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QUANTUM GLASS TRANSITION IN A PERIODIC LONG-RANGE JOSEPHSON ARRAY

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We show that the ground state of a periodic long-range Josephson array frustrated by a magnetic field is a glass for sufficiently large Josephson energies despite the absence of quenched disorder. Like superconductors, this glass state has non-zero phase stiffness and Meissner response; for lower Josephson energies the glass «melts» and the ground state loses its phase stiffness and becomes insulating. We find the critical scaling behavior near this quantum phase transition: the excitation gap vanishes as $(J - J_c)^2$, and the frequency-dependent magnetic susceptibility behaves as $\chi(\omega) \propto \sqrt{\omega} \ln \omega$.

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1. INTRODUCTION

Glass formation in the absence of intrinsic disorder is a long-standing problem, but recent years have witnessed rapid progress [1–7] in the qualitative understanding of this phenomenon. Mostly this progress is due to the solution of periodic models that assume a mapping between the periodic model and the appropriate random model [1–3]. The validity of this assumption is still an open question in the general case, but it was shown that at least one periodic model allows direct study of the phase transition [5] and non-ergodic behavior below the transition [7] without any reference to a disordered model. This model describes a long-range Josephson array in a magnetic field, and another reason for the interest in this model is that it can be realized experimentally (cf. Refs. [6] and [8] for a discussion of experimental conditions).

All these results were obtained in the framework of classical statistical mechanics; the glass formation in regular quantum systems has not been addressed. The goal of this paper is to fill this gap. The problem of glass formation in disordered quantum systems is discussed in a number of papers [9–12], which studied critical behavior near the quantum vitrification transition [9, 10] and the properties of the glassy phase itself [11] using the replica approach. They found that the glass phase transition at $T = 0$ indeed exists; further, it strongly resembles the classical (high-T) phase transition in the same system. The main difference is in the critical exponent of the correlation function, which decays faster than at the classical critical point: $D(t) = \overline{\langle S_j(0) S_j(t) \rangle} \propto t^{-1}$ at $T = 0$ (cf. $D(t) \propto t^{-1/2}$ at non-zero T). A surprising result

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obtained in Ref. [11] is that at zero temperature, no replica symmetry breaking (RSB) is needed for the description of the glassy state, i.e. the replica-symmetric solution is stable at $T = 0$. Since RSB is usually believed to be a signature of non-ergodicity, this result means either absence of non-ergodic behavior at $T = 0$ or violation of the usual relation between RSB and non-ergodicity. We believe that the latter case is more likely because of the following. The non-ergodicity affects the full thermodynamic averaging only if higher metastable states contribute to the partition sum. It is more likely that in a typical situation the gap between the ground state and the next state remain finite at $T \to 0$, so in this limit only the ground state contributes to the full thermodynamic average and the RSB does not occur, although the system retains its non-ergodic behavior; note that a similar phenomena is believed to occur in the classical $T = 0$ limit [13]. We feel that in order to clarify this important question, an approach that is free from the ambiguities of the replica method should be employed.

Understanding quantum glass formation in a system with a regular Hamiltonian is important for the general problem of quantum computating [14]. The reason is that a quantum computer is also a quantum system with an exponential number of states, and the process of computation can be viewed as an almost adiabatic change of the external parameters, resulting in a different state. The crucial issue is how to ensure that such a process does not lead to the collapse of the density matrix due to coupling to the environment. This issue is relevant to the spin glass system as well, and one can learn about decoherence in a generic large system with an exponential number of states by addressing it.

Here we study the quantum version of a long-range Josephson array in a frustrating magnetic field, as suggested in Refs. [4–7]. We consider here only the problem of glass formation, approaching the glass from the «liquid» (i.e., insulating) side. We show that the quantum version of this problem can be described by the same dynamical equations as the quantum disordered p-spin model studied in Ref. [15]. Thus, we explicitly prove that this frustrated quantum system can be mapped onto a quantum disordered system, in complete analogy with the situation for classical problems. Further, we provide a direct numerical proof that the transition in this model is indeed continuous as conjectured in Ref. [15] and we calculate the anomaly of the diamagnetic response associated with this transition.

Another (more physical) justification of the model is the following. It is well established, both experimentally [16, 17] and theoretically [18] that usual nearest-neighbor Josephson arrays made of small superconducting islands exibit a zero-T superconductor—insulator transition as the ratio of the Josephson coupling E_J between superconductive islands to the Coulomb energy cost $E_C = (2e)^2/2C$ for the transfer of the Cooper pair between the islands decreases. At small values of $x = E_J/E_C$, the ground state is an insulator with nonzero Coulomb gap in the excitation spectrum. At nearly critical values of $x \approx x_{cr}$, the transition between insulating and superconducting states can be triggered by application of a weak magnetic field, producing frustration of the Josepson interaction. Moreover, this transition can be split [16] into a sequence of two different transitions: superconductor → metal → insulator. Although the main qualitative features of these phenomena are understood, there is still no quantitative theory that describes quantum phase transitions in two-dimensional short-range systems, especially in the presence of frustration. Therefore, in our attempt to study the origin of a quantum glass state, we have to turn to the simplest (theoretical) model of a Josephson array with long-range interaction, which consists of long superconducting wires (instead of islands), which will enable us to employ some version of mean-field theory and reduce the problem to a zero-dimensional quantum theory with an interaction that is non-local in time.

The system that we study is a stack of two mutually perpendicular sets of N parallel thin

superconducting wires with Josephson junctions at each node, located in an external transverse field H. Macroscopic quantum variables of this array are the $2N$ superconducting phases associated with each wire (e.g., the phase of the superconducting order parameter at the center of each wire). We will always assume that excitations within individual wires can be neglected, so that the whole wire is characterized by a single phase ϕ_m. In the absence of an external field the phase differences would be zero at each junction, but this is not possible at finite H, so the phases are frustrated. Here we assume that the Josephson currents are sufficiently small so that the induced fields are negligible in comparison with H (this imposes an important constraint on experimental realization of this network [6]).

The array can therefore be described by the Hamiltonian[1)]

$$\mathscr{H} = \mathscr{H}_J + \mathscr{H}_C = -E_J \sum_{m,n} \cos\left(\phi_n - \phi_m - \frac{2e}{\hbar c}\int \mathbf{A}\cdot d\mathbf{l}\right) + \frac{(2e)^2}{2}\sum_{m,n} \hat{C}^{-1}_{m,n}\frac{\partial}{\partial\phi_m}\frac{\partial}{\partial\phi_n}, \quad (1)$$

where $\mathscr{H}_J$ and $\mathscr{H}_C$ represent the Josephson and Coulomb parts of the Hamiltonian, and $\hat{C}_{m,n}$ is the capacitance matrix. There are several different contributions to $\hat{C}$: self-capacitances of the wires C_l (with respect to the substrate), the junction capacitances C_J, and the mutual capacitances of wires C_{ll}. Below we assume that the self-capacitance is the largest of all: $C_l \gg C_{ll}, NC_J$ (the factor N accounts for the fact that there are N junctions along each wire). These conditions enable us to neglect all mutual capacitances and consider the matrix $C_{m,n}$ to be diagonal, with eigenvalues C_l.

It is convenient to rewrite the Hamiltonian in terms of «spin» variables $s_m = e^{i\phi_m}$. Choosing the Landau gauge for the vector potential and introducing J_0 via $E_J = J_0/\sqrt{N}$ so that the transition temperature remains constant in the limit $N \to \infty$ at fixed J_0, we obtain

$$\mathscr{H} = -\sum_{m,n}^{2N} s_m^* \mathscr{J}_{mn} s_n + \frac{E_C}{2}\sum_n Q_n^2, \quad (2)$$

where $Q_n \equiv -i\partial/\partial\phi_n$ is the charge operator conjugate to the phase ϕ_n, $E_C = 4e^2/C_l$, and $\mathscr{J}_{mn}$ is the coupling matrix

$$\hat{\mathscr{J}} = \begin{pmatrix} 0 & \hat{J} \\ \hat{J}^\dagger & 0 \end{pmatrix} \quad (3)$$

with

$$J_{jk} = \frac{J_0}{\sqrt{N}}\exp(2\pi i\alpha jk/N), \quad 1 \le (j,k) \le N,$$

where $j(k)$ is the index labeling the horizontal (vertical) wires; $s_m = e^{i\phi_m}$, where the ϕ_m are the superconducting phases of the $2N$ wires, and $\alpha = NHl^2/\Phi_0$ is the flux per unit strip (l is the inter-node spacing and Φ_0 is the flux quantum).

[1)] Random version of the same Hamiltonian was considered in Ref. [12], where an approximate dependence of the transition temperature $T_g(E_C, E_J)$ on the ratio of charging to Josephson energy E_C/E_J was calculated. Unfortunately, even in the classical limit $E_C \to 0$ the result of [12] for T_g differs from the one obtained in [18] by the method which is free from any approximations or assumptions.

Because every horizontal (vertical) wire is coupled to every vertical (horizontal) wire, the connectivity in this model is high (N) and it is accessible to a mean-field treatment (its classical version was developed in Refs. [19] and [5]). For $1/N \ll \alpha < 1$ there exist many metastable solutions that minimize the Josephson («potential») part of the Hamiltonian (2); these minima are separated by barriers that scale [4] with N. A similar (classical) long-range network with disorder was previously found to display a spin glass transition [19] for $\alpha \gg 1/N$; in the absence of short-range phase coherence between wires ($\alpha \gg 1$), it is equivalent to the Sherrington—Kirkpatrick model [20]. Physically this glassy behavior occurs because the phase differences associated with the couplings, J_{jk}, acquire random values and fill the interval $(0, 2\pi)$ uniformly. For the periodic case, this condition is satisfied in the «incommensurate window» $1/N \ll \alpha \leq 1$ for which the magnetic unit cell is larger than the system size, so that the simple «crystalline» phase is inaccessible [4].

There are thus no special field values for which there are only a few minima of the potential energy, in contrast to the situation for $\alpha > 1$. Below we consider the case $1/N \ll \alpha \ll 1$ only. As follows from previous studies [4–7], the characteristic energy scale related to the potential energy $\mathscr{H}_J$ is of the order of the glass transition temperature of the classical system, $T_G \approx \approx J_0/\sqrt{\alpha}$. The zero-$T$ transition we study here is driven by the competition between Josephson and Coulomb energies, the scale of the latter being $E_C = 4e^2/C_l$. Thus, we expect that the quantum transition occurs at $J_0/\sqrt{\alpha} \sim E_C$. Our goal is to show that such a (continuous) phase transition indeed occurs, and to study the critical behavior near the transition point. Below, in the main part of the paper, we measure all energies in units of E_C, and return to physical units only in the final expression for the critical behavior of the ac diamagnetic susceptibility.

2. QUANTUM LOCATOR EXPANSION

We develop a diagram technique for the Hamiltonian that is (2) very similar to the one employed previously [5] for the classical Langevin dynamics of the same array. The idea is to treat the Coulomb part of the Hamiltonian as the zero-level approximation, and construct an expansion in powers of the Josephson coupling constant J_0, keeping all terms of lowest order in the coordination number $1/N$. Thus, our approach can be considered a quantum version of the Thouless—Anderson—Palmer [21] method.

The diagram technique for the Matsubara Green function

$$G_{m,n}(\tau) = -\langle T_\tau s_m(\tau) s_n^\dagger(0)\rangle, \quad s(\tau) = e^{-\tau\mathscr{H}} s e^{\tau\mathscr{H}} \tag{4}$$

is closely related to the one developed in Ref. [5]. The Dyson equation for the frequency-dependent matrix Green function reads (note that in our units $E_C = 1$)

$$\mathbf{G}_\omega = \frac{1}{\tilde{\mathbf{G}}_\omega^{-1} - (\mathbf{J}\mathbf{J}^\dagger)\tilde{\mathbf{G}}_\omega}, \tag{5}$$

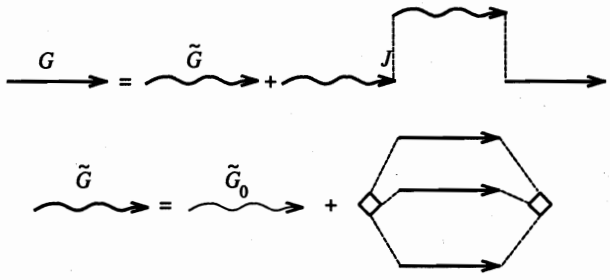
where we have introduced the local Green functions $\tilde{G}_\omega$ that is irreducible with respect to the J_{ij} lines. The matrix $(JJ^\dagger)_{ij}$ depends only on the «distance» $i-j$ and acquires a simple form in Fourier space:

$$(JJ^\dagger)_p = (J_0^2/\alpha)\theta(\alpha\pi - |p|);$$

therefore in this representation

$$G_\omega(p) = \frac{\theta(\alpha\pi - |p|)}{\tilde{G}_\omega^{-1} - \dfrac{J_0^2}{\alpha}\tilde{G}_\omega} + \frac{\theta(|p| - \alpha\pi)}{\tilde{G}_\omega^{-1}}. \tag{6}$$

Diagrammatically, Eq. (5) and the equation for the irreducible function $\tilde{G}\omega$ are represented by the graphs shown below.



Note that the equation for $\tilde{G}$ is written in the lowest nontrivial order in α. Indeed, it is seen from Eq. (6) that the nontrivial part of the Green function, which contains critical slowing down, is of relatively small weight $\sim\alpha$. It is this long-time part of G_ω that enters into the 3-line diagram and makes it proportional to α^3; more complicated diagrams either contain even higher powers of α, or are small and go as $1/N$. Since the second diagram contains single-site functions only, the whole system of equations can be written in the form

$$G(\omega) = (1-\alpha)\tilde{G}(\omega) + \hat{G}(\omega), \quad \hat{G}(\omega) = \frac{\alpha\tilde{G}(\omega)}{1 - J_0^2\tilde{G}^2(\omega)/\alpha}, \tag{7}$$

$$\tilde{G}(\omega) = \tilde{G}_0(\omega) + \Sigma(\omega), \quad \Sigma(\omega) = \left(\frac{J_0^2}{\alpha}\right)^3 \chi_3^2 \int \hat{G}^3(t)\exp(i\omega t)\, dt. \tag{8}$$

Here $\chi_3 \sim 1$, as in Ref. [5], is the static value of four-point vertex denoted by a square box in the diagram (we assume that, as in Ref. [5], the main critical anomaly is contained in the 2-point Green function alone). Equations (7) and (8) must be solved with obvious initial condition

$$G(t=0) = \int \frac{d\omega}{2\pi} G(\omega) = 1. \tag{9}$$

A similar normalization condition in the classical problem was sufficient to determine $\tilde{G}(\omega = 0)$ exactly [4]. The same calculation is difficult in the present quantum problem, and we will not carry it out here. Instead, we use general properties of the function $\tilde{G}_0(\omega)$, namely: i) $\tilde{G}_0(0) \sim 1$, and ii) $\tilde{G}_0(\omega)$ is analytic at low ω, and has a characteristic frequency scale of the order of 1. In doing so, we do not determine the exact position of the phase transition (i.e., the critical value J_c of the coupling strength J_0), but we demonstrate the existence of a continuous transition and find the form of critical scaling.

We first analyze equations (7)–(9), omitting the term containing Σ, and using the simplest interpolation

$$\tilde{G}_0(\omega) = (\lambda + \omega^2)^{-1}.$$

Then initial condition (9) yields an equation for λ:

$$1 = \frac{1}{2\sqrt{\lambda}} + \frac{\alpha}{4\sqrt{\lambda - g}}, \tag{10}$$

where $g = J_0/\sqrt{\alpha}$. Thus $\lambda \sim 1$ as long as $g \leq 1$. On the other hand, at $g \gg 1$ the solution is

$$\lambda - g \equiv a \approx (\alpha/4)^2.$$

The value of a determines the asymptotic decay rate of the Green function

$$G(t) = \frac{\alpha}{\sqrt{a}} \exp\left(-|t|\sqrt{a}\right) \tag{11}$$

with Σ being neglected. It will be seen below that $a \sim \alpha$ and thus $\lambda \sim 1$ near the phase transition point $g \doteq g_c$ (we will also see that $\Sigma \sim \alpha$, and thus it is much smaller than the ω^2 term at high frequencies $\omega \gg \alpha^{1/2}$). This means that the parameter a can be considered a smooth function of g in the vicinity of g_c. Clearly, this conclusion does not depend on the model of $\tilde{G}_0(\omega)$ used in the above analysis.

Now we reintroduce $\Sigma(\omega)$ into the equations for $\hat{G}(\omega)$ and focus on its low-frequency behavior at $\omega \leq \sqrt{\alpha}$:

$$\hat{G}(\omega) = \frac{\alpha}{a - 2\Sigma(\omega) + \omega^2}, \tag{12}$$

$$\Sigma(\omega) = \tilde{g}^6 \int \hat{G}^3(t) \exp(i\omega t)\, dt, \tag{13}$$

where

$$\tilde{g} = g\chi_3^{1/3} \sim g.$$

Strictly speaking, Eqs. (12) and (13) do not form a complete set, since a should be determined with the use of Eq. (9) which contains high-frequency contributions. However, in this high-frequency region (which produces the main contribution to the normalization condition (9)) the contribution of $\Sigma(\omega)$ can be neglected and thus a can be treated as an external control parameter that governs the transition.

The Green function defined by Eqs. (12) and (13) acquires a singularity when $2\Sigma(0) = a$. To find the form of this singularity, we make use of the scaling ansatz $G(t) = qt^{-\nu}$ and neglect the ω^2 term in the denominator of Eq. (12). Then we find $\nu = 1/2$ and $q \sim \tilde{g}^{-1}\alpha^{1/4}$. This critical-point solution matches the short-time asymptotic behaviour (11) at $t \sim \alpha^{-1/2}$. The estimation of $\Sigma(0)$ that follows from the above scaling ansatz,

$$\Sigma(\omega = 0) \approx \tilde{g}^6 q^3 \int_{\sqrt{a}}^{\infty} \frac{dt}{t^{3/2}} \approx \tilde{g}^3 q^3 a^{1/4}$$

yields $\Sigma(0) \approx a$ at $\tilde{g} \sim 1$ and $a \sim \alpha$, as expected. These estimates show that second-order phase transition with critical slowing down can indeed occur in the above range of parameters. In the next section we study the vicinity of the critical point in more detail.

3. GREEN FUNCTION NEAR THE $T = 0$ TRANSITION POINT

To study the form of the critical singularity, it is convinient to define universal scaling functions $\mathcal{G}(\omega)$ and $\sigma(\omega)$ that do not contain the small parameter $\alpha \ll 1$, and a parameter b, which measures proximity to the critical point:

$$\hat{G}(\omega) = \mathcal{G}(\tilde{\omega}), \quad \alpha\Sigma(\omega) = \sigma(\tilde{\omega}), \quad \tilde{\omega} = \omega/\sqrt{\alpha}, \quad b = (a - 2\Sigma(0))/\alpha. \tag{14}$$

Equations (12) and (13) acquire then the following form:

$$\mathcal{G}(\tilde{\omega}) = \frac{1}{b + 2(\sigma(0) - \sigma(\tilde{\omega}))}, \qquad \sigma(\tilde{\omega}) = \tilde{g}^6 \int \mathcal{G}^3(\tilde{t}) \exp(i\tilde{\omega}\tilde{t})\, d\tilde{t}. \tag{15}$$

Exactly at the critical point $b = 0$, the solution of Eq. (15) is

$$\mathcal{G}(\tilde{\omega}) = \left(\frac{\pi}{8}\right)^{1/4} \tilde{g}^{-3/2} |\tilde{\omega}|^{-1/2}. \tag{16}$$

Consider now the vicinity of the critical point, $0 < b \ll 1$. It is clear from the form of the solution (16) that a similar result should be valid at $\tilde{\omega} \gg b^2$. Next we focus on the long-time, low-ω region, $\tilde{\omega} \ll b^2$, and look for the purely exponential solution

$$\mathcal{G}(\tilde{t}) = \mathcal{G}_1 \exp\left(-\tilde{t}/\tau_0\right). \tag{17}$$

This type of asymptotic solution is known to exist in the classical version of the same model (cf. Refs. [5] and [7]). In the present problem one can show, considering the analytic structure of (15), that at $b > 0$ the singularity of $\mathcal{G}(\tilde{\omega})$ closest to the real ω axis, is necessarily a simple pole at some $\tilde{\omega} = i/\tau_0$; the next singularity may exist at $\tilde{\omega} \geq 3i/\tau_0$. Solving (15) with the ansatz (17) in the region $\tilde{t} \gg \tau_0$ determines parameters τ_0 and $\mathcal{G}_1$ as functions of b:

$$\tau_0 = \frac{\sqrt{32/27}}{\tilde{g}^3} \frac{1}{b^2} \qquad \mathcal{G}_1 = \sqrt{\frac{27}{2}} \tilde{g}^3\, b. \tag{18}$$

This solution is similar to the one found in Ref. [5]; however, an important difference is that in the present case the factor $\mathcal{G}_1$ scales to zero at the critical point $b = 0$.

The full solution in the vicinity of the transition point should contain both (16) and (18) as asymptotic solutions, and can be written in the form

$$\mathcal{G}(\tilde{t}) = \frac{1}{\sqrt{\tilde{t}}} f\left(\frac{\tilde{t}}{\tau_1}\right) + \mathcal{G}_1 \exp\left(-\frac{\tilde{t}}{\tau_0}\right), \tag{19}$$

where $f(x)$ is some scaling function that approaches a constant at $x = 0$ and decays rapidly as $x \to \infty$; $\tau_1 \leq \tau_0/3$. To confirm an existence of this type of solution, we solved Eqs. (15) numerically for several values of $b \ll 1$. The results of this computation are shown in Fig. 1. Clearly, all three functions $\mathcal{G}(\omega)$ coincide in the high-ω region, where they are close to the square-root asymptotic behaviour (16). The low-frequency parts (for $\omega \leq 0.08$) of these solutions can be made to coinside by a proper rescaling of their arguments, $\omega^* = \Lambda\omega$. Figure 2 demonstrates the linear relationship between b^{-2} and the scaling coefficient Λ, as suggested by Eqs. (18) and (19).

These results confirm the existence of $T = 0$ critical behavior of the type of Eq. (19).

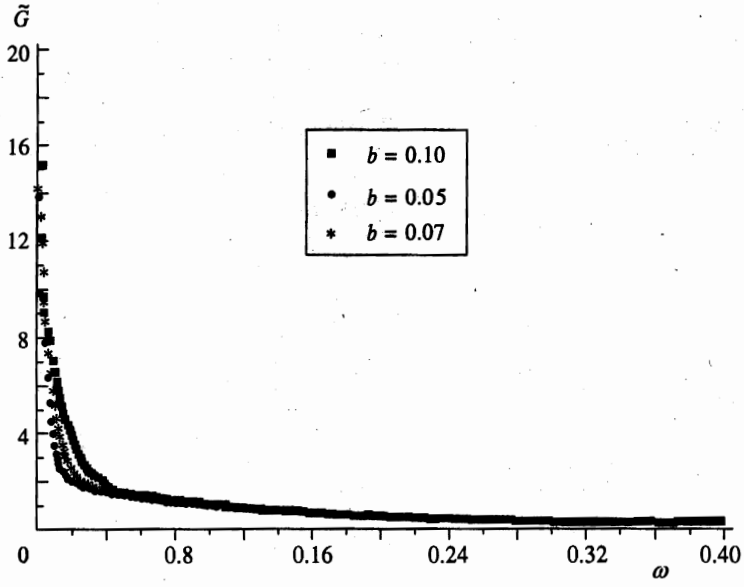


Рис. 1. Low-frequency asymptotic behavior of $\tilde{G}(\omega)$ for various b at $T = 0$

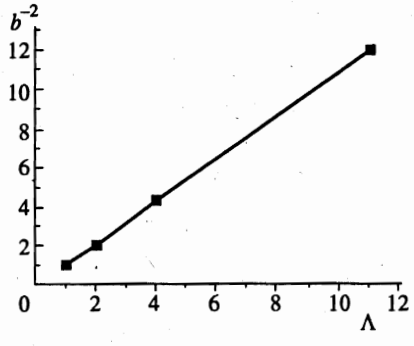


Рис. 2. The relation between scaling parameter Λ and proximity to the transition point b at $T = 0$

4. CRITICAL BEHAVIOR AT $T > 0$

The above results refer to the zero-T phase transition controlled by the single parameter $g = J_0/\sqrt{\alpha}$. We found that this phase transition is a continuous one, and the corresponding critical behavior differs considerably from the behaviour found in an analogous classical model [5]. In particular, at the $T = 0$ critical point $g = g_c$, there is no «plateau» solution with approximately constant $G(t)$ at $t \to \infty$, which is known to be a distinctive property of regular classical glasses. Now we consider low but non-zero temperatures $T = \beta^{-1}$, and find out how «classical» critical scaling «grows up» from the «quantum» background; we also find the low-temperature shape of the phase transition curve in the (T, g) plane.

The Green function is now defined at discrete frequencies $\omega_n = 2\pi nT$, and Equations (12) and (13) can be written in the form

$$\hat{G}(\omega_n) = \frac{\alpha}{a - 2\Sigma(\omega_n) + \omega^2}, \qquad \Sigma_M(\omega_n) = \tilde{g}^6 \int_0^\beta \hat{G}^3(t) \exp(i\omega_n t)\, dt\ . \tag{20}$$

It will be convenient now to perform analytic continuation of Eqs. (20) and rewrite them in terms of real-time correlation function

$$D(t) = \langle [S(t), S(0)]_+ \rangle$$

and response function

$$\chi(t) .= i[S(t), S(0)]_- \theta(t).$$

The functions $G(\omega_n)$, $D(\omega)$ and $\chi(\omega)$ are related as follows:

$$G(-i\omega + \eta) = \chi(\omega), \quad \eta \to +0, \quad D(\omega) = \mathrm{Im}\chi(\omega) \coth(\omega/2T) \tag{21}$$

After analytic continuation Eqs. (20) can be written

$$\chi(\omega) = \frac{\alpha}{\tilde{a} - 2\Sigma(\omega)}, \quad \Sigma(\omega) = 8\tilde{g}^6 \int_0^\infty D^2(t)\chi(t)\,(\exp(i\omega t) - 1)\ dt, \quad \tilde{a} = a - 2\Sigma(\omega = 0), \tag{22}$$

where we omitted the ω^2 term, which is irrelevant in the vicinity of the critical point. Equations (22) form (together with the fluctuation-dissipation relation (second of Eqs. (21)) a complete set that determines the critical singularity at $T > 0$. Formally, Eqs. (22) coincide with the corresponding «classical» equations from Ref. [5], the only difference being the form of the fluctuation-dissipation relation.

We now consider the low-temperature region $T \ll \sqrt{\alpha}$. As long as we are interested in the long-time behaviour $t \gg 1/T$, the correlation and response functions are related by the classical FDT: $D(\omega) = 2T/\omega\ \mathrm{Im}\chi(\omega)$. Characteristic times relevant to (22) also belong to classical region $t \gg 1/T$. Therefore the correlation function at the transition point has the same critical behavior as in the classical case:

$$\lim_{t\to\infty} D(t) = q.$$

However, the parameter $a \equiv \lambda - g$ is determined by the «quantum» frequency range $\omega \gg T$, i.e., by Eq. (10). Substituting this expression into (22) yields

$$q \propto \alpha^{1/4} T^{1/2} \qquad \tilde{a} \propto \alpha^{3/4} T^{1/2}. \tag{23}$$

In the short-time domain $t \ll T^{-1}$, the zero-T critical solution with $D(t) \propto \alpha^{1/4} t^{-1/2}$ is valid. Equation (23) demonstrates how the «classical» solution with nonzero $\lim_{t\to\infty} D(t)$ grows up with the temperature increase.

5. DIAMAGNETIC RESPONSE NEAR THE TRANSITION POINT

Correlation and response functions $D(t)$ and $\chi(t)$ are not directly measurable in our system, but they can be used to calculate a measurable physical quantity, the dynamic diamagnetic

susceptibility $\chi_{\mathscr{M}}(\omega)$, as was done previously for the classical problem [5]. The total magnetic moment induced by a time-dependent external magnetic field is

$$\mathscr{M} = \frac{1}{2}\left(\frac{2e}{\hbar c}\right) l^2 \sum_{mn} S_m^{\dagger} \tilde{\mathscr{J}}_{mn} S_n \ , \tag{24}$$

where $\tilde{\mathscr{J}}_{mn} = imn\mathscr{J}_{mn}$ [5]. Then the magnetic susceptibility $\chi_{\mathscr{M}}$ can be found via the Kubo formula, $\chi_{\mathscr{M}}(t-t') = i\left[\mathscr{M}(t),\ \mathscr{M}^{\dagger}(t')\right]\theta(t)$, which leads to the expression

$$\chi_{\mathscr{M}}(\omega) = \left(\frac{2e}{\hbar c}\right)^2 l^2 \int_0^{\infty} \left(e^{i\omega t} - 1\right) \mathrm{Re}\,\mathrm{Tr}\tilde{\mathscr{J}}\hat{\chi}(t)\tilde{\mathscr{J}}\hat{D}(t)dt \ . \tag{25}$$

Here we omit the term containing the irreducible four-spin correlator (of the order of $1/N$), and take into account that $\mathscr{M}(H = 0) = 0$. Note, that Eq. (25) formally coincides with the classical formula for magnetic response [5]. The matrix functions $\hat{D}(t)$ and $\hat{\chi}(t)$ contain elements (denoted by superscript $^{(0)}$) belonging to the same (horizontal or vertical) sublattice of our array, as well as «off-diagonal» elements (with superscript $^{(1)}$) that describe correlation of phases on wires of different type (horizontal/vertical). These functions are related by

$$\hat{\chi}(\omega)^{(1)} = J\tilde{G}(\omega)\hat{\chi}(\omega)^{(0)}.$$

Thus, the expression for magnetic susceptibility has the form

$$\chi_{\mathscr{M}}(\omega) = \left(\frac{2e}{\hbar c}\right)^2 \left(\frac{l^2}{12}\right)^2 N^5 \frac{J_0^2}{\alpha^2} I(\omega), \tag{26}$$

where

$$I(\omega) = \int \left(\delta(t-t_1) - \frac{J_0^2 \tilde{G}^2(t-t_1)}{\alpha}\right) \chi(t_1)D(t_1)\left(e^{i\omega t} - 1\right)\theta(t)dt\,dt_1. \tag{27}$$

Near the transition point, only the long-time parts of the functions $\chi(t)$ and $D(t)$ in (27) are relevant, and this expression can be reduced to the form

$$I(\omega) = (\Sigma(\omega) - \Sigma(0)) \int \chi(t)D(t)e^{i\omega t}dt \ , \tag{28}$$

where the first factor came from the first brackets in (27); note that this vanishes in the limit $\omega \to 0$.

Using the solution (16), we obtain at the quantum critical point $J = J_c$

$$I(\omega) = \frac{\alpha}{2\pi}\left(\frac{\alpha\pi}{8\tilde{g}^6}\right)^{1/4} \sqrt{i\omega}\ln\omega \tag{29}$$

Near the $T = 0$ transition point at high enough frequencies

$$\omega \gg (J/J_c - 1)^2 \alpha^{-3/2},$$

Eq. (29) still holds. In the opposite case of low frequencies,

$$I(\omega) = \frac{8J_c^3\alpha^3}{81(J_c - J)^3}\,\frac{\omega^2\alpha^{1/2}}{g}\,. \tag{30}$$

Note that the parameter $\tilde{g}$ (which is known only up to factors of order 1) does not enter into the low-ω asymptotic behaviour of $I(\omega)$.

Making use of Eqs. (26), (29) and (30), and returning to the original units of frequency, we finally obtain the ac diamagnetic susceptibility near the quantum transition point:

$$\chi_{\mathscr{M}}(\omega) \approx \left(\frac{2e}{\hbar c}\right)^2 l^4 N^5 \frac{(J_c C_l)^{1/2}}{2e}\sqrt{\frac{i\omega C_l}{e^2}}\ln\left(\frac{\omega C_l}{e^2}\right), \quad \omega \gg \frac{C_l(J - J_c)^2}{e^2\alpha^{5/2}}, \tag{31}$$

$$\chi_{\mathscr{M}}(\omega) = \left(\frac{2e}{\hbar c}\right)^2 \left(\frac{l^2}{12}\right)^2 N^5\,\frac{2C_l\alpha^{7/2}}{81e^2J_c}\,\frac{J_c^3}{(J_c - J)^3}\,\omega^2, \quad \omega \ll \frac{C_l(J - J_c)^2}{e^2\alpha^{5/2}}. \tag{32}$$

These expressions are valid at frequencies $\omega \gg T/\hbar$, otherwise the «classical» asymptotic behaviour of the Green functions should be used and will lead to frequency dependencies like those in Ref. [5].

6. CONCLUSIONS

We have shown that a regular frustrated long-range Josephson array has a quantum (zero-temperature) phase transition between the Coulomb-dominated insulator phase and a superconductive state. This transition happens when the nearest-neighbor Josephson coupling exceeds the critical value:

$$J_{ij} \sim N^{-1/2}\sqrt{\alpha}e^2/C_l,$$

where C_l is the self-capacitance of an individual wire.

We found that quantum critical behavior of the model at $J \to J_c$ is different from that of an analogous classical system [5]: at the quantum critical point

$$D(t) \propto t^{-1/2},$$

while at the classical critical point

$$q = \lim_{t\to\infty} D(t).$$

However, at any non-zero temperature a «classical» type of asymptotic behavior is recovered at the longest times, $t \gg \hbar/T$, leading to $q \propto T^{1/2}$. Near the $T = 0$ critical point, the gap in the excitation spectrum decreases as $\tau_0^{-1} \propto (J_c - J)^2$.

Near the phase transition, the effective inductance $\mathscr{L}$ of the array, defined by

$$\mathscr{L} \propto \partial^2\chi_{\mathscr{M}}(\omega)/\partial\omega^2|_{\omega\to 0},$$

diverges as $(J_c - J)^{-3}$; this shows that the glass state has macroscopic phase rigidity (cf. also Ref. [19]). Right at the critical point we find unusual frequency behavior of the complex diamagnetic susceptibility:

$$\chi_{\mathscr{M}}(\omega) \propto \sqrt{i\omega}\ln\omega.$$

The frustrated nature of couplings in our array and comparison with previous results [7] in the classical version of the same model indicates that the high-J state is a quantum glassy superconductor. The $T = 0$ nonergodic properties (irreversibility, aging) remain an open question; note here that a recent study [15] of nonequilibrium glassy behavior in a p-spin spherical quantum model assumed strongly dissipative (overdamped) dynamics, whereas the dynamics relevant to a Josephson array at $T = 0$ must be underdamped.

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