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MANIFESTATION OF DISSIPATION AND COMPRESSIBILITY IN THE EXPERIMENTS ON QUANTUM CAVITATION

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Cavitation in the liquid helium isotopes of ^{3}He and ^{4}He is considered. It is shown that the dissipative processes play an important role in the growth of the stable phase nucleus in the normal liquid ^{3}He. This leads to the lack of the quantum behavior of cavitation in this system down to 2 mK, which is in contrast to the thermal-quantum crossover in the cavitation of the superfluid ^{4}He at 600 mK. Below 180 mK, the dissipative ^{3}He kinetics is of the Knudsen type. The high value 600 mK for the transition into the quantum kinetic behavior in ^{4}He is related to the compressibility of a liquid, which, in particular, leads to a noticeable emission of sound with cavitation. The recent experiments on quantum cavitation in the liquid helium isotopes ^{3}He and ^{4}He are discussed.

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1. INTRODUCTION

At low temperatures, the first-order phase transition associated with the formation of droplets of a stable phase occurs via quantum tunneling through the potential barrier originating from the positive interfacial tension between the phases. Such macroscopic underbarrier motion is associated with the flow of the metastable fluid towards the droplet due to the mass difference between the stable and metastable liquids [1]. The growth of the droplet in the metastable phase is also accompanied by the dissipative effects due to the lack of equilibrium in the medium during the droplet evolution [2]. The direct experimental observations of the dissipation effects accompanying the kinetics of the first-order phase transition are absent so far.

Recently, there was performed an experiment on the kinetics of the first-order phase transition at the cavitation in superfluid ^{4}He [3] and normal liquid ^{3}He [4]. In the experiment [5], the large pressure oscillations in helium are produced by focusing ultrasonic waves at the center of the cell that has four windows providing an optical access in the two perpendicular directions. The method is used to obtain the negative pressure region in the bulk of the liquid in order to avoid the surface nucleation. The cavitation is found to be a stochastic process. A significant cavitation rate is observed near the spinodal pressure.

The investigation of cavitation has a long history. First of all, this involves the investigation of the crossover from thermal to quantum behavior. According to the first estimates [6] of the cavitation rates at which bubbles nucleate in a liquid ^{4}He, it has been expected that quantum nucleation should dominate over the thermally activated one at temperatures below ≈ 0.3 K and that for this temperature range, the pressure providing a noticeable nucleation rate or the tensile strength should be about $P \approx -15$ atm. Later, Maris and Xiong [7] attracted one's attention to the possibility that before this pressure can be achieved, the liquid ^{4}He becomes unstable against the long wavelength density fluctuations once the square of the sound velocity becomes negative. The extrapolations of the sound velocity into the negative pressure range and some numerical calculations suggest that the sound velocity at the pressure P vanishes as

$$c(P) \propto (P - P_c)^\nu$$

with the exponent ν close to $1/3 \div 1/4$. The critical

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pressure P_c, i.e., the pressure at the spinodal point, was estimated as $P_c = -(8\text{–}9)$ atm at absolute zero for ^{4}He. For liquid ^{3}He, it was expected that $P_c = -(2\text{–}3)$ atm [8]. According to [9], the crossover temperature T^* from thermal to quantum behavior must be about 125 mK for ^{3}He and 220 mK for ^{4}He. The analysis is based only on the thermodynamic properties of ^{3}He and ^{4}He, i.e., on the chemical potentials and surface energies. The kinetic properties of the system (relaxation processes) were not involved.

The above result for T^* was also supported by the description of homogeneous and unhomogeneous states of liquid helium within the density-functional method [10]. In addition, the liquid-vapor phase diagram in ^{3}He and ^{4}He was analyzed with the help of this method [10] in the vicinity of the spinodal line. The spinodal pressures $P_c = -9$ atm for liquid ^{4}He and $P_c = -(2\text{–}3)$ atm for liquid ^{3}He were found. A Monte Carlo simulation of the critical behavior of liquid ^{4}He at negative pressure in the vicinity of the spinodal curve [11] was performed.

The dissipation [2] and sound emission [12] strongly affect the underbarrier nucleation kinetics [13]. Recently, the dissipation effects in liquid ^{3}He have also been considered within the density-functional method using the hydrodynamic description for the metastable fluid [14].

The nucleation of bubbles in ^{4}He at negative pressures and temperatures down to 65 mK is studied experimentally [5]. The results are consistent with the idea that the nucleation is a result of the quantum tunneling through the potential barrier below 0.6 K. The quantum nucleation of bubbles occurs at the negative pressure $P_c = -9.23$ bar, which is close to the spinodal pressure, i.e., although only 0.29 bar above. For the normal ^{3}He, although the observed cavitation threshold is smaller and agrees with the prediction $P_c = -3.1$ bar, the results are too preliminary to assert the quantum regime of the cavitation above 40 mK.

Here, we emphasize two points that permit us to understand the disagreement between theory [9] and experiment [5]. First of all, it is the energy dissipation during the underbarrier motion of a nucleus of the stable phase in the normal ^{3}He that reduces the quantum nucleation rate. The second point is that the experiments are performed near the spinodal line and the sound velocity vanishes at $P = P_c$. In this case, the kinetic energy of a growing bubble K falls down and the crossover temperature T^* increases [12, 13]. This may be one of reasons why T^* equals 0.6 K in ^{4}He [4, 5] and not 0.2 K as predicted in [7].

2. DISSIPATION AND SOUND EMISSION IN THE THEORY OF QUANTUM CAVITATION

We discuss the rate at which the bubbles can nucleate via quantum fluctuations in the normal ^{3}He and superfluid ^{4}He at negative pressures and sufficiently low temperatures. The energy dissipation due to viscosity and the sound emission due to compressibility of a fluid are involved in the bubble growth kinetics. Owing to viscosity, the quantum cavitation kinetics in ^{3}He differs qualitatively from that in ^{4}He and corresponds to the dissipative tunneling in the overdamped regime. The compressibility results in increasing the cavitation rate and is essential in both liquids, especially, for the small critical bubbles that are responsible for the experimentally observable rates of the quantum cavitation.

Usually, all the calculations of the cavitation rate and tensile strength in the region of the quantum tunneling regime have been performed within the framework of the Lifshits–Kagan theory [1] of the first-order phase transitions. However, this theory neglects the compressibility of the metastable liquid; in other words, the sound velocity is taken to be infinite in the liquid. Clearly, a more realistic theory of the quantum cavitation must involve the effect of the finite compressibility, especially, in the closest vicinity of the instability point at which the sound velocity vanishes.

To investigate quantum-mechanical tunneling between the metastable and stable states of a condensed medium and to calculate the rate at which cavities nucleate, we use the formalism based on the finite action solutions (instantons) of equations continued to the imaginary time. This approach [15], elaborated for describing quantum-mechanical tunneling in the systems with a macroscopic number of degrees of freedom, was used for incorporating the influence of energy dissipation in a metastable condensed medium on the quantum kinetics of the first-order phase transitions at low temperatures [2].

The rate of the quantum nucleation can be written as

$$\Gamma(T) = \Gamma_0(T) \exp\left(-S(T)/\hbar\right) \qquad (1)$$

where the prefactor Γ_0 is the rate at which cavitation is attempted per unit volume and unit time. According to the general notions of the nucleation kinetics, the prefactor Γ_0 can be evaluated approximately as an attempt frequency ν_0 multiplied by the number of centers at which the independent cavitation events can occur.

In its turn, the exponent S is the critical value of the effective Euclidean action [2]

$$S_{eff}[R_\tau] = \int\limits_{-\beta\hbar/2}^{\beta\hbar/2} d\tau \left[U(R_\tau) + \frac{1}{2}M(R_\tau)\left(\frac{dR}{d\tau}\right)^2\right] + \\ + \frac{1}{4\pi}\iint\limits_{-\beta\hbar/2}^{\beta\hbar/2} d\tau\, d\tau' \frac{(\pi T)^2}{\hbar^2 \sin^2\left[\pi T(\tau-\tau')/\hbar\right]} \times \\ \times \left[D(R_\tau, R_{\tau'}) - C(R_\tau, R_{\tau'})\right], \quad (2)$$

where $\beta = T^{-1}$ is the inverse temperature. The path $R(\tau)$, which is defined in the imaginary time τ, satisfies the periodic boundary conditions $R(-\beta\hbar/2) = R(\beta\hbar/2)$. It should be emphasized that all parameters of the effective action are unambiguously associated with the corresponding parameters in the classical equation of growth,

$$U'(R) + \mu_1(R)\dot{R} + M(R) \times \\ \times \left[\ddot{R} + \frac{1}{2}\frac{M'(R)}{M(R)}\dot{R}^2\right] - \mu_3(R) \times \\ \times \left[\dddot{R} + \frac{3}{2}\frac{\mu_3'(R)}{\mu_3(R)}\ddot{R}\dot{R} + \frac{1}{2}\left(\frac{\mu_3''(R)}{\mu_3(R)} - \frac{\mu_3'^2(R)}{2\mu_3^2(R)}\right)\dot{R}^3\right] + \\ + \ldots = 0. \quad (3)$$

The correspondence can readily be settled by analytically continuing ($|\omega_n| \to -i\omega$) the Euler–Lagrange equation ($\delta S_{eff}/\delta R_\tau = 0$) for the effective action to real time, which gives the classical equation of growth. The substitution $|\omega_n| \to -i\omega$ of the Matsubara frequencies with the real ones must be performed in the frequency representation of the corresponding equations.

In the classical equation (3) for the growth of the supercritical droplet, we assume the limit of a low growth rate of the droplet and expand the rate in powers of the growth rate. The growth rate-independent term corresponds to the potential energy of the bubble. The term that is linear in the growth rate describes the energy dissipation, the second-order term corresponds to the kinetic energy of the droplet, and the third-order term can be attributed to the sound emission with the change of the bubble volume. Thus, it is clear that the first two terms in Eq. (2) can be referred to as the potential energy $U(R_\tau)$ and the kinetic energy with the mass $M(R_\tau)$ of the bubble. The other terms are nonlocal in time and are due to the energy dissipation $D(R_\tau, R_{\tau'})$ during the bubble growth and the sound emission $C(R_\tau, R_{\tau'})$ originating from the finite compressibility. The energy dissipation is connected with the vortex $\gamma_1(R)$ [2,13] as

$$D(R_\tau, R_{\tau'}) = (\gamma_1(R_\tau) - \gamma_1(R_{\tau'}))^2 \quad (4)$$

and $\gamma_1(R)$ is unambiguously determined by the friction coefficient

$$\mu_1(R) = (\partial\gamma_1(R)/\partial R)^2. \quad (4a)$$

We would like to make an important remark concerning the behavior of the friction coefficient $\mu_1(R)$ as a function of the bubble radius and temperature. In the hydrodynamic approximation, the bubble radius must be much larger than the mean free path $l(T)$ of excitations in the medium surrounding the bubble. Since the mean free path increases rapidly at low temperatures, in particular, $l(T) \propto 1/T^2$ for ^{3}He, the crossover from the hydrodynamic $R \gg l$ regime to the ballistic or Knudsen regime with $R \ll l$ must occur. Depending on whether the hydrodynamic or ballistic regime occurs, we arrive at the general expression for the friction coefficient $\mu_1(R)$,

$$\mu_1(R) = 16\pi\eta R f(R/l),$$

where η is the viscosity coefficient and

$$f(x) = \begin{cases} 1, & x \gg 1, \\ ax, & x \ll 1 \end{cases} \quad (4b)$$

is a dimensionless function of the ratio of the bubble radius to the mean free path of excitations in the liquid. The numerical factor a is of the order of unity, depends on the specific features of the interaction of excitations with the bubble surface, and can be calculated explicitly using the kinetic equation.

It should be noted that the friction coefficient $\mu_1(R)$ in the ballistic $R \ll l$ regime is independent of the mean free path $l(T)$ since $\eta \sim \rho c l$, where ρ is the density of the liquid. In this case, $\gamma_1(R)$ does not depend on the temperature

$$\gamma_1(R) = \sqrt{4\pi a\eta/l}\, R^2. \quad (5)$$

This ballistic regime with the temperature-independent $\gamma_1(R)$ is the only possible underbarrier motion of the nucleus because the opposite case where $R > l$ implies a large critical radius R_c within the whole range of temperatures outside a close vicinity of T_λ. The large critical radius R_c leads to a negligible decay rate of the metastable liquid and to the impossibility of recording it experimentally. Thus, for the underbarrier motion of the cavity, we can always assume the validity of Eq. (5).

The second nonlocal term in (2),

$$C(R_\tau, R_{\tau'}) = \left[\frac{\partial\gamma_3(R_\tau)}{\partial\tau} - \frac{\partial\gamma_3(R_{\tau'})}{\partial\tau'}\right]^2, \qquad (6)$$

is responsible for the excitation and emission of sound waves in the course of the underbarrier growth of a bubble and $\gamma_3(R)$ is determined by the kinetic coefficient $\mu_3(R)$,

$$\mu_3(R) = (\partial\gamma_3(R)/\partial R)^2.$$

The corresponding coefficient $\mu_3(R)$ is given by

$$\mu_3(R) = \frac{4\pi\rho}{c}R^4,$$

which leads to

$$\gamma_3(R) = \frac{2}{3}\sqrt{\frac{\pi\rho}{c}}R^3. \qquad (7)$$

It is interesting to note that in contrast to the term with the Ohmic dissipation $D(R_\tau, R_{\tau'})$ related to the dissipative function that is proportional to the square of the first-order time derivative, the term $C(R_\tau, R_{\tau'})$ due to the finite compressibility of the fluid medium gives a negative contribution into the effective action (Eq. (2)). This results in enhancing the quantum nucleation rate compared with the one calculated in the framework of the Lifshits–Kagan model [1] of an incompressible fluid. Some hints for this conclusion can be seen from the fact that the finiteness of the sound velocity confines the region of the bubble environment that can be disturbed and set into motion. The size of this region is approximately equal to $\Lambda = c\tau$, where τ is a typical time of growth. In some sense, one can say that the total kinetic energy of the fluid flowing away from the expanding bubble becomes smaller than for the incompressible fluid where the perturbation induced by the formation of the bubble extends instantaneously to the infinity.

The kinetic energy can be described in terms of the variable mass of the bubble

$$M(R) = 4\pi\rho R^3. \qquad (8)$$

It can be attributed to the kinetic energy of the fluid that flows away from the bubble. In the case of cavitation, the potential energy can be represented as

$$U(R) = \frac{4\pi}{3}PR^3 + 4\pi\alpha R^2, \qquad (8a)$$

where α is the liquid–gas surface tension.

Equations (1) and (2) with the coefficients (4)–(8) allow us to calculate the rate of the underbarrier motion of the bubble. It should be emphasized that all parameters of the effective action are unambiguously associated with the corresponding parameters in the classical equation of growth (3).

3. THE MANIFESTATION OF COMPRESSIBILITY IN THE EXPERIMENT ON QUANTUM CAVITATION IN SUPERFLUID ^{4}He

Recently [5], cavitation was studied in superfluid ^{4}He and normal liquid ^{3}He experimentally. The investigation of cavitation in these liquids is related to the possibility of avoiding impurities, which usually manifest themselves as centers of cavitation. On the other hand, these liquids have essentially different properties, namely, ^{4}He is a superfluid liquid and ^{3}He is a normal viscous Fermi liquid in the experimental range of temperatures from 40 to 1000 mK. The cavitation process is induced by sound pulses at a frequency ω close to 1 MHz and is focused in the center of the experimental cell. The pulses create oscillations of the local pressure about several bars around the static pressure. The typical size of the acoustic focus is ~ 0.12 mm and the size of the experimental cell is 8 mm. The above-mentioned limiting temperature 40 mK is connected with thermal radiation due to these sound pulses with a short duration between 30 and 70μs and the repetition rate within the range 0.1–1 Hz.

The cavitation process is observed to be stochastic. For the invariant temperature and pressure parameters, some sound pulses of a given amplitude produce the cavitation and some pulses of the same amplitude do not. Applying several sound pulses and counting the number of cavitation events, one can determine the probability Σ of cavitation as a function of the applied voltage and temperature. According to [5], it is obtained that the cavitation probability in ^{4}He depends on temperature only above about 400 mK. One of the difficulties in interpreting the experiment is related to the fact that the maximum of the sound attenuation exists in the system in this temperature range. An increase of the sound attenuation has the consequence that a larger voltage is needed to produce the same pressure swing at the acoustic focus where cavitation occurs. After the correction, the cavitation voltage is found to be independent of the temperature up to 600 mK. Above this temperature, the voltage decreases as T increases, corresponding to a thermally activated nucleation. This experimental result can be interpreted as a crossover from the quantum cavitation below 600 mK to the thermally activated cavitation. It should be emphasized [5] that the stochastic behavior of

the nucleation process combined with the temperature-independent behavior of the voltage at which the cavitation occurs is in contrast to the assumption that the spinodal pressure is achieved, because this pressure cannot result in the stochastic behavior of the cavitation process.

There is one more difficulty in interpreting the experiment. The temperature of the cell that is measured in the experiment [5] may be different from the temperature in the focus where the cavitation occurs. The point is that the acoustic wave is adiabatic in the first approximation, and consequently, the temperature and the pressure oscillates at the focus. Within the adiabaticity assumption, the temperature in the focus of the sound wave can readily be estimated [5]. At the temperatures below 0.7 K, phonons make the dominant contribution into the entropy per unit mass [16],

$$S \approx S_{ph} = \frac{2\pi^2 T^3}{45\rho^3 c^3}. \tag{9}$$

In the isentropic process, the temperature is therefore proportional to the sound velocity c. It is found experimentally [5] that near the spinodal at $P = -9.23$ bar, the sound velocity is 74 m/s, which is by the factor 3 lower than at zero pressure ($c = 238$ m/s). As a result, the local instantaneous temperature T must be reduced at the focus by the same factor 3 with respect to the static temperature T_{stat} of the cell [5].

This interpretation is consistent only if the following two conditions are fulfilled. The first condition is related to the well-known fact that the nonlinear effects arise very early in an alternating field [17]. The typical field in which the nonlinear effects arise is proportional to the exponential

$$V = \tilde{V} \exp\{\omega\tau_0\}, \tag{10}$$

where ω is the sound frequency, τ_0 is a typical time of the underbarrier motion and V is the sound amplitude. In the experiment [5], $\omega = 1$ MHz and in the experimentally analyzed vicinity of the spinodal line, τ_0 can be estimated as $\tau_0 = 10^{-10}$ s^{-1}. Thus, in the experiment range where $\omega\tau_0 \ll 1$, the nonlinear effects can be neglected. The second condition is much more severe, meaning that T entering Eq. (9) follows local variations of the pressure in space and time in the sound wave. The conditions can be represented as

$$\omega\tau \ll 1 \quad \text{or} \quad l \ll \lambda. \tag{11}$$

These conditions are essentially equivalent. The second inequality can be obtained from the first by multiplying it by the sound velocity c. The second condition means that the sound wavelength λ must be much larger than the mean free path l. In the experiment [5], the opposite condition is fulfilled within the entire temperature range. The size of the acoustic focus is ~ 0.12 mm and this distance is much less than the mean free path l_{ph} for the phonon–phonon scattering, which equals 1.3 mm at 0.7 K. According to [16], $\tau_{ph}^{-1} \approx 6 \cdot 10^6 T^7$ s (with T measured in K), $l_{ph} \simeq c\tau_{ph}$, with the value $c = 74$ m/s near the spinodal line used in the estimate. Moreover, l_{ph} increases as T^{-7} with lowering the temperature and becomes about 15 mm at $T = 0.5$ K, which exceeds the size of the experimental cell of 8 mm. The other scattering processes, in particular, the phonon–roton and roton–roton scatterings, are inefficient at low temperature for the relaxation to local equilibrium because of freezing rotons. Thus, the local temperature T in the sound wave cannot follow the variations of the pressure in the sound wave in this range of temperatures, with the entropy S in Eq. (9) being conserved. The local temperature in the focus of the sound wave is therefore equal to the temperature outside the focus, i.e., $T^* = 0.6$ K at the crossover point from thermal to quantum behavior of the kinetics of bubble nucleation.

We are now able to compare the thermal–quantum crossover temperature T^* obtained experimentally with the calculations. We start from the simplest estimate that can be obtained from the first two terms of Eq. (2). These two terms are the potential and kinetic energy of the growing cavity and correspond to the Lifshits–Kagan analysis [1]. In the case of cavitation the crossover temperature reduces to the following equation with the known parameters:

$$T^* = \frac{256\hbar}{405\pi\sqrt{6}} \frac{|P|^{3/2}}{\alpha\sqrt{\rho}}. \tag{12}$$

The substitution of the ^{4}He data $\rho = 0.095$ g/cm^3, $\alpha = 0.37$ erg/cm^2, and the experimental value $P = -9.5$ bar near the spinodal line gives $T^* = 0.15$ K. The estimate used is a thin-wall approximation where the bubble is assumed to have a sharp surface of the radius R forming the boundary between an empty interior and the bulk liquid surrounding the bubble. A more elaborate calculation for the bubbles of the radius that is comparable with the interface thickness [9] uses the density-functional approximation for the energy of the metastable liquid and gives $T^* = 0.2$ K. The insignificant difference between these two approximations is not surprising because they both are based on the same value of the surface energy α [1, 9]. The difference between these two opposite estimates is less than the experimental value, which, as is emphasized, should be taken as $T^* = 0.6$ K instead of $T^* = 0.2$ K that was

assumed in [5]. The results are insensitive to the inclusion of the third term $D(R_\tau, R_{\tau'})$ in Eq. (2) that describes dissipation because the dissipation in ^{4}He is negligible at low temperatures. Moreover, this leads to lowering T^* and to a deviation of its value from the experimental result $T^* = 0.6$ K. But the term involving $C(R_\tau, R_{\tau'})$ in Eq. (2) leads to the opposite and important effect of increasing T^*. If we consider the term with $C(R_\tau, R_{\tau'})$ in Eq. (2) as a perturbation, we obtain the following expression for effective action S_{eff} in Eq. (2) in the low-temperature limit:

$$S_{eff} = \frac{5\sqrt{2}\pi^2}{16}(\alpha\rho)^{1/2} R_c^{7/2}\left(1 - \frac{4}{9c}\sqrt{\frac{2\alpha}{\rho R_c}}\right), \qquad R_c = \frac{3\alpha}{|P|}. \tag{13}$$

This expression differs from that for S_{eff} in [1] only by the factor $(1 - \dot{R}/c)$, where $\dot{R} = (2/3)^{5/2}(|P|/\rho)^{1/2}$ is the rate of the underbarrier growth of the cavity. Substituting the data $\rho = 0.095$ g/cm^3, $P = -9.5$ bar, and $c = 74$ m/s, we obtain $\dot{R}/c = 0.48$ and hence T^* increases approximately twice and equals $T^* = 0.4$ K. We assert that the tendency of increasing T^* due to a finite compressibility of ^{4}He and the underbarrier sound emission during cavitation is a reason for the high value of T^* observed experimentally. The manifestation of the phenomenon is strongly related to a high value of the ratio $\dot{R}/c$, which is about one half. The high value is directly related with the experimental conditions [3, 5] of the cavitation taking place in the vicinity of the spinodal pressure. For $P = 0$, this ratio is only about 0.1. In any case, more elaborated considerations should be used in analysing the phenomenon because the leading approximation in $\dot{R}/c \ll 1$ is assumed for the derivation of Eq. (2).

4. THE MANIFESTATION OF DISSIPATION IN THE EXPERIMENT ON QUANTUM CAVITATION IN NORMAL FLUID ^{3}He

We now turn to the analysis of the experiments on quantum cavitation in liquid ^{3}He. ^{3}He is a normal viscous Fermi liquid within the experimental range of temperatures from 40 to 1000 mK [4]. The simplest estimate for T^* in Eq. (12) gives $T^* = 0.09$ K for $\rho = 0.054$ g/cm^3, $\alpha = 0.16$ erg/cm^2, and the pressure $P = -3.1$ bar near the spinodal of ^{3}He. A more accurate calculation for the bubbles of the radii comparable with the interface thickness uses the density-functional approximation for the energy of the metastable liquid and gives $T^* = 0.125$ K [9]. However, the crossover to the quantum behavior is not observed experimentally down to $T = 0.04$ K [4].

Both the above estimates are based on the first two terms in the effective action S_{eff} in Eq. (2), which include only the potential and kinetic energies in different approximations and ignore the fact that liquid ^{3}He is a viscous liquid. The viscosity η behaves as T^{-2} with the temperature since ^{3}He is a Fermi liquid and $\eta \sim \rho v_F l_F$, where ρ is the density of ^{3}He, v_F is the Fermi velocity, and $l_F \sim v_F \tau_F$ is the mean free path. Here τ_F is the collision time for excitations in the Fermi liquid and $\tau_F \sim D\hbar\varepsilon_F/T^2$, where ε_F is the Fermi energy, $D \sim (p_F a_0/\hbar)^{-2}$ is a dimensionless coefficient, and a_0 is the scattering length [18]. Using the expression for the Fermi momentum $p_F = \hbar(3\pi^2\rho/m)^{1/3}$ and $\varepsilon_F = p_F^2/2m^*$, with m being the mass of the ^{3}He atom and m^* being the effective mass such that $m^*/m = 3.08$, and substituting $\rho = 0.054$ g/cm^3 near the spinodal, we obtain $p_F/\hbar = 0.68 \cdot 10^8$ cm^{-1}. If we put $D \sim 0.15$, we obtain $\tau_F = 1.2 \cdot 10^{-12} T^{-2}$ s (with T expressed in K), which differs from the value for τ_F obtained from the viscosity [16] only by the factor 1.3 due to the difference between the density of ^{3}He near the spinodal line ($\rho = 0.054$ g/cm^3) and the density $\rho = 0.082 g/cm^3$ at pressure $P = 0$. For l_F, we have

$$l_F \simeq A/T^2, \qquad A = \hbar^4 \frac{3\pi^2}{2}\left(\frac{m^*}{m}\right)^{-2}\frac{D\rho}{m^3}, \qquad l_F \simeq 0.5 \cdot 10^{-8} T^{-2}\text{cm} \quad (T \text{ in K}). \tag{14}$$

We see that l_F is about 0.5 Å at the temperature $T = 1$ K. In this case, $l_F \ll R_c$, because $R_c \sim 10$ Å according to [1] (see also the introduction to [19]). With lowering the temperature, the mean free path l_F grows drastically as T^{-2} and l_F becomes ~ 50 Å at $T = 0.1$ K for $l_F \gg R_c$. Thus, within the temperature range 1–0.1 K, the behavior of the nucleation of bubbles varies from the hydrodynamic type to the ballistic one and we can introduce [2] the temperature T_l at which the mean free is path $l_F \simeq R_c$ and the hydrodynamic nucleation type is replaced by the ballistic one,

$$T_l = \sqrt{A/R_c}. \tag{15}$$

For cavitation in ^{3}He near the spinodal, $T_l = 0.18$ K. At this point, we go over from one type of the dissipation in the system to another one. Above $T > T_l$, the nucleation is governed by the hydrodynamic flow of viscous Fermi liquid (4b) (with $R_c/l_F \gg 1$) and for $T < T_l$ the ballistic propagation of excitations in the Fermi liquid

occurs (4b) (with $R_c/l_F \ll 1$). For $T > T_l$, the viscosity η enters the dissipation at the bubble nucleation. Inserting the above estimate into the expression for η, we obtain from [16] $\eta = \alpha_0/T^2$, with $\alpha_0 \sim 10^{-6}$ pois. For $T < T_l$, the dissipation is governed by η/l_F.

We see that near the spinodal in ^{3}He, T_l is higher than T^*, which is about 0.1 K according to the estimate without the dissipation processes at nucleation are taken into account. The involvement of dissipation only reduces T^*. In any case, therefore, the quantum cavitation is accompanied by the dissipation of the Knudsen type. We can now compare the value of the effective action S_{eff}, Eq. (2), in the dissipationless case (13) with the one involving the dissipation, $S_{diss} \approx \mu_1 R_c^2$. For $T < T_l$ ($R_c/l_F \ll 1$), this ratio is $q = S_{kin}/S_{diss} \sim 0.05(P/\rho)^{1/2} v_F^{-1} \sim 0.08$ for the above-mentioned values P and ρ near the spinodal of ^{3}He. This estimate means that the growth of the bubble is accompanied by a strong dissipation corresponding to the overdamped quantum regime and the thermal-quantum crossover temperature is $T^* = \hbar U_0/S_{diss}$, with $U_0 = (16/27)\alpha R_c^2$ being the height of the potential barrier. For the dissipation of the Knudsen type, we thus obtain T^* as

$$T^* = s^{-1} \frac{m^*}{3^5 \pi} \frac{\hbar}{p_F} \frac{P^2}{\rho\alpha}, \tag{16}$$

where s is a coefficient that depends only on the dissipation type. In what follows, we see that for the ballistic propagation of excitations in the metastable environment, $s = s_b \approx 1.2$ and T^* becomes about 2 mK. Note that $\hbar$ does not enter the expression for T^* in Eq. (16) because $p_F/\hbar$ depends only on the density ρ, i.e., approximately, $p_F/\hbar = (3\pi^2\rho/m)^{1/3}$. The absence of $\hbar$ in T^* is related to the overdamped ballistic regime of dissipation. In this case, the dissipation is proportional to η/l, which is of the order ρv_F and is proportional to $\hbar$. Thus, $\hbar$ does not enter T^* because of a purely quantum nature of the dissipation in the Fermi liquid. A formal reduction of the dissipation ($\hbar \to 0$) leads to a dissipationless behavior where $\hbar$ enters again in T^*.

To determine s_b, we can use only two terms of the effective action $S_{eff}[R_\tau]$ in Eq. (2), namely, the potential energy $U(R_\tau)$ and the nonlocal dissipative term $D(R_\tau, R_{\tau'})$, because the term with the kinetic energy is small in the case of a strong dissipation and is proportional to $q \sim 0.08$. We can reduce $S_{eff}[R_\tau]$ to the dimensionless action $s_b(t)$, cf. [2],

$$s_b[x_\tau] = \int_{-1/2t}^{1/2t} d\tau x_\tau^2 (1 - x_\tau) + \frac{\pi t^2}{4} \times \\ \times \int_{-1/2t}^{1/2t} d\tau' \frac{(x_\tau^2 - x_{\tau'}^2)^2}{\sin^2 [\pi t(\tau - \tau')]}. \tag{17}$$

The numerical calculation of (17) gives $s = s_b \approx 1.2$.

5. CONCLUSIONS

We would like to emphasize the qualitative feature whereby the normal liquid ^{3}He differs from the superfluid ^{4}He [2], namely, the dissipation of energy in the course of quantum cavitation for normal ^{3}He. In the absence of the dissipation, only the kinetic energy K of the motion of the metastable liquid governs the underbarrier dynamics of the growing bubble. The kinetic energy can be described in terms of the variable mass of the bubble $M(R) = 4\pi\rho R^3$ as $K = M\dot{R}^2/2$ [1]. The underbarrier motion of the bubble in ^{4}He corresponds to the dynamic motion indicated above if we disregard phonon excitations. Anyway, this is true at low temperatures. In ^{3}He, we must also take the viscous motion of the normal Fermi liquid into account. This leads to the appearance of the term with the energy dissipation in the bubble expansion equation.

The growth rate of the bubble is determined by the interplay of the kinetic energy K and the energy dissipation. It should be emphasized that there are no free parameters in ^{3}He that can determine the relative contribution of these two terms. The kinetic energy K is of the order $M(R)\dot{R}^2/2$ and should be compared with $\mu(R)R\dot{R}$ from the energy dissipation. The ratio of K to the energy dissipation is of the order $\dot{R}/v_F \ll 1$. The last condition is connected with the approximate relation $v_F \sim c$ and v_F differs from the sound velocity c only by a numerical coefficient of about $2/(\sqrt{3}\pi^{1/3})$. Our consideration assumes the slow growth rate of the bubble, $\dot{R} \ll c$. We would like to emphasize that the dynamics of the underbarrier motion of the bubble is governed by the energy dissipation power, rather than by the kinetic energy K. This implies the overdamped regime of quantum cavitation. Accordingly, we have the exponent $\mu(R_c)R_c^2$ in the growth rate instead of $\sqrt{2M(R_c)U(R_c)}$ as in the absence of dissipation. In addition,

$$\mu(R_c)R_c^2 \gg \sqrt{2M(R_c)U(R_c)}.$$

Thus, because of the dissipation, the crossover temperature T^* falls down and becomes lower than the temperature of the ^{3}He transition into superfluidity. This manifests itself as T^* because of the lack of dissipation in the superfluid state. That is why the crossover from thermal to quantum behavior was not found [4].

For the understanding of the experiments on quantum cavitation in superfluid ^{4}He [3], it is important to incorporate the compressibility and sound emission into the equation of the bubble growth. The point is that the experiments are performed near the spinodal line and the sound velocity vanishes at $P = P_c$. In this case, the kinetic energy K falls down and T^* increases. This is the reason why T^* equals 0.6 K in ^{4}He [3, 5], and not 0.2 K as predicted in [9].

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