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ELECTRIC CURRENTS OF EXCITATIONS IN THE ONE-DIMENSIONAL ATTRACTIVE HUBBARD MODEL

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We have calculated electric currents of various types of excitations in the Hubbard model. Both spin and charge excitations carry the electric current. The the electric charge is a continuous function of the band filling and the single-site repulsion potential.

1. INTRODUCTION

Electron-electron interactions have a great effect on properties of one-dimensional systems. This results in a critical behavior at $T = 0$ with algebraic decay of correlation functions. The particle momentum distribution differs from a Fermi-liquid step function. As distinct from a Fermi liquid, such systems are called Luttinger liquids. In the framework of the one-dimensional (1d) Hubbard model with attractive interaction we investigate the electrical conductivity properties of the excitations. Interest in this model arises because to that the Hubbard model with attraction is the simplest one with dominant superconductivity fluctuations.

In contrast to Fermi systems, where the electric charge of the quasiparticles can have integer values e or $2e$, we will show that the value of the electron charge of one-particle states can depend continuously on the system parameters. The charge of the one-particle excitations is defined by $q = j/v$, where $j = -\partial\epsilon/\partial A$ is the electric current, defined as the derivative of the energy with respect to the magnetic vector-potential, and $v = \partial\epsilon/\partial p$ is the velocity of the excitation. We will find that the spin and particle-hole excitations near the Fermi surface carry the electric current. This effect is absent for models with an exactly linear electron spectrum, so that in the weak coupling regime the current is proportional to the Fermi-velocity dispersion.

Some time ago unusual electrical properties of excitations were found in $1d$ electron-phonon Peierls systems. As a result of the electron-phonon interactions a transition from a metallic to an insulating state with creation a charge density wave takes place. It was found that excitations in the Peierls system (solitons, polarons) [1, 2] may have fractional charges depending continuously on the band filling and the electron-phonon coupling constant. But it is obvious that after integrating the Peierls model over the phonon degrees of freedom we will have an electron model with some effective electron-electron interaction (attraction). For example, we get in the quantum limit (the ion mass tends to zero) an effective «g-ology» Hamiltonian with a backscattering term due to the electron-phonon interaction. Therefore excitations in the two systems may have common properties. We have found indeed that both model excitations carry noninteger electric charges.

The plan of the paper is as follows. In Section 2 we quote known results that are needed. In Section 3 we calculate the currents and charges of excitations for the model with $U < 0$ and generalize some results to the $U > 0$ case.

2. PROPERTIES OF THE MODEL

2.1 Attractive interaction

The Hamiltonian for the Hubbard ring in a magnetic flux Φ is

$$H = -\sum_{j,\sigma}\{c^{\dagger}_{j,\sigma}c_{j+1,\sigma}\exp(i\nu) + \text{H.c.}\} + 4U\sum_{j} n_{j,\uparrow}n_{j,\downarrow} - \\ -\frac{h}{2}\sum_{j}(n_{j,\uparrow} - n_{j,\downarrow}) - \mu\sum_{j,\sigma} n_{j,\sigma}, \tag{1}$$

where N_a is the number of sites, $c^{\dagger}_{n,\sigma}, c_{n,\sigma}$ are the creation and annihilation operators for electrons with spins $\sigma =\uparrow, \downarrow$, $U < 0$ is the onsite attraction amplitude of particles with opposite spins, h is the spin magnetic field, μ is the chemical potential,

$$\nu = 2\pi\Phi/N_a\Phi_0, \quad \Phi = AN_a, \quad \Phi_0 = hc/e$$

is the magnetic unit flux, and A is the vector potential of the orbital magnetic field.

The electric current by definition is

$$j = -\left.\frac{\partial H}{\partial A}\right|_{A=0}. \tag{2}$$

The ground state and excitations are described by sets of quasimomenta k_j and rapidities λ_α, which are solutions of the Bethe ansatz (BA) equations [3]

$$N_a k_j - N_a\nu - \sum_{\beta=1}^{M}\theta(2\sin k_j - 2\lambda_\beta) = 2\pi I_j, \qquad j = 1, \dots, N, \tag{3}$$

$$N_a P_0(\lambda_\alpha) - 2N_a\nu - \sum_{j=1}^{X}\theta(2\lambda_\alpha - 2\sin k_j) - \sum_{\beta=1}^{M}\theta(\lambda_\alpha - \lambda_\beta) = 2\pi J_\alpha, \quad \alpha = 1, \dots, M, \tag{4}$$

where

$$\theta(x) = 2\,\text{arctg}(x/2u), \; u = |U|,$$

$N = 2M+X$ is the number of particles, M is the number of pairs, X is the number of unpaired particles, and $P_0(\lambda_\alpha) = 2\text{Re}\arcsin(\lambda_\alpha - iu)$ is the bare momentum of a pair. We treat states with a number of singlet bound pairs and a number of electrons with uncompensated up-spins. In contrast to the case $U > 0$, where all wavenumbers are real in the ground state, all particles are paired and the wavenumbers are complex. If the external magnetic field exceeds some critical value h_c unpaired electrons are formed. Equations (3) with real numbers k_j describe electrons with uncompensated up-spins. Singlet bound pairs are characterized by a pair of complex wavenumbers $k^{\pm}_\alpha$ and a rapidity λ_α connected through the relation

$$\sin k^{\pm}_\alpha = \lambda_\alpha \pm iu, \quad \alpha = 1, \dots, M.$$

Eguations (4) are obtained from the equations of Lieb and Wu [4] by eliminating the complex wavenumbers $k^{\pm}_\alpha$.

In the ground state the numbers I_j, J_α are distributed symmetrically about zero. They satisfy

$$I_j = \frac{1}{2}(1 - N + 2M) + j - 1, \quad J_\alpha = \frac{1}{2}(1 - M) + \alpha - 1.$$

In the case of moderate fields $h < h_c$ all spins are paired ($X = 0$).

The system energy is

$$W = \sum_j \left[e_0(k_j) - \mu\right] + \sum_\alpha \left[E_0(\lambda_\alpha) - 2\mu\right], \tag{5}$$

where

$$E_0(\lambda_\alpha) = -4\mathrm{Re}\sqrt{1 - (\lambda_\alpha - iu)^2}, \quad e_0(k_j) = -2\cos k_j - \frac{h}{2}$$

are the bare energies of pairs and unpaired electrons.

The momentum is

$$P = \sum_j (k_j - \nu) + \sum_\alpha \left[P_0(\lambda_\alpha) - 2\nu\right].$$

The density functions of the k and λ distributions are usually introduced as

$$\rho(k) = \frac{1}{2\pi}\frac{dp(k)}{dk}, \quad \sigma(\lambda) = \frac{1}{2\pi}\frac{dp(\lambda)}{d\lambda},$$

where

$$p(k) = k - \frac{1}{N_a}\sum_{\beta=1}^{M} \theta(2\sin k - 2\lambda_\beta),$$

$$p(\lambda) = P_0(\lambda_\alpha) - \frac{1}{N_a}\sum_{j=1}^{X} \theta(2\lambda_\alpha - 2\sin k_j) - \frac{1}{N_a}\sum_{\beta=1}^{M} \theta(\lambda_\alpha - \lambda_\beta).$$

In the thermodynamic limit Eqs. (3), (4) can be written in the matrix form [3]

$$\rho(k, \lambda) = \rho_0(k, \lambda) + K \otimes \rho(k', \lambda'), \tag{6}$$

where

$$\rho(k, \lambda) = (\rho(k), \sigma(\lambda))^T, \quad \rho_0(k, \lambda) = \left(\frac{1}{2\pi}, \frac{1}{2\pi}\frac{dP_0(\lambda)}{d\lambda}\right)^T,$$

$$K(k, \lambda | k', \lambda') = \begin{pmatrix} 0 & -2\cos k K\left(2(\sin k - \lambda')\right) \\ -2K(2(\lambda - \sin k')) & -K(\lambda - \lambda') \end{pmatrix},$$

$$K(x) = \frac{1}{2\pi}\frac{d\theta(x)}{dx} = \frac{1}{2\pi}\frac{4u}{4u^2 + x^2}.$$

The product $\otimes$ indicates the usual matrix product and integration over the common variables, from $-Q$ to Q over k and from $-\Lambda$ to Λ over λ, respectively; the superscript «T» means matrix transposition.

Equation (5) acquires the form

$$W = \epsilon_0^T \otimes \rho = \rho_0^T \otimes \epsilon,$$

where

$$\epsilon = \epsilon_0 + K^T \otimes \epsilon, \tag{7}$$

$\epsilon = (e(k), \epsilon(\lambda))^T$ are the excitation energies of unpaired electrons and of the pairs, $\epsilon_0 = (e_0(k) - \mu,\ E_0(\lambda) - 2\mu)^T$.

It is known that for magnetic fields less than the critical value $h < h_c$ the spectrum of paired excitations is gapless ($\epsilon(\pm\Lambda) = 0$), while unpaired electron states have a gap $e(k = 0) \neq 0$.

The matrix of dressed charges is defined by [3, 5]

$$\xi(k, \lambda) = \begin{pmatrix} \xi_1^1(k) & \xi_1^2(k) \\ \xi_2^1(\lambda) & \xi_2^2(\lambda) \end{pmatrix}, \tag{8}$$

where

$$\xi(k, \lambda) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + K^T \otimes \xi. \tag{9}$$

For the case $h < h_c$ only the function $\xi_2^2(\lambda) = Z(\lambda)$ is relevant, and it satisfies the equation [6]

$$Z + \int_{-\Lambda}^{+\Lambda} K(\lambda - \lambda') Z(\lambda') d\lambda' = 1, \tag{10}$$

$$\frac{1}{2} \le Z(\lambda) \le 1, \quad \frac{\partial Z}{\partial \Lambda} < 0, \quad \frac{\partial Z(\lambda)}{\partial u} > 0.$$

The solution of this equation is known in some limits

$$\rho \to 1, \quad 2Z^2(\Lambda) = 1 - \frac{1}{2\ln\left[C/(1-\rho)\right]}, \quad C = \sqrt{\frac{8}{\pi e}} I_0\left(\frac{\pi}{2u}\right),$$

$$\rho = 1, \qquad 2Z^2(\Lambda) = 1,$$

$$\rho \to 0, \quad \frac{\rho}{u} \to 0, \quad 2Z^2(\Lambda) = \frac{1}{2}\left(1 + \frac{\rho}{2}\sqrt{1 + \frac{1}{u^2}}\right).$$

For magnetic fields $h > h_{c1}$ the gap in the spectrum of unpaired excitations closes ($e(0) = 0$) and for $h = h_{c2}$ the system undergoes a transition into the saturated ferromagnetic ground state. In the region $h_{c2} > h > h_{c1}$ the dressed charge matrix has been found in [7] as

$$\xi(Q, \Lambda) = \begin{pmatrix} 1 + \dfrac{\kappa k_0}{2} & 0 \\ -\dfrac{1}{2} - \kappa k_0 & \left(1 + \dfrac{u}{2\pi\lambda_0}\right)\dfrac{1}{\sqrt{2}} \end{pmatrix},$$

where

$$\kappa = \frac{\ln 2}{2u}, \quad k_0 = \sqrt{\frac{h - h_c}{\eta}}, \quad \eta = 1 - 2\int_0^\infty \frac{tdtJ_1(t)}{1 + \exp(2ut)} > 0, \quad \lambda_0 = \frac{2u}{\pi} \ln \frac{C}{1-\rho}.$$

2.2. Repulsive interaction

Similar equations are valid for the model with $U > 0$. The matrix K differs from (6) only in the signs of the nondiagonal terms. A density function $\rho(k)$ describes the distribution of particle quasimomenta and the function $\sigma(\lambda)$ describes the rapidities of spin-down particles. The energy and the momentum are

$$W = -\sum 2\cos k_j, \quad p = \sum (k_j - \nu).$$

The dressed charge matrix is [5]

$$\xi = \begin{pmatrix} \xi(k) & \xi(k)/2 \\ 0 & \sqrt{2}/2 \end{pmatrix}, \tag{11}$$

where $\xi(k)$ satisfies the equation

$$\xi(k) = 1 + \int_{-Q}^{Q} dk' \cos k' \tilde{K}(\sin k - \sin k')\xi(k'), \tag{12}$$

$$\tilde{K} = \frac{1}{2\pi} \int_0^\infty \frac{e^{-\omega u}}{\operatorname{ch}(\omega u)} \cos(\omega\lambda) d\lambda.$$

The solution $1 \le \xi(Q) \le \sqrt{2}$ is known in some limiting cases [5]

$\sin Q/u \ll 1$, $\xi(Q) = 1 + \sin Q \ln 2/(\pi u)$;

$\rho = 1, Q = \pi$, $\xi(Q) = 1$, $0 < u < \infty$;

$\rho \to 1$, $\xi(Q) = 1 + \ln 2(1-\rho) f(u)/u$, $f(u) = 1 - \int_0^\infty dx J_0(x) \exp(-ux)/\operatorname{ch} xu$;

$\rho \ll 1, u$, $\xi(Q) = 1 + \rho \ln 2/u$;

$u \gg 1$, $\xi(Q) = 1 + \sin \pi\rho \ln 2/(\pi u)$;

$u \ll \sin Q$, $\xi(Q) = \sqrt{2}\left[1 - u/(2\pi \sin Q)\right]$.

3. ELECTRIC CURRENTS

3.1. Attractive interaction

3.1.1. Gapless paired excitations ($h < h_{c1}$)

In the ground state all particles are bound into singlet pairs. The lowest excitations are gapless excitations of bound pairs. For a sufficiently strong magnetic field ($h > h_c$) unpaired electron excitations are gapless too.

The particle-hole excitations of pairs are described by a set of numbers $J_\alpha = J^0_\alpha + \Theta(\alpha - \alpha_0)$ (for hole type excitations), where $\{J^0_\alpha\}$ and $\{\lambda^0_\alpha\}$ are the ground-state sets. Here $\Theta(x)$ is the usual step-function.

From (3) we find an equation for the function $\tilde{\sigma}(\lambda_\alpha) = N_a \sigma(\lambda_\alpha)\delta\lambda_\alpha$:

$$\tilde{\sigma}(\lambda) + \int_{-\Lambda}^{\Lambda} K(\lambda - \lambda')\tilde{\sigma}(\lambda')d\lambda' = \Theta(\lambda - \lambda_0) + \frac{2\Phi}{\Phi_0}, \tag{13}$$

where $\delta\lambda = \lambda_\alpha - \lambda^0_\alpha$ is the shift of a number λ_α due to the excitation and $\sigma(\lambda_0)$ is the ground-state solution.

Taking the derivative with respect to ν we have $\partial\tilde{\sigma}/\partial\nu = N_a \xi_2(\lambda)/\pi$. The energy is

$$W = \sum E_0(\lambda^0_\alpha) + \sum E_0'(\lambda_\alpha)\delta\lambda_\alpha + \frac{1}{2}\sum E_0''(\lambda^0_\alpha)(\delta\lambda)^2 = \\ = \int E_0'\tilde{\sigma}(\lambda)d\lambda + \frac{1}{2}\int E_0'' \frac{\tilde{\sigma}^2}{N_a\sigma(\lambda)}d\lambda. \tag{14}$$

Then the current $j = -\partial W/\partial A$ equals

$$j = \frac{2}{\Phi_0}\int \frac{E_0''\tilde{\sigma}(\lambda)\xi_2(\lambda)}{\sigma(\lambda)}d\lambda. \tag{15}$$

The momentum is $p = \int P_0'(\lambda)\tilde{\sigma}(\lambda)d\lambda$, where $\tilde{\sigma}$ is the solution of Eq. (13) for $\Phi = 0$.

For excitations with small momenta $p \propto (\Lambda - \lambda_0)$ we can easily see that the current $j \propto pE_0''(\Lambda)$ is proportional to the dispersion of the Fermi velocity. In the limit $u \to 0$ we find that

$$\tilde{\sigma} = \Theta(\lambda - \lambda_0)/2, \qquad \sigma(\lambda) = 1/2, \qquad Z = 1/2, \qquad j = (\pi/\Phi_0)E_0(p_\Lambda)p.$$

As we showed in the case of repulsion [8–10], the current would be absent in the linear spectrum approximation.

3.1.2. States with added particles

We calculate now the currents of states obtained by adding an electron pair or unpaired electron. The simplest way to calculate the current is to include the magnetic vector-potential in the $1/N_a$ corrections of the energy, found in [11]:

$$\delta W = \frac{2\pi}{N_a}\sum_{n=1,2} v_n \left\{(Z^{-1}\Delta\mathbf{N})^2_n + (Z^T\mathbf{D})^2_n + I^+_n + I^-_n\right\}, \tag{16}$$

where $v_1 = \partial\epsilon(Q)/\partial p(Q)$ and $v_2 = \partial\epsilon(\Lambda)/\partial p(\Lambda)$ are the Fermi velocities of unpaired and pair excitations, respectively, and $Z_{ij} = \xi^i_j(Q, \Lambda)$. Note that for $h < h_c$ unpaired excitations have a gap, so for this case $v_1 = 0$. In the case $h > h_c$ both singlet and pair excitations are gapless, that is, $v_1, v_2 \neq 0$. Here $\Delta\mathbf{N}^T = (\Delta N_1, \Delta N_2)$, $\Delta N_1, \Delta N_2$ are the numbers of added unpaired electrons and bound pairs, respectively; $\mathbf{D}^T = (D_1, D_2)$, $2D_1, 2D_2$ are differences in the numbers of positive and negative I_j, J_α numbers, respectively; $I^\pm_1 = \sum I^\pm_j$, $I^\pm_2 = \sum J^\pm_\alpha$

are sums of quantum numbers of particle-hole states near to the right (+) and left (−) Fermi points of the k and λ seas. To include the vector-potential we substitute

$$D_1 \to D_1 + \frac{N_a \nu}{2\pi}, \qquad D_2 \to D_2 + \frac{N_a \nu}{\pi}.$$

By varying (16) over the vector-potential A we obtain for the electric current

$$j = 2 \sum_{n=1,2} v_n (Z_{1n} D_1 + Z_{2n} D_2)(Z_{1n} + 2Z_{2n}). \tag{17}$$

For pair excitations we have $2D_2 = 1, D_1 = 0$,

$$j = 2v_2 Z_{22}^2(\Lambda). \tag{18}$$

Therefore the charge is equal to

$$q = 2Z_{22}^2(\Lambda). \tag{19}$$

As follows from (10) the charge may acquire any value in the interval $1/2 \le q \le 2$. In the case $\rho \to 1$ we have from (10)

$$q = 1 - \frac{1}{2\ln\left[C/(1-\rho)\right]},$$

$$\rho = 1,\ q = 1,\ 0 < u < \infty.$$

In the limit $\rho \to 0$, $\rho/u \to 0$

$$q = \frac{1}{2}\left(1 + \frac{\rho}{2}\sqrt{1 + \frac{1}{u}^2}\right).$$

In the limit $u \ll 1$, $\Lambda \gg u$ $K(x) \approx \delta(x)$, $Z \to 1/\sqrt{2}$ and $q \to 1$. In the limit $u \to \infty$, $\Lambda \ll u$ we have $Z \approx 1$, $q \approx 2$. In strong magnetic fields $h > h_c$ we obtain

$$q = \left(1 + \frac{u}{2\pi\lambda_0}\right)^2.$$

For unpaired added particles the current and charge are found substituting $2D_1 = 1, D_2 = 0$ to Eq. (16):

$$j = v_1 q, \quad q = Z_{11}(Q)\left[Z_{11}(Q) + 2Z_{21}(Q)\right] = 1 - 4\int K\left(2(\sin k - \lambda)\right)\xi_{22}(\lambda)d\lambda.$$

In the case $u \to 0$ we have $q \to 0$, that is, the excitations do not carry an electric current. By using the Wiener–Hopf technique we find in the limit $\Lambda/u \gg 1$, $u \ll 1$ that

$$q \approx \sqrt{\frac{8}{\pi e}} \exp\left(-\frac{\Lambda\pi}{2u}\right).$$

In the opposite limit $u \to \infty$, $\Lambda \ll u$ we obtain $q \approx 1 - 4\Lambda/(\pi u)$.

Note that in this linear-spectrum approximation we find that particle-hole excitations do not carry current ($j = 0$), in accordance with previous results.

3.2. Repulsive interaction

3.2.1. Hole and particle states

A similar expressions can be derived for the case $U > 0$. For spin singlet-triplet excitations or particle-hole states we derived [8]

$$\tilde{\rho}(k) = \frac{N_a \nu}{2\pi} + f(k) + \int\limits_{-Q}^{Q} \tilde{\rho}(k') \cos k' \tilde{K}(\sin k - \sin k') dk', \tag{20}$$

where $f(k) = \sum (1/\pi) \operatorname{tg}^{-1}\{\exp[2\pi(\sin k - \lambda_i)/u]\}$ for spin excitations and $f(k) = \Theta(k - k_i)$ for hole states, $\tilde{\rho}(k_j) = N_a \rho(k_j) \delta k_j$, $\rho(k)$ is a known function for the ground state, and δk_j is the shift of a number k_j due to the excitation. For the energy, momentum and current we find

$$W = \sum \epsilon_0(k_j) = \sum \epsilon_0(k_j^0) + \sum \epsilon_0'(k_j)\delta k_j + \frac{1}{2}\sum \epsilon_0''(k_j)(\delta k_j)^2 \to \frac{1}{2}\int \frac{\epsilon_0'' \tilde{\rho}^2}{N_a \rho(k)} dk,$$

where $\epsilon_0 = -2\cos k_j$, and $\delta k = \tilde{\rho}(k)/N_a \rho(k)$, and

$$p = \int \tilde{\rho}(k) dk,$$

$$j = \frac{1}{\Phi_0} \int_{-Q}^{Q} dk \frac{\epsilon_0'' \tilde{\rho} \xi(k)}{\rho(k)}, \tag{21}$$

where $\xi(k)$ is the solution of Eq. (12). In accordance with the results [8–10], we have $j \propto p\epsilon_0''(p_F)$. In the limit $u \to 0$ we again find $j = 2p\cos(\pi\rho/2)$. Equation (21) supplements the results of [8, 9].

3.2.2. States with added particles

A similar treatment can be carried out for the repulsive model $u > 0$. In this case the subscripts 1, 2 in Eq. (16) correspond to charge and spin degrees of freedom, respectively, so v_1 and v_2 are the charge and spin excitation velocities. In contrast to the $u < 0$ case both types of excitations are gapless. To include the orbital magnetic field we substitute

$$D_1 \to D_1 + \frac{N_a \nu}{2\pi}.$$

By varying with respect to the vector potential we obtain for the electric current

$$j = 2v_1(Z_{11}D_1 + Z_{21}D_2)Z_{11} = v_1(2D_1 + D_2)\xi^2(Q). \tag{22}$$

Here $2D_1$ ($2D_2$) denotes the difference in the numbers of positive and negative I_j (J_α), respectively. Changing the number of spin-up and spin-down electrons by $\Delta N_\uparrow$ and $\Delta N_\downarrow$, i.e., changing the total number of particles by $\Delta N_1 = \Delta N_\uparrow + \Delta N_\downarrow$ and the number of spin-down particles by $\Delta N_2 = \Delta N_\downarrow$, is equivalent to adding (removing) ΔN_1 extra I_j and ΔN_2 extra J_α values. The values I_j, J_α may be integer or half-integer, depending on the parity of the numbers N_1, N_2. Therefore the numbers D_1, D_2 depend on ΔN_1, ΔN_2 nontrivially, so that

$$D_1 = \frac{\Delta N_1 + \Delta N_2}{2} (\text{mod } 1), \quad D_2 = \frac{\Delta N_1}{2} (\text{mod } 1).$$

Adding a spin-up particle to the system $\Delta N_1 = 1$, $\Delta N_2 = 0$ we obtain $D_1 = \pm 1/2$, $D_2 = \mp 1/2$. Similarly, adding a spin-down particle corresponds to $\Delta N_1 = \Delta N_2 = 1$, $D_1 = 0$, $D_2 = \pm 1/2$. In both cases we find for the electric current

$$j = qv_1, \quad q = \xi^2(Q)/2. \tag{23}$$

Substituting solutions of Eq. (12) into (23) we find $1/2 \le q \le 1$ and
$\sin Q/u \ll 1$, $q = 1/2 + \sin Q \ln 2/(\pi u)$;
$\rho = 1$, $Q = \pi$, $q = 1/2$, $0 < u < \infty$;
$\rho \to 1$, $q = 1/2 + \ln 2(1-\rho)/uf(u)$;
$\rho \ll 1, u$; $q = 1/2 + \rho \ln 2/u$;
$u \gg 1$, $q = 1/2 + \sin \pi\rho \ln 2/(\pi u)$;
$u \ll \sin Q$, $q = 1 - u/(4\pi \sin Q)$.
These results complement our earlier results [8–10].

4. CONCLUSIONS

We have considered the electron currents of excitations in the one-dimensional Hubbard model. We found that for particle-hole excitations with a small momentum the current is proportional to the momentum and to the dispersion of the Fermi velocity. Therefore this current is absent in the linear spectrum approximation. We have calculated the currents and charges of states with added particles (unpaired or paired bound electrons in the case $u < 0$). We found that the charge is noninteger and continuously depends on the band filling and the onsite potential u. Note that a spin-charge decoupling has no place in the case of the Hubbard model in the magnetic field. The contributions of spin- and charge-density waves cannot be described by two independent effective Hamiltonians. Charge- and spin-density waves interact. The physical values (the spectrum of conformal operator dimensions, the electric charges in question, etc.) are determined by the 2×2 dressed charge matrix rather than two scalar coupling constants. As a result both spin and charge excitations cary the electron current.

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