlinear pulse compression occurs when $L_k = L_d$. This gives the value of the one-soliton solution amplitude of a single nonlinear Schrödinger equation

$$A_{NLS} = \sqrt{\frac{\sigma n\lambda_0}{\pi t_0^2\chi_{eff}}} \approx 0.5 \cdot 10^4 \text{ esu}$$

for the 0.1 ps pulse duration. The corresponding length scale is $L_k^{(NLS)} \approx 70$ cm. The nonlinear Schrödinger one-soliton peak intensity can be estimated as

$$I_{NLS} = \frac{c(A_{NLS})^2}{8\pi} \approx 4 \cdot 10^9 \text{ W/cm}^2.$$

For comparison, the amplitude of a 0.1 ps 2π-pulse is $A_{2\pi} = 2\hbar d^{-1}t_0^{-1} \approx 4 \cdot 10^6$ esu. The peak intensity of the pulse reaches the magnitude $I_{2\pi} \approx 2 \cdot 10^{15}$ W/cm^2.

Another balance equality $L_k = L_c$ yields the electric field strength $A_c = (2n\Delta n\chi_{eff}^{-1})^{1/2}$ known as the characteristic light wave field for a continuous wave (cw) of a nonlinear directional coupler [21], $A_c \approx 10^4$ esu, the intensity $I_c \approx 1.5 \cdot 10^{10}$ W/cm^2. This broadly means that for the input field amplitude values higher than A_c, the nonlinear birefringence initiated by Kerr processes begins to have a noticeable effect.

The quantity

$$L_r^{(2\pi)} = \frac{n\hbar\lambda_0}{2\pi^2 n_a d^2 t_0} \approx 5 \cdot 10^2 \text{ cm}$$

is the distance in the sample over which the reciprocal reaction of the medium in the form of polarization and population differences develops to produce coherent transients, e.g., the self-induced transparency [13], photon echoes [22, 23], optical nutations, breather waves [24], etc. For signals with a small pulse area θ [24],

$$\theta = d\hbar^{-1}\int_{-\infty}^{\infty} R(z,t)dt,$$

the parameter $L_r^{(2\pi)}$ serves as the absorption length. The pulse area of the NLS soliton $\theta_{NLS} = \pi d\hbar^{-1}t_0A_{NLS} = 3 \cdot 10^{-3}\pi$ is extremely small in comparison with $\theta_{SIT} = 2\pi$.

4. EVOLUTION OF POLARIZATION STATES IN A FIBER. NUMERICAL ANALYSIS

We can now proceed to examine typical numerical results. We focus on the diagnostic of the temporal profile of the field amplitude and polarization parameters ε and θ at every cross-section of the nonlinear birefringent fiber. We assume the light wave to be in exact resonance with the homogeneously broadened atomic transition, i.e., $\nu = 0$. In order not to overcomplicate the problem, we also ignore the walk-off effect in this paper, although we observed some of its obvious results in our preliminary computations. In the numerical simulations demonstrated in Figs. 1–5 below, amplitudes (11) of the input pulses were chosen as $e_{m1} = \sqrt{3}/2$ and $e_{m2} = 1/2$, while the respective initial phases were 0 and π. We also set $f = 0.0015$, thus assuming that the resonance interaction process is not a strong perturbation to the fiber effects.

The propagation of a light pulse in a birefringent fiber is accompanied by the two-way coupling between the orthogonal counter-rotating polarization modes with the spatial beat period $2\pi\Delta\beta^{-1}$. No dispersion is involved in the numerical simulation at this stage. For the linear undoped fiber (i.e., when the contribution of the Kerr self- and cross-modulation effect can be neglected) the solution of Eqs. (7) and (8) is quite simple (Fig. 1*a, b*). The period of the partial energy transfer between the modes is $\ell_b = \pi\ell_c$. It is seen from the $3D$ plot of the azimuth θ and the ellipticity ε (Fig. 1*c, d*) that both functions are uniform across the pulse and oscillate in the course of propagation inside the fiber [18]. It is worth to note that if the launched pulse amplitudes were $e_{m1} = 1$, $e_{m2} = 0$, the azimuth angle θ would change from $-\pi/4$ to $\pi/4$ and the polarization state would change from the linear polarization ($\varepsilon = 0$) to a circular polarization of the opposite direction ($\varepsilon = \pm 1$).

When both polarizations are excited in an asymmetric manner, the ellipticity oscillates between the elliptical clockwise and elliptical anticlockwise polariza-

Fig. 1. a, b — normalized field strengths in counter-rotating polarization modes in birefringent ($\ell_c = 0.25$), linear ($\ell_k = \infty$), and dispersionless ($\ell_d = \infty$) fiber; c, d — space–time evolution of the azimuthal angle θ and the ellipticity ε; e, f — gray scale surfaces of the functions $\theta(\tau, \zeta)$ and $\varepsilon(\tau, \zeta)$; g — phase trajectories of θ vs ε taken at $\tau = \tau_0$ for a different $\eta = (e_{m1}e_{m2}^{-1})^2$ ratio (see in text)

tions. This is clearly seen from the gray scale modular surface of $\theta(\zeta,\tau)$ and $\varepsilon(\zeta,\tau)$ (Figs. 1*e* and 1*f*). The dark gray up to black corresponds to the maxima of the plotted function, while the light gray down to white to the minima. The phase trajectories on the ε vs θ plane (with ε and θ calculated at the moments of peak intensity of the pulse) parameterized by ζ are closed circles (Fig. 1*g*). In this picture, each trajectory is associated with a different $\eta = (e_{m1}e_{m2}^{-1})^2$ ratio. The outer curve pertains to $\eta = 999$. The subsequent cycles correspond to η = 99, 9, 3, 1.5, 1.22. The biggest circle is the ultimate trajectory related to a nearly net circularly right-hand polarized light and small circles correspond to a nearly linearly polarized light. The circle in open dots corresponds to the case that was numerically investigated: $e_{m1} = \sqrt{3}/2$ and $e_{m2} = 1/2$. This numerical picture is in good agreement with the one presented in [18] for the cw-waves.

With the Kerr and walk-off effects ignored, the combined action of the linear birefringence ($\ell_c = 0.25$) and dispersion ($\ell_d = 1.0$) provides a well interpretable effect of the intensity hump spreading (Fig. 2*a, b*) in the depth of the fiber, as is clearly seen in the gray scale map (Fig. 2*c, d*). In this case, the polarization properties of the travelling field (Fig. 2*e, f*) are quite similar to those in Fig. 1. The spikes on both sides of the central area in Fig. 2*e, f* are the result of numerical fluctuations provoking random switchovers of the ellipticity ε and the azimuth θ on the wings of the propagating pulse, where the field is extremely weak in both polarizations. In Fig. 2*g*, we display the phase plane (ε vs θ) for the coupling + dispersion case for the parameters $\eta = 999$, 9, 3, 1.22. The value $\eta = 3$ corresponds to the case under numerical simulations. The plotted curves are quite similar to those in Fig. 1*g*.

The interplay between the linear coupling and the Kerr nonlinear phase modulation yields the picture that was not immediately evident (Fig. 3). We injected the pulses of the counter rotating polarization in the fiber, with the amplitudes of the pulses expressed in physical units satisfying the conditions $A_{m1} \approx 2A_c$ and $A_{m2} \approx A_c$. This corresponds to choosing $\ell_c = 0.25$ and $\ell_k = 0.05$ for the characteristic lengths. In this case, one could expect the Kerr compressing every time the energy couples back to the mode from the conjuncted polarization state. Instead, we observe the interference between the coupling processes with different beat periods. The coupling is revealed in the form of the inserted cycles when every new growth of the amplitude begins while the previous one has not yet finished. The physical explanation may be found if one notes that both input amplitudes are chosen to be of the order of the critical strength of the electric field A_c for cw switching. For such intensities, the Kerr processes become sufficiently strong to make the birefringence a nonlinear process and the beat period can even grow unlimitedly [25–28]. Attention must be drawn to the fact that the periodicity of the onsets of the back and forth coupling cycles approaches the value prescribed by the choice $\ell_c = 0.25$ (compare with Fig. 1). The linear behavior occurs only on the slopes of the pulse envelope, where the field intensity has not reached the critical value. The further growth of the pulse field strength in a pulse envelope forces the beat period to increase as well. The result is seen in Fig. 3*a, b* and *c, d* showing the $3D$ picture and the gray scale map of the polarization mode dynamics. The envelopes of the field in both polarization modes experience a temporal counter-phase modulation in the central part of the propagating waveform (Fig. 3*h*). The modulation of the amplitudes of the counter rotating polarization modes leads to the oscillation of ε and θ over τ in the propagating light wave that is clearly seen from the gray scale maps in Fig. 3*e, f* and on the comparative plots of the fields, ellipticity, and azimuth at the exit from fiber placed in Fig. 3*h*. Picture *g* in Fig. 3 shows the (ε,θ) phase plane for the same values of the parameter η as above. Unlike the previous cases, the ultimate circle is distorted, which agrees with the analysis in [18]. This cycle is smeared because the spatial modulation of the peak intensity of polarization components is complex.

Our calculations presented in Fig. 4 illustrate the combined action of the linear birefringence, Kerr nonlinearity, and dispersion. The dispersion length $\ell_d = 1$ serves as a scale length, while the coupling length ℓ_c and the Kerr length ℓ_k are shorter, $\ell_k = 0.1$ and $\ell_c = 0.25$. The choice of parameters dictates the values of the polarization mode amplitudes at the entrance to the fiber: $A_{m1} \approx \sqrt{2}\,A_c \approx 2.7A_{NLS}$ and $A_{m2} = 0.8A_c \approx 1.6A_{NLS}$. Weak ripples at the edges of the computational grid are due to the time boundary conditions.

The current case is not a completely integrable problem because of the inter-mode coupling. The propagating pulse cannot find a stable form at least over the distance considered here. In one of our preliminary computations under the conditions similar to those in Fig. 4, but with $\ell_k \approx 0.05$ (i.e., for a greater amplitude), we observed the breaking up of the input pulses of both polarizations into two separate subpulses subsequently scattering aside.

The periodical squeezing of the pulse shape, a feature of a high-order NLS solution, produces new oscillations on the wings of the pulse (Fig. 4*a, b*) be-

Fig. 2. a, b — the same as in Fig. 1 for the parameters chosen as $\ell_k = \infty$, $\ell_c = 0.25$, and $\ell_d = 1.0$; c, d — gray scale maps of the central parts of pictures a and b; e, f — gray scale maps of the azimuthal angle θ and the ellipticity ε; g — the same as in Fig. 1

Fig. 3. a, b — normalized field strengths in polarization components of the pulse propagating in birefringent nonlinear and dispersionless fiber with $\ell_c = 0.25$, $\ell_k = 0.05$; c, d, e, and f — the same as in Fig. 2; g — the same as in Fig. 1; h — from top to bottom: the azimuthal angle θ, the ellipticity ε, and the polarization mode modules at the exit of the fiber

Fig. 4. a, b — normalized field strengths in polarization components of the pulse for $\ell_c = 0.25$, $\ell_d = 1.0$, and $\ell_k = 0.1$; c, d, e, and f — the same as in Fig. 2; g — polarization mode shapes (absolute values) at the entrance (dashed line) and at the exit (solid line) of the fiber

cause the Kerr processes and dispersion are spatially mismatched. The dispersion spreading is noticeable at several ($\zeta \sim 4$) normalized lengths when the dispersion chirp fills almost the entire time window (Fig. 4*c, d*). It is then natural that polarization properties of the light wave (i.e., the alternation of dark and light shades of the gray) map the broadening area of the spatial–temporal oscillation of the polarization components (Fig. 4*e, f*) caused by the dispersion, thereby making the entire picture rather complicated. As in Fig. 3*h*, the oscillations of the field remain out of phase in polarization modes (Fig. 4*g*). We note that there are fewer coupling periods in Fig. 4*a, b* than in Fig. 1 or Fig. 2. Clearly, the nonlinear narrowing and peak amplification drive the propagation of the pulse into a nonlinear birefringent regime. A further growth of the pulse input amplitudes strengthens the inequality $\ell_k < \ell_d$, thereby making the process somewhat analogous to that in Fig. 3, plus the dispersion-originated oscillations spreading away of the sharp central peak.

The resonance interaction of a short pulse with the ensemble of resonance atoms is now added to the conventional fiber effects as indicated in Eqs. (7)–(8) and (12)–(14). The evolutionary behavior of the counter circularly polarized components with the input amplitudes $A_{m1} \approx 2A_c \approx 4A_{NLS}$, $A_{m2} \approx A_c \approx 2A_{NLS}$ is plotted in Fig. 5*a, b* and *c, d*. We assume the resonance interaction to be weak by setting $f = 0.0015$. Under this condition, the population differences insignificantly deviate from their initial values. The spatial scale of the process is $\ell_d = 1.0$, whereas $\ell_k = 0.05$, $\ell_c = 0.25$, and $\ell_r = 0.01$. The value of the resonance interaction length $L_r^{(2\pi)}$ can be estimated as $L_r^{(2\pi)} \approx 7L_d$ (see (6)). This means that the total length of the fiber in Fig. 5 is about $0.6L_r^{(2\pi)}$ or $4L_d$. The resonance interaction process transfers energy more effectively than the dispersion off the pulse to the radiation born by the reciprocal reaction of the medium in the pulse after the action region. It is then clear that in comparison with Fig. 4, the amplitudes of the humps rapidly decrease in the propagation direction (Fig. 5*a, b*).

Attention should be drawn to two humps in the center of Fig. 5*g*. These are the above-mentioned relics of the NLS N-soliton break up. The visible asymmetry of the pattern relative to the initial pulse position results from the delayed response of the resonance subset. Generally, we can predict that at longer distances inside the doped fiber, the well-evolved dispersion and coherent «ring» effects can hardly be distinguished.

The polarization properties of the light pulse are displayed in the gray scale maps in Fig. 5*e, f*. It is interesting to note that these pictures preserve the periodic alternation of the regions with the opposite ellipticity and azimuthal angle owing to the linear coupling (see Fig. 2*e, f*). In our further computations (not shown), when we set $\ell_r = 0.001$ for the ten times larger concentration of impurities, we saw the resonance oscillations already filling the entire (ζ, τ) computational area at an early stage of the pulse propagation. It was interesting to observe how the increase of the dopant concentration developed the generic picture of the periodic azimuth and ellipticity variations with the same beat period seen in Figs. 1 and 2. Qualitatively, this can be regarded as a result of the resonance absorption when the progressive dumping of the field humps decreases the field amplitude below the critical value of the electric field strength A_c, thereby driving the process back into the linear regime, when ℓ_c begins to be shorter than ℓ_k. Anyway, because the dispersion and the resonance interaction are time-dependent processes, they introduce a temporal modulation to the basic polarization picture consistently with the linear birefringence.

The comparison of our numerical simulation with the known results [25] is displayed in Figs. 6 and 7. The parameters introduced in this paper correspond to the analogous quantities in [25, Fig. 1*b*] if we set $\ell_d = 2$, $\ell_c = 4$, $\ell_k = 0.33$, and $e_{m1}^2 = 1.25$. This yields the estimate $A_{m1} \approx 4A_c \approx 3A_{NLS}$, $A_{m2} = 0$ in physical units for the input field amplitudes in the cases depicted in Figs. 6 and 7. In both pictures, we kept the original [25] length of the fiber, although it corresponds to $\ell_{fiber} = 8\ell_d$ in our conventions. The discrepancy originates from renormalizing the factor in the dispersion term in (5) by the coefficient 1/2. In Fig. 6, we reproduce the results of [25] observing the formation of two distinct periodicities of the coupling process between the modes. The pulse shape dynamics (Fig. 6*a, b* and *c, d*) can be physically interpreted in terms of the ℓ_c/ℓ_k ratio, which in the current case is the biggest of all those described above, namely $\ell_c/\ell_k = 12$. The coupling process between the modes must then reveal a nonlinear behavior because of the power dependence of the energy exchange period. In the L_d units, the linear coupling length (i.e., at low power) should have been $\ell_b = L_b L_d^{-1} = \pi L_c L_d^{-1} = 2\pi$. As a matter of fact, the visual estimate of the beat period in Fig. 6*a, b* and *c, d* yields $\ell_b^{nl} = L_b^{nl} L_d^{-1} \approx 4.5\pi > \ell_b$. We note that in the case under consideration, the period of the typical higher-order soliton compressions became power dependent. This manifests the difference between our approach and the ideal completely integrable model [29].

Figure 6*e, f* shows the gray scale (τ, ζ) maps of the azimuthal angle θ and the ellipticity parameter ε. It is seen that the time–space features of the dispersion pro-

Fig. 5. a, b — normalized field strengths in polarization components of the pulse propagating in a fiber with impurities ($\ell_r = 0.01$). Other parameters are $\ell_c = 0.25$, $\ell_d = 1.0$, and $\ell_k = 0.05$. c, d, e, and f — the same as in Figs. 2–4; g — polarization mode modules at the entrance (dashed line) and at the exit (solid line) of the fiber

Fig. 6. Pictures a–d show the same as Fig. 4a–d with the parameters $\ell_d = 2.0$, $\ell_c = 4.0$, $\ell_k = 0.33$

cess are distinctly reproduced. The brightest lines and spots in the region of pulse slopes demonstrate abrupt changes of the polarization state due to a rapid growth or drop of the field in one polarization mode compared to the other.

The reciprocal reaction of resonance impurities on the field of the propagating short pulse can noticeably change the space–time picture of polarization dynamics in the analysis of nonlinear effects in a pure fiber by Trillo et al. [25]. For the computational variant presented in Fig. 7, all the fiber parameters and input pulse amplitudes remain unaltered with respect to the case in Fig. 6. But in contrast to the variant in Fig. 6, the field of a short propagating pulse is now coupled to the resonance subsystem, and therefore, the complete system in Eqs. (7)–(8) and (12)–(14) must be solved numerically. The value of the normalized resonance length was set as $\ell_r = 0.01$, with the corresponding physical length of the resonance interaction $L_r^{(2\pi)} \approx 3L_d$ and the parameter $f = 0.0015$. The population difference between the resonance levels (which was computed but is not shown here) remains practically unchanged because the areas

Fig. 7. Pictures a–d show the same as Fig. 5a–d with the parameters $\ell_d = 2.0$, $\ell_c = 4.0$, $\ell_k = 0.33$, $\ell_r = 0.01$

of the coherent pulses are small.

The resonance response changes the waveform of the polarized light. In the current case, the intensity of the interaction process is higher than in Fig. 5 because the resonance interaction length is shorter. The energy of the input pulse is rapidly transferred to oscillations of the resonance polarization (Fig. 7*a, b*) in the region of the retarded action of the propagating pulse (Fig. 7*c, d*). The resonance absorption noticeably weakens the humps for greater ζ, moving them out of the calculation window. It is interesting that the intensity damping leads to the restoration of the linear beat period $\ell_b \approx 2\pi$, and the propagation process is therefore converted from a nonlinear regime to the linear one. Thus, the nonlinear phase modulation and dispersion do not play the leading role in the dynamics of the pulse. This explains why the maps of polarization parameters (Fig. 7*e, f*) are so flat. In fact, the slow variations in the form of dark and light stripes due to the linear coupling process period (Fig. 7*e, f*) are the only prominent feature of the displayed plots. It is interesting that the oscillations of the field enve-

lope caused by the retarded reciprocal reaction of the medium, being in phase, do not produce a modulation of the polarization parameters ε and θ except in the vicinities of especially rapid changes of the field.

Passing to a higher concentration ($\ell_r = 0.001$) demonstrates the typical features of the coherent phenomena in a resonance medium. In the depth of the fiber, there is no solitary wave; instead, we have a waveform with the oscillating envelope. This wave packet rapidly shifts towards the later times, leaving the calculation grid somewhere at $\zeta = 6$. The polarization properties remain indifferent to the complete destruction of the pulse and the linear birefringence beat period is preserved.

5. CONCLUSIONS

In this paper, we have tried to give an indication of a rich space–time dynamics arising from the propagation of an elliptically polarized light pulse in a nonlinear birefringent doped fiber. The resonance impurities in the form of two-level atoms were included in the model in addition to the full set of nonlinear fiber effects. We have concentrated on the case of a weak input field, for which the amplitude of the pulse is about the amplitude of a single NLS pulse and the coupling to the resonance system is therefore not strong. A trivial account of the weak effect of the resonance system on the propagating optical pulse leads to a linear absorption. Generally speaking, the coherent interaction of short pulses with resonance atoms is a non-Markovian process [24, 30]. Moreover, the degeneration of resonance levels gives the contribution to birefringence that is nonlocal in time. With the exception of big detunings off the resonance, the analytical consideration of all these effects is extremely difficult. Therefore, a direct numerical simulation of the pulse evolution is preferable. But even within the weak-interaction approximation, the general picture proved to be sufficiently complex. The polarization properties of the pulsed light are nonstationary across the pulse width and can also drastically change in space. Our numerical simulations show that the polarization dynamic is basically featured by the interplay between the Kerr nonlinear self- and cross-phase modulation and dispersion, while the linear birefringence leads to a spatial modulation of the azimuthal angle and the ellipticity. There is a range of the input amplitudes where the birefringence becomes a nonlinear power dependent process because of the Kerr cross-phase modulation, and the power beat period can therefore grow. At the same time, when the spatial scale of the resonance interaction becomes less than or about the characteristic lengths of the fiber effects, the propagating pulse experiences a strong distortion and a resonance absorption. The intensity damping leads to the restoration of the linear beat period, thereby converting the propagation process from the nonlinear regime to the linear one. To observe perceptible coherent effects such as the SIT or the photon echo, one must take many times more powerful pulses, which in their turn excite a higher-order N-soliton effect. Therefore, the problem requires a special treatment.

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