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INDUCED ELECTRON CAPTURE IN FIELD-ASSISTED ENERGETIC ATOMIC COLLISIONS

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We present model consideration for the process of the electron capture in energetic nonrelativstic collisions of light atomic particles in the presence of a relatively weak low-frequency external electromagnetic field. The field is treated as an elliptically polarized quantum single-mode field. Establishing the validity of the dipole approximation to the electron transfer where the total momentum of all the emitted or absorbed photons can be well above the typical inneratomic momenta of the electron in its initial and final states and neglecting the Doppler and aberration effects, we give a fully nonrelativistic treatment for the field-assisted collisions and show that the capture cross section is invariant under Galilean transformations. The model consideration suggests that the field can substantially influence the capture dynamics and considerably change the capture cross section compared to the field-free collisions. This is especially the case if the «resonance» conditions $n\omega \approx \pm v^2/2$ are satisfied, with $n\omega$ being the energy transferred to or absorbed from the electromagnetic field and v the collision velocity.

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1. INTRODUCTION

Electron transfer in nonrelativistic atomic collisions is one of the fundamental problems in atomic physics that has been studied in great detail (see, e.g., [1–3] and references therein). The inclusion of an electromagnetic field into atomic collisions introduces new degrees of freedom and can substantially influence the collision process under certain conditions. A good example of this influence is represented by the radiative electron capture process, where the interaction with the radiation field (with the QED photon vacuum field) dramatically changes the capture process at high collision velocities (see, e.g., [4, 5], and references therein). The present paper is an attempt of a preliminary analysis of the possibility to influence the electron transfer process in fast nonrelativistic collisions by an external monochromatic electromagnetic field. We consider nearly symmetrical collisions of light atomic particles, $Z_2 \sim Z_1 \sim 1$, one of which (Z_1) initially carries an electron in the ground state and the second is a bare nucleus. We assume that the collision velocity v is sufficiently high, $v \gg Z_{1,2}$, but not relativistic, $v \ll c$, where $c = 137$ a.u. is the speed of light. The electromagnetic field is treated as a quantized single-mode field that initially contains a definite number of photons. This field is assumed to be elliptically polarized in general and to have a frequency that is small compared to the minimal excitation energy of the electron bound in the ground state of the particles Z_1 or Z_2. The electric field strength F_0 is regarded to be small compared to a typical inner atomic field in the ground state,

$$F_{at} \sim \frac{Z_{1,2}}{a_0^2} = Z_{1,2}^3 \text{ a.u.}, \quad F_0 \ll F_{at}.$$

We also assume that there are no multiphoton resonances between the ground and excited states in the particles 1 and 2. Using such a low-frequency electromagnetic field, we pursue two objects. First, a relatively weak low-frequency field allows us to avoid substantial depleting of the electron ground states in collision-free atomic systems. Second, as we see below, the coupling of the electron to a field in the charge exchange process is effectively stronger for lower frequencies.

Atomic units are used throughout the paper unless otherwise stated.

2. GENERAL CONSIDERATION

2.1. Preliminary remarks

Because the collision velocity v is supposed to be sufficiently high, we can use the impact parameter approximation. We assume that the electron is initially in the ground state of particle 1 moving along a straight-line trajectory

$$\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$$

in an inertial reference frame K. Particle 1 collides with particle 2 that rests at the origin in K. As the result of the collision in the presence of an electromagnetic field, the electron undergoes a transition into the ground state of particle 2 simultaneously with the induced emission or absorption of $m = 0, 1, 2, \ldots$ photons with the frequency ω.

To describe the system consisting of an electron subjected to the Coulomb interaction with two colliding Coulomb centers and the electromagnetic field, we take the Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi\rangle = (H_{col} + H_{int} + H_{ph})\,|\Psi\rangle, \tag{1}$$

where $|\Psi\rangle$ is the state vector of the system,

$$H_{col} = -\frac{\Delta}{2} + V_1(\mathbf{r} - \mathbf{R}(t)) + V_2(\mathbf{r}) \tag{2}$$

is the Hamiltonian of the electron in the fields of the two colliding centers, and

$$H_{int} = \frac{1}{c}\mathbf{A}\cdot\hat{\mathbf{p}} + \frac{A^2}{2c^2} \tag{3}$$

is the interaction of the electron with the electromagnetic field, with $\hat{\mathbf{p}}$ being the electron momentum operator. In the Schrödinger picture, the vector potential $\mathbf{A}$ of the quantized electromagnetic field is given by (see, e.g., [6, 7])

$$\mathbf{A} = \lambda\left(\mathbf{e}a\exp(i\mathbf{k}\cdot\mathbf{r}) + \mathbf{e}^*a^\dagger\exp(-i\mathbf{k}\cdot\mathbf{r})\right), \tag{4}$$

where

$$\lambda = c\sqrt{\frac{2\pi}{\omega V}},$$

V is the quantization volume, $\mathbf{k}$ is the photon momentum, and a and $a^\dagger$ are the time-independent annihilation and creation operators, respectively. We assume that these operators are space-independent, i.e., that vector potential (4) corresponds to a plane wave. The polarization vectors $\mathbf{e}$ and $\mathbf{e}^*$ are given by

$$\begin{aligned} \mathbf{e} &= \mathbf{e}_1\cos(\xi/2) + i\mathbf{e}_2\sin(\xi/2), \\ \mathbf{e}^* &= \mathbf{e}_1\cos(\xi/2) - i\mathbf{e}_2\sin(\xi/2), \end{aligned} \tag{5}$$

where $\mathbf{e}_1$ and $\mathbf{e}_2$ are the unit vectors that are perpendicular to the photon momentum $\mathbf{k}$ and to each other,

$$\mathbf{e}_{1,2}\cdot\mathbf{k} = 0, \quad \mathbf{e}_1\cdot\mathbf{e}_2 = 0.$$

The vectors $\mathbf{e}$ and $\mathbf{e}^*$ satisfy the relations

$$\begin{aligned} &\mathbf{e}\cdot\mathbf{e}^* = 1, \\ &\mathbf{e}\cdot\mathbf{e} = \mathbf{e}^*\cdot\mathbf{e}^* = \cos\xi. \end{aligned} \tag{6}$$

The angle ξ determines the degree of polarization, e.g. $\xi = 0$ and $\xi = \pi/2$ correspond to the linear and circular polarizations, respectively.

The term H_{ph} in Eq. (1) describes the free electromagnetic field. It can be written as (see the appendix)

$$H_{ph} = \omega(N_a - N), \tag{7}$$

where

$$N_a = \frac{1}{2}(aa^\dagger + a^\dagger a)$$

and N is the initial number of photons in the electromagnetic field.

With the ansatz

$$|\Psi\rangle = \exp\left(-i\mathbf{k}\cdot\mathbf{r}(N_a - N)\right)|\Psi_1\rangle, \tag{8}$$

the Schrödinger equation can be rewritten as

$$\begin{aligned} i\frac{\partial}{\partial t}|\Psi_1\rangle = H_{col}|\Psi_1\rangle - \left(\mathbf{k}(N_a - N) - \frac{1}{c}\mathbf{A}_0\right)\cdot\hat{\mathbf{p}}|\Psi_1\rangle + \\ + \left(\frac{k^2(N_a - N)^2}{2} + \omega(N_a - N) + \frac{A_0^2}{2c^2}\right)|\Psi_1\rangle, \end{aligned} \tag{9}$$

where

$$\mathbf{A}_0 = \lambda\left(\mathbf{e}a + \mathbf{e}^*a^\dagger\right) \tag{10}$$

is independent of the electron coordinates. Equation (9) can be simplified by noting two points. First, the term

$$\frac{k^2(N_a - N)^2}{2} = \omega(N_a - N)\frac{\omega(N_a - N)}{2c^2}$$

represents a relativistic correction to the term $\omega(N_a - N)$ and must be dropped within the accuracy of the nonrelativistic Schrödinger equation (see the appendix). Second, a typical change of the electron momentum in the electron transfer process is approximately equal to $\mathbf{v}$ and the term $\mathbf{k}(N_a - N)\hat{\mathbf{p}}|\Psi_1\rangle$ can be roughly estimated as

$$\mathbf{k}(N_a - N)\hat{\mathbf{p}}|\Psi_1\rangle \sim \mathbf{k}\cdot\mathbf{v}(N_a - N)|\Psi_1\rangle \sim \frac{v}{c}\omega(N_a - N)|\Psi_1\rangle.$$

Thus, it is seen that the main effect of the term $\mathbf{k}(N_a - N)\hat{\mathbf{p}}|\Psi_1\rangle$ is related to the Doppler shift. For nonrelativistic collisions, one has

$$\mathbf{k}(N_a - N)\hat{\mathbf{p}}|\Psi_1\rangle \ll \omega(N_a - N)|\Psi_1\rangle$$

and the term $\mathbf{k}(N_a - N)\hat{\mathbf{p}}|\Psi_1\rangle$ can also be omitted. The Schrödinger equation then becomes

$$i\frac{\partial}{\partial t}|\Psi_1\rangle = H_{col}|\Psi_1\rangle + \left(\frac{1}{c}\mathbf{A}_0\cdot\hat{\mathbf{p}} + \frac{A_0^2}{2c^2}\right)|\Psi_1\rangle + \omega(N_a - N)|\Psi_1\rangle. \quad (11)$$

This equation looks like the Schrödinger equation with the electromagnetic field taken in the dipole approximation. A remark about the validity of the dipole approximation for the field-assited electron transfer may now be in order. Although the momentum $k = \omega/c$ of one low-frequency photon is much less than a typical electron momentum in the ground state of the target ($p_1 \sim Z_1$) or of the projectile ($p_2 \sim Z_2$), the total momentum of all the emitted or absorbed photons can be well above $p_{1,2}$ (e.g., in the «resonance» case, see Sec. 3). We have analyzed the role of the photon momentum in the field-assisted electron transfer. The analysis shows that in general, the corrections to the capture cross section due to the photon momentum are of the order of v_e/c, where $v_e \sim v$ is a characteristic electron velocity in the process. Thus, with the electron assumed to be nonrelativistic in the capture process, the corrections to the capture cross section are of minor importance.

Now, with the validity of the dipole approximation for the electron transfer in nonrelativistic collisions being established, we can neglect the Doppler and aberration effects and give fully nonrelativistic treatment for the field-assisted electron transfer process where capture cross sections must be invariant under Galilean transformations [4].

Equation (11) can be further simplified. To this end, we consider the interaction term

$$\frac{1}{2c^2}A_0^2 = \frac{\lambda^2}{2c^2}\left((a^2 + a^{\dagger 2})\cos\xi + aa^\dagger + a^\dagger a\right)$$

in more detail. The quadratic terms a^2 and $a^{\dagger 2}$ can be removed from the Schrödinger equation by applying the so called «squeezed light» transformation (see, e.g., [8])

$$\begin{aligned} a &= b\,\mathrm{ch}\,\chi + b^\dagger\,\mathrm{sh}\,\chi, \\ a^\dagger &= b^\dagger\,\mathrm{ch}\,\chi + b\,\mathrm{sh}\,\chi, \end{aligned} \quad (12)$$

where

$$\mathrm{th}(2\chi) = -\frac{\lambda^2\cos\xi}{c^2\omega + \lambda^2}. \quad (13)$$

The corresponding Schrödinger equation for the electron interacting with «b-photons» is given by

$$i\frac{\partial}{\partial t}|\Psi_1\rangle = H_{col}|\Psi_1\rangle + \frac{\lambda}{c}\left(\mathbf{e}_b b + \mathbf{e}_b^* b^\dagger\right)\cdot\hat{\mathbf{p}} + \omega_{eff}N_b|\Psi_1\rangle, \quad (14)$$

where

$$N_b = \frac{1}{2}(b^\dagger b + bb^\dagger),$$

$$\omega_{eff} = \sqrt{(\omega + \lambda^2/c^2)^2 - (\lambda^4/c^4)\cos^2\xi}.$$

As follows from (13), the difference between «a-photons» and «b-photons» is determined by the factor

$$\frac{\lambda^2}{\omega c^2} = \frac{2\pi}{V\omega^2}.$$

Since the quantization volume V of a laser field is usually of a macroscopic dimension, we can assume that

$$\frac{2\pi}{V\omega^2} \to 0$$

except for extremely low frequences that are not considered in this paper. Therefore, we have $\chi \approx 0$ and the difference becomes very small. Disregarding this difference and replacing «b-photons» by «a-photons» in Eq. (14), we finally arrive at the Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi_1\rangle = H_{col}|\Psi_1\rangle + \frac{\lambda}{c}\left(\mathbf{e}a + \mathbf{e}^* a^\dagger\right)\cdot\hat{\mathbf{p}} + \omega(a^\dagger a - N)|\Psi_1\rangle. \quad (15)$$

In Eq. (15), we have also neglected the difference between

$$N_a = 0.5(aa^\dagger + a^\dagger a) = a^\dagger a + 1/2$$

and

$$N_a' = a^\dagger a,$$

which is inessential for the electron transfer process.

2.2. Model one-center electron states dressed by the interaction with an electromagnetic field

We regard the charge transfer process as an electron transition, due to a collision with the second center, between the field-dressed electron states centered on the target and the projectile. We first consider the problem of an electron bound to center 1, that moves in the frame K with a constant velocity $\mathbf{v}$ and is subjected to the electromagnetic field. As shown in the previous subsection, the corresponding Schrödinger equation can be written as

$$i\frac{\partial}{\partial t}|\Phi_{i,0}\rangle = \left(H_{at,1}+\frac{\mathbf{A}_0}{c}\cdot\hat{\mathbf{p}}+\omega(a^\dagger a-N)\right)|\Phi_{i,0}\rangle, \quad (16)$$

where

$$H_{at,1} = -\frac{\Delta}{2}+V_1(\mathbf{r}-\mathbf{R}(t)) \quad (17)$$

is the Hamiltonian of the electron in the Coulomb field of the moving center.

The state vector $|\Phi_{i,0}\rangle$ of the system consisting of the electron bound to the moving center and of the electromagnetic field containing initially N photons with the frequency ω can be expanded as

$$|\Phi_{i,0}(t)\rangle = \\ = \sum_\alpha\sum_n \exp\left(-in\omega t\right) a_{\alpha,n}(t)\psi_\alpha(t)|N+n\rangle, \quad (18)$$

where the unknown time-dependent coefficients $a_{\alpha,n}$ must be determined. In (18), the summation runs over all the electron states $\{\psi_\alpha\}$ including the continuum and over the photon states with different numbers of additional photons ($n=0,\pm1,\pm2,\dots$). The states ψ_α of the electron in the field of binding center 1 moving along a straigh-line trajectory

$$\mathbf{R}(t)=\mathbf{b}+\mathbf{v}t$$

are given by

$$\psi_\alpha(t)=\varphi_\alpha^{(1)}(\mathbf{r}-\mathbf{R}(t))\exp(-i\varepsilon_\alpha^{(1)}t)\times \\ \times\exp(i\mathbf{v}\cdot\mathbf{r})\exp\left(-i\frac{v^2}{2}t\right), \quad (19)$$

where $\varphi_\alpha^{(1)}(\mathbf{r})$ is the atomic state (discrete or continuous) of the electron at center 1 with the energy $\varepsilon_\alpha^{(1)}$.

Inserting (18) in (16), we obtain the system of differential equations for the unknown coefficients $a_{\alpha,n}$

$$i\frac{da_{\alpha,n}}{dt}=\frac{1}{c}\sum_m\sum_\beta a_{\beta,m}\exp(i(n-m)\omega t)\times \\ \times\langle N+n|\mathbf{A}_0|N+m\rangle\langle\psi_\alpha|\hat{\mathbf{p}}|\psi_\beta\rangle. \quad (20)$$

Because

$$\langle\psi_\alpha|\hat{\mathbf{p}}|\psi_\beta\rangle=\exp(i\omega_{\alpha\beta}t)\left(\mathbf{v}\delta_{\alpha\beta}+\langle\varphi_\alpha|\hat{\mathbf{p}}|\varphi_\beta\rangle\right), \quad (21)$$

where

$$\omega_{\alpha\beta}=\varepsilon_\alpha^{(1)}-\varepsilon_\beta^{(1)},$$

we obtain

$$i\frac{da_{\alpha,n}}{dt}=\frac{\mathbf{v}}{c}\sum_m a_{\alpha,m}\exp(i(n-m)\omega t)\times \\ \times\langle N+n|\mathbf{A}_0|N+m\rangle+ \\ +\frac{1}{c}\sum_m\sum_{\beta\neq\alpha}a_{\beta,m}\exp(i((n-m)\omega+\omega_{\alpha\beta})t)\times \\ \times\langle N+n|\mathbf{A}_0|N+m\rangle\langle\varphi_\alpha|\hat{\mathbf{p}}|\varphi_\beta\rangle. \quad (22)$$

The first and the second terms on the right-hand side of (22) correspond to different mechanisms of the dressing of the electron by the electromagnetic field. The double sum in (22) describes the part of the electron dressing that is accompanied by transitions of the electron into excited atomic states, including the atomic continuum. In our model treatment applying this part of the dressing can be neglected, which corresponds to taking the so-called diagonal dressing into account (see e.g. [9]). Because the electron initially occupies the state ψ_0, we then have

$$i\frac{da_{0,n}}{dt}=\frac{\mathbf{v}}{c}\sum_m a_{0,m}\exp(i(n-m)\omega t)\times \\ \times\langle N+n|\mathbf{A}_0|N+m\rangle,\quad a_{\alpha,n}=0,\quad \alpha\neq0. \quad (23)$$

Equations (23) together with the assumption that the coupling between the electron and the electromagnetic field is adiabatically switched on and off at $t\to-\infty$ and $t\to+\infty$, respectively, form the basis for the model of the system «bound electron + electromagnetic field».

The matrix elements $\langle N+n|\mathbf{A}_0|N+m\rangle$ are not equal to zero only for $m=n\pm1$. We assume the initial number of photons N to be very large, $N\gg|n|$ and $N\gg|m|$, which corresponds to regarding the electromagnetic field as an inexhaustible source and sink of photons. The matrix elements $\langle N+n|\mathbf{A}|N+n-1\rangle$ and $\langle N+n|\mathbf{A}|N+n+1\rangle$ can then be assumed to be n-independent and the system of equations (23) reduces to

$$i\frac{da_{0,n}}{dt}=\frac{\lambda\sqrt{N}}{c}\left(\mathbf{e}\cdot\mathbf{v}a_{0,n+1}\exp(-i\omega t)+ \\ +\mathbf{e}^*\cdot\mathbf{v}a_{0,n-1}\exp(i\omega t)\right). \quad (24)$$

In order to solve (24), it is convenient to rewrite the scalar products $\mathbf{e}\cdot\mathbf{v}$ and $\mathbf{e}^*\cdot\mathbf{v}$ as

$$\begin{aligned} \mathbf{e}\cdot\mathbf{v} &= v_0\exp(i\phi), \\ \mathbf{e}^*\cdot\mathbf{v} &= v_0\exp(-i\phi), \end{aligned} \tag{25}$$

where v_0 and ϕ are given by

$$\begin{aligned} v_0 &= \sqrt{(\mathbf{v}\cdot\mathbf{e}_1\cos(\xi/2))^2 + (\mathbf{v}\cdot\mathbf{e}_2\sin(\xi/2))^2}, \\ \phi &= \operatorname{arctg}\left(\frac{\mathbf{v}\cdot\mathbf{e}_2}{\mathbf{v}\cdot\mathbf{e}_1}\operatorname{tg}(\xi/2)\right). \end{aligned} \tag{26}$$

Making the ansatz

$$a_{0,n}(t) = f_n\exp(in(\omega t - \phi)), \tag{27}$$

where f_n are time-independent, and inserting (27) in (24), we obtain the simple relation

$$f_{n+1} + f_{n-1} = \frac{2n}{G}f_n, \tag{28}$$

where

$$G = -\frac{2\lambda v_0\sqrt{N}}{c\omega}. \tag{29}$$

The absolute value of G determines the effective strength of the electron–field coupling. Solutions of the recurrence relation (28) are the Bessel functions (see, e.g., [10]). Therefore,

$$a_{0,n} = C\zeta_n(G)\exp(in(\omega t - \phi)), \tag{30}$$

where ζ_n denotes the Bessel functions J_n, Y_n, $H_n^{(1)}$, $H_n^{(2)}$, or any linear combination thereof and C is n-independent.

Taking Eqs. (30) and (18) into accout, we rewrite the state vector as

$$|\Phi_{i,0}(t)\rangle = C\psi_0(t)\sum_n \zeta_n(G)\exp(-in\phi)|N+n\rangle. \tag{31}$$

In order to determine C and to find which of the Bessel functions corresponds to ζ_n, we note that in the absence of the coupling between the electron and the electromagnetic field,

$$\mathbf{A}_0\cdot\mathbf{v} = 0,$$

the state vector has the form

$$|\Phi_{i,0}(t)\rangle = \psi_0(t)|N\rangle. \tag{32}$$

Therefore, in order to recover Eq. (32) from Eq. (31), one must set $C = 1$ and $\zeta_n(G) = J_n(G)$ in Eq. (31) with J_n being the Bessel function of the first kind. Then the initial state vector becomes

$$|\Phi_{i,0}(t)\rangle = \psi_0(t)\sum_n J_n(G)\exp(-in\phi)|N+n\rangle. \tag{33}$$

This state describes the moving electron bound in the ground state and dressed by the interaction with the electromagnetic field. Because the coupling to the field is switched off as $t\to+\infty$, we have

$$|\Phi_{i,0}(t\to+\infty)\rangle = \psi_0(t).$$

Within the adopted approximation, therefore, if no collision event occurs, the state vector of the «electron + electromagnetic field» system is finally the same as initially. Thus the dressing given by (33) does not result in any electron transitions within the same center and can therefore be viewed, to some extent, as «hidden». It is the collision that can display the «hidden» dressing.

The final state vector $|\Phi_{f,m}\rangle$ describes the electron (finally) bound in the ground state $\varphi_0^{(2)}$ of particle 2 and the presence of $N+m$ photons. Within the approximation similar to that used to obtain the state vector $|\Phi(t)_{i,0}\rangle$, we obtain

$$|\Phi_{f,m}\rangle = \varphi_0^{(2)}\exp(-i(\varepsilon_0^{(2)} + m\omega)t)|N+m\rangle. \tag{34}$$

2.3. Transition amplitudes and cross sections

Because the collision velocity is supposed to be sufficiently high, one can use perturbation theory in the Coulomb interaction to consider the charge exchange. It is known (see, e.g., [1–3] and references therein) that the boundary-corrected Born approximation must be employed in order to obtain reliable results for the nonradiative charge exchange processes in energetic Coulomb collisions. However, in order to obtain just a preliminary insight into the field-assisted electron capture, we use a simpler approach that does not take the Coulomb-corrected boundary conditions into account and corresponds to the OBK approximation for the field-free collisions. It is known (see, e.g., [1]) that for external field-free collisions, the second-order terms (representing the Thomas double scattering mechanism) are of a minor practical importance for the total $1s$-$1s$ capture cross sections. For example, in the

$$p + H(1s) \to H(1s) + p$$

collisions, the second-order term dominates over the first-order one at $v \gtrsim 80$ a.u. At these velocities, however, the radiative electron capture dominates over the nonradiative one and in addition, the relativistic effects cannot be ignored in general. In the region of the collision velocities of interest in the present paper ($v\sim 10$ a.u.), the first-order term dominates in the $1s$-$1s$ cross sections. In this paper, we consider the $1s$-$1s$

capture and use the first-order approach[1)], which corresponds to the first-order OBK (OBK1) approximation for the field-free collisions (see, e.g., [12, 1]).

2.3.1. Prior form of the cross section of the field-assisted charge exchange

In the first order of perturbation theory in the prior form for the field-assisted electron capture accompanied by the emission or absorption of $|n|$ photons, the transition amplitude is given by

$$a_{prior}^{(n)} = -i \int_{-\infty}^{\infty} dt \langle \Phi_{f,n} | V_2(\mathbf{r}) | \Phi_{i,0} \rangle, \qquad (35)$$

where V_2 is the interaction of the electron with the second center and the initial and final state vectors are given by Eqs. (33) and (34), respectively.

After some straighforward but lengthy algebra, we rewrite the transition amplitude as

$$a_{prior}^{(n)} = 2\pi i J_n(G) \int d^3q \chi_0^{(1)}(\mathbf{q}+\mathbf{v}) \left(\varepsilon_0^{(2)} - 0.5q^2\right) \times \\ \times \left(\chi_0^{(2)}(\mathbf{q})\right)^* \exp(i\mathbf{q}\cdot\mathbf{b}) \times \\ \times \delta\left(\varepsilon_0^{(2)} + n\omega - \varepsilon_0^{(1)} + \frac{v^2}{2} + \mathbf{q}\cdot\mathbf{v}\right), \qquad (36)$$

where $\chi_0^{(1)}(\mathbf{q})$ and $\chi_0^{(2)}(\mathbf{q})$ are the Fourier transforms of the respective wavefunctions $\varphi_0^{(1)}(\mathbf{r})$ and $\varphi_0^{(2)}(\mathbf{r})$.

The cross section for the electron transfer accompanied by net emission ($n > 0$) or net absorption ($n < 0$) of $|n|$ photons is given by

$$\sigma_{prior}^{(n)} = \int d^2\mathbf{b} |a_{prior}^{(n)}(\mathbf{b})|^2 = J_n^2(G) \frac{16\pi^4}{v} \times \\ \times \int d^3q |\chi_0^{(1)}(\mathbf{q}+\mathbf{v})|^2 \left(\varepsilon_0^{(2)} - 0.5q^2\right)^2 \times \\ \times |\chi_0^{(2)}(\mathbf{q})|^2 \delta\left(\varepsilon_0^{(2)} + n\omega - \varepsilon_0^{(1)} + \frac{v^2}{2} + \mathbf{q}\cdot\mathbf{v}\right). \qquad (37)$$

The charge exchange cross section is given by

$$\sigma_{prior} = \sum_n \sigma_{prior}^{(n)}. \qquad (38)$$

In this equation, the different terms in the sum describe the electron transfer cross sections accompanied by the induced emission ($n > 0$) or absorption ($n < 0$) of different numbers of photons. The term with $n = 0$ corresponds to the capture where the net number of exchanged photons is zero.

2.3.2. Post form of the cross section of the field-assisted charge exchange

In the post form of the field-assisted electron capture accompanied by the emission or absorption of $|n|$ photons, the transition amplitude is represented by

$$a_{post}^{(n)} = - \int_{-\infty}^{\infty} dt \langle \Phi_{f,n} | V_1(\mathbf{r} - \mathbf{R}(t)) | \Phi_{i,0} \rangle, \qquad (39)$$

where V_1 is the interaction of the electron with the first center and the initial and final state vectors are again given by Eqs. (33) and (34), respectively.

In the post form, the cross sections for the electron transfer are given by

$$\sigma_{post}^{(n)} = \int d^2\mathbf{b} |a_{post}^{(n)}(\mathbf{b})|^2 = J_n^2(G) \frac{16\pi^4}{v} \times \\ \times \int d^3q |\chi_0^{(1)}(\mathbf{q}+\mathbf{v})|^2 \left(\varepsilon_0^{(1)} - 0.5(\mathbf{q}+\mathbf{v})^2\right)^2 \times \\ \times |\chi_0^{(2)}(\mathbf{q})|^2 \delta\left(\varepsilon_0^{(2)} + n\omega - \varepsilon_0^{(1)} + \frac{v^2}{2} + \mathbf{q}\cdot\mathbf{v}\right) \qquad (40)$$

and

$$\sigma_{post} = \sum_n \sigma_{post}^{(n)}. \qquad (41)$$

2.3.3. Galilean invariance of the charge exchange cross sections

One can give a slightly more general treatment for the field-assisted electron transfer by considering the process in an inertial reference frame K' where both particles 1 and 2 move with the respective velocities $\mathbf{v}_1$ and $\mathbf{v_2}$ along the trajectories

$$\mathbf{R}_1(t) = \mathbf{b}_1 + \mathbf{v}_1 t$$

and

$$\mathbf{R}_2(t) = \mathbf{b}_2 + \mathbf{v}_2 t,$$

with

$$\mathbf{v} = \mathbf{v}_1 - \mathbf{v_2}$$

[1)] It is worthwhile to note the following. The dominance of the Thomas double scattering mechanism at asymptotically high collision velocities is directly related to the kinematics of the electron transfer in field-free collisions. However, it is not clear how an external field can influence the kinematics. In [11], for example, the radiative electron capture was considered as collision-stimulated transitions between one-center electron states dressed by the interaction with the radiation field, i.e., using an approach quite similar to that applied in the present paper. In [11], the Coulomb interaction with the other center was taken into account only in the first order. Nevertheless, this approach was shown to yield the correct velocity dependence for the radiative capture cross section $\sigma_{REC} \sim 1/v^5$ at asymptotically high velocities.

being the collision velocity. If one assumes that these trajectories are parallel lines, the collision impact parameter $\mathbf{b}$ is given simply by

$$\mathbf{b} = \mathbf{b}_1 - \mathbf{b}_2.$$

In the frame K', the initial state vector is described by Eq. (33) with the evident replacements $\mathbf{b} \to \mathbf{b}_1$ and $\mathbf{v} \to \mathbf{v}_1$ in the electron state, in the argument of the Bessel functions, and in the phase $\phi(=\phi_1)$. The final state vector is now represented by

$$|\Phi_{f,m}(t)\rangle = \psi_0^{(2)}(t)\exp(-im\omega t) \times \times \sum_{m'} J_{m'}(G_2)\exp(im'\phi_2)|N+m+m'\rangle, \quad (42)$$

where $\psi_0^{(2)}(t)$, ϕ_2, and G_2 are given by Eqs. (19), (26) and (29) with evident replacements. Because the electromagnetic field adiabatically switches off as $t \to \infty$, state vector (42) asymptotically reduces to

$$|\Phi_{f,m}(t)\rangle = \psi_0^{(2)}(t)\exp(-im\omega t)|N+m\rangle, \quad (43)$$

which describes the electron and the field with $N+m$ photons that are decoupled as $t \to \infty$.

Using Graf's addition theorem for the Bessel functions (see [10, p. 363, № 9.1.79]), one can show that this more general treatment yields cross sections that depend only on $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ and are exactly equal to those given by Eqs. (37) and (38) or Eqs. (40) and (41).

The derivation briefly outlined above stresses the Galilean invariance of the cross sections.

3. RESULTS AND DISCUSSION

Analyzing the form of dressed state (33), transition amplitude (36), and cross sections (37) and (40), one can conclude that the effective strength of the coupling between the electron and the electromagnetic field occurring in the process of the electron transfer is determined by the factor $|G|$. The effective strength of this coupling is determined not only by the field parameters themselves but also by the change in the electron velocity. For high collision velocities, this coupling can therefore be strong even for relatively weak electromagnetic fields.

In what follows, we consider the electromagnetic field to be linearly polarized for definiteness, although similar conclusions can also be drawn for a more general case of the elliptical polarization. For a linearly polarized field, the coupling factor reduces to

$$|G| = \frac{|\mathbf{F}_0 \cdot \mathbf{v}|}{\omega^2},$$

where

$$\mathbf{F}_0 = \sqrt{\frac{2\pi\omega N}{V}}\mathbf{e}_1$$

is the electric component of the electromagnetic field.

3.1. Weak coupling with the electromagnetic field

If the factor $|G|$ is much smaller than 1, the term with $n = 0$ dominates in the total charge exchange cross section. For $n \neq 0$, only the terms with $n \pm 1$ in (38) (or (41)) can reach noticeable values. In this case,

$$\sigma^{(0)} = J_0^2\left(\frac{\mathbf{F}_0 \cdot \mathbf{v}}{\omega^2}\right)\sigma_{OBK1} \approx \approx \left(1 - \left(\frac{\mathbf{F}_0 \cdot \mathbf{v}}{2\omega^2}\right)^2\right)^2 \sigma_{OBK1} \approx \sigma_{OBK1}, \quad (44)$$

where

$$\sigma_{OBK1} = \frac{16\pi^4}{v}\int d^3q|\chi_0^{(1)}(\mathbf{q}+\mathbf{v})|^2\left(\varepsilon_0^{(2)} - 0.5q^2\right)^2 \times \times|\chi_0^{(2)}(\mathbf{q})|^2\delta\left(\varepsilon_0^{(2)} - \varepsilon_0^{(1)} + \frac{v^2}{2} + \mathbf{q}\cdot\mathbf{v}\right) \quad (45)$$

is the cross section of the nonradiative charge exchange obtained in the OBK1 approximation.

In accordance with (37), cross sections for the charge exchange accompanied by the emission and absorption of one photon are given by

$$\sigma^{(\pm1)} \approx \left(\frac{\mathbf{F}_0 \cdot \mathbf{v}}{2\omega^2}\right)^2 \frac{16\pi^4}{v}\int d^3q|\chi_0^{(1)}(\mathbf{q}+\mathbf{v})|^2 \times \times \left(\varepsilon_0^{(2)} - 0.5q^2\right)^2 |\chi_0^{(2)}(\mathbf{q})|^2 \times \times \delta\left(\varepsilon_0^{(2)} - \varepsilon_0^{(1)} \pm \omega + \frac{v^2}{2} + \mathbf{q}\cdot\mathbf{v}\right). \quad (46)$$

Because ω is small, the terms $\pm\omega$ do not play an essential role in the integrands in Eq. (46) and can be dropped. Therefore, the processes accompanied by the emission and the absorption of one photon give practically identical contributions to the capture cross section and are related to the OBK1 cross section (45) by

$$\sigma^{(\pm1)} \approx \left(\frac{\mathbf{F}_0 \cdot \mathbf{v}}{2\omega^2}\right)^2 \sigma_{OBK1}. \quad (47)$$

From (44) and (47), it follows that

$$\sigma^{(-1)} + \sigma^{(0)} + \sigma^{(+1)} \approx \sigma_{OBK1}, \quad (48)$$

i.e., that the total electron transfer is only very slightly influenced by the field in the case of a weak coupling.

We note that for a weak coupling, the post form of the cross section (Eqs. (40) and (41)) yields results that are identical to (45) and (47).

3.2. Strong coupling with the electromagnetic field

The ratio between the contributions of the charge transfer processes involving different numbers of emitted or absorbed photons to the total capture cross section becomes entirely different in the «intermediate» ($|\mathbf{F_0}\cdot\mathbf{v}|/\omega^2 \gtrsim 1$) and especially, in the strong-coupling ($|\mathbf{F_0}\cdot\mathbf{v}|/\omega^2 \gg 1$) limits. For a strong coupling, $J_0(x) \ll 1$ for $x \gg 1$, and the charge transfer process without net emission or absorption of photons is therefore strongly suppressed compared to the weak coupling limit. The main contribution to the total charge exchange cross section is now due to the electron transfer accompanied by the absorption and emission of large numbers of photons. It follows from the properties of the Bessel function $J_n(x)$ [10, 13] that in order to obtain a noticeable contribution of the $|n|$-photon process, x must be at least of the order of $|n|$. Therefore, one can estimate that the maximum number of photons involved in the field-assisted charge exchange process is of the order of $|\mathbf{F_0}\cdot\mathbf{v}|/\omega^2$.

3.2.1. «Resonance» conditions, the post-prior discrepancy, and the correspondence to different physical mechanisms of the charge exchange

The factors $|\chi_0^{(1)}(\mathbf{q}+\mathbf{v})|^2$ and $|\chi_0^{(2)}(\mathbf{q})|^2$ entering the integrands in (37) and (40) imply that at high velocities, each integrand (excluding the delta-function) has two peaks centered around $\mathbf{q} = -\mathbf{v}$ and $\mathbf{q} = 0$. Therefore, the integrals over the momentum transfer in (37) and (40) can be relatively large only if the argument of the delta-function can be equal to zero at $\mathbf{q} \approx -\mathbf{v}$ or $\mathbf{q} \approx 0$. This can occur if there is a considerable probability for many-photon processes where the numbers $|n|$ of the photons involved satisfy the «resonance» conditions given by

$$n\omega = \frac{v^2}{2} + \varepsilon_0^{(1)} - \varepsilon_0^{(2)} \approx \frac{v^2}{2} \tag{49}$$

for the emission and by

$$n\omega = -\frac{v^2}{2} + \varepsilon_0^{(1)} - \varepsilon_0^{(2)} \approx -\frac{v^2}{2} \tag{50}$$

for the absorption. As in the radiative electron transfer, the «resonance» condition for the charge exchange stimulated by emission looks more transparent if we view the charge exchange in the rest frame of the projectile: the electron with the initial energy $\varepsilon_0^{(1)} + v^2/2$ undergoes a transition to the bound state of the projectile with the energy $\varepsilon_0^{(2)}$, transferring the energy difference to the electromagnetic field by means of a photon emission. On the other hand, the «resonance» condition for the electron transfer accompanied by absorption looks more natural if we take the target frame as a reference frame: the electron with the initial energy $\varepsilon_0^{(1)}$ undergoes a transition to the bound state of the moving projectile, where its energy is equal to $\varepsilon_0^{(2)} + v^2/2$, absorbing the energy difference from the electromagnetic field. If the «resonance» conditions are satisfied, the collision kinematics for the electron transfer can be substantially improved in the same way as for the radiative electron capture, where only one high-energy photon with the frequency $\omega \approx v^2/2$ is spontaneously emitted.

Analyzing the strong-coupling case, we encounter a difficulty related to the fact that the charge exchange cross sections obtained in the prior and post forms become drastically different. The integrands in (37) and (40) are strictly equal to each other only for $n = 0$ and approximately equal for low $|n|$. As $|n|$ increases, the difference between the integrands in (37) and (40) increases. This difference becomes especially large when the «resonance» conditions are satisfied. The latter case is of a particular interest, however, and the rest of this section is mainly devoted to the analysis of the «resonance» case.

In the integrand in (37) (excluding the delta-function), the ratio between the peaks at $\mathbf{q} \approx -\mathbf{v}$ and $\mathbf{q} \approx 0$ is proportional to v^4, which means that the peak at $\mathbf{q} \approx 0$ is negligible compared to the one at $\mathbf{q} \approx -\mathbf{v}$. This results in the conclusion that in accordance with (37), the electron capture is favored if it is accompanied by the emission of a large number of photons.

On the other hand, in the integrand in (40) (excluding the delta-function) the ratio of the peak at $\mathbf{q} \approx -\mathbf{v}$ to that at $\mathbf{q} \approx 0$ is proportional to v^{-4}, and the peak at $\mathbf{q} \approx -\mathbf{v}$ is therefore negligible compared to the one at $\mathbf{q} \approx 0$. This means that in accordance with (40), the electron capture is favored if it is accompanied by the absorption of a large number of photons.

As a first example, we now consider the capture cross sections for the

$$p + H(1s) \to H(1s) + p$$

collisions assisted by the electromagnetic field with $F_0 = 2.15\cdot 10^{-2}$ a.u. and $\omega = 0.117$ eV $= 4.3\cdot 10^{-3}$ a.u. at the collision velocity $v = 10$ a.u. In this example and in all other examples, we assume that $\mathbf{F_0}$ is parallel (or antiparallel) to $\mathbf{v}$. Further, we take all the reported values of the cross sections to be multiplied by the factor 0.3 that is known to bring the OBK1 cross sections to a reasonable agreement with experimental data at intermediately high collision velocities. At the collision velocity $v = 10$ a.u., the cross section for the nonradiative capture in electromagnetic field-free collisions calculated in the OBK1 approximation (and multiplied by 0.3) is equal to $\sigma_{OBK1} = 1.14$ b. Using the prior form of the cross sections, we obtain $\sigma^{(0)} = 6.2\cdot 10^{-5}$ b, $\sum_{n=1}^{1000}\sigma_{prior}^{(n)} = 6.4\cdot 10^{-2}$ b, $\sum_{n=1}^{10000}\sigma_{prior}^{(n)} = 38.24$ b, and $\sum_{n=1}^{12000}\sigma_{prior}^{(n)} = 367.3$ b for the collisions assisted by the electromagnetic field. Adding the higher-n terms does not noticeably change the prior cross section[2)]. In accordance with the prior form, the main contribution comes from the terms with $10000 < n < 12000$ and the contribution from negative n is negligible. Using the post form of the cross sections, we have

$$\sigma_0 = 6.2\cdot 10^{-5}\ \text{b},\quad \sum_{n=-1}^{-10000}\sigma_{post}^{(n)} = 38.24\ \text{b},$$

$$\sum_{n=-1}^{-12000}\sigma_{post}^{(n)} = 367.3\ \text{b}$$

for the same collisions. In accordance with the post form, the main contribution is given by the terms with $-12000 < n < -10000$ and positive n contribute negligibly. Although the prior and the post forms yield the same transfer cross sections for symmetrical collisions, the physics that they describe is totally different. The prior form stresses the electron transfer due to emission (the induced multiphoton bremsstrahlung) and the post form supports the transfer process due to absorption (the multiphoton ionization).

As further examples, we consider the

$$\mathrm{p} + \mathrm{He}^+ \to \mathrm{H} + \mathrm{He}^{2+}$$

and

$$\mathrm{He}^{2+} + \mathrm{H} \to \mathrm{He}^+ + \mathrm{p}$$

collisions at $v = 10$ a.u. assisted by the electromagnetic field with the same parameters as in the first example. For the field-free collisions, one has $\sigma_{OBK1} = 27.5$ b for both colliding systems. For the field-assisted collisions, in accordance with the prior form, we obtain the respective cross section $\sigma_{prior} = 304$ b and $\sigma_{prior} = 9280$ b for the $\mathrm{p} - \mathrm{He}^+$ and $\mathrm{He}^{2+} - \mathrm{H}$ collisions. In both reactions, in the prior form, the terms with negative n (absorption) contribute negligibly. In accordance with the post form, we have $\sigma_{post} = 9280$ b and $\sigma_{post} = 304$ b for the $\mathrm{p} - \mathrm{He}^+$ and $\mathrm{He}^{2+} - \mathrm{H}$ collisions, respectively, and we find that the terms with positive n (emission) have a negligible impact on the cross section. Analyzing Eqs. (37) and (40), we conclude that the above correspondence between the prior and the post cross sections is a particular case of the relation

$$\sigma_{post}(Z_1, Z_2) = \sigma_{prior}(Z_2, Z_1) \tag{51}$$

that holds for the capture cross sections obtained in the prior and post forms.

Comparing the capture cross sections in the above three examples, we see that

$$\sigma_{prior} \propto Z_2^{\nu} \tag{52}$$

with $\nu \approx 5$, while the dependence of the cross section on Z_1 is relatively weak. On the other hand,

$$\sigma_{post} \propto Z_1^{\mu} \tag{53}$$

with $\mu \approx 5$ and the dependence of the cross section on Z_2 is relatively weak. Our calculations for other «target–projectile» pairs show similar dependences on Z_1 and Z_2. It is worthwhile to note that the dependence $\propto Z_1^5$ on the charge of the target nucleus is a signature of the photoeffect (see, e.g., [6]), while the dependence $\propto Z_2^5$ on the charge of the projectile is a signature of two closely related processes: the radiative recombination and the radiative electron capture [2].

In a rigorous theory, evidently, there must be no discrepancy between cross sections calculated in the prior and post forms. It is also clear that the large discrepancy between the prior and post forms in our case originates from the fact that the dressed states (33) and (34) do not exactly represent the system «electron in the field of a nucleus + field»[3)]. One way to deal with the difficulty is to try to remove the prior–post discrepancy by implementing exact solutions for

2) Although the numbers of the emitted or absorbed photons are very large, simple estimates show that they are still much smaller than the huge number of photons available in the «coherence» volume $V_c \sim \lambda^3 = (2\pi c/\omega)^3$ of the field with $F_0 = 2.15\cdot 10^{-2}$ a.u. and $\omega = 4.3\cdot 10^{-3}$ a.u. Therefore, the assumption that the field is an inexhaustible source and sink of photons, which has been used in deriving (33), is not violated.

3) The problem of the post–prior discrepancy is also known in the theory of the field-free electron transfer based on the eikonal approximation, where additional physical arguments are necessary in order to decide which form is more suitable (see, e.g., [1] and references therein).

the dressed states, which would yield identical results in both forms. However, this is very difficult to achieve and in addition, one can then encounter the problem related to the overcomplete representation of the electron Hilbert space by two complete sets of states centered on the target and on the projectile. We choose another way instead. In what follows, we argue that one can still obtain physically reasonable results with the approximate dressed states (33) and (34) keeping in mind that with these states, the prior and post forms of the transition amplitude describe the electron transfer due to different physical processes. To analyze this, we first consider a collision-free system consisting of an atom that is initially in the ground state and a low-frequency relatively weak electromagnetic field. We are interested in the ionization probability due to the interaction with the field and in particular, in the probability to finally find the electron in high-energy continuum states $\{\varphi_{\mathbf{p}}^{(1)}\}$. For a high-energy continuum state, we can neglect its distortion due to the interaction with the target nucleus and write the state in the presence of the electromagnetic field as

$$\psi_{\mathbf{p}}^{(1)} \propto \exp(i\mathbf{p}\cdot\mathbf{r}) \sum_n J_n\left(\frac{\mathbf{F}_0\cdot\mathbf{p}}{\omega^2}\right)|N+n\rangle. \qquad (54)$$

For the amplitude for transitions from the ground state $\varphi_0^{(1)}$ to the high-energy state, we then have

$$a_{\mathbf{p},n}^{ion} \propto J_n\left(\frac{\mathbf{F}_0\cdot\mathbf{p}}{\omega^2}\right)\langle e^{i\mathbf{p}\cdot\mathbf{r}}|V_1|\varphi_0^{(1)}\rangle \times \\ \times\,\delta\left(\frac{p^2}{2}-n\omega-\varepsilon_0^{(1)}\right). \qquad (55)$$

For high $|\mathbf{p}|$, the magnitude $|a_{\mathbf{p},n}^{ion}|$ is very small. Still, in accordance with (55) and with the properties of the Bessel functions, we can expect the continuum states to be populated with a small but nonzero probability for all $|\mathbf{p}|$ up to

$$\frac{p_{max}^2}{2} \approx n_{max}\omega \approx \frac{F_0 p_{max}}{\omega^2}\omega,$$

i.e., up to

$$p_{max} \approx \frac{2F_0}{\omega}.$$

We now assume that the atom collides with a projectile having the velocity $-\mathbf{v}$. Because the ground state of the projectile can be represented by the continuum states of the target and the momentum of the ionized electron matches the momenta of the electron bound in the moving projectile, we see that if $p_{max} \gtrsim v$, the atomic electron can finally be captured by the projectile as the result of ionization. It is now not difficult to see that using states (33) and (34) (or their counterparts in the rest frame of the target or in any other reference frame) and applying the post form with the interaction V_1 amounts to calculating the contribution to the electron transfer that is represented by the part of the target multiphoton ionization[4)] where the final electron states in the continuum of the target match the ground state of the projectile. For $v \gg Z_2$, the high-energy continuum states with $\mathbf{p} \approx -\mathbf{v}$ can easily «cover» the ground state of any light projectile, independently of Z_2. Therefore, in applying the post form, we see that the capture cross section is almost independent of Z_2.

A similar analysis can also be done for the contribution to the charge transfer described by the prior form of the transition amplitude. We consider a system consisting of a nucleus at rest (representing the projectile in its rest frame) and an incoming free electron moving in the presence of the electromagnetic field. The initial state of the electron with the momentum $\mathbf{q}$ is given by an expression similar to (54),

$$\psi_{\mathbf{q}}^{(2)} \propto \exp(i\mathbf{q}\cdot\mathbf{r}) \sum_n J_n\left(\frac{\mathbf{F}_0\cdot\mathbf{q}}{\omega^2}\right)|N+n\rangle. \qquad (56)$$

As the result of the collision with the nucleus, the electron can emit photons (the induced multiphoton bremsstrahlung) and undergo a transition to another state. One of the possible final states of the electron can be the ground state of the projectile, $\varphi_0^{(2)}$. The amplitude of this transition is given by

$$a_{\mathbf{q},n}^{br} \propto J_n\left(\frac{\mathbf{F}_0\cdot\mathbf{q}}{\omega^2}\right)\langle\varphi_0^{(2)}|V_2|e^{i\mathbf{q}\cdot\mathbf{r}}\rangle \times \\ \times\,\delta\left(\frac{q^2}{2}-n\omega-\varepsilon_0^{(2)}\right). \qquad (57)$$

We now note that the state of the electron initially bound in the target moving with the velocity $\mathbf{v}$ in the rest frame of the projectile can be represented by a superposition of states given by Eq. (56). Taking that into account and comparing Eq. (57) with the transition amplitude in the prior form, Eqs. (35) and (36), we arrive at the following conclusion. Using states (33) and (34) (or their counterparts in any other reference frame) and applying the prior form with the interaction

4) In the case under consideration, $F_0/\omega \gg 1$. We note that the ionization of an atom by a classical electromagnetic field with $F_0/\omega \gg 1$ can be viewed as a tunneling effect [14].

V_2 amounts to calculating the contribution to the electron transfer that is due to that part of the induced multiphoton bremsstrahlung of the electron, initially bound in the ground state of the target, where the final electron state is the ground state of the projectile.

The radiative electron capture that proceeds with a spontaneous emission of one high-energy photon is known to be weakly dependent on the charge of the target, Z_1, provided $Z_1 \ll v$, where v is the collision velocity [2]. According to our calculations, a similar situation is encountered in the case under consideration where the electron capture proceeds with the induced emission of a large amount of low-frequency photons.

Summarizing the above analysis, we can make the following important conclusions. First, for the field-assisted electron capture in the strong-coupling case, one can still use the approximate state vectors given by Eqs. (33) and (34) in order to describe the capture. Second, using these approximate states, one must keep in mind that the prior and post forms of the capture cross sections are drastically different in general. Third, this difference is related to the fact that in the adopted approximation for the dressed electron states, the prior and post forms describe the electron transfer due to different physical processes: the multiphoton ionization and the induced multiphoton bremsstrahlung. Fourth, the total cross section for the electron capture in the case of a strong coupling with the electromagnetic field can be evaluated as the sum of cross sections corresponding to different physical processes mentioned above,

$$\sigma_{tot} = \sum_{n<0} \sigma_{post}^{(n)} + \sigma_0 + \sum_{n>0} \sigma_{prior}^{(n)} \approx \approx \sum_{n<0} \sigma_{post}^{(n)} + \sum_{n>0} \sigma_{prior}^{(n)}, \quad (58)$$

where $\sigma_{prior}^{(n)}$ and $\sigma_{post}^{(n)}$ are given by Eqs. (37) and (40), respectively, and describe the electron transfer due to the induced bremsstrahlung and photoionization, and

$$\sigma_0 = \sigma_{prior}^{(0)} = \sigma_{post}^{(0)} = J_0^2\left(\frac{|\mathbf{F_0}\cdot\mathbf{v}|}{\omega^2}\right)\sigma_{OBK1}$$

is negligible.

We have already mentioned that the problem of a large prior–post discrepancy is also known in eikonal calculations for the field-free electron capture, where additional physical arguments must be used in order to decide which form should be applied. The same problem is encountered in first-order calculations for the electron capture in field-free collisions with multielectron targets. In the latter case, however, it is very difficult to decide which form should be given preference and sometimes one introduces the average transition amplitude

$$a_{if} = 0.5(a_{if}^{prior} + a_{if}^{post})$$

(see, e.g., [15]). This is different from the situation with the field-assisted collisions in the strong-coupling case, where one can argue that the post and the prior forms correspond to different electron transfer channels that do not interfere and that the capture cross section must be evaluated in accordance with Eq. (58). One can add that because of the different dependences $\sigma_{prior} \propto Z_2^5$ and $\sigma_{post} \propto Z_1^5$, only one of these forms is of practical importance for calculations of the electron capture in asymmetric field-assisted collisions, where the ratio Z_2/Z_1 considerably differs from unity.

To conclude this discussion, we briefly compare our calculations for the electron transfer in collisions assisted by an external field to the radiative electron capture (REC) [11]. In the latter process, the «resonance» condition for the electron transfer $\omega_{sp} \sim v^2/2$ is satisfied due to a spontaneous emission of one high-energy photon and, naturally, the electron transfer with a photon absorption is not possible because there are no photons in the initial state of the free radiation field. In [11], the REC was considered as the collision-stimulated transitions between one-center electron states dressed by the interaction with the radiation field. It was found in [11] that the prior form of the REC cross section obtained within the approach that is obviously very similar to that employed here yields an excellent agreement with the well established results for the radiative capture cross section. However, the post form of the theory in [11] leads to REC cross sections that are smaller by many orders of magnitude. The reason is as follows. With the approximate one-center electron states dressed by the free radiation field as in [11], the prior and post forms are «responsible» (similarly to the present approach) for the electron transfer due to bremsstrahlung and photoionization, respectively. However, the coupling to the free radiation field, which can produce spontaneous bremsstranhlung, cannot result in photoionization.

3.2.2. Some additional examples of the field-assisted charge exchange

In the remaining part of this section, we consider several examples of the electron exchange in field-assisted and field-free collisions. We first consider

$$\mathrm{He}^{2+} + \mathrm{He}^{+}(1s) \to \mathrm{He}^{+}(1s) + \mathrm{He}^{2+}$$

at the collision velocity $v = 10$ a.u. assisted by the electromagnetic field with $F_0 = 2.15 \cdot 10^{-2}$ a.u. and $\omega = 4.3 \cdot 10^{-3}$ a.u., i.e., with the same parameters as in the examples considered in 3.2.1. We obtain $\sigma_{OBK1} = 0.68$ kb and $\sigma_{tot} = 17.5$ kb in the field-free and field-assisted collisions, respectively (we remind the reader that all the reported values for cross sections are multiplied by the factor 0.3).

In contrast to the p-H collisions, where we have $\sigma_{OBK1} = 1.14$ b and $\sigma_{tot} = 2 \cdot 367$ b $= 734$ b, the field-free and field-assisted cross sections for the He^{2+}-He^{+} collisions differ only by a factor about 26. The difference in the cross section enhancement in the p-H and He^{2+}-He^{+} collisions can probably be attributed to the fact that the momentum distribution of the electron in the He^{+} ground state is considerably broader than that in the hydrogen ground state. The broader momentum distribution makes the charge exchange process less «resonant» and therefore diminishes the effect of a possible improvement of the kinematic conditions due to the interaction with the electromagnetic field.

As further examples, we also consider the reactions

$$\mathrm{He}^{2+} + \mathrm{He}^{+}(1s) \to \mathrm{He}^{+}(1s) + \mathrm{He}^{2+},$$

$$\mathrm{Li}^{3+} + \mathrm{Li}^{2+}(1s) \to \mathrm{Li}^{2+}(1s) + \mathrm{Li}^{3+},$$

$$\mathrm{He}^{2+} + \mathrm{Li}^{2+}(1s) \to \mathrm{He}^{+}(1s) + \mathrm{Li}^{3+},$$

$$\mathrm{Li}^{3+} + \mathrm{He}^{+}(1s) \to \mathrm{Li}^{2+}(1s) + \mathrm{He}^{2+}$$

assisted by the electromagnetic field with $F_0 = 0.1$ a.u. and $\omega = 0.01$ a.u. at the collision velocity $v = 20$ a.u. The cross section for the nonradiative capture in electromagnetic field-free collisions calculated in the OBK1 approximation is then equal to $\sigma_{OBK1} = 0.29$ b for the He^{2+}-He^{+} collisions, $\sigma_{OBK1} = 13$ b for the Li^{3+}-Li^{2+} collisions, and $\sigma_{OBK1} = 1.92$ b for the He^{2+}-Li^{2+} and Li^{3+}-He^{+} collisions. In the collisions assisted by the electromagnetic field, one obtains $\sigma_{tot} = 182$ b for the He^{2+}-He^{+} collisions, $\sigma_{tot} = 1164$ b for the Li^{3+}-Li^{2+} collisions, and $\sigma_{tot} = 711$ b for the He^{2+}-Li^{2+} and Li^{3+}-He^{+} collisions. One sees again that the enhancement effect in the capture cross sections rapidly decreases with increasing the width of the momentum distribution of the electron in the ground states.

Comparing the He^{2+}-He^{+} collisions at $v = 10$ and $v = 20$, one can note that the cross section enhancement effect in field-assisted collisions is much stronger in the latter case. This can be attributed to the fact that the peaks in the momentum distributions of the electron bound to the target and that bound to the projectile are more separated in the collisions at $v = 20$ than those at $v = 10$. Therefore, the kinematic conditions for the electron transfer are more «resonant» at the higher velocity and the improvement of these conditions due to the emission and absorption of photons results in a stronger effect. One can note that the ratio between the cross sections in the field-free and the field-assisted collisions is nearly the same for the p-H collisions at $v = 10$ considered in the first example and the He^{2+}-He^{+} collisions at $v = 20$. In both cases, the ratio between the collision velocity and the typical width of the momentum distribution of the electron in the ground state ($\sim Z$) is the same.

In calculating the field-assisted capture, we assumed that $\mathbf{F_0}$ is parallel (or antiparallel) to $\mathbf{v}$. For $\mathbf{F_0} \perp \mathbf{v}$, the total cross section for the electron transfer in the field-assisted collisions is reduced to the OBK1 cross section in accordance with Eqs. (37), (38), (40), and (41). Therefore, comparing the values of the cross sections obtained in the field-free and field-assisted collisions, one can conclude that the effect of a linearly polarized electromagnetic field on the electron transfer can be very sensitive to the angle between $\mathbf{F_0}$ and $\mathbf{v}$. This effect is favored for $\mathbf{F_0} \parallel \mathbf{v}$ or $\mathbf{F_0} \parallel -\mathbf{v}$ and is negligible if $\mathbf{F_0} \perp \mathbf{v}$.

4. CONCLUSIONS

The electron transfer process in fast collisions assisted by a relatively weak low-frequency electromagnetic field can represent an interesting example of an effectively strong coupling between the electron and the electromagnetic field. A key consequence of the strong electron–field coupling in the charge exchange collisions is the emission and absorption of a very large number of photons that can substantially improve the electron transfer kinematics under certain conditions.

The effect of a low-frequency electromagnetic field on the electron transfer process discussed in the present paper is closely related to some well studied processes. We have already discussed the connection with the multiphoton ionization and the induced multiphoton bremsstrahlung. In addition, we now note the relation to a particular case of the induced bremsstrahlung— free electron-atom collisions in a laser field. Free electron-atom collisions assisted by the electromagnetic field were studied in some detail, mainly theoretically (see, e.g., [16] for a review). One of the conclusions of these studies that is relevant to the topic of the present paper is that the external electromagnetic field can substantially increase the magnitude of the scattering cross section if there are some quasi-stationary states that can be resonantly populated during the scattering via the stimulated emission or absorption of photons.

The analysis of the present paper suggests that the capture cross sections can be substantially influenced by the electromagnetic field. In addition, one can also expect the effect of the field to be reflected by noticeable changes in the spectra of high-energy photons that are spontaneously emitted during the radiative electron capture in field-assisted collisions.

In the examples considered in the previous section, very large values of the ratio $\sigma_{tot}/\sigma_{OBK1}$ were predicted for the capture cross sections in the field-assisted collisions compared to the field-free ones. If one compares the radiative electron capture to the nonradiative capture, one sees that the corresponding ratio σ_{REC}/σ_{NR} can also be very large at high velocities.

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APPENDIX

To derive a suitable form of the nonrelativistic Schrödinger equation for the electron interacting with a quantized electromagnetic field, we start with the Dirac equation

$$i\frac{\partial}{\partial t}|\Psi\rangle = c\boldsymbol{\alpha}\cdot\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)|\Psi\rangle + W|\Psi\rangle + \\ + \beta m_e c^2|\Psi\rangle + \omega N_a|\Psi\rangle, \quad \text{(A.1)}$$

where W describes the Coulomb interaction with the nuclei and $\boldsymbol{\alpha}$ and β are the Dirac matrices. Decomposing $|\Psi\rangle$ into major and minor components denoted by φ and χ respectively, we rewrite Eq. (A.1) as

$$\begin{aligned} &\left(i\frac{\partial}{\partial t} - W - \omega N_a - m_e c^2\right)|\varphi\rangle = \\ &\quad = c\boldsymbol{\sigma}\cdot\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)|\chi\rangle, \\ &\left(i\frac{\partial}{\partial t} - W - \omega N_a + m_e c^2\right)|\chi\rangle = \\ &\quad = c\boldsymbol{\sigma}\cdot\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)|\varphi\rangle, \end{aligned} \quad \text{(A.2)}$$

where $\boldsymbol{\sigma}$ is the Pauli matrix. A common way to derive the nonrelativistic equation from a relativistic one is to assume that all other energies in the system are much less than the electron rest energy $m_e c^2$. In our case, this assumption does not hold because, as simple estimates easily show, even for a very weak electromagnetic field occupying a macroscopic volume, its total energy is much larger than the electron rest energy. It is clear, however, that the relevant guantity is the amount of the electromagnetic energy that can be transferred between the electromagnetic field and the electron, rather than the total amount of the field energy. Making the ansatz

$$\begin{aligned} |\varphi\rangle &= \exp(-i(m_e c^2 + N\omega)t)|\varphi_1\rangle, \\ |\chi\rangle &= \exp(-i(m_e c^2 + N\omega)t)|\chi_1\rangle, \end{aligned} \quad \text{(A.3)}$$

where N is the initial number of photons in the quantum field, we remove the irrelevant part of the total field energy and obtain

$$\begin{aligned} &\left(i\frac{\partial}{\partial t} - W - \omega(N_a - N)\right)|\varphi_1\rangle = \\ &\quad = c\boldsymbol{\sigma}\cdot\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)|\chi_1\rangle, \\ &\left(i\frac{\partial}{\partial t} - W - \omega(N_a - N) + 2m_e c^2\right)|\chi_1\rangle = \\ &\quad = c\boldsymbol{\sigma}\cdot\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)|\varphi_1\rangle. \end{aligned} \quad \text{(A.4)}$$

Assumung that the energy transfer between the electromagnetic field and the electron is nonrelativistic,

$$\omega(N_a - N) \ll m_e c^2,$$

we can now approximate

$$|\chi_1\rangle = \frac{1}{2m_e c}\boldsymbol{\sigma}\cdot\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)|\varphi_1\rangle.$$

Inserting this expression into the first equation in (A.4) and neglecting the spin term, we obtain Schrödinger equation (1) for the major component.

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