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QED RADIATIVE CORRECTIONS TO IMPACT FACTORS

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We consider radiative corrections to the electron and photon impact factors. The generalized eikonal representation for the e^+e^- scattering amplitude at high energies and fixed momentum transfers is violated by non-planar diagrams. The additional contribution to the two-loop approximation appears from the Bethe—Heitler mechanism of the fermion pair production with the identity of the fermions in the final state taken into account. The violation of the generalized eikonal representation is also related to the charge parity conservation in QED. The one-loop correction to the photon impact factor for small virtualities of the exchanged photon is obtained using the known results for the cross-section of the e^+e^- production at photon–nuclei interactions.

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1. INTRODUCTION

It is well known (see [1]) that the QED scattering amplitude for the process

$$a + b \to a' + b'$$

in the Regge kinematics

$$\begin{gathered} A(p_A, a) + B(p_B, b) \to A(p'_A, a') + B(p'_B, b'), \\ s = (p_A + p_B)^2 \gg -t = -(p_A - p'_A)^2 \propto m^2 \end{gathered} \tag{1}$$

has the impact factor representation

$$A(s,t) = \frac{is}{(2\pi)^2} \int \frac{d^2k\, \tau^A(k,r)\, \tau^B(k,r)}{[(k+r)^2+\lambda^2][(k-r)^2+\lambda^2]} \times \left(1 + O\left(\frac{t}{s}\right)\right), \quad 4r^2 = -t > 0, \tag{2}$$

that is valid in the first non-trivial order of the perturbation theory. Here, λ is the photon mass and the two-dimensional vectors r and k are orthogonal to the initial particle momenta p_A and p_B. The impact factors τ describe the inner structure of colliding particles. For the electron, we have

$$|\tau^e| = 4\pi\alpha\delta_{ij},$$

where the indices i, j enumerate the electron polarization states. The expression for the impact factors of the photon on its mass shell can be written as [1]

$$\tau_{ij}^{\gamma} = 8\alpha^2 \int_0^1 dy \int_0^1 dx_+ dx_- \times \delta(x_+ + x_- - 1)(A_{ij} - B_{ij}), \tag{3}$$

with

$$A_{ij} = \frac{1}{4r^2x_+^2y(1-y)+m^2}\left[8x_+^3x_-y(1-y)r_ir_j - - x_+^2r^2\left(1 - 8x_+x_-\left(y-\frac{1}{2}\right)^2\right)\delta_{ij}\right],$$

$$B_{ij} = \frac{1}{4Q^2y(1-y)+m^2}\left[8x_+x_-y(1-y)Q_iQ_j - - Q^2\left(1 - 8x_+x_-\left(y-\frac{1}{2}\right)^2\right)\delta_{ij}\right],$$

$$Q = \frac{1}{2}(k+r) - x_+r,$$

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where i, j refer to the photon polarization states.

According to the Regge theory, the impact factor is proportional to the residue of the pole $\propto (j-1)^{-1}$ of the t-channel partial wave f_j^+, with the positive signature describing the t-channel transition of two particles into a nonsense state of two virtual photons [1],

$$\tau^A = \lim_{j\to 1}(j-1)\int\limits_{s_{th}}^{\infty}\frac{d\,s'}{\pi}\,\mathrm{Im}\,A^{\gamma^*}_{\mu_1\mu_2}(q,k;s')\times \\ \times\frac{p_B^{\mu_1}}{s}\frac{p_B^{\mu_2}}{s}Q_{j-2}(z'),\quad s'=-2kp_A, \tag{4}$$

where $z' = \cos\theta$ is the cosine of the scattering angle θ in the t-channel. It is a linear function of s'. Here and below s_{th} means the threshold value of s'. Higher orders of perturbation theory involve the poles $f_j \propto 1/(j-1)^n$ that must be subtracted from τ to provide sums of all the logarithmic cotributions $\propto \log^n(s)$ using the Bethe—Salpeter equation [1].

For $t = 0$, the impact factor is proportional to the s' integral of the total cross-section for the scattering of the photon with the virtuality $-k^2$ off the target a with the mass m,

$$\tau = k^2\int\limits_{s_{th}}^{\infty}\frac{ds'}{\pi}\,\frac{\sigma_{a\gamma^*}(s',k^2)}{s'}f(s',\,k^2), \\ f(s',\,k^2)=\frac{\sqrt{s'^2-4k^2m^2}}{s'}, \tag{5}$$

where $f(s', k^2)$ accounts for the virtual photon flux factor. This multiplier equals unity in the limit as $k^2 \to 0$, which corresponds to the Weizsäcker—Williams approximation:

$$\lim_{k^2\to 0}\frac{\tau}{k^2}=\int\limits_{s_{th}}^{\infty}\frac{ds'}{\pi}\,\frac{\sigma_{a\gamma}(s')}{s'}\,. \tag{6}$$

The motivation for our calculation of radiative corrections to impact factors is the high-precision experiments performed on colliders where some interesting physical quantities (for example, the BFKL pomeron intercept) are measured [2]. In this case, one must know the impact factors of the virtual photon [2]. Generally, impact factors describe the coupling of particles with the pomeron in QED or in QCD. For colliders with electron (positron) beams, radiative corrections to impact factors can be used to calculate the QED-part of cross-sections with a good accuracy.

For small-angle e^+e^- scattering, the amplitude for the diagrams with the multi-photon exchange has the eikonal representation

$$A(s,t)=A_0(s,t)e^{i\delta(t)}, \\ A_0(s,t)=4\pi\alpha\frac{2}{st}\bar{u}(p_1')\hat{p}_2u(p_1)\times \\ \times\bar{v}(p_2)\hat{p}_1v(p_2')=4\pi\alpha\frac{2s}{t}N_1N_2, \\ |N_i|=1,\quad \delta(t)=-i\alpha\ln\left(\frac{-t}{\lambda^2}\right), \tag{7}$$

where we used that only the longitudinal (nonsense) polarizations of the t-channel virtual photons are essential at high energies,

$$\bar{u}(p_1')\gamma_\mu u(p_1)\bar{v}(p_2)\gamma_\nu v(p_2')G^{\mu\nu}(q), \\ G^{\mu\nu}(q)=\frac{1}{q^2}\,\frac{2\,p_2^\mu p_1^\nu}{s}\,. \tag{8}$$

The radiative corrections to A_0 appear from the so-called «decorated boxes». These Feynman diagrams were assumed to lead to a generalized eikonal representation

$$A = A_0(s,t)[\Gamma_1(t)]^2e^{i\delta(t)}, \tag{9}$$

where $\Gamma_1(t)$ is the Dirac form-factor of electron,

$$V^\mu(t)=\gamma^\mu\Gamma_1(t)+\frac{\sigma^{\mu\nu}q_\nu}{2m}\Gamma_2(t),\quad q^2=t, \\ \Gamma_1(t)=1+\gamma\Gamma_1^{(2)}(t)+\ldots,\quad \gamma=\frac{\alpha}{\pi}. \tag{10}$$

We note that $\delta(t)$ must also include corrections to the virtual photon Green function, leading in particular to the electric charge renormalization.

In the next section, we verify the generalized eikonal representation for the decorated boxes.

2. ONE-LOOP CORRECTION TO THE ELECTRON IMPACT FACTOR

Keeping in mind that the amplitude for the near-forward scattering with the two-photon exchange is purely imaginary (with the corrections of the order m^2/s omitted), we can calculate its s-channel discontinuity. The radiative corrections to this discontinuity originate from the virtual photons and from the emission of the real photon in the intermediate state. We split the last contribution into two parts corresponding to the emission of soft and hard photons.

The virtual photon contribution contains the electron vertex function for the on-shell initial and final electrons,

$$
\begin{aligned}
\Delta\tau_e^{virt} &= \frac{\alpha}{\pi}\tau_e(0)\left[F_1^{(2)}(-k^2)+F_1^{(2)}(-k^{'2})\right],\\
F_1^{(2)}(t) &= -G(t)\ln\frac{m}{\lambda}-G_1(t)-T(t),\\
G(t) &= \frac{1+a^2}{2a}\ln b-1,\\
G_1(t) &= 1-\frac{1+2a^2}{4a}\ln b,\\
T(t) &= \frac{1+a^2}{2a}\left[-\frac{1}{4}\ln^2 b+\ln b\ln(1+b)-\right.\\
&\left.-\int\limits_1^b\frac{dx}{x}\ln(1+x)\right],\\
a &= \sqrt{1-\frac{4m^2}{t}},\quad b=\frac{a+1}{a-1},\quad t<0.
\end{aligned}
\tag{11}
$$

The contribution from the emission of a soft photon has the classical form

$$
\begin{aligned}
&-\frac{\alpha}{4\pi^2}\left(\frac{p_1}{p_1k_1}-\frac{p}{pk_1}\right)\left(\frac{p_1}{p_1k_1}-\frac{p_1'}{p_1'k_1}\right)\tau_e^{(0)}\times\\
&\times\left.\frac{d^3k_1}{\omega_1}\right|_{\omega_1<\delta E},\quad \delta E\ll E=\frac{\sqrt{s}}{2},
\end{aligned}
\tag{12}
$$

where p and p_1' are the momenta of the initial and final electrons and p_1 is the electron momentum in the intermediate state. Because the energies of these particles are approximately equal (but large compared to the electron mass), we can use the relations

$$
\begin{aligned}
&\frac{1}{2\pi}\int\frac{d^3k_1}{\omega_1}\frac{m^2}{(p_ik_1)^2}=2L_e,\\
&\frac{1}{2\pi}\int\frac{d^3k_1}{\omega_1}\frac{p_ip_j}{(p_ik_1)(p_jk_1)}=\frac{1+a^2}{a}\left[L_e\ln b-\right.\\
&\left.-\frac{1}{4}\ln^2 b+\ln b\ln(1+b)-\int\limits_1^b\frac{dx}{x}\ln(1+x)\right],\\
&L_e=\ln\Delta+\ln\frac{m}{\lambda},\quad t=(p_i-p_j)^2,\\
&\Delta=\frac{\delta E}{E}\ll 1,
\end{aligned}
\tag{13}
$$

with the quantities a and b defined above. Thus, we obtain

$$
\begin{aligned}
\Delta\tau_e^{soft} &= \frac{\alpha}{\pi}\left[\left(G(-k^2)+G(-k'^2)-G(t)\right)L_e+\right.\\
&\left.+T(-k^2)+T(-k'^2)-T(t)\right],
\end{aligned}
\tag{14}
$$

where $T(t)$ was defined above.

We next consider the hard photon emission. Its contribution to the imaginary part of the electron–electron scattering amplitude can be written as

$$
\begin{aligned}
\mathrm{Im}_s A(s,t) &= -s\frac{\alpha^3}{2\pi^2}\int\frac{d^2k}{k^2k'^2}N_1N_2\frac{d^2k_1dx}{x(1-x)}\times\\
&\times I(x,k_1,k),\quad \Delta<x<1,
\end{aligned}
\tag{15}
$$

where x is the energy fraction of the hard photon. We obtain

$$
\begin{aligned}
I(x,k_1,k) &= \frac{1}{d_1d_2}(-4m^2+2t_1z)+\\
&+\frac{1}{d_1d_1'}\left(-4m^2x^2(1-x)+2tz(1-x)\right)+\\
&+\frac{1}{d_2d_1'}(-4m^2+2t_2z)-2z\frac{1}{d_1}-4z\frac{1}{d_2}+\\
&+\frac{8m^2}{d_2^2}-2z\frac{1}{d_1'},\quad z=1+(1-x)^2,
\end{aligned}
\tag{16}
$$

where

$$
\begin{aligned}
d_1 &= (p-k_1)^2-m^2=-\frac{1}{x}[m^2x^2+\mathbf{k}_1^2],\\
d_2 &= (p_1+k_1)^2-m^2=\frac{1}{x(1-x)}\left[m^2x^2+(x\mathbf{k}-\mathbf{k}_1)^2\right],\\
d_1' &= (p_1'-k_1)^2-m^2=-\frac{1}{x}\left[m^2x^2+(x\mathbf{q}-\mathbf{k}_1)^2\right],\\
2pp_1 &= t_1=\frac{1}{1-x}\left[m^2z+(\mathbf{k}-\mathbf{k}_1)^2\right],\\
2p_1p_1' &= t_2=\frac{1}{1-x}\left[m^2z+(x\mathbf{q}+\mathbf{k}_1-\mathbf{k})^2\right].
\end{aligned}
\tag{17}
$$

The subsequent integration is straightforward and gives the result

$$
\begin{aligned}
\Delta\tau_e^{hard} &= \tau_e^{(0)}\frac{\alpha}{\pi}\left[\left(G(-k^2)+G(-k^{'2})-G(t)\right)\ln\frac{1}{\Delta}+\right.\\
&\left.+G_1(-k^2)+G_1(-k^{'2})-G_1(t)\right],
\end{aligned}
\tag{18}
$$

where $G(t)$ and $G_1(t)$ were defined above.

The interference of two amplitudes with the photon emitted by two initial particles is small $\sim O(t/s)$. This fact is known in the literature as the up-down cancellation. The contribution of the diagrams with the two-photon exchange is purely imaginary and, consequently, does not interfere with the real Born amplitude. Adding all the contributions, we obtain the final result for one-loop radiative corrections to the electron impact factor,

$$
\Delta\tau_e=\frac{\alpha}{\pi}\tau_e^{(0)}F_1^{(2)}(t),\quad \tau_e^{(0)}=4\pi\alpha.
\tag{19}
$$

This result agrees with the generalized eikonal form of the small-angle scattering amplitude. But in the higher orders, the eikonal representation is violated, as shown below.

3. GENERALIZED EIKONAL REPRESENTATION

The above result for the radiative corrections to the electron impact factor can be obtained in a simple way. We consider again the decorated box with the positron block corresponding to the Born diagram and the electron one containing the set of four Feynman graphs with a virtual photon. We express the components of the exchanged photon momentum in terms of the squared invariant energies s_1 and s_2 for the electron and positron blocks using the Sudakov parameters

$$k = \alpha p_2 + \beta p_1 + k_\perp,$$

$$d^4k = \frac{1}{2s} ds_1 ds_2 d^2k_\perp, \quad k_\perp^2 = -\mathbf{k}^2,$$

$$s_1 = (k - p_1)^2 = -s\alpha - \mathbf{k}^2 + m^2,$$

$$s_2 = (k + p_2)^2 = s\beta - \mathbf{k}^2 + m^2 .$$

Performing the s_2-integration by taking the residue of the intermediate positron propagator (which also takes the diagram with crossed photon lines into account), we obtain the total radiative corrections

$$\frac{4\,\alpha}{s\,(2\pi)^2} \int \frac{d^2\mathbf{k}}{(\mathbf{k}^2 + \lambda^2)((\mathbf{q} - \mathbf{k})^2 + \lambda^2)} \times \\ \times \int_C ds_1 p_2^\mu p_2^\nu \bar{u}(p_1') A_{\mu\nu} u(p_1), \quad (20)$$

where $\bar{u}(p_1') A_{\mu\nu} u(p_1)$ is the Compton scattering amplitude corresponding to the Feynman diagrams with only the s-channel singularities and the contour C is situated above these singularities. The amplitude has the pole at $s_1 = m^2$, which corresponds to the electron intermediate state, and the right-hand cut starting from $s_1 = (m + \lambda)^2$, which corresponds to the one-electron and one-photon intermediate state.

Using the Sudakov parametrization for the photon momentum k and omitting the small contribution $\propto 1/s$ proportional to βp_1, we can represent p_2^μ as

$$p_2^\mu = \frac{1}{\alpha}(k - k_\perp - \beta p_1)^\mu \approx -\frac{s}{s_1 + \mathbf{k}^2}(k - k_\perp)^\mu. \quad (21)$$

We now consider the product of two terms in the right-hand side of this equation with the Compton amplitude $A_{\mu\nu}$. The contribution of the term $\propto k_\perp$ is zero,

$$|\mathbf{k}| s p_2^\nu \int_C \frac{ds_1 k_\perp^\mu}{(s_1 + \mathbf{k}^2)|\mathbf{k}|} \times \\ \times \bar{u}(p_1') A_{\mu\nu}(s_1, k, k') u(p_1) = 0. \quad (22)$$

This follows from the convergence of the integral over the large circle in the s_1 plane and the absence of the left cut. The second property is valid for planar Feynman graphs. The integral converges because for the physical (transverse) polarizations of the virtual photon, the quantity $e^\mu p_2^\nu A_{\mu\nu}, \mathbf{e} = \mathbf{k}_\perp/|\mathbf{k}|$ behaves as m^2/s_1 at large s_1.

Applying the Ward identity for the first contribution $\propto k^\mu$, we obtain

$$p_2^\mu p_2^\nu \bar{u}(p_1') A_{\mu\nu}(s_1) u(p_1) = \\ = -\frac{se^2}{s_1} p_2^\nu \bar{u}(p_1') \Gamma^\nu(q) u(p_1), \quad s_1 \gg m^2. \quad (23)$$

The integral over the large semi-circle gives the generalized eikonal result $\propto \Gamma_\nu$, which means, in particular, that the total contribution of the various intermediate states is not zero for physical $t < 0$. In particular, we see that radiative corrections to the impact factor of the electron contain infrared divergences cancelled only in the total cross-section with the contribution of the inelastic process (the photon emission).

For the n-photon exchange, the eikonal result for the scattering amplitude corresponds to the classical picture where all the intermediate fermions are on their mass shell. This is so because the Born amplitude for the t-channel photon interactions with external particles tends to zero as $(p_A\, k_i)^{-2}$ for $(p_A\, k_i) \to \infty$, which allows us to calculate all the integrals over $(p_A\, k_i)$ by taking residues. For the radiative corrections corresponding to the decorated diagrams with one additional virtual photon, we can use the arguments similar to those applicable in the two-photon case. The physical reason for the generalized eikonal result for the total contribution is that the integration over the invariant s_i (corresponding to the virtuality of the inner fermion line to which the virtual gluon line is attached) gives zero because after the cancellation of the renormalization effects in accordance with the Ward identity, the amplitude behaves as $1/s_i^2$ at large s_i. The non-vanishing result is obtained only from the diagrams where the virtual gluon line is attached to the external fermion lines, but we then obtain the generalized eikonal result. This argument is not valid for nonplanar diagrams because they have left and right singularities in the s_i planes [6].

4. IMPACT FACTORS IN THE TWO-LOOP APPROXIMATION

In the radiative corrections to the photon impact factor, the infrared divergences are cancelled in the sum of contributions from the $e^+e^-\gamma$ and e^+e^- intermediate states. Using the crossing relations for $t=0$ [7], one can express the contribution of the $e^+e^-\gamma$ intermediate state to τ^γ in terms of the contribution of the $e\gamma\gamma$ intermediate state to τ^e, which is investigated better (see [4–6]). We here estimate the radiative corrections for $t=0$ only at small virtualities of the exchanged photon $\mathbf{k}^2$. Their value can be extracted from the results of [3], where the one-loop correction to the cross-section of pair production by photon on the Coulomb field of nuclei was calculated as

$$\sum_{i=1}^{i=2}[\tau+\Delta\tau]^\gamma_{ii}(k,0)=\frac{28\mathbf{k}^2\alpha^2}{9m^2}[1+\delta_p],\quad \mathbf{k}^2\ll m^2,$$
$$\delta_p=\frac{\alpha}{\pi}\frac{9}{14}\left(\frac{1128}{35}\zeta(3)-\frac{6971}{210}\right)=0.009. \qquad (24)$$

The radiative corrections to the photon impact factor can be easily found also in the region $k^2\gg m^2$, where one can use the DGLAP evolution equations [10].

We now consider the radiative corrections to the electron impact factor. The generalized eikonal hypothesis is violated in the two-loop approximation. (This fact was verified explicitly for $t=0$ [6].) Indeed, if the generalized eikonal hypothesis were valid, the complete compensation of contributions from the transition of the initial electron to the intermediate states e, $e\gamma$, and $e\gamma\gamma$ would occur. However, it was shown that the total contribution is not zero and is equal to the interference term for the e^+e^- pair production amplitudes.

To clarify this result, we write the impact factor as

$$\tau^A=\int\limits_C\frac{ds_1}{2\pi i}\,\frac{1}{s^2}J^{(A)}_{\mu\nu}p^\mu_B p^\nu_B, \qquad (25)$$

where the quantity $(1/s^2)J^{(A)}_{\mu\nu}p^\mu_B p^\nu_B$ is expressed in terms of the amplitudes $J^{(A)}$ for the scattering of the virtual photon off the initial particles and does not depend on s as $s\to\infty$.

In contrast to the planar amplitude $A_{\mu\nu}$ discussed in the previous section, $J^e_{\mu\nu}$ corresponds to contributions of all possible diagrams. The integration contour C is displaced in accordance with the Feynman prescription between the right- and left-hand side singularities of the amplitude. The right singularities are the poles at $s_1=m^2$ and the cuts at

$$s_1>(m+\lambda)^2,\quad s_1>(m+2\lambda)^2,\quad s_1>9m^2.$$

There also exist left singularities at the same points for the crossing variable

$$u_1=-s_1-t-2m^2+\mathbf{k}^2+(\mathbf{q}-\mathbf{k})^2.$$

The additional e^+e^- pair can be produced in accordance with the Bethe—Heitler or bremsstrahlung mechanisms. There also exist interference terms taking the identity of the final electrons into account. The most important contribution is from the Bethe—Heitler mechanism corresponding to the e^+e^- pair production by two virtual photons. The corresponding impact factor contains the divergence in s_1 related to the presence of two-photon intermediate states in the crossing channel. (For $t=0$, this contribution was calculated in [11].) We write it here only in the Weizsäcker—Williams approximation, where it has the form of the sum rule for the Borselino formulas for the total cross-section $\sigma(s_1)$ of the e^+e^- pair production in the electron–photon collisions through the Bethe—Heitler mechanism,

$$\tau^e_{BH_e}=k^2\int\limits_{s_{th}}^{s}\frac{ds_1}{\pi}\,\frac{\sigma(s_1)}{s_1}=\frac{\alpha^3k^2}{\pi m^2}\left(a\ln^2\frac{s}{m^2}+b\ln\frac{s}{m^2}+c\right), \qquad (26)$$

$$a=\frac{14}{9},\quad b=-\frac{218}{27},\quad c=\frac{418}{27}-\frac{13}{2}\zeta(2). \qquad (27)$$

As discussed above, the logarithmic dependence on the upper limit s in the integral over s_1 must be subtracted in a self-consistent way to avoid the double counting, because the logarithmic contributions are summed by the Bethe—Salpeter equation for the pomeron in QED (cf. a similar procedure for the BFKL pomeron in the next-to-leading approximation [12]). For the muon production, we have

$$\tau^e_{BH_\mu}=\frac{\alpha^3k^2}{\pi M^2}\left(a\ln^2\frac{s}{M^2}+b\ln\frac{s}{M^2}+c\right), \qquad (28)$$

$$a=\frac{14}{9},\quad b=-\frac{218}{27}+\frac{28}{9}\ln\frac{M}{m},$$
$$c=\frac{3011}{324}-\frac{28}{9}\zeta(2)-\frac{107}{9}\ln\frac{M}{m}, \qquad (29)$$

where m and M are the respective masses of electron and muon.

The contribution of the bremsstrahlung mechanism to e^+e^- pair production must be added with the corresponding two-loop radiative corrections to the electron form-factor for the elastic intermediate state; the resulting expression corresponds to the generalized eikonal approximation because the corresponding diagrams are planar [6].

Among many Feynman graphs obtained from the interference between the various amplitudes for the pair production, there are only four non-planar diagrams corresponding to the identity of electrons in the final state in the Bethe—Heitler mechanism. Only they give a non-vanishing result for τ^e at $t = 0$. In the Weizsäcker—Williams approximation, the corresponding contribution was calculated in [7],

$$\tau_{int}^{(e)} \approx \frac{k^2}{m^2}\frac{\alpha^3}{\pi}\left(\frac{221}{315} + \frac{41549}{6300}\zeta(2) - \right.$$
$$\left. - \frac{216}{105}\zeta(3) - \frac{792}{105}\xi(2)\ln 2\right) \approx \frac{k^2}{m^2}\frac{\alpha^3}{\pi}(-3.57). \quad (30)$$

It leads to the sum rules for the integrals of the one- and two-photon bremsstrahlung cross-sections and the slope of the Dirac form-factor at $t = 0$ [6].

Finally, the total two-loop contribution to the electron impact factor can be written as

$$\tau^e = \frac{\alpha^2}{\pi^2}\,\tau_e^0\,F_1^{(4)} + \tau_{BH}^e, \quad (31)$$

where $F_1^{(4)}$ is the full two-loop correction to the Dirac form-factor (including the non-planar diagrams and the diagrams with the inner fermion loop). The term τ_{BH}^e is the total contribution of the imaginary part corresponding to the Bethe—Heitler mechanism of the pair production including the interference effects related to the identity of the produced electrons ($\tau_{BH}^e = \tau_{BH_e}^e + \tau_{BH_\mu}^e + \tau_{int}^{(e)}$ for $t = k^2 = 0$).

The physical meaning of this formula is obvious: the non-trivial corrections to impact factors are related only to the charge particle production in the intermediate states.

5. CONCLUSION

In the three-loop approximation, the most important contribution to the photon impact factor corresponds to the diagram with two fermionic loops connected in the t-channel by two photons. It contains the logarithmic divergence $\propto \ln s$ because the imaginary part of the corresponding amplitude is proportional to s_1 for large s_1. In particular, for $t = \mathbf{k}^2 = 0$, the impact factor can be expressed as the integral of the cross-section for the transition of two real photons into two e^+e^- pairs. Again, the ultraviolet divergence in s_1 is compensated by the infrared divergence in the relative rapidities of the produced pairs in the Bethe—Salpeter equation for the pomeron in QED. The virtual photon actually interacts with the electric dipoles inside the initial photon [13]. The growth of the impact factor $\propto \ln s$ is related to the logarithmic increase of the number of dipoles at large energies. The fermion identity effects in the intermediate state do not have any influence on this growth. The contribution of the diagrams with one e^+e^- pair and several photons gives a finite contribution to the photon impact factor.

We now consider three-loop corrections to the electron impact factor. The most important contribution $\propto \ln^2 s$ comes from the one-loop radiative corrections to the Bethe—Heitler mechanism of the e^+e^- production. Other diagrams lead to finite terms. The generalized eikonal representation is violated by non-planar diagrams related to the e^+e^- pair production, but there is another reason for its violation. It is related to the charge parity conservation in QED. Indeed, two external photons with the momenta k and $q - k$ cannot pass through the fermion loop to the three-photon intermediate state in the t-channel. Therefore, the generalized eikonal representation, containing in particular the form-factor corresponding to the transition of the external photon through the fermion loop into the three-photon state, cannot be valid in the three-loop approximation.

The methods developed above for QED can also be used in QCD, where we urgently need to calculate the radiative corrections to impact factors of the virtual photon and other particles to find the energy region of applicability of the BFKL theory in the next-to-leading approximation [11].

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