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INELASTIC LIGHT SCATTERING BY ELECTRON EXCITATIONS WITH LARGE WAVE VECTORS IN A 2*D* MAGNETOPLASMA

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Microscopic mechanisms of inelastic light scattering in an interacting electron plasma in semiconductor heterostructures are considered. In the dipole limit, the cross section consists of two main contributions: the first is related to a disorder-induced mechanism and the second arises from the Coulomb interaction. The spectra of disorder-induced light scattering are described in terms of correlation functions of a random potential. The spectrum induced by the Coulomb interaction arises from two-quasiparticle excitations. The mechanisms which are studied in this paper result in the appearance of large wave vector excitations in the spectra of resonant light scattering. These results can be used to model the experimentally observed appearance of the roton density of states in light scattering spectra in the integer quantum Hall regime of a two-dimensional system. Furthermore, we show that the lineshape of spectra strongly depends on the character of disorder and, in particular, on the spatial positions of impurities with respect to a quantum well.

1. INTRODUCTION

Raman studies of two-dimensional ($2D$) systems in a high magnetic field are currently an active area of research [1]. In particular, inelastic light scattering (LS) permits the observation of the roton excitations in the regimes of the integer and fractional quantum Hall effects (QHE's) [2–4]. The magnetoroton excitations in the integer QHE regime have characteristic wave vectors of the order of $1/l_c$, where l_c is the magnetic length. At the same time, in-plane momentum transfers of the order of $1/l_c$ are not easily accessible in experiments. Magnetorotons in LS spectra were interpreted as arising from breakdown of momentum conservation in the presence of residual disorder [1, 2].

The feature of the experiments mentioned above is that LS spectra contain excitations with relatively large wave vectors. In this paper, we consider specific mechanisms of LS, which allow the observation of such excitations.

The effect of disorder on LS spectra in the integer QHE regime was investigated theoretically in Ref. [5] in the framework of a phenomenological approach. The cross section, calculated in Ref. [5], reflects the density of states of electron excitations and is written as

$$\frac{d^2\sigma}{d\Omega d\omega} \propto \int S_0(\omega, q) f(q) d^2q, \tag{1}$$

where ω is the photon energy transfer in the LS process, q is the in-plane wave vector of an excitation, and $S_0(\omega, q)$ is the structure factor of the system. The function $f(q)$ describes

breakdown of momentum conservation and is written in Ref. [5] in the Lorentzian form: $f(q) = (\alpha/\pi)/(q^2 + \alpha^2)$, where α is the phenomenological broadening parameter.

In the fractional QHE regime, LS was studied theoretically in a work of Platzman and Song He [6], where the authors have obtained numerical data for the intra-Landau-level spectra. They considered LS as a shake-up process, which results in two-excitation lines. This process is directly connected with the Coulomb interaction and was described in Ref. [6] by phenomenological matrix elements. Shake-up processes for the case of two-phonon LS from the Wigner crystal were considered in Ref. [7].

In the present paper, we study resonant LS in the dipole limit ($k \to 0$, where k is the light momentum transfer). To calculate the LS spectra, we find the Hamiltonian responsible for dipole-allowed LS by using a general formalism developed in Ref. [8]. The cross section is expanded into a series of the parameter $1/(E_g - \omega_1)$, where E_g is the optical-gap energy, and ω_1 is the laser frequency. We obtain analytical expressions for the amplitudes of LS induced by disorder and by the Coulomb interaction. The cross section of disorder-induced LS is expressed in terms of correlation functions of a random potential, which determines the characteristic wave vectors of excitations in Raman spectra. The Coulomb interaction in an electron system results in two-quasiparticle excitations in LS spectra. The mechanisms of LS, which we consider here, were studied earlier in Refs. [5, 6] by using various phenomenological approaches. In this paper, we develop from first principles a theory describing such LS mechanisms. The results obtained by us can be applied both to bulk and $2D$ systems. We focus on $2D$ semiconductor heterostructures because LS mechanisms, which involve magnetoexcitations with large wave vectors, play the most important role in these systems. In particular, we calculate the LS spectra of a $2D$ magnetoplasma with the filling factor $\nu = 2$.

2. RESONANT LIGHT SCATTERING IN AN ELECTRON SYSTEM

Resonant LS is connected with two virtual interband processes of absorption and emission which are induced by the incident and scattered photons, respectively. In the following, we will consider resonance between the lowest $2D$ subbands in the conduction and heavy-hole valence bands. Resonances with the light-hole and split-off valence bands can be described similarly. In addition, we assume that electrons occupy only the lowest $2D$ subband in the conduction band.

The cross section of LS and the structure factor $S(\omega)$ are given by (see Ref. [8])

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{\omega_2}{\omega_1}\frac{e^4}{c^4 m_0^4} S(\omega),$$

$$S(\omega) = \sum_F |\langle F|\hat{V}_{eff}|0\rangle|^2 \delta(E_0 - E_F + \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle 0|\hat{V}_{eff}^{+}\hat{V}_{eff}(t)|0\rangle e^{i\omega t} dt, \qquad (2)$$

where $|0\rangle$ and $|F\rangle$ are the initial and final states of the many-electron system, E_0 and E_F are their energies, and m_0 is the free-electron mass; the operator $\hat{V}_{eff}$ is the interaction Hamiltonian describing LS, and

$$\hat{V}_{eff}(t) = \exp\left(-i\hat{H}_{tot}t\right)\hat{V}_{eff}\exp\left(i\hat{H}_{tot}t\right),$$

where $\hat{H}_{tot}$ is the Hamiltonian of the crystal; $\omega_{1(2)}$ are energies of incident (scattered) photons, $\omega = \omega_1 - \omega_2$ is the energy transfer, and $\hbar = 1$. The temperature is assumed to be zero. Assuming resonance, the matrix elements $\langle F|\hat{V}_{eff}|0\rangle$ are (see Ref. [8])

$$\langle F|\hat{V}_{eff}|0\rangle = \sum_N \frac{\langle F|\hat{j}_2|N\rangle\langle N|\hat{j}_1|0\rangle}{\omega_1 + E_0 - E_N} = -i\langle F|\int_0^\infty \hat{j}_2\hat{j}_1(t)e^{i\omega_1 t}dt|0\rangle, \tag{3}$$

where $|N\rangle$ are intermediate many-electron states. The operators $\hat{j}_1$ and $\hat{j}_2$ describe the interband optical processes assisted by the incident and scattered photons, respectively. The states $|N\rangle$ are characterized by a single hole in the valence band and one additional electron in the conduction band.

Single-electron states of the conduction band are given by $|\alpha\rangle = e^{i\mathbf{pr}}\phi_c(z)|\sigma\rangle$, where $\mathbf{r}$ and z are the in-plane and normal coordinates, respectively; $\mathbf{p}$ is the electron momentum, $\phi_c(z)$ is the wave function describing size-quantization and $\sigma = \pm 1/2$ are the spin indexes. Single-electron states of the valence band can be written similarly: $|\gamma\rangle = e^{i\mathbf{pr}}\phi_v(z)|\mathbf{J}\rangle$, where $\phi_v(z)$ is the wave function connected with size-quantization, and $\mathbf{J}$ is the angular momentum of heavy holes. For simplicity, we disregard mixing between the valence bands.

The Hamiltonian of the electron system is $\hat{H}_{tot} = \hat{H}_b + E_c\hat{n}_c + E_v\hat{n}_v$, where the subscripts c and v refer to the first subbands in the conduction and valence bands, respectively; $\hat{H}_b$ is the Hamiltonian describing intraband energies and direct Coulomb interaction between electrons of different bands, $\hat{n}_c(\hat{n}_v)$ are the operators of electron numbers in the conduction (valence) bands, and $E_c(E_v)$ are the energies of electrons at zero in-plane momentum in corresponding $2D$ subbands. The operator $\hat{H}_b = \hat{H}_c + \hat{H}_v + \hat{H}_{cv}^{int}$, where $\hat{H}_{cv}^{int}$ is the operator of direct Coulomb interaction between electrons of the conduction and valence bands. Exchange interaction between electrons of different bands is ignored. The operators $\hat{H}_c$ and $\hat{H}_v$ describe the intraband energies of electrons: $\hat{H}_c = \hat{T}_c + \hat{u}_c^{def} + \hat{H}_{cc}^{int}$ and $\hat{H}_v = \hat{T}_v + \hat{u}_v^{def}$. Here $\hat{T}_c$ and $\hat{T}_v$ are the operators of kinetic energy; $u_c^{def}(\mathbf{r})$ and $u_v^{def}(\mathbf{r})$ are random potentials in the conduction and valence bands, respectively; and $\hat{H}_{cc}^{int}$ is the operator of the Coulomb interaction in the conduction band. Here we include in the Hamiltonian $\hat{H}_{tot}$ only the terms related to the conduction and valence subbands, which are coupled by interband resonant transitions. The Coulomb potentials are given by the matrix elements:

$$\begin{aligned} U_{cc}(\mathbf{r}-\mathbf{r}') &= \langle\phi_c(z)\phi_c(z')|U\left(\sqrt{(z-z')^2+(\mathbf{r}-\mathbf{r}')^2}\right)|\phi_c(z)\phi_c(z')\rangle, \\ U_{cv}(\mathbf{r}-\mathbf{r}') &= \langle\phi_c(z)\phi_v(z')|U\left(\sqrt{(z-z')^2+(\mathbf{r}-\mathbf{r}')^2}\right)|\phi_c(z)\phi_v(z')\rangle, \end{aligned} \tag{4}$$

where $U(R) = e^2/\epsilon R$, and ϵ is the dielectric constant.

A method to simplify the operator $\hat{V}_{eff}$ was proposed in Refs. [7, 9]. According to this method, we find the commutator $[E_c\hat{n}_c + E_v\hat{n}_v, \hat{j}_1] = (E_c - E_v)\hat{j}_1 = E_g\hat{j}_1$. At the same time, the operator $\hat{H}_b$ does not change the number of particles in any band; i.e., the commutator $[E_c\hat{n}_c + E_v\hat{n}_v, \hat{H}_b] = 0$. Thus, the effective Hamiltonian of resonant LS can be written as

$$\hat{V}_{eff} = -i\int_0^\infty \hat{j}_2\hat{j}_1(t)\exp(i\omega_1 t)\,dt = -i\int_0^\infty \hat{j}_2\exp\left(-i\hat{H}_b t\right)\hat{j}_1\exp(i\hat{H}_b t)\exp\left[i(\omega_1 - E_g)t\right]dt. \tag{5}$$

We now expand Eq. (5) in a series of the operator $\hat{H}_b$ using the equation

$$e^{\hat{a}}\hat{b}e^{-\hat{a}} = \hat{b} + \frac{[\hat{a},\hat{b}]}{1!} + \frac{[\hat{a},[\hat{a},\hat{b}]]}{2!} + ...$$

After integration in Eq. (5), we have

$$\hat{V}_{eff} = \hat{V}_1 + \hat{V}_2 + \hat{V}_3 + ..., \tag{6}$$

$$\hat{V}_1 = \frac{\hat{j}_2\hat{j}_1}{\Delta}, \quad \hat{V}_2 = \frac{\hat{j}_2[\hat{H}_b,\hat{j}_1]}{\Delta^2}, \quad \hat{V}_3 = \frac{\hat{j}_2[\hat{H}_b,[\hat{H}_b,\hat{j}_1]]}{\Delta^3}, \ ....,$$

where $\Delta = \omega_1 - E_g$. The expansion (6) is valid if the value Δ is much larger than the energies of electron excitations in a plasma; i.e., $|\Delta| \gg \epsilon_k, \epsilon_C, \Gamma$, where ϵ_k and ϵ_C are the characteristic kinetic and Coulomb energies, and Γ is the broadening of electron levels due to a random potential. In a high magnetic field, $\epsilon_k \sim \omega_c$ and $\epsilon_C \sim e^2/(\epsilon l_c)$, where ω_c is the cyclotron energy. At the same time, we assume $|\Delta| \ll E_g$. Below we will focus only on two first terms in the expansion (6).

2.1. The operator $\hat{V}_1$

The first term in Eq. (6) was calculated by Hamilton and McWhorter [8]. In the Kane model, the operator $\hat{V}_1$ is written as (see Ref. [8])

$$\hat{V}_1 = -\frac{1}{\Delta}[f_e\hat{\rho}_e(\mathbf{k}_\parallel) + f_s\rho_s(\mathbf{k}_\parallel)], \tag{7}$$

$$\hat{\rho}_e(\mathbf{k}_\parallel) = \sum_i \exp\left(i\mathbf{k}_\parallel\mathbf{r}_i\right), \quad \hat{\rho}_s(\mathbf{k}_\parallel) = \sum_i \exp\left(i\mathbf{k}_\parallel\mathbf{r}_i\right)\hat{\sigma}_{iz}, \tag{8}$$

where $\mathbf{k}_\parallel = \mathbf{k}_{1\parallel} - \mathbf{k}_{2\parallel}$, $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors of incident and scattered photons, respectively; $\mathbf{r}_i$ is the in-plane coordinate of the i-electron, $\hat{\sigma}_{iz}$ is the Pauli matrix, and $\hat{\rho}_e$ and $\hat{\rho}_s$ are the operators of electron and spin densities, respectively. The parameters $f_{e(s)}$ in Eq. (7) show selection rules of LS in resonance with the heavy-hole valence band [8, 10]: $f_e = D(\mathbf{e}_{1\parallel}\mathbf{e}^*_{2\parallel})$ and $f_s = iD[\mathbf{e}_1\mathbf{e}^*_2]_z$, where $\mathbf{e}_1(\mathbf{e}_2)$ are the polarization vectors of incident (scattered) photons,

$$D = \frac{|P_{cv}|^2}{2}\langle\phi_v(z)|\exp(-ik_{2\perp}z)\,|\phi_c(z)\rangle\langle\phi_c(z)|\exp(ik_{1\perp}z)\,|\phi_v(z)\rangle,$$

and P_{cv} is the interband matrix element.

2.2. The operator $\hat{V}_2$

The second term in Eq. (6) can be written as

$$\hat{V}_2 = -\frac{1}{\Delta^2}\left[f_e\hat{Q}_e(\mathbf{k}) + f_s\hat{Q}_s(\mathbf{k}) + \hat{C}(\mathbf{q})\right], \tag{9}$$

$$\hat{Q}_e(\mathbf{k}) = \sum_i \left\{\frac{\hat{\mathbf{p}}_i^2}{2m_c} + \frac{(\hat{\mathbf{p}}_i - \mathbf{k}_{1\parallel})^2}{2m_v} + u_c^{def}(\mathbf{r}_i) - u_v^{def}(\mathbf{r}_i)\right\}\exp\left(i\mathbf{k}_\parallel\mathbf{r}_i\right),$$

$$\hat{Q}_s(\mathbf{k}) = \sum_i \hat{\sigma}_{iz}\left\{\frac{\hat{\mathbf{p}}_i^2}{2m_c} + \frac{(\hat{\mathbf{p}}_i - \mathbf{k}_{1\parallel})^2}{2m_v} + u_c^{def}(\mathbf{r}_i) - u_v^{def}(\mathbf{r}_i)\right\}\exp\left(i\mathbf{k}_\parallel\mathbf{r}_i\right),$$

where $m_{c(v)}$ are the effective masses in the conduction (valence) bands ($m_v > 0$). Here, the single-electron momentum $\hat{\mathbf{p}}_i$ should be written with allowance for the perpendicular magnetic field. The operator $\hat{C}(\mathbf{q})$ arises from the Coulomb interaction in the intermediate states. In the dipole limit, we have

$$\hat{C} = \sum_{\mathbf{q}} [U_{cc}(q) - U_{cv}(q)] \{ f_e \hat{\rho}_e(\mathbf{q}) \hat{\rho}_e(-\mathbf{q}) + f_s \hat{\rho}_s(\mathbf{q}) \hat{\rho}_e(-\mathbf{q}) \}. \tag{10}$$

The matrix elements $U_{cc}(q)$ and $U_{cv}(q)$ in Eq. (10) have opposite signs because the interband exciton in intermediate states is neutral. The value $U_{cc}(q) - U_{cv}(q)$ is nonzero if the wave functions $\phi_c(z)$ and $\phi_v(z)$ differ. Hence, the contribution $U_{cc} - U_{cv}$ can be essential in tilted quantum wells.

The operator $\hat{V}_1$ induces LS by charge- and spin-density excitations in corresponding geometries. The cross section connected with the operator $\hat{V}_1$ was calculated in a number of works for the case of bulk semiconductors [8, 11, 12] and for the case of quantum wells (for instance, see Ref. [13]). It is essential that in Refs. [8, 11–13] the cross section $d^2\sigma/d\Omega d\omega \to \delta(\omega)$, when $k \to 0$; i.e., the operator $\hat{V}_1$ leads to dipole-forbidden inelastic LS.

The operator $\hat{V}_2$ includes the contributions proportional to the operator of kinetic-energy density

$$\hat{T}_c(k_{\|}) = \sum_i \exp\left(i\mathbf{k}_{\|}\mathbf{r}_i\right) \frac{\hat{\mathbf{p}}_i^2}{2m_c}.$$

The operator $\hat{T}_c(k)$ results in so-called LS by fluctuations of kinetic-energy density, which was considered in Refs. [11, 12, 14]. In the approaches of Refs. [11, 12, 14], the cross section induced by $\hat{T}_c(k)$ vanishes if $\omega \neq 0$ and $k = 0$. Therefore, the dipole-forbidden contributions, which can, in principle, play an important role in semiconductors, were taken into account in Refs. [11, 12, 14]

2.3. Dipole-allowed inelastic light scattering

We now consider the operators $\hat{\rho}_e(\mathbf{k})$, $\hat{\rho}_s(\mathbf{k})$, $\hat{Q}_e(\mathbf{k})$, and $\hat{Q}_s(\mathbf{k})$ in the dipole limit $k \to 0$. At zero wave vector we have $\hat{\rho}_e(0) = \hat{n}_c$, $\hat{\rho}_s(0) = 2\hat{s}_z$ and, consequently, $[\hat{H}_c, \hat{n}_c] = 0$ and $[\hat{H}_c, \hat{s}_z] = 0$, where $\hat{s}_z$ is the total spin. These equalities mean that the operators $\hat{\rho}_e(0)$ and $\hat{\rho}_s(0)$ induce elastic LS.

Inelastic LS in the dipole limit can be connected with the operators $\hat{Q}_e(0)$ and $\hat{Q}_s(0)$, because the commutators $[\hat{Q}_e(0), \hat{H}_c]$ and $[\hat{Q}_s(0), \hat{H}_c]$ are nonzero. Usually, the Coulomb interaction and a random potential play the role of perturbations, i.e., $\epsilon_k \gg \epsilon_C, \Gamma$. Hence, it is convenient to exclude the kinetic energy from the operator $\hat{V}_2$. For instance, the operator $\hat{Q}_e(0)$ can be written as

$$\begin{aligned} \hat{Q}_e(0) &= \sum_i \left\{ \frac{\hat{\mathbf{p}}_i^2}{2\mu} + u_c^{def}(\mathbf{r}_i) - u_v^{def}(\mathbf{r}_i) \right\} = \\ &= \frac{m_c}{\mu}\hat{H}_c - \sum_i \left\{ \frac{m_c}{m_v} u_c^{def}(\mathbf{r}_i) + u_v^{def}(\mathbf{r}_i) \right\} - \frac{m_c}{\mu}\hat{H}_{cc}^{int}, \end{aligned} \tag{11}$$

where $1/\mu = 1/m_c + 1/m_v$. The operator $(m_c/\mu)\hat{H}_c$ in Eq. (11) contributes to the elastic LS process. Using similar transformations for all terms of $\hat{V}_2$, we have in the dipole limit

$\hat{V}_1 + \hat{V}_2 = \hat{V}_{elas} + \hat{W}$, where the operator $\hat{V}_{elas}$ induces elastic LS. The contributions of kinetic energy remain now in the operator $\hat{V}_{elas}$. Inelastic processes arise from the operator $\hat{W} = \hat{W}_{def} + \hat{W}_{Coul}$, where

$$\hat{W}_{def} = \frac{1}{\Delta^2} \sum_i u_{eff}(\mathbf{r}_i) [f_e + f_s \hat{\sigma}_{iz}] = \frac{1}{\Delta^2} \sum_{\mathbf{q}} u_{eff}(\mathbf{q}) [f_e \hat{\rho}_e(\mathbf{q}) + f_s \hat{\rho}_s(\mathbf{q})], \tag{12}$$

and

$$\hat{W}_{Coul} = \frac{1}{\Delta^2} \sum_{i,i';i \neq i'} \left[\left(\frac{m_c}{2\mu} - 1 \right) U_{cc}(\mathbf{r}_i - \mathbf{r}_{i'}) + U_{cv}(\mathbf{r}_i - \mathbf{r}_{i'}) \right] \left(f_e + f_s \left(\frac{\hat{\sigma}_{iz} + \hat{\sigma}_{i'z}}{2} \right) \right) =$$

$$= \frac{1}{\Delta^2} \sum_{\mathbf{q}} \left[\left(\frac{m_c}{2\mu} - 1 \right) U_{cc}(q) + U_{cv}(q) \right] \{ f_e \hat{\rho}_e(\mathbf{q}) \hat{\rho}_e(-\mathbf{q}) + f_s \hat{\rho}_s(\mathbf{q}) \hat{\rho}_e(-\mathbf{q}) \}. \tag{13}$$

Here

$$u_{eff}(\mathbf{r}_i) = u_v^{def}(\mathbf{r}_i) + \frac{m_c}{m_v} u_c^{def}(\mathbf{r}_i). \tag{14}$$

The functions $u_{eff}(\mathbf{q})$ and $U_{cc}(\mathbf{q})$ are the Fourier transforms of the corresponding potentials. The operators $\hat{W}_{def}$ and $\hat{W}_{Coul}$ describe LS induced by disorder and by the Coulomb interaction, respectively. We note that the operator $\hat{W}$ includes terms proportional to the small parameters Γ and ϵ_C. The latter is essential because we may use first-order perturbation theory in this case (assuming $\epsilon_k \gg \Gamma,\ \epsilon_C$) to calculate the cross section.

3. LIGHT SCATTERING FROM THE 2D MAGNETOPLASMA

In this section, we intend to consider a $2D$ electron system in the perpendicular magnetic field B at the filling factor $\nu = 2$ ($\nu_\uparrow = \nu_\downarrow = 1$) and inter-Landau-level excitations with $\Delta l = 1$ (Δl is the change of Landau-level number). We assume that the Coulomb energy is much less than the cyclotron frequency; $\epsilon_C = e^2/(\epsilon l_c) \ll \omega_c$, that allows us to use perturbation theory [15, 16]. The creation operators for charge-density and spin-density excitations with the wave vector $\mathbf{q}$ are (see Ref. [15])

$$\hat{A}_e^+(\mathbf{q}) = \frac{1}{\sqrt{N_s}} \hat{\rho}_e(\mathbf{q}), \quad \hat{A}_s^+(\mathbf{q}) = \frac{1}{\sqrt{N_s}} \hat{\rho}_s(\mathbf{q}),$$

where N_e is the density of $2D$ electrons. The commutators with the Hamiltonian $\hat{H}_c$ are $[\hat{H}_c, \hat{A}_m^+(\mathbf{q})] = \omega_m \hat{A}_m^+(\mathbf{q})$, where the index $m = e(s)$ for charge-density (spin-density) excitations. The dispersions of magnetoexcitations (magnetoexcitons) ω_m were calculated in Refs. [15, 16] and are shown in the inset of Fig. 1. The wave functions of magnetoexcitons can now be written as $|\mathbf{q}; m\rangle = \hat{A}_m^+(\mathbf{q})|0\rangle$.

The cross section can be written in terms of correlation functions

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \langle 0 | \hat{A}_{m'}(\mathbf{q}') \exp\left(-i\hat{H}_c t\right) \hat{A}_m^+(\mathbf{q}) \exp\left(i\hat{H}_c t\right) |0\rangle e^{i\omega t} dt =$$

$$= -\frac{|L_{01}|^2}{\pi} \operatorname{Im}[G_m(\omega, p)] \delta_{m,m'} \delta_{\mathbf{q},\mathbf{q}'}, \tag{15}$$

where $\omega > 0$, and

$$|L_{01}|^2 = \frac{l_c^2 q^2}{2} \exp\left(-\frac{l_c^2 q^2}{2}\right).$$

The Green's function is

$$G_m(\omega, q) = \frac{1}{\omega - \omega_m(q) + i\Gamma_m(q)}, \tag{16}$$

where $1/\Gamma_m(q)$ is the lifetime of a magnetoexciton.

3.1. Wave-vector-dependent light scattering

First, we discuss dipole-forbidden LS induced by the operator $\hat{V}_1$, which can be written as

$$\hat{V}_1 = -\frac{\sqrt{N_e}}{\Delta}\left[f_e \hat{A}_e^+(\mathbf{k}_\parallel) + f_s \hat{A}_s^+(\mathbf{k}_\parallel)\right] + \text{c.c.} \tag{17}$$

The structure factor of LS (2) is

$$\begin{aligned} S(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle 0|\hat{V}_1^+ \hat{V}_1(t)|0\rangle \exp(i\omega_1 t)\, dt = \\ &= -\frac{N_e}{\pi\Delta^2}|L_{01}(k_\parallel)|^2 \left\{|f_e|^2 \operatorname{Im}\left[G_e(\omega, k_\parallel)\right] + |f_s|^2 \operatorname{Im}\left[G_s(\omega, k_\parallel)\right]\right\}. \end{aligned} \tag{18}$$

We see that the structure factor has the Lorentzian form and $\propto k_\parallel^2/\Gamma_m$, when $k_\parallel l_c \ll 1$ and $|\omega - \omega_m(k_\parallel)| \simeq \Gamma_m$. The integrated intensity of dipole-forbidden LS

$$I = \int S(\omega) d\omega \propto k_\parallel^2/B,$$

when $k_\parallel l_c \ll 1$.

3.2. Light scattering in the dipole limit

We now calculate the cross section of LS in the dipole limit $k \to 0$. At zero temperature we can retain in the operator $\hat{W}$ [Eqs. (12) and (13)] only the terms with $\hat{A}_m^+(\mathbf{q})$, $\hat{A}_m^+(-\mathbf{q})\hat{A}_{m'}^+(\mathbf{q})$. Thus, we have

$$\begin{aligned} \hat{W}_{def} &= \frac{\sqrt{N_e}}{\Delta^2}\sum_q u_{eff}(\mathbf{q})\left\{f_e \hat{A}_e^+(\mathbf{q}) + f_s \hat{A}_s^+(\mathbf{q})\right\}, \\ \hat{W}_{Coul} &= \frac{m_c N_e}{2\mu\Delta^2}\sum_q U(q)\left\{f_e \hat{A}_e^+(-\mathbf{q})\hat{A}_e^+(\mathbf{q}) + f_s \hat{A}_e^+(-\mathbf{q})\hat{A}_s^+(\mathbf{q})\right\}. \end{aligned} \tag{19}$$

Here we consider the strictly $2D$ system, where $U_{cc} = U_{cv} = U(q) = 2\pi e^2/(\epsilon q)$.

3.3. Light scattering induced by disorder (defects)

The operator $\hat{W}_{def}$ (Eq. (19)) results in LS by magnetoexcitons with arbitrary wave vectors q. This effect can be understood in a single-electron picture. Consider the inelastic LS process, in which $\psi_{c1}(r)(\psi_{c2}(r))$ are the initial (final) single-electron states in the conduction band and $\psi_v(r)$ is the intermediate single-electron state in the valence band. The amplitude of LS is proportional to

$$\langle\psi_{c2}(r)|\psi_v(r)\rangle\langle\psi_v(r)|\psi_{c1}(r)\rangle.$$

The wave functions $\psi_{c1(2)}(\psi_v)$ are solutions of the usual equations

$$\left[\frac{\hat{p}^2}{2m_c}+u_c^{def}(r)\right]\psi_c=\epsilon_c\psi_c,\quad \left[-\frac{\hat{p}^2}{2m_v}+u_v^{def}(r)\right]\psi_v=\epsilon_v\psi_v.$$

We see that the wave functions ψ_c and ψ_v coincide if $u_c^{def}m_c=-u_v^{def}m_v$. The latter means $u_{eff}=0$ and $\langle\psi_{c2}(r)|\psi_v(r)\rangle\langle\psi_v(r)|\psi_{c1}(r)\rangle=0$ if $\psi_{c1}(r)\neq\psi_{c2}(r)$. Thus, defect-induced LS arises from the difference between the wave functions of electrons and holes. Light scattering induced by a quasi-classical smooth electric field was considered earlier in Ref. [17]. In contrast to Bechstedt et al. [17], we treat electron scattering by defects with a quantum-mechanical approach.

Using the operator $\hat{W}_{def}$ [Eq. (19)], we find the structure factor of defect-induced LS

$$S^{def}(\omega)=-\frac{N_e}{\pi\Delta^4}\int\frac{d^2q}{(2\pi)^2}|L_{01}(q)|^2\langle u_{eff}^2\rangle_q\left\{|f_e|^2\,\mathrm{Im}\,G_e(\omega,q)+|f_s|^2\,\mathrm{Im}\,G_s(\omega,q)\right\},\qquad(20)$$

where the correlation function $\langle u_{eff}^2\rangle_q$ is given by

$$\langle u_{eff}^2\rangle_q=|u_{eff}(q)|^2=\iint d\mathbf{r}d\mathbf{r}'u_{eff}(\mathbf{r})u_{eff}(\mathbf{r}+\mathbf{r}')e^{i\mathbf{q}\mathbf{r}'}.\qquad(21)$$

In the backscattering geometry ($\mathbf{k}_{1(2)}\parallel\mathbf{z}$), the polarized spectra of LS ($\mathbf{e}_1\parallel\mathbf{e}_2$, $f_e\neq 0$) arise from charge-density excitations. The depolarized spectra ($\mathbf{e}_1\perp\mathbf{e}_2$, $f_s\neq 0$) relate to spin-density excitations. These selection rules are similar to those for k-dependent LS [8]. We see from Eq. (20) that characteristic wave vectors of magnetoexcitons are determined by random potentials (the function $\langle u_{eff}^2\rangle_q$) and by the magnetic length (the function L_{01}). In the limit $\Gamma_m\to 0$, the structure factor is proportional to the density of states of magnetoexcitons. Near the critical points of dispersions ω_{cr}, where $d\omega_m(q)/dq=0$, the density of states is proportional to $|\omega-\omega_{cr}|^{-1/2}$. The polarized spectrum S_{pol}^{def} in the limit $\Gamma_e\to 0$ has two peaks, which correspond to the excitations with q_{rot} and q_{max}, where q_{rot} and q_{max} are the wave vectors of the roton minimum and maximum, respectively (see the inset in Fig. 1). In the limit $\Gamma_s\to 0$, the depolarized spectrum S_{dep}^{def} has one singularity which is related to the roton minimum.

Light scattering processes with $\Delta l=2,3,\ldots$ can be described in the same way. The structure factor for the process with $\Delta l=N$ is given by Eq. (20) with a correction $L_{01}\to L_{0N}$.

We now discuss the mechanisms of electron scattering by disorder in quantum wells.

3.3.1. Interface defects

In the case of imperfect interfaces, the size-quantization energies of particles depend on the in-plane coordinate: $W_c=\pi^2/(2m_c^0L^2(\mathbf{r}))$ and $W_v=-\pi^2/(2m_v^0L^2(\mathbf{r}))$, where $L(\mathbf{r})$ is the

width of a quantum well, and $m^0_{c(v)}$ are the effective masses in a bulk semiconductor. The size-quantization energies of electrons and holes W_c and W_v play the role of the potentials $u_c^{def}(\mathbf{r})$ and $u_v^{def}(\mathbf{r})$. In this simplest model, we have:

$$u_{eff} = \frac{\pi^2}{2L^2(\mathbf{r})}\left(-\frac{1}{m_v^0} + \frac{m_c}{m_c^0 m_v}\right).$$

Typically, for GaAs–AlAs quantum wells we can write: $m_v^0 \neq m_v$ and $m_c^0 \simeq m_c$. Consequently, a reasonable approximation for the effective potential is

$$u_{eff} = \frac{\pi^2}{2L^2(\mathbf{r})}\left(\frac{1}{m_v} - \frac{1}{m_v^0}\right).$$

In the case of GaAs–AlAs quantum wells, we have $m_v^0 \simeq 0.4m_0$ and $m_v \simeq 0.17m_0$ for the first heavy-hole subband [18]. For weak fluctuations of the width of a quantum well, we can write

$$u_{eff} = u_{eff}(L_0) + \left(\frac{1}{m_v^0} - \frac{1}{m_v}\right)\frac{\pi^2 \delta L(\mathbf{r})}{4L_0^3},$$

where L_0 is the average width of a quantum well, $\delta L(\mathbf{r}) = L(\mathbf{r}) - L_0$, and $|\delta L| \ll L_0$.

In the limit $|\delta L(\mathbf{r})| \ll L_0$, the correlation function (21) is written as

$$\langle u_{eff}^2 \rangle_q = \left[\frac{(1/m_v^0 - 1/m_v)\pi^2}{4L_0^3}\right]^2 \langle \delta L^2(\mathbf{r}) \rangle_q. \tag{22}$$

In the case of short-range fluctuations of $\delta L(\mathbf{r})$, we may assume that the characteristic wave vector q_0 of the correlation function $\langle \delta L^2(\mathbf{r}) \rangle_q$ is much larger then l_c^{-1} and $\langle \delta L^2(\mathbf{r}) \rangle_q \simeq$ const for $q \simeq l_c^{-1}$.

3.3.2. Impurities

Another mechanism of electron scattering is connected with impurities. In this case, the potentials are

$$u_c^{def}(\mathbf{r}) = u_v^{def}(\mathbf{r}) = \sum_n u_t(\mathbf{r} - \mathbf{R}_n)$$

and

$$u_{eff} = u_c^{def}(1 + m_c/m_v),$$

where $\mathbf{R}_n$ are the positions of impurities, and $u_t(\mathbf{r})$ is their potential. The correlation function in the cross section (20) is written as

$$\langle u_{eff}^2 \rangle_q = N_t \left(1 + \frac{m_c}{m_v}\right)^2 |u_t(q)|^2, \tag{23}$$

where N_t is the $2D$ density of the impurities, and $u_t(q)$ is the Fourier transform of a single-impurity potential. The impurity potential is taken in the form

$$u_t(r) = \frac{e^2}{\epsilon\sqrt{r^2 + z_0^2}},$$

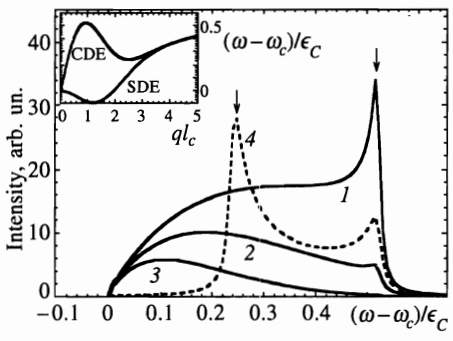

Fig. 1. Light scattering spectrum induced by disorder for the case of charge-density excitations, $\Gamma_e/\epsilon_C = 0.01$; curves *1*, *2*, and *3* correspond to the impurity-induced mechanism with the parameters $z_0 = 200, 300$, and 500 Å, respectively; the magnetic length is $l_c = 100$ Å. Curve *4* shows the spectrum of light scattering induced by a short-range random potential (interface defects). The arrows correspond to the critical points of the excitation dispersions. Inset: the dispersions of charge-density (CDE) and spin-density (SDE) excitations in the $2D$ electron plasma with the filling factor $\nu = 2$ (see Ref. [16])

where z_0 is the distance between the center of a quantum well and the δ-layer of the impurities.

Figure 1 shows the charge-density-excitation spectrum $S_{pol}^{def}(\omega)$ calculated for various mechanisms of disorder-induced scattering. The lineshape strongly depends on the correlation function $\langle u_{eff}^2 \rangle_q$. Curves *1*, *2*, and *3* in Fig. 1 represent impurity-induced LS in the systems with $z_0 = 200, 300$, and 500 Å, respectively. The magnetic length $l_c = 100$ Å corresponds to $B = 7$ T. The parameter Γ_e can be estimated from the width of the cyclotron peak in high-mobility heterostructures; it is of the order of 0.1 meV. Hence, for $B = 7$ T we have $\epsilon_C \simeq 10$ meV and $\Gamma_e/\epsilon_C \simeq 0.01$. In the case $z_0 = 200$ Å, the main contributions to the spectra arise from the critical points of the dispersion, i.e., from excitations with $q = q_{max}$ and q_{rot}. The spectrum in the case $z_0 = 500$ Å is strongly shifted to low energies and includes mostly excitations with $q < q_{max}$. This fact is connected with the exponential function

$$u_i^2(q) = U^2(q) \exp(-2qz_0)$$

in Eq. (20). In the case $z_0 = 500$ Å, the impurity potential is too smooth to induce roton excitations. In the spectrum for $z_0 = 300$ Å, the structure is shifted to low energies, but the contribution of rotons is still essential. Curve *4* in Fig. 1 shows the spectrum of LS induced by a short-range random potential ($\langle \delta L^2(\mathbf{r}) \rangle_q = \text{const}$). We see that the main contributions to the spectrum *4* in Fig. 1 are related to the critical points of the dispersion. In Fig. 1 the peak intensities strongly depend on the parameter Γ_e/ϵ_C, while the lineshape of the spectra away from the peaks is relatively insensitive to this parameter.

In their experiment Pinczuk et al. [2] observed a broad structure at energies above the cyclotron frequency, which was interpreted as the roton density of states. For the multiple quantum wells studied by them the distance z_0 was about 300 Å. Our calculations show that for the distance $z_0 = 300$ Å the spectrum is shifted to energies below the roton energy. It can be assumed, therefore, that in the experiment [2] the LS spectrum arises, in part, from a short-range random potential (interface defects). In addition, the experimental spectra depend essentially on the laser frequency, which is the signature of strong interband resonance. Our theoretical results are valid away from strong resonance and, consequently, a detailed comparison between theoretical and experimental spectra is not possible. Our theory, nevertheless, makes it possible to estimate the characteristic wave vectors of excitations in Raman spectra and to understand the mechanism of LS.

3.4. Light scattering by two elementary excitations

Light scattering induced by the Coulomb interaction is connected with the operator $\hat{W}_{Coul}$ (see Eq. (19)). This operator results in LS by two charge-density excitations (the polarized spectrum) and by combined excitations $\omega = \omega_s + \omega_e$ (the depolarized spectrum). Light scattering by two spin-density excitations is absent here, because the operator $\hat{V}_2$ contains the first power of the spin operator $\hat{\sigma}_z$. We can assume that the next terms in the expansion (6) can lead to LS by two spin-density excitations.

The structure factors for polarized and depolarized spectra are

$$S^{Coul}_{pol(dep)}(\omega) = -|f_{e(s)}|^2 \frac{N_e^2}{\pi\Delta^4}\left(\frac{m_c}{2\mu}\right)^2 \int \frac{d^2q}{(2\pi)^2}|L_{01}(q)|^4 U^2(q)\,\mathrm{Im}\left[G_{ee(es)}(\omega, q)\right], \qquad (24)$$

where $G_{ee(es)}(\omega, q) = 1/(\omega - \omega_e - \omega_{e(s)} + 2i\Gamma)$ are the two-magnetoexciton Green's functions. For simplicity, we assume $\Gamma_e = \Gamma_s = \Gamma$. In the limit $\Gamma \to 0$, the structure factors (24) are proportional to the density of states of two magnetoexcitons. We see from Eq. 24, that the characteristic wave vectors of magnetoexcitons in LS spectra are of the order of l_c^{-1}. In the limit $\Gamma \to 0$, the functions $S^{Coul}_{pol(dep)}$ have peaks at critical-point energies (Fig. 2).

The matrix elements $\hat{W}_{Coul}$ (Eqs. (13) and (19)) originate from Coulomb correlations. Such a process can be considered as a «shakeup». In other words, interband optical transitions are accompanied by shakeup of an electron system with emission of elementary excitations.

Light scattering by low frequency excitations in the regime of the fractional QHE was reported in Refs. [3,4]. The spectra observed in Ref. [4] were interpreted in terms of two-roton excitations, which have low energies (about $0.2\epsilon_C$). The roton excitations in the fractional QHE are connected with intra-Landau-level transitions. Until now, we do not know of any publications devoted to studies of the structure near the frequency $2\omega_c$. In an experimental situation, the Raman spectrum near the frequency $\omega = 2\omega_c$ can consist of two contributions: the first can be the defect-induced structure related to the magnetoexciton $\Delta l = 2$ and the second can be the contribution of the two-magnetoexciton process. These contributions can be separated because the critical-point energies of two types of excitations are different.

Here we do not consider the spin-flip inter-Landau-level spectra, which occur in our approach if the geometry deviates from backscattering and the incident light is in resonance with the light-hole valence band [10]. A theoretical description for these processes is similar to that for the case of charge-density (spin-density) excitations.

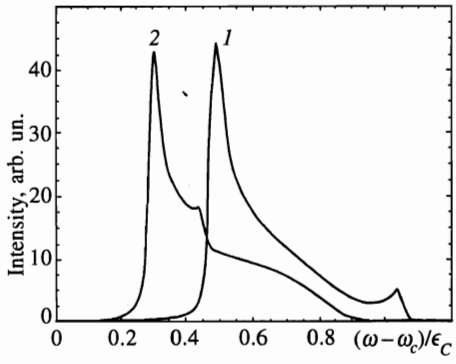

Fig. 2. The spectrum of light scattering by two magnetoexcitons: curves *1* and *2* show the excitations $2\omega_e$ and $\omega_e + \omega_s$, respectively; the parameter $\Gamma/\epsilon_C = 0.01$

4. RESONANT STRUCTURE OF THE CROSS SECTION

In Sec. 2, we consider the operator $\hat{V}_{eff}$ in the limit $|\Delta| = |E_g - \omega_1| \gg \omega_c$. We now discuss the case of strong resonance between Landau levels in the conduction and valence bands when $|\Delta_{ll'}| \sim \omega_c(\omega_{hh})$, where $\Delta_{ll'} = \omega_1 - E_g - \omega_c(1/2+l) - \omega_{hh}(1/2+l')$, and ω_{hh} is the cyclotron frequency of heavy holes. The energies $E_{gll'} = E_g + \omega_c(1/2+l) + \omega_{hh}(1/2+l')$ correspond to the interband optical resonances. The effective g-factors are neglected. At the same time, we assume that $|\Delta_{ll'}| \gg \epsilon_C \gg \Gamma$ and $\omega_c(\omega_{hh}) \gg \epsilon_C$. Thus, the expansion parameters are $\epsilon_C/\Delta_{ll'}$, ϵ_C/ω_c and Γ/ω_c.

The resonant contributions to the amplitude in third-order perturbation theory are shown in Fig. 3. In these diagrams, the interband virtual transitions are optical, while the intraband transitions are assisted by a random potential. Using these diagrams, we rewrite the operator $\hat{W}_{def}$ (Eq. (12)) with the substitution:

$$\frac{u_{eff}(q)}{\Delta^2} \to \left(\frac{1 + m_c/m_v}{\Delta_{10}\Delta_{21}} - \frac{1}{\Delta_{11}\Delta_{21}} \right) u_c^{def}(q) + \frac{u_v^{def}(q)}{\Delta_{11}\Delta_{10}}.$$

The diagrams for two-magnetoexciton processes are shown in Fig. 4. The correction for the operator $\hat{W}_{Coul}$ (Eq. (13)) is

$$\frac{1}{\Delta^2}\left[\left(\frac{m_c}{2\mu} - 1\right) U_{cc} + U_{cv}\right] \to \left(\frac{m_c}{2\mu} \frac{1}{\Delta_{20}\Delta_{31}} - \frac{1}{\Delta_{11}\Delta_{31}} \right) U_{cc} + \frac{1}{\Delta_{11}\Delta_{20}} U_{cv},$$

which shows a fine structure of interband resonances.

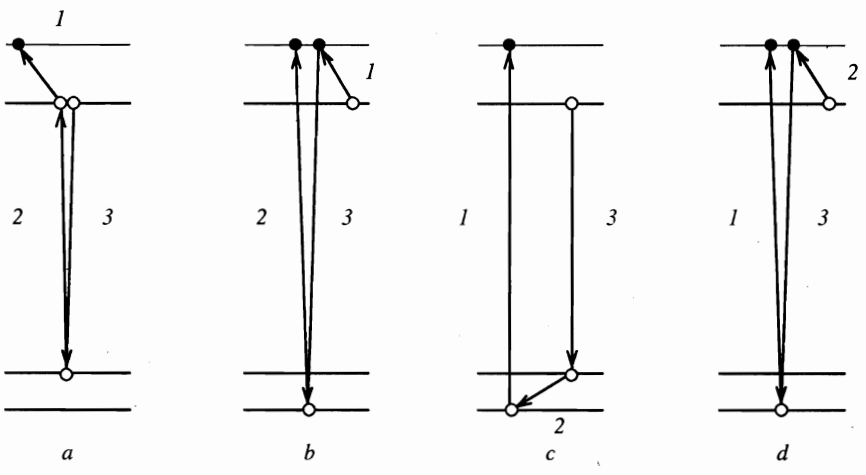

Fig. 3. Diagrams for light scattering induced by defects, in which the interband transitions are optical and the intraband transitions are assisted by defects. Scattering by defects occurs in the initial electron states (diagrams *a* and *b*) and in the intermediate states of the light scattering process (diagrams *c* and *d*). For the case $\nu = 2$, the contributions connected with defect-induced scattering in the final states in third-order perturbation theory are absent

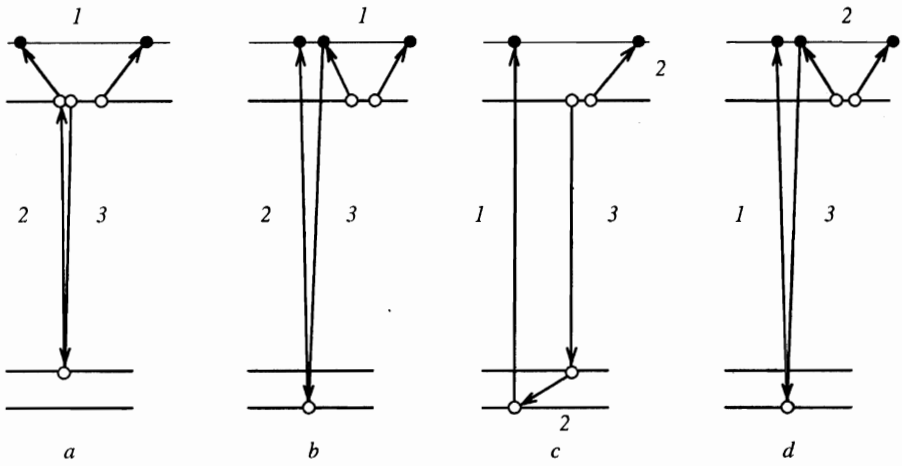

Fig. 4. Diagrams for light scattering induced by the Coulomb interaction, in which the interband transitions are optical and the intraband transitions are assisted by the Coulomb coupling. In the diagrams *a* and *b*, the Coulomb interaction induces virtual transitions in the initial states. The diagrams *c* and *d* include similar transitions in the intermediate states

5. DISCUSSION

In this paper, we have shown that resonant LS from an electron plasma in the dipole limit can be described by effective operators of two types (see Eqs. (12) and (13)):

$$\hat{W}_{def} = \frac{1}{\Delta^2}\sum_{\mathbf{q}} u_{eff}(\mathbf{q})\,[f_e\hat{\rho}_e(\mathbf{q}) + f_s\hat{\rho}_s(\mathbf{q})]\,,$$
$$\hat{W}_{Coul} = \frac{1}{2\Delta^2}\sum_{\mathbf{q}} U(q)\,[f_e\hat{\rho}_e(-\mathbf{q})\hat{\rho}_e(\mathbf{q}) + f_s\hat{\rho}_e(-\mathbf{q})\hat{\rho}_s(\mathbf{q})]\,, \tag{25}$$

where $\hat{\rho}_e(\mathbf{q})$ and $\hat{\rho}_s(\mathbf{q})$ are the Fourier transforms of the charge and spin densities, respectively; the functions $f_{e(s)}$ determine the selection rules of LS. The operators (25) are the leading terms in the expansion of the LS amplitude in terms of the parameter $1/\Delta$.

The operator $\hat{W}_{def}$ describes LS processes in the presence of disorder (defects). The matrix element u_{eff} is a combination of Fourier transforms of random potentials in the conduction and valence bands (Eq. (14)):

$$u_{eff}(\mathbf{q}) = u_v^{def}(\mathbf{q}) + \frac{m_c}{m_v}u_c^{def}(\mathbf{q}).$$

The cross section of disorder-induced LS is given by Eq. (1) with $f(q) \propto |u_{eff}(\mathbf{q})|^2$. The characteristic wave vectors of excitations in these spectra are connected with the correlation functions of disorder.

The operator $\hat{W}_{Coul}$ relates to LS induced by the Coulomb interaction. This operator leads to two-quasiparticle spectra in an ideal system. The characteristic wave vectors of excitations in LS spectra in this case are determined by the electron-electron interaction potential and in a high magnetic field are of the order of $1/l_c$.

The results of this paper can be used for a description of light scattering in semiconductor plasmas and in laterally modulated electron systems (quantum wires and dots). Our approach is valid when $E_g \gg |\Delta| = |E_g^{'} - \omega_l| \gg \epsilon_{exc}$, where ϵ_{exc} is the characteristic energy of electron excitations in the LS process.

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