CRITICAL VELOCITY IN ^{3}He-B VIBRATING WIRE EXPERIMENTS AS ANALOG OF VACUUM INSTABILITY IN A SLOWLY OSCILLATING ELECTRIC FIELD

A. Calogeracos [a,b], *G. E. Volovik* [a,c]

[a] *Low Temperature Laboratory, Helsinki University of Technology*
02150, Espoo, Finland
[b] *NCA Research Associates*
Maroussi 151 01, Athens, Greece
[c] *L. D. Landau Institute for Theoretical Physics*
117940, Moscow, Russia

The Lancaster experiments [5] with a cylindrical wire moving in superfluid ^{3}He-B are discussed, where the measured critical velocity of pair creation was much below the Landau critical velocity. The phenomenon is shown to be analogous to the instability of the electron-positron vacuum in an adiabatically alternating strong electric potential of both signs, where the positive- and negative-root levels cross and thus the instability treshold is twice less than in the conventional case of a single static potential well.

1. INTRODUCTION

In superfluid Fermi system the pairs of quasiparticles are created by the uniformly moving object, if its velocity exceeds the Landau critical velocity, $v_L = \Delta_0/p_F$. Here p_F is the Fermi momentum, Δ_0 is the superfluid gap in bulk liquid. The critical velocity v_L is also called the pair-breaking velocity; it marks the threshold of the instability of the superfluid vacuum: breaking of Cooper pairs which form the superfluid condensate. In the vacuum of the high energy physics a similar situation can occur: (i) In a strong electric field [1–3]; (ii) In a strong gravitational filed, say, in the presence of the event horizon [4]; (iii) If the hypothetical object, which is external to the physical vacuum, moves with the superluminal speed. Here we consider the pair creation in superfluid ^{3}He-B, which is analogous to the production of the electron-positron pairs in a strong electric field.

Such experiments have been conducted in Lancaster [5,6], where a cylindrical wire vibrating in superfluid ^{3}He-B has been used as a moving object. It appeared that the measured critical velocity, at which a large extra dissipation of the wire was observed due to particle creations, was essentially less than v_L. It was about $0.25 v_L$ independent of the material and radius of the wire.

It was originally suggested in [5], that such reduction has two origins: a geometrical factor $1/2$ results from the local enhancement of the velocity near the wire, while the other reduction is related to the suppression of the gap in the vicinity of the surface of the wire, $\Delta < \Delta_0$. As a result, the Landau criterium for the filling of the surface bound states is essentially smaller than v_L. However to provide the momentum loss by the wire, the quasiparticles must escape to infinity. That is why the production of the scattering states at subcritical velocity has to be explained. This scenario was developed by Lambert [7], who showed that the adiabatic

oscillation could do this job, if the velocity amplitude of the wire exceeds some value, which was estimated as $v^* = (1/5)v_L$.

We develop further these arguments taking into account that in ^{3}He-B (1) the surface leads to the splitting of the gap, $\Delta_\parallel$ and $\Delta_\perp$; and (2) the classical description of the bound state in the surface layer should be substituted by the quantum mechanical one. We obtain the modified value for v^*, which depends on the gap suppression. Since the Bogoliubov-Nambu fermions in ^{3}He-B are in many respects similar to the Dirac electrons, we connect the critical radiation of the quasiparticles by a slowly vibrating wire with the instability of the electron-positron vacuum in the presence of a strong electric field. Our case corresponds to a slowly alternating electric potential of both signs, which allows the electron-positron production at essentially weaker field than in the conventional mechanism discussed by Gershtein and Zel'dovich [2]. In this scenario the classical positive- and negative-root solutions cross, which leads to the particle-antiparticle production (see also discussion in Ref. [8]). We constructed a simple time dependent potential for Dirac electrons, which allows us to model the proposed scenario.

2. FERMIONS IN THE VIBRATING WIRE

2.1. Fermionic spectrum in ^{3}He-B

In bulk superfluid ^{3}He-B the fermionic spectrum is defined by the following 4×4 matrix Hamiltonian (Bogoliubov-Nambu Hamiltonian) [9, 10]:

$$H(\mathbf{p}) = \beta M(p) + c\mathbf{p}\boldsymbol{\alpha}, \quad M(p) = v_F(p - p_F), \quad c = \Delta_0/p_F . \tag{1}$$

Here β and α are Dirac matrices, composed from the 2×2 Pauli matrices τ describing the Bogoliubov-Nambu spin in particle-hole space and 2×2 Pauli matrices σ for conventional spin:

$$\beta = \tau_3, \quad \boldsymbol{\alpha} = \tau_1 \boldsymbol{\sigma} . \tag{2}$$

The energy spectrum is

$$E_\pm(p) = \pm\sqrt{M^2(p) + c^2 p^2} . \tag{3}$$

The quantity c plays the part of speed of light, however as distinct from the relativistic case the mass M depends on the momentum p. Since $v_F \gg c$ the minimum of the positive energy occurs not at $p = 0$ but at $p = p_F$ with min $E_+(p) = \Delta_0$.

According to the Landau criterium, if the external body moves with the velocity larger than $v_L = \min(E_+(p)/p) = c$ it will radiate the quasiparticles. As distinct from the relativistic case, where the minimum is realized at $p \to \infty$, in ^{3}He-B it occurs at $p = p_F$.

In the reference frame of the body the energy spectrum is Doppler shifted:

$$H(\mathbf{p}) = \mathbf{p}\mathbf{v}_s + \beta M(p) + c\mathbf{p}\boldsymbol{\alpha} ,$$
$$E_\pm(p) = \mathbf{p}\mathbf{v}_s \pm \sqrt{M^2(p) + c^2 p^2}, \tag{4}$$

where $\mathbf{v}_s$ is the superfluid velocity in the body frame. If $v_s(\infty) > c$, the positive square-root continuum merges with the negative square-root continuum and thus the production from the vacuum of pairs of quasiparticles with momentum p_F becomes possible. Here we discuss the
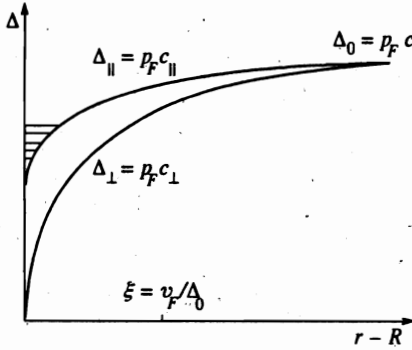
71

Fig. 1.   Schematic illustration of gaps, «speeds of light» and bound states near the surface of the wire

situation when the particle production is possible even well below the Landau criterium. It is the combined effect of (i) the enhancement of the local superfluid velocity in the vicinity of the surface of the object; (ii) the decrease of the «speed of light» near the surface; and (iii) adiabatic oscillation of the velocity of the body.

2.2. Fermions in the surface layer

In the experimental situation [5,6] the external body moving in ^{3}He-B is the cylindrical wire of the radius R from 2 to 50 μm, which is much larger than the coherence length $\xi \sim v_F/\Delta_0$. The velocity of the wire performs the oscillating motion, $\mathbf{u}(t) = \hat{\mathbf{x}}u(t)$, $u(t) = u_0 \cos(\omega t)$, with frequency $\omega \sim 10^2 - 10^3$Hz, which is much smaller than the characteristic quasiparticle energy of order Δ_0 and thus the motion is extremely adiabatic. The presence of the moving external object disturbs the vacuum state of the superfluid. First, the velocity field is modified by the moving wire. In the reference frame of the wire the superfluid performs an ideal dipole flow around the wire:

$$\mathbf{v}_s(\mathbf{r}, t) = -\mathbf{u}(t) + \frac{R^2}{r^2}[2\hat{\mathbf{r}}(\hat{\mathbf{r}}\mathbf{u}(t)) - \mathbf{u}(t)], \quad r > R, \tag{5}$$

where $\mathbf{r} = (x, y)$ is the $2D$ radius vector in the plane perpendicular to the wire counted from the center of the wire; $\hat{\mathbf{r}} = \mathbf{r}/r$. At two lines at the surface of the wire the superfluid velocity is twice larger than at infinity: $\mathbf{v}_s(\pm R\hat{\mathbf{y}}) = -2\mathbf{u}(t)$.

The second effect is that the order parameter (gap) is suppressed near the surface of the wire in the layer of the thickness of the coherence length size. In ^{3}He-B this suppresion is anisotropic, which leads to the two «speeds of light» in the region $r - R \sim \xi \sim v_F/\Delta_0$ (see Fig. 1):

$$H = \beta M(p) + \left[c_\parallel(\delta_{ij} - \hat{n}_i\hat{n}_j) + c_\perp \hat{n}_i\hat{n}_j\right] p_i\alpha_j ,$$

$$E_\pm(p) = \mathbf{p}\mathbf{v}_s \pm \sqrt{M^2(p) + c_\perp^2(\hat{\mathbf{n}}\mathbf{p})^2 + c_\parallel^2[\hat{\mathbf{n}}\mathbf{p}]^2} , \tag{6}$$

where $c_\perp = \Delta_\perp/p_F$ and $c_\parallel = \Delta_\parallel/p_F$ are the «speeds of light» along the normal $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ to the surface of the wire and parallel to the surface correspondingly. According to [11], where the diffusive boundary conditions were considered, the transverse speed of light is completely suppressed, $c_\perp(r = R) = 0$, while $c_\parallel(r = R) \approx 0.4c$ at $T = 0$. Due to the suppression of the order parameter the surface layer serves as a potential well for quasiparticles, which contains the bound states with energies below the gap [12] (see Fig. 1).

72

3. CRITICAL VELOCITIES AND NUCLEATION OF QUASIPARTICLES

3.1. Excitations of the bound states

Let us consider first the uniformly moving wire with constant velocity u. The filling of the bound states can occur at a velocity smaller than the Landau velocity v_L for creation of the fermions in the continuous spectrum. This velocity can be estimated from the Landau criterium for the classical spectrum in Eq. (6) for the surface fermions. Since near the wall the superfluid velocity is tangential, the Landau velocity for the nucleation of the quasiparticles in the surface states is $v_L^{surf} = \min (E_+(p)/p_\parallel) = c_\parallel(r = R)$. The minimum first occurs at $p_\parallel = p_F$ and $E_+ = p_F c_\parallel(r = R)$; note that the transverse speed of light $c_\perp(r)$ does not enter the criterium. Taking into account the enhancement of the superfluid velocity near the wall, one obtains that the negative energy levels in the surface layer appear if the velocity u exceeds

$$v_0^* = \frac{1}{2} c_\parallel(r = R) = v_L \frac{\Delta_\parallel(r = R)}{2\Delta_0}. \tag{7}$$

Here we used the Lambert notations for different critical velocities, see Ref. [7] (in his paper however he did not take into account the splitting of the gap and assumed that v_0^* is very small).

The situation does not change if instead of the classical consideration of the energy spectrum in the surface layer, one takes into account the quantization of the quasiparticle motion along the normal to the wall. According to [12] the quasicontinuum of the subgap bound states starts above the energy $p_F c_\parallel(r = R)$ with $p_\parallel \approx p_F$, which again gives $(1/2)c_\parallel(r = R)$ for the Landau critical velocity for nucleation of the surface fermions.

Can the negative energy levels in the surface layer be filled by quasiparticles? For this it is necessary to have the connection with the reservoir of quasiparticles. It appears that this always occurs in our situation. The negative square-root branch E_- of the quasiparticles in Eq. (4) is always occupied. When the velocity u exceeds v_0^*, the energy of branch E_- can be positive, while the energy of branch E_+ can be negative, so the branches overlap and the quasiparticle from the filled branch E_- can jump to the empty level on E_+. Since momenta p_x of these states are opposite, this can happen only if the momentum p_x is not conserved, which is always the case because of the surface roughness.

3.2. Analog of Zel'dovich mechanism of positron nucleation

However when the surface Landau velocity is reached, the created surface quasiparticles, which have zero energy in the wire reference frame, cannot escape to infinity where the minimal energy of the scattering state is $\Delta_0 - p_F u = \Delta_0[1 - (1/2)(c_\parallel(r = R)/c)] > 0$. For quasiparticles to escape to infinity the velocity of wire must be essentailly higher. This happens when the lowest energy of the bound state $p_F c_\parallel(r = R) - 2p_F u_0$ merges with the continuum of the negative root states, whose upper edge is at $-\Delta_0 + p_F u$. This gives the criterium for the emission of the quasihole, $u > v_1^*$

$$v_1^* = [c + c_\parallel(r = R)]/3 . \tag{8}$$

This is equivalent to the production of the positron by the strong electrostatic potential well discussed by Zel'dovich, when the created electron fills the bound state, while the positron is emitted to infinity.

It may be helpful to remind the reader of the essential features of the Zel'dovich mechanism [2] (see also [13] for a detailed review). Consider an electron-attractive potential
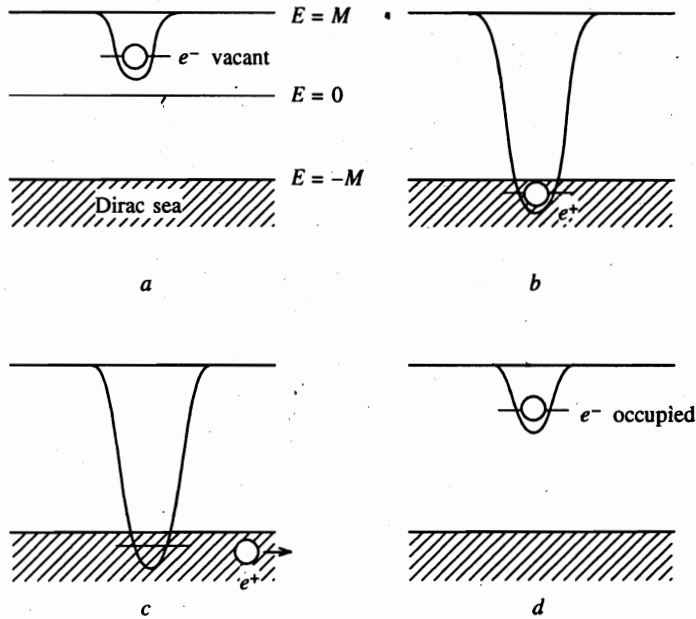
Fig. 2. Zel'dovich mechanism of positron creation

with a vacant discrete level (Fig. 2a). Suppose that the potential adiabatically increases in strength. The level will cross $E = 0$ for some value V_1 of the potential ($V_1 = \pi/2$ for a δ-function potential). There is nothing critical happening during the crossing. For some greater value V_2 the level crosses $E = -M$ and thus merges with the negative energy continuum ($V_2 = \pi$ for a δ-function potential). The original electron vacancy is now interpreted as the presence of the positron; and since the positron occupies a scattering state it can escape to infinity (Fig. 2c). If the potential now becomes weak again we go back to the situation of a discrete energy level (Fig. 2d) which however now is electron-filled. The whole cycle clearly conserves charge; however the positron escapes when the potential is strong and the electron is observed when the potential returns to its original weak value.

If the velocity of the object is kept constant, the emission of quasiparticles at $u > v_1^*$ will finally stop after all the negative levels become occupied. Then the object will move without dissipation, but its mass will be larger due to the quasiparticles which occupied the negative energy bound states. In the case of moving vortices in superfluids and superconductors a similar enhancement of the mass due to the trapped quasiparticles is the origin of the so-called Kopnin mass of the vortex (see Ref. [14]).

Thus for the uniformly moving object the dissipation is absent even if its velocity exceeds v_1^*, and nothing happens until the Landau velocity $v_L = c$ is reached, if however the hydrodynamic instability does not develop earlier [15]. The source of this instability can be the following: the filling of the bound state leads to increase of the normal component density and thus to the rearrangement of the whole superflow pattern because of the mass conservation law (see Ref. [14] for the effect of the backflow due to the normal component in the vortex core). At some velocity the superflow pattern becomes unstable, being unable to satisfy the mass conservation law. Such

hydrodynamic instability leads usually to the formation of vortices by the moving object.

Eq. (8) is analogous to the criterium obtained by Lambert [7], and it transforms to his result if $c_{\parallel}(r = R)$ is neglected. However, in the real situation $c_{\parallel}(r = R)/c$ is not small: it is close to unity for the specular boundary conditions, while for diffusive conditions it is about $c_{\parallel}(r = R)/c = 0.4$ [11]. Thus the most optimistic estimation gives $v_1^* = 0.47v_L$ which is too large compared with the experment, which shows that the supercritical dissipation starts at $\sim 0.25v_L$. Thus it appeared that the Zel'dovich mechanism in its simplest form is not responsible for the supercritical behavior. The modification of this mechanism is required according to another scenario, also suggested by Lambert [7], who exploited the adiabatic oscillations of the wire.

3.3. Radiation by adiabatically oscillating potential

The idea of this mechanism explores the fact that in the oscillating wire $u = u_0 \cos(\omega t)$ the velocity changes sign after half a period. Let us consider the case when the amplitude of the velocity $u_0 > v_0^*$ in Eq.(7). After the maximal velocity, say, $+u_0$ is reached, the bound state with the energy $E_+ = \Delta_0 c_{\parallel}(r = R)/c - 2p_F v_0^* = 0$ will be filled by the quasiparticle. Now, if the vibration of the wire is slow, which is the case since $\omega \ll \Delta_0$, then after the half of the period the energy of this quasiparticle will become $E_+ = \Delta_0 c_{\parallel}(r = R)/c + 2p_F v_0^*$. We must compare this energy with the minimal energy of the scattering states, which occurs for the opposite direction of the momentum: $E_+(\text{min scattering}) = \Delta_0 - p_F v_0^*$. So, if

$$v_0^* > v_L/5, \quad \text{i.e.} \quad c_{\parallel}(r = R)/c > 2/5 , \tag{9}$$

the continuum (conducting) energy band is achieved and the quasiparticles will be emitted by the vibrating wire. If, however $c_{\parallel}(r = R) < (2/5)c$, then the same mechanism starts to work at higher velocity, when $u_0 > (1/5)v_L$. The latter case corresponds to the Lambert result obtained under the assumption that the quantity v_0^* is very small. Thus the criterium for the radiation of the quasiparticles by the vibrating wire is $u_0 > v^*$, with

$$\begin{aligned} v^* &= v_0^* , \quad \text{if} \quad v_0^* > v_L/5, \\ v^* &= v_L/5, \quad \text{if} \quad v_0^* < v_L/5 . \end{aligned} \tag{10}$$

The general scheme of the particle production at $u_0 > v^*$ is shown in Fig. 3. In the supercritical regime (b) in a time evolution the two branches, $E_+(p_x = p_F)$ and $E_-(p_x = -p_F)$, of bound states cross each other, if the momentum p_x is conserved. In a real situation the surface roughness mixes $+p_F$ and $-p_F$ states, which leads to repulsion of levels. The time evolution of levels and one of the trajectories of the quasiparticle in supercritical regime are shown in Fig. 3c during the whole period of oscillation. The transition of the quasiparticle from the branch $E_-(-)$ to the branch $E_+(+)$ occurs either by scattering or Zener tunneling. During one cycle the particle moves from the Dirac sea to the positive energy continuum via the bound states. This corresponds to the production of the electron-positron pair via the bound states.

This mechanism is different from the Zel'dovich mechanism, in which the bound state energy touches the continuum spectrum of the Dirac sea, the electron occupies the bound state, and the positron is emitted. In our case the criticality occurs when the bound state energy of the branch E_+ reaches the zero energy and thus touches the occupied bound states of the branches E_-. In this process two particles in the scattering states are created («electron» and «positron»), resulting in the production of the momentum $2p_F$ from the vacuum. The level
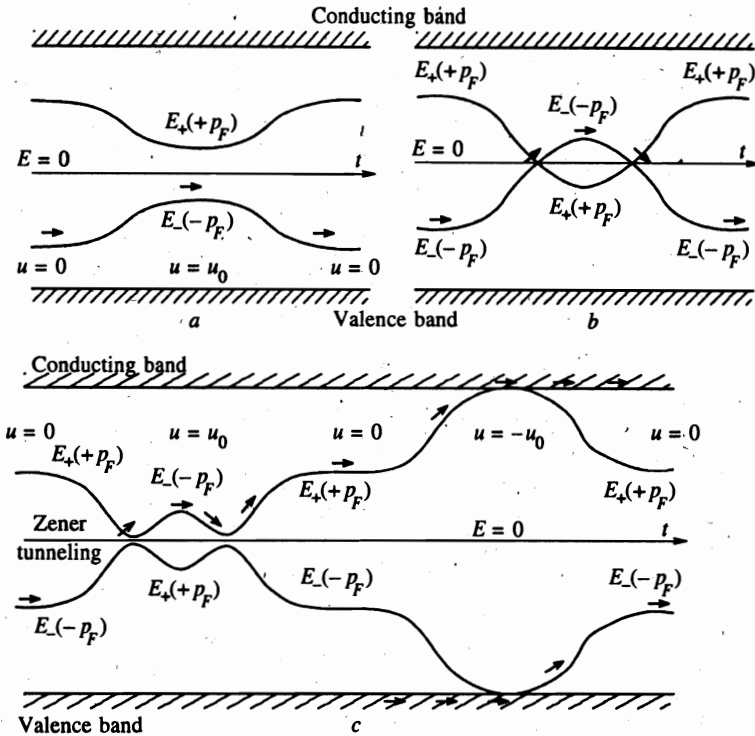
Fig. 3. Time evolution of two branches, $E_+(p_x = p_F)$ and $E_-(p_x = -p_F)$, of bound states. (a) Subcritical regime. The half of the period is shown when the velocity increases till u_0 and then decreases till zero. (b) In the supercritical regime the two branches cross each other, but the evolution of the levels does not change if momentum p_x is conserved. (c) Level flow in the presence of mixing of $+p_F$ and $-p_F$ states. The whole period of oscillations is shown, in which «electron-positron» pair is created

flow along two other branches, $E_-(p_x = p_F)$ and $E_+(p_x = -p_F)$, is similar but is shifted by half a period. As a result in this process an opposite momentum, $-2p_F$, can be produced during a cycle.

4. ANALOGY WITH FERMION PRODUCTION IN A STRONG ELECTRIC FIELD

Since close to the threshold velocity the relevant quasiparticle momentum p_x is maximal, $p_x = \pm p_F$, the term $\mathbf{pv_s}$ in Eq. (4) serves as the time like component of the 4-vector electromagnetic potential: $\mathbf{pv_s} = \pm p_F v_{sx}(x, t) = eA_0(x, t)$. Here the sign of the momentum plays the part of the electric charge. Thus we have a problem of Dirac particles in a strong electric field. The above mechanism of particle creation requires 4 ingredients:

(1) Bound states.

(2) For the filling of the negative energy levels above v_0^* it is necessary to have the mirror image branch of quasiparticles with opposite momentum (i.e. with opposite e).

(3) There should be the interaction which mixes the momenta p_F and $-p_F$ and thus allows
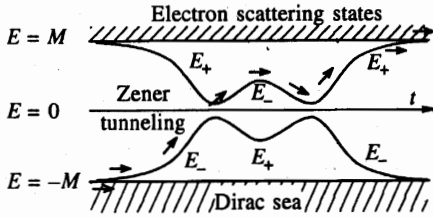
Fig. 4. Spectral flow and pair production in the system of potentials alternating in counterphase in supercritical regime

to change the sign e.

(4) The potential A_0 should be strong enough for the positive-root and negative-root branches to cross.

(5) The potential A_0 should slowly oscillate in time. During one cycle the positive-root and negative-root levels cross and then return to their respective (positive/negative) continua.

That is why, in the mapping to the Dirac problem we would need the particles with both negative and positive charges, which can transform to each other. One possibility is to use instead of the time-like component of the 4-vector electromagnetic potential a time and space dependent mass term. In this case the spectrum is symmetric, so that the positive and negative energy bound states can in principle approach each other [3], in a similar manner as in Fig. 3b.

The other possibility is to have the conventional electromagnetic field A_0, but in the form of two spatially separated potentials with the opposite sign of A_0. In this case one has the required mirror image of states. This can be modelled by the conventional Dirac Hamiltonian with potential

$$A_0(x,t) = U\cos(\omega t)[\delta(x+a) - \delta(x-a)] , \tag{11}$$

Assuming that the Dirac mass $M = 1$, and oscillations are adiabatic, $\omega \ll 1$, one obtains the time dependent bound states energy levels

$$E^2 = \cos^2\lambda + e^{-4ka}\sin^2\lambda, \quad \lambda = U\cos(\omega t), \quad k^2 = 1 - E^2 . \tag{12}$$

If U exceeds the critical value $U_1 = \pi/2$ the first (positive energy) bound state crosses $E = 0$. If the δ-potentials are well separated, $a \gg 1$, the time-dependent energy levels are in Fig. 4. Here E_+ and E_- denote the bound state levels in the right and in the left δ-function potential correspondingly. The probability of nucleation of electron-positron pair is determined by the transition between the E_- and E_+ branches. For U slightly above but not very close to $U_1 = \pi/2$ one obtains the result similar to that for the Landau-Zener tunneling problem [16], with the probability for pair creation per one cycle

$$2P(1-P), \quad P = \exp\left(-\frac{2\sqrt{\pi}\mathscr{T}^2}{\omega\sqrt{U-U_1}}\right) , \tag{13}$$

$$\mathscr{T}^2 = e^{-4a} \ll U - U_1 . \tag{14}$$

If ω is large enough, the transition between the E_- and E_+ states is given by the matrix element $\mathscr{T}$, while for small ω the process is determined by Zener tunneling across the gap $2\mathscr{T}$ between the repulsing levels.

The similar effect in nuclear physics would correspond to the case, different from that suggested by Gershtein and Zel'dovich. In their case the positron production is possible during

collision of two heavy bare nuclei with the total charge Z greater then supercritical Z_c, at which the electron bound state with energy $E = -M$ appears. This would correspond to the critical strength $U_2 = \pi$ of the δ-function potential. In our case the critical strength is $U_1 = \pi/2$. This means that we need essentially less total charge Z, at which the negative energy bound state for electron appears, $E_+ < 0$. But in addition nearby one should have a similar hypothetical collision of the anti-nuclei, which produces the potential of the opposite sign. If the latter contains the bound state with $E_- = E_+$, the electron occupying this bound state can tunnel to the bound state of the positively charged nucleus. As a result the electron-positron pair will appear after such collision.

5. DISCUSSION

According to the discussed scenario the observed critical velocity for the pair nucleation by a vibrating wire, $v_0^* \approx 0.25 v_L$ [5], is determined by the bound states near the surface of the wire and thus by the suppression of the parallel gap at the surface of the wire in Eq. (7). This gives an experimental estimation for the suppressed gap, $\Delta_\parallel(r = R) \approx 0.5\Delta_0$, which is comparable to the theoretical estimation $\Delta_\parallel(r = R) \approx 0.4\Delta_0$ [11]. This consistency provides the experimental evidence for the modified Zel'dovich mechanism of pair creation in a strong field, in which the particles can be created by the subcritical electric potential because of the level crossing.

The other objects, whose motion can be used to simulate the particle production from the vacuum, are the topological objects, vortices and domain walls. On the production of the momentum from the vacuum by the moving vortex, caused by the axial anomaly phenomenon, see review [17]. The quasiparticle production by the moving soliton in superfluid ^{3}He-A due to the combined effect of the Schwinger pair production, event horizon and ergoregion is dicussed in Ref. [18].

References

1. I. Pomeranchuk and Ya. Smorodinsky, J. Fiz. USSR **9** 97 (1945).
2. S. S. Gershtein and Ya. B. Zel'dovich, ZhETF **57**, 674 (1969) (JETP **30**, 358 (1970)).
3. W. Greiner, B. Muller, and J. Rafelski, *Quantum Electrodynamics of Strong Fields*, Springer-Verlag, Berlin (1985), Sec. 3.2.
4. W. T. Zaumen, Nature **247**, 530 (1974).
5. C. A. M. Castelijns, K. F. Coates, A. M. Guenault, S. G. Mussett, and G. R. Pickett, Phys. Rev. Lett. **56**, 69 (1986).
6. J. P. Carney, A. M. Guenault, G. R. Pickett, and G. F. Spencer, Phys. Rev. Lett. **62**, 3042 (1989).
7. C. J. Lambert, Physica B, **165–166**, 653 (1990).
8. P. Jetzer, P. Liljenberg, and B.-S. Skagerstam, Astroparticle Phys. **1**, 429 (1993).
9. D. Vollhardt, P. Wölfle, *The superfluid phases of helium 3*, Francis & Taylor, London (1990).

10. G. E. Volovik, *Exotic properties of superfluid* ^{3}He, World Scientific, Singapore, New Jersey, London, Hong Kong (1992).

11. N. B. Kopnin, P. I. Soininen, and M. M. Salomaa, J. Low Temp. Phys. **85**, 267 (1991).

12. I. A. Privorotskii, J. Low Temp. Phys. **26**, 379 (1977).

13. A. Calogeracos, N. Dombey, and K. Imagawa, Yad. Fiz. **59**, 1331 (1995).

14. G. E. Volovik, Pis'ma ZhETF **67**, 502 (1998) [JETP Lett. **67**, 528 (1998)].

15. Ü. Parts, V. M. H. Ruutu, J. H. Koïvuniemi, Yu. N. Bunkov, V. V. Dmitriev, M. Fogelström, M. Huebner, Y. Kondo, N. B. Kopnin, J. S. Korhonen, M. Krusius, O. V. Lounasmaa, P. I. Soininen, and G. E. Volovik, Europhys. Lett. **31**, 449 (1995).

16. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Pergamon Press (1977), §90.

17. G. E. Volovik, Physica B **255**, 86 (1998).

18. T. A. Jacobson and G. E. Volovik, Phys. Rev. D **58**, 064021 (1998).