Supertransparency

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(Submitted 28 March 1996)
Zh. Éksp. Teor. Fiz. **110**, 1688–1711 (November 1996)

A complete theoretical and experimental description of supertransparency, an effect which we discovered and which occurs when a light pulse propagates coherently through an optically dense ($k_0L \geqslant 60$), resonantly absorbing, two-level medium in a convergent light-beam geometry, is presented. It is found that the absorption of the pulse, which is accompanied by pushing of the spectrum of the pulse outside the confines of an inhomogeneously broadened absorption line, is anomalously weak compared with classical self-induced transparency. The maximum shift reached $1.7\delta\nu_D = 2400$ MHz for a pulse spectrum of input width ≈ 1000 MHz, and the carrier frequency was always pushed onto the red wing of the absorption line. The magnitude of the shift depended on the intensity of the incident pulse and on the optical density of the absorber and, over wide limits, it was independent of the initial magnitude and sign of the detuning. It is shown that the experimental results cannot be explained on the basis of existing theories. A solution is obtained analytically in the form of a stationary, transversely bounded, 2π self-induced transparency pulse. The spatial dynamics of a three-dimensional pulse is investigated and a dispersion–diffraction stabilization mechanism is proposed. The detailed qualitative and quantitative analysis made of the main features of the supertransparency effect on the basis of dispersion–diffraction stabilization mechanism gives good agreement with experiment. © *1996 American Institute of Physics.* [S1063-7761(96)01011-6]

1. INTRODUCTION

One of the most striking manifestations of the coherent interaction of a light pulse with a two-level resonantly absorbing medium is the soliton regime of the propagation of a radiation pulse when the threshold condition $\theta > \pi$ for the input pulse area is satisfied. As a result of a transient process, the initial radiation pulse is converted into a pulse (or sequence of pulses) of stationary shape that then propagates with no absorption.[1–3] The propagation of a pulse without loss of energy, a phenomenon with no analog in the classical theory of the interaction of radiation with matter, has been termed self-induced transparency.

The unusual features of the temporal dynamics of the field were investigated intensively for a number of years and are reflected in review articles.[3–7] Nonetheless, very little attention has been devoted to questions concerning the transverse dynamics of self-induced-transparency pulses, though such questions are the main obstacle standing in the way of complete utilization of the properties of the self-induced transparency effect. The instability of pulses against transverse perturbations, which was noted experimentally in Ref. 3 and proved theoretically in Refs. 8–10, destroys the beam structure after the pulse has transversed 10–15 absorption lengths. To weaken the instability and to observe self-induced transparency in its pure form, beams with high spatial uniformity and a planar wavefront were employed.

In the experiments described below the classical conditions for observing self-induced transparency were intentionally violated: A lens, which strongly focused the beam outside the cell next to the exit window, was placed in front of the entrance into the absorbing cell. In consequence, new characteristics that cannot be explained on the basis of the existing theories of self-induced transparency were detected. Specifically, the anomalously large shifts ≈ 2400 MHz of the pulse carrier frequency into the red region of the spectrum are at least an order of magnitude larger than any theoretically predicted and experimentally recorded shifts due to phase modulation. This "expulsion" of the pulse spectrum onto the red wing of the absorption line and the associated anomalously long propagation paths, $k_0L \geqslant 60$ (k_0 is the linear absorption coefficient for the field at the center of the absorption line and L is the length of the cell containing the absorbing atoms), are the heart of the phenomena which we term supertransparency.[11]

The properties of the experimental arrangement dictated the choice of theoretical model, which is based on the solution of the self-consistent system of Maxwell–Bloch equations taking account of the radial variation of the field. The existence of stationary solutions in the form of three-dimensional light pulses is ensured by the formation of a resonant waveguide in the absorbing medium:[12] The transverse distribution of the density of absorbing atoms must match the transverse distribution of the field. When this matching is present, the classical condition for the pulse area ($\theta = 2\pi$) holds over the entire cross section of the beam. At the same time, this makes it possible to equalize the velocity in all parts of the beam, so that a three-dimensional pulse is a natural extension of a self-induced transparency soliton with a planar wavefront to the case of three spatial variables. Despite the obvious analogy, a three-dimensional self-induced transparency pulse exhibits new properties which are associated with the interplay of nonlinear diffraction mixing of different rays in the beam and the temporal dynamics of energy transfer between the medium and the field

 1063-7761/96/110927-13$10.00

and are therefore absent in the classical theory of self-induced transparency.

The convergent light-beam geometry employed in the experiments prevents rapid breakup of the pulse. This makes it possible to observe the characteristics predicted theoretically for stationary propagation of a three-dimensional self-induced transparency pulse in a resonant waveguide.

In the present paper it is shown that the supertransparency effect is fundamental and can be regarded as an extension of the self-induced transparency effect to the case of three spatial dimensions. Together with the well-known dynamics of a self-induced transparency soliton, associated with the excitation of the atoms of the medium into the upper excited state and subsequent induced emission of the stored energy back into the field of the pulse, a new mechanism of dispersion–diffraction stabilization of the self-induced transparency pulse also appears in the supertransparency effect. A detailed description of the new physical mechanism and its theoretical justification are given below, and it is shown that this mechanism plays a decisive role in the interpretation of the observed characteristics of supertransparency.

2. THREE-DIMENSIONAL CHARACTERISTICS OF SELF-INDUCED TRANSPARENCY

The phenomena discussed in the present paper are based on an extension of the self-induced transparency effect to the case of three spatial dimensions. Correspondingly, the Maxwell–Bloch equations are written down taking account of the radial variations of the field:

$$\varepsilon\Delta_\perp \mathscr{E} - i\sigma \frac{\partial}{\partial u}\mathscr{E} + i\frac{\partial}{\partial v}\mathscr{E} = -\frac{1}{\vartheta}\mathscr{P},$$

$$\frac{\partial}{\partial u}\mathscr{P} = -i\vartheta\mathscr{E}\mathscr{N}, \qquad (1)$$

$$\frac{\partial}{\partial u}\mathscr{N} = \frac{i}{2}\vartheta(\mathscr{E}\mathscr{P}^* - \mathscr{E}^*\mathscr{P}),$$

where

$$\sigma = L_{nl}\frac{V^{-1}-c^{-1}}{\tau}, \quad \vartheta = A_0\tau,$$

$$\varepsilon = \frac{L_{nl}}{L_d}, \quad \Delta_\perp = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho}, \qquad (2)$$

$$L_d = 2kr_0^2, \quad L_{nl} = c\hbar\eta_0/2\pi\omega_0 d^2 n\tau.$$

Here $u=(t-z/V)/\tau$ is the wave coordinate; $v=z/L_{nl}$ is the longitudinal coordinate; $\rho=r/r_0$ is the transverse coordinate normalized to the beam radius; A_0, V, and τ are the amplitude, velocity, and duration of the pulse (the amplitude and duration and, therefore, the wave coordinate can depend on the transverse coordinate); r_0 is the beam radius; ω_0 is the frequency of the resonance transition; ω_p is the carrier frequency of the pulse; n is the density of absorbing particles; d is the transition dipole moment; η_0 is the linear refractive index; and, L_{nl} and L_d are the nonlinear and diffraction lengths,[1)] respectively. The total field and polarization are written in the form

$$E = A_0\frac{\hbar}{d}\mathscr{E}\exp[-i(\omega_p t - kz) + i\phi],$$

$$P = n\mathscr{P}\exp[-i(\omega_p t - kz) + i\phi].$$

In discussing the transverse dynamics of a self-induced transparency pulse it is convenient to distinguish two limiting cases: weak diffraction ($\varepsilon \ll 1$) and strong diffraction ($\varepsilon \gg 1$). The first case describes the dynamics of wide light pulses, in which the nonlinear coherent pulse–medium interaction plays the dominant role, while diffraction distorts only very little the ray trajectories within the space occupied by the pulse. On the other hand, the transverse dynamics of narrow light beams must be described in the strong-diffraction limit, where the rays disperse so rapidly on account of linear diffraction that there is not enough time for the nonlinear mechanism of the interaction of the field with the resonance medium to come into play.

In contrast to the one-dimensional case, the system of equations (1) has no solutions in the form of stationary pulses if special conditions which stabilize the transverse distribution of the beam (for example, a waveguide or resonator) are not established. An ultrashort pulse in dense resonantly absorbing media which are uniform in the transverse direction breaks up due to the self-focusing instability, termed resonant self-focusing and investigated intensively in Refs. 3, 9, 10, and 13–15.

The mechanism of the self-focusing instability is due to the dependence of the propagation velocity of the pulse on the intensity of the pulse, as a result of which the periphery of the beam moves with a lower velocity than the near-axis part of the beam (the intensity distribution in the transverse cross section is assumed to be bell-shaped). As a result, after the pulse has transversed in the medium a distance equal to several absorption lengths, the energy in the trailing edge of the pulse is concentrated in the outer rings. Light is diffracted from the ring zone to the center, resulting in a higher intensity on the axis. Self-focusing of the beam as a whole occurs, and in the process a stable spatial shape of the pulse is not established, since photon transfer between the elements of the transverse structure of the light beam destroys coherent energy transfer from the field into the medium and back: The excitation region remains inverted, but it does not radiate in a matched manner because the excitation phases in neighboring regions are different. This diffraction-mixing process plays the role of effective transverse relaxation and transfers energy irreversibly from the pulse to the medium. As a result of instability development, after traversing a distance $L \approx (10-15)L_{ab}$ the pulse is completely dispersed in the medium even in the absence of dissipation. This greatly limits the propagation path length of a self-induced transparency soliton in any experimental investigation of the effect.

Pulse breakup can be avoided by suppressing the instability by an external action, for example, by allowing the pulse to propagate in a fiber-optic waveguide, as demonstrated in Refs. 16 and 17. Stabilization of the transverse structure of the beam through conservation of the form of the waveguide mode occurs when the condition $\varepsilon \gg 1$ is satisfied, i.e., when rays in the beam react to a change in the index

profile much more rapidly than to the slower process of energy transfer between the field and the resonant medium.

Self-induced transparency experiments are ordinarily performed with gases or metal vapors, in which it is very difficult to achieve the required index profile and conditions for strong diffraction. The experiments of Ref. 18 indicated a new method for stabilizing self-induced transparency pulses under weak-diffraction conditions. The method is based on the use of a convergent light-beam geometry — a lens, placed immediately in front of an absorbing cell containing Ne, focused the beam on the exit from the medium. Under such conditions, a pulse whose duration is only three times less than the transverse relaxation time propagated over the distance $L \approx 60 L_{ab}$ (where $L_{ab} = c\hbar\eta/2\pi\omega_{ab}d^2nT^*$ is the absorption length and $(T^*)^{-1}$ is the half-width of the inhomogeneously broadened absorption line) without any strong indications of breakup and the pulse was frequency-shifted into the red wing of the absorption line by the amount ≈ 2400 MHz (approximately by two Doppler line widths). The observed effect was termed supertransparency. Its characteristic features are as follows:

1) A pulse propagates over anomalously long distances, exceeding both the known limits for self-induced transparency: $L \approx (T_2/\tau)L_{ab}$ (T_2 is the transverse relaxation time of the absorber), the limit associated with the presence of incoherent energy losses due to the finiteness of T_2, and $L \approx (10-15)L_{ab}$, the limit associated with pulse breakup on account of resonance self-focusing.

2) The propagation of a self-induced transparency pulse is accompanied by a large shift of its carrier frequency. This shift was not observed in previous experiments and it was not predicted theoretically: $\max(\Delta\nu) \approx 2\Delta\nu_D$.

3) The frequency shift is asymmetric — the frequency shifts only in the red direction from line center.

4) Finally, the frequency shift is independent of the initial detuning.

The experiments of Ref. 18 initiated theoretical investigations that revealed the reasons for the stabilization of the shape of a self-induced transparency pulse propagating in dense resonantly absorbing media[2)] and for the large shifts of the carrier frequency.[11,12,19] The diffraction nature of the effect was first suggested in Refs. 11, 12, and 19. It should be noted that in the experiments of Ref. 18 stabilization of the self-induced transparency pulse was obtained with[3)] $\varepsilon < 1$ and therefore the nonlinearity of the field–medium interaction strongly influences the transverse dynamics. Therefore the idea of a strict conservation of the form of the transverse mode, determined by purely geometric parameters, which is acceptable when analyzing the propagation of self-induced transparency solitons in one-mode fiber-optic waveguides,[16,20] cannot be used in our case. On the other hand, an important difference between the cases of weak and strong diffraction is that when $\varepsilon \ll 1$ holds one cannot talk about the formation of an absolutely stable soliton structure, since the beam remains sensitive to small-scale disturbances of the wavefront. For this reason, we shall talk only about a tendency for the spatiotemporal structure of the pulse to stabilize.

In Refs. 12 and 19 we performed a detailed investigation of the spatial dynamics of the self-induced transparency pulse for the case of weak diffraction. This investigation will be employed below as a basis for formulating the principle of stabilization of light pulses which are bounded in the transverse direction and interact coherently with an absorbing medium. We term this mechanism dispersion–diffraction stabilization[4)] (Ref. 21). The solution found in Ref. 12 remains unstable with respect to the growth of small-scale disturbances of the field,[5)] just as in the experiments of Ref. 18 which we are discussing, but from the very fact that such a solution can exist it can be concluded that new physical mechanisms, whose analysis points to a way for achieving beam stabilization, come into play.

The operation of the new mechanism can be briefly described as follows. It is easy to show that the factorized dependence of the field on the transverse coordinate, $\mathscr{E}(u, v, \rho) = \mathscr{U}(u, v)\mathscr{R}(\rho)$, cannot satisfy the fundamental property of a self-induced transparency soliton — conservation of the envelope area, which equals 2π. Therefore the formation of a stable solitary wave is necessarily due to ray mixing, which is of a purely nonlinear origin and gives rise to the same effect as in the case of the propagation of a pulse in a medium with a nonlinear refractive index. A phase self-modulation arises whose dependence on the field intensity is approximately the same as in Kerr media. From the theory of nonlinear Schrödinger solitons it is known that the formation of stable solitary waves is possible through the balancing of phase self-modulation and dispersion compression of pulses in the region of anomalous dispersion of the group velocity. Therefore, in order to compensate the phase self-modulation of a three-dimensional self-induced transparency pulse and thereby ensure stabilization of the propagation regime, the spectrum of the pulse must be located in the region of anomalous dispersion of the group velocity. The dispersion of the resonant medium is induced by the passing pulse and exhibits a complicated frequency dependence. The region of anomalous dispersion lies in the low-frequency range, and therefore the formation of a soliton structure becomes possible if the pulse carrier frequency is red-shifted away from resonance.

Our goal in the present paper is to explain the basic laws of supertransparency from the standpoint of the dispersion–diffraction stabilization mechanism. In addition to a qualitative interpretation of the results, quantitative estimates which agree well with the experimental data are given.

3. SUPERTRANSPARENCY EFFECT

We investigated experimentally the characteristic features of the coherent propagation of a pulse of superradiance of neon through an optically dense ($L/L_{ab} > 60$) resonantly absorbing medium in a convergent light-beam geometry.[6)] The plasma in the positive column of a glow discharge in neon, containing a large number of metastable atoms in the state $1s_5(2p^5 3s^3P_2)$, played the role of the absorbing medium. As estimates show, the interaction of the neon atoms with the superradiance pulse can be effectively described in the two-level approximation.

3.1. Pulse source

In the experiment, a nanosecond, high-voltage, pulsed discharge in neon, accompanied by superradiance at a wavelength of 614.3 nm (a transition between the $2p_6$ and $1s_5$ states of the $2p^53p$ and $2p^53s$ configurations, respectively) served as a source of short coherent light pulses. The discharge occurred in a thick-wall glass capillary. A pulse from the standard power supply for a nitrogen laser was applied to the discharge tube. The rate of rise of the voltage pulse was $\approx 6 \cdot 10^{11}$ V/s. In such a system the gas is excited by an ionizing wave of the gradient of the potential.[22]

The experiment was performed in the pressure range 0.7–5 Torr, where generation was unidirectional. The geometric parameters of the capillary (0.3 mm inner diameter and 20 cm length) were chosen so that the source would operate in the single-mode regime (Fresnel number $F \sim 1$).

The duration of the superradiance pulse was ≈ 4 ns and the maximum peak power was ≈ 0.7 W. The characteristics of the superradiance pulse depended on the conditions for excitation of a discharge in the capillary. The pulse carrier frequency could be varied with respect to the resonance transition frequency over the range $\delta\nu = \nu_{ab} - \nu_p = -200$–$1400$ MHz. The width of the spectrum of the superradiance pulse was equal to 700–2000 MHz and was greater than the inverse pulse duration, which could be due to chirping of the frequency in the source. The maximum light-flux density at the exit from the source was equal to ≈ 700 W/cm^2. When a lens which focused the light flux was used, the light flux density at the waist of the caustic could reach $\approx 10^4$ W/cm^2.

3.2. Absorbing medium and the two-level approximation

The superradiance pulse from the source enters the absorbing medium — the plasma in the positive column of a glow discharge in neon, containing metastable neon atoms in the $1s_5$ state with maximum density $n \approx 10^{12}$ cm^{-3}. The discharge occurred in a glass tube 30 cm long and 10 mm in diameter. The working pressure was equal to 1.66 Torr. Prior to the arrival of the pulse, a definite distribution of populations between the states of the two configurations indicated above already exists in the medium. The arriving superradiance pulse interacts only with the $2p_6 - 1s_5$ transition, since the detuning between the pulse carrier frequency and the frequency of the closest spectral lines ($\lambda = 616.3$ nm, 612.8 mm) is much greater than the Rabi frequency. The spontaneous decay time of the $2p_6$ state to levels of the bottom configuration equals 19.7 ns. The reaction of these levels due to population mixing on the pulse propagation dynamics is negligibly small because the spontaneous decay and mixing times (microsecond range) are long compared with the pulse duration (≈ 4 ns).

The neon atom has zero nuclear spin and the levels have no hyperfine structure. However, there is a fine structure, which results in degeneracy of the resonant transition ($J_1 = 2$, $J_2 = 2$).

On the whole, it can be concluded that the population dynamics of the resonant transition, in which we are interested, accompanying the propagation of a superradiance pulse can be described in the two-level approximation taking account of the degeneracy of the levels.

The experimental conditions made it possible to vary the parameters of the absorber. A hydrogen getter was soldered in the tube. Hydrogen, which substantially decreased the density of metastable neon atoms, could be released by specially heating the getter. In this manner, the optical density L/L_{ab} of the resonantly absorbing layer could be varied in the range 3–60. The optical density was controlled by varying the discharge excitation conditions (different currents in the range 7–70 mA, introduction of small additions of H_2).

The light beam passing through the absorbing cell was focused with the aid of a lens with focal length $F = 20$ cm in a manner so that the beam converged directly outside the absorbing medium.

For the interaction of the superradiance with the resonantly absorbing plasma to be coherent, the condition $\tau < T_2$ must be satisfied. According to our estimates,[23] the phase memory time, or the polarization relaxation time T_2 of the absorbing medium, is $T_2 \approx 11$ ns for the case being described ($p = 1.66$ Torr, $i = 7 - 70$ mA). In calculating T_2 the radiation process and the collisions of atoms in the $1s_5$ and $2p_6$ states with neon atoms in the ground state were taken into consideration. The collisions of excited neon atoms with electrons were neglected, since under the discharge conditions indicated above the electron density was less than 10^{12} cm^{-3}.

In summary, under the conditions of our experiment $T_2/\tau = L_{cr}/L_{ab} \approx 3$, where L_{cr} is the maximum optical thickness of the sample to which a pulse with a planar wavefront can penetrate into the medium according to existing theories of self-induced transparency.

3.3. Basic results

The experimental study of pulse propagation through a resonantly absorbing medium showed that when a convergent light-beam geometry is used, self-induced transparency occurs for absorption lengths much greater than the classical limits ($L/L_{ab} > 60$).

Above a definite energy threshold ($I \approx 0.2$ W, $J \approx 6$ W/cm^2), the superradiance pulse passed through the absorbing medium with very little attenuation. The central frequency of the superradiance pulse at the exit from the absorbing layer was found to be shifted into the red region of the spectrum. The magnitude $\Delta\nu$ of the red shift was equal to 1500-2400 MHz, and the maximum of the pulse spectrum was found to be located outside the Doppler width $\delta\nu_D = 1400$ MHz of the absorption line. The spectral width of the shifted spectrum was less than that of the spectrum of the superradiance pulse entering the absorbing medium. Variation of the initial frequency detuning $\delta\nu = \nu_0 - \nu_p$ in the range -200–1400 MHz had no effect on the frequency of the pulse leaving the absorbing cell.

As the pulse passed through the absorber, self-focusing was observed simultaneously with a reddening of the pulse spectrum. When the power at the entrance exceeded 0.5 W, breakup into small-scale filaments of self-focusing occurred. The number of filaments at the exit from the absorbing medium increased with the source power. A superradiance pulse

passing through an absorber ($L/L_{ab} \geq 60$) under the conditions of small-scale self-focusing was attenuated by a factor of ≈ 4.

A decrease in the optical density of the absorbing medium to $L/L_{ab} \approx 10$ as a result of a decrease in the density of absorbing metastable atoms substantially decreased the observed spectral shift. The shift of the pulse spectrum was 3–4 times less than for the case of high optical density ($L/L_{ab} = 60$). In experiments with variable optical density it was observed the superradiance intensity at the entrance into the absorbing medium, the density of absorbing particles, and the expulsion of the pulse spectrum into the red region are all related.

3.4. Satisfaction of the self-induced transparency conditions

The choice of the pulse source and absorber combination for demonstrating self-induced transparency effects is a nontrivial problem. In practice, it is difficult to match the frequency of the absorption line with the pulse carrier frequency because the transition frequencies rarely coincide and substantial technical contrivances are required to solve this problem.[24–26] An alternative approach is to use the same transition for generation and absorption, in which case frequency tuning does not present any problems. This is the alternative used in our experiment, so that the conditions for resonant interaction are satisfied. As shown above, the absorber employed is satisfactorily described in the two-level approximation. Moreover, when necessary, our source makes it possible to control the detuning of the pulse frequency within the limits of the absorption line. It should be noted that experiments on the observation of self-induced transparency in a transition that does not include the ground state were first described in Refs. 27 and 28.

In self-induced transparency experiments, the conditions for coherent pulse–absorber interaction must be satisfied. In rarefied atomic gases and in low-pressure discharges the conditions $\tau \ll T_2 < T_1$ are satisfied for nanosecond durations. The population-difference relaxation constant T_1 measured by Bennett[29] and the computed phase memory time T_2 for the absorbing transition employed satisfy these inequalities.

We shall now discuss in greater detail the basic characteristics of self-induced transparency, and we shall compare them with the experimental dependence. It has already been noted that there exists an energy threshold at which supertransparency was observed. This threshold is associated with the existence of a threshold for observing self-induced transparency. It is well known that at the entrance plane of the absorbing medium the pulse area must exceed the value $\theta = \pi$. This condition imposes a limit on the light-flux density, which at the entrance into the absorber must exceed 6.12 W/cm^2 ($\theta = d/\hbar\tau_p$, $d = 5 \cdot 10^{-18}$ cgse), which corresponds to radiation power $P \approx 0.17 - 0.21$ W at the entrance with beam diameter $D_l = 2.0 \pm 0.1$ mm at the surface of the lens. The fact that the experimentally measured transmission threshold agrees with the theoretically computed value is a direct indication that the observed effect is a self-induced transparency effect. For the transition studied here in neon, the dependence of the transmission of media which are not too long ($k_0 L \approx 2$) on the input pulse area was investigated systematically in Ref. 23. The transmission curve shows rapid growth of transmission with input area equal to π and an oscillatory behavior for input areas greater than 2π, indicating coherent pulse–medium interaction.

Another characteristic manifestation of self-induced transparency is the decrease in the pulse propagation velocity.[1] Temporal measurements in supertransparency experiments have shown that a pulse propagates more slowly in optically dense absorbing media. For $L/L_{ab} = 60$ and average power level at the entrance ≈ 0.3 W, the corresponding time delay equaled ≈ 2 ns. The delay is smaller than the values estimated using the classical self-induced transparency formulas for a 2π-pulse because the spectrum is pushed out from under the absorption-line contour, where the pulse velocity is determined mainly by the refractive index at the shifted frequency and is virtually identical to the velocity of light. Moreover, McCall and Hahn[3] as well as Hopf and Scully[30] investigated the time delay as a function of pulse area in the coherent case, and they showed that the delay for input areas close to π is smaller because of the finiteness of the times T_1 and T_2 (see also Ref. 5). The same results were confirmed experimentally in Ref. 28, and in addition a large decrease was recorded in the delay in a wide range of input areas for transitions with degenerate levels. The latter remark is important, since the $2p_6 - 1s_5$ absorbing transition is degenerate with respect to energy.

In addition, the on-axis energy density increases as a result of the focusing achieved in the experiment, and the pulse area can exceed the stationary value of 2π. It is well known that as the area increases, the delay decreases.

For these reasons, in supertransparency experiments the absence of large delays cannot be used as an argument against a coherent character of the pulse–absorber interaction.

The temporal measurements also showed that the pulse duration decreased very little — down to 3 ns — as the pulse travelled through the absorber. This effect can be explained on the basis of the theory of self-induced transparency: It is known[31] that a pulse whose area satisfies the inequalities $2\pi < \theta < 3\pi$ tends to transform into a pulse with area $\theta = 2\pi$ and its duration decreases at the same time.

Finally, there is the fact that as a pulse propagates in a two-level medium, the pulse spectrum is transformed and the transformation can be attributed only to phase, i.e. coherence, effects. The interpretation of the spectrum shift on the basis of burning-out of spectral components close to the resonance transition frequency cannot be adopted as a hypothesis, since the maximum shift is more than twice the width of the pulse spectrum.

We conclude on the basis of our analysis of the experimental relations that the experimental conditions correspond to coherent interaction of the pulse with a degenerate two-level resonance transition.

3.5. Analysis of mechanisms of the shift of the pulse carrier frequency

Attempts to observe the expulsion of the spectrum of a nonresonance pulse from the absorption line[32] without focus-

ing of the radiation were made in Ref. 25 (the pulse area at the entrance into the absorbing medium was $\theta \leqslant 3\pi$, with $L \approx 5L_{ab}$). The detuning between the pulse carrier frequency and the central frequency of the absorbing transition varied by the Doppler width of the line both above and below resonance. For input pulse area $\theta \approx \pi$, a shift of the pulse carrier frequency was recorded, but the shift was less than 10–20 MHz, which is much less than both the inhomogeneous width of the absorption line and the initial detuning.

A detailed theoretical investigation of phase effects accompanying coherent interaction of a pulse with a two-level medium is presented in Refs. 33 and 34. Chirped pulses with the average carrier frequency equal to the transition frequency once again remain in exact resonance during propagation. In the case of chirped pulses which are initially detuned, it is observed that a 2π-pulse tends to a hyperbolic secant form at the same (entrance) frequency. In both cases, the excess phase modulation is carried off by a precursor — a secondary pulse formed as a result of the splitting of the initial pulse into two parts. The phase modulation of the precursor is much greater than the initial modulation and grows with distance. The precursor propagates with a velocity close to the velocity of light and sustains anomalously small losses, since its Fourier components overlap very little with the absorption line.

In the supertransparency experiments, the pulses emanating from the source exhibited large phase modulation and therefore similar effects can be expected to appear. However, irrespective of the initial detuning, no large splitting of the pulses was observed, pulses did not form at the input frequency, and the width of the output pulse spectrum was less than that of the input pulse spectrum, regardless of the optical density of the absorber. Therefore we have no grounds for associating the observed pushing out of the frequency with the action of the initial phase modulation.

Another process resulting in the motion of the pulse carrier frequency is the relaxation of the elements of the density matrix of the two-level system. Depending on the ratio of the spontaneous relaxation times of both levels and the dephasing time, either pulling of the pulse toward resonance or, conversely, pushing of the pulse away from resonance can be observed.[3,6] Although relaxation processes play a very important role in our experiments, because the action on both wings of the absorption line is symmetric they cannot be responsible for the anomalously large red shift, which was independent of both the magnitude and sign of the detuning.

Therefore the large changes in the carrier frequency which were recorded in the supertransparency experiments cannot be satisfactorily explained in terms of the existing theories of self-induced transparency for pulses with a planar wavefront.

Coherent resonance self-focusing during the propagation of short, spatially nonuniform, light pulses in absorbing media was investigated experimentally and numerically in Refs. 8, 14, and 35–38. In our opinion, this model is closest to the phenomenon which we are discussing, and it can incorporate the main features of supertransparency, but in the cited work attention was directed mainly on the aspects of the problem which are associated with the transfer and exchange of energy between elements of the beam in the transverse direction. The spectral characteristics of the radiation leaving the absorbing cell were not analyzed either in the experimental or the numerical investigations.

We shall now briefly discuss the possibility of interpreting the spectrum shift which we observed from the standpoint of specific properties of the absorbing medium as an active plasma object. For example, the possibility of nonlinear resonant pumping of plasma oscillations by the self-induced transparency pulse — a process accompanied by energy transfer from the pulse to the plasma and resulting in a corresponding reddening of the pulse spectrum — cannot be ruled out *a priori*.

Estimates showed that under the experimental conditions ($i \approx 7$ mA, $n_e \approx 10^{11}$ cm^{-3}) the plasma frequency is $\nu_p \approx 1000$ MHz, and the width and shift of the pulse spectrum are very close in magnitude. Since the plasma frequency satisfies $\nu_p \propto \sqrt{n_e}$, when nonlinear buildup of the plasma oscillations occurs the spectrum shift should depend on the electron density in the absorbing medium (plasma). However, special experiments performed to check this hypothesis did not show any changes in the magnitude of the shift of the pulse spectrum when the strength of the current was varied over wide limits (7–70 mA). This indicates that the mechanism considered here for the frequency shift does not participate in the supertransparency effect which we are studying.

4. DYNAMICS OF A THREE-DIMENSIONAL SELF-INDUCED TRANSPARENCY PULSE

The foregoing arguments attest convincingly to the impossibility of explaining from the standpoint of the classical theory of self-induced transparency all of the experimental results on supertransparency. A theory claiming to explain the new effect must satisfy at least two basic requirements:

1) It must explain the mechanism of the shift or, in other words, the mechanism of energy transfer from one region of the spectrum into another and

2) it must give results which are in quantitative agreement with the experimental results.

We expound below the basic principles of the theory of a three-dimensional self-induced transparency pulse, and the focus of the investigations is transferred to the experimental context. In Ref. 11 we proved for the case of weak diffraction ($\varepsilon \ll 1$) that solutions can exist in the form of pulses with a stationary shape:

$$\mathcal{E} = \tau^{-1}(\rho)\,\mathrm{sech}[u(\rho)]\exp(i\phi), \quad \vartheta = 2, \tag{3}$$

and the phase of the field is found from the equation

$$\frac{\partial^2}{\partial u^2}\phi - \tanh[u(\rho)]\frac{\partial}{\partial u}\phi = -\frac{1}{|\mathcal{E}|}\frac{\partial}{\partial u}\Delta_\perp|\mathcal{E}|. \tag{4}$$

A solution in the form of the stationary pulse (3) exists when the transverse distribution $\max(|\mathcal{E}|)$ of the field matches the transverse distribution $n(\rho)$ of the density of resonance atoms, which in the simplest case of a homogeneously broadened absorption line and with the carrier frequency of the field tuned to exact resonance with the atomic transition has the form (see Ref. 11)

$$n(\rho)\propto\tau^{-2}(\rho). \tag{5}$$

Under this condition the pulse velocity does not depend on the transverse coordinate.

For further analysis, we shall require the concept of an average pulse carrier frequency. This concept is introduced according to the formula (see Ref. 33)

$$\langle\dot{\phi}(u,\rho)\rangle_u=\left\langle\frac{\partial\phi(u,\rho)}{\partial t}\right\rangle_u = \int_{-\infty}^{\infty}\frac{\partial\phi}{\partial t}\mathscr{E}^2(u,\rho)du\left[\int_{-\infty}^{\infty}\mathscr{E}^2(u,\rho)du\right]^{-1}. \tag{6}$$

The expression for the phase is found by solving Eq. (4). If $\tau^{-1}(\rho)$ is chosen as a Gaussian function

$$\tau^{-1}(\rho)=\tau_0^{-1}\exp(-\rho^2/2),$$

then we obtain for the frequency shift

$$\langle\dot{\phi}(u,\rho)\rangle_u=\frac{2}{3}\varepsilon\tau_0\left(1-\frac{\rho^2}{4}\right)\exp\left(-\frac{\rho^2}{2}\right). \tag{7}$$

Without analyzing in detail the dependence of the frequency shift on the transverse coordinate, we average the expression (7) over ρ and finally obtain the following expression for the average shift:

$$\Delta\nu=\frac{\langle\dot{\phi}(u,\rho)\rangle_{u,\rho}}{2\pi}=\frac{\nu_0}{(kr_0)^2}S. \tag{8}$$

Here we have introduced the new parameter

$$S=\frac{\max\langle\mathscr{E}(u,\rho)^2\rangle_\rho/4\pi}{\hbar\omega n_0}, \tag{9}$$

where the averaging is performed over the transverse coordinate ρ.

The physical meaning of the parameter S is quite obvious — S is proportional to the fraction of the energy stored in the radiation field relative to the maximum energy which the medium can absorb.

From the formula for the frequency shift (8) it can be concluded that the correction to the phase as a result of the diffractional mixing of rays in the beam determines the red shift of the frequency away from the resonance line.

We shall now examine in greater detail the dynamics of the propagation of the pulse (3) in a medium possessing waveguide properties (5). When diffraction of the beam is taken into account, the trajectories of the rays in the region of space occupied by the pulse become curved. The path of the rays in the energy-carrying part of the pulse ($u<1$) with the transverse distribution of the amplitude of the field taken in the form $\tau^{-1}=\tau_0^{-1}\exp(-\rho^2/2)$ is shown in Fig. 1a. From Fig. 1a it is evident that the beam radius is maximum on the wings of the pulse and minimum at the peak of the pulse. This dynamical behavior of the rays follows from an analysis of the expression for the transverse energy flux

$$J=\frac{c^2}{2\pi\omega}\frac{\partial\phi}{\partial\rho}|\mathscr{E}|^2. \tag{10}$$

Near the axis the flux is directed inwards on the leading edge of the pulse and outwards on the trailing edge. In addition, as expected for stationary propagation, the integrated energy flux

$$\int_{-\infty}^{\infty}Jdt$$

equals zero, and the beam radius averaged over the pulse remains constant.

One can see from Fig. 1b that within the pulse envelope the ray trajectories curve in a complicated manner, but for the energy-carrying part there is a tendency for monotonic compression of the beam to occur with increasing instantaneous intensity.

There is an obvious analogy to self-focusing of a beam in Kerr media. We shall show that this analogy has a formal basis, and elaboration of the analogy gives an effective tool for describing the physics of the processes which we are studying. We rewrite the equation for the field in the form

$$\frac{\partial}{\partial v}\mathscr{E}=\sigma\frac{\partial}{\partial u}\mathscr{E}+i\frac{1}{\vartheta}\mathscr{P}+i\varepsilon\Delta_\perp|\mathscr{E}|, \tag{11}$$

which is correct to within ε, inclusively. Let us treat Eq. (11) as a formal extension of the nonlinear Schrödinger equation, where $\mathscr{P}$ plays the role of the dispersion of the group velocity and $\varepsilon\Delta_\perp|\mathscr{E}|$ plays the role of the phase self-modulation. Then the nonlinear correction to the refractive index is formally written as

$$\delta\eta=\frac{1}{2(kr_0)^2}\frac{\Delta_\perp|\mathscr{E}|}{|\mathscr{E}|}. \tag{12}$$

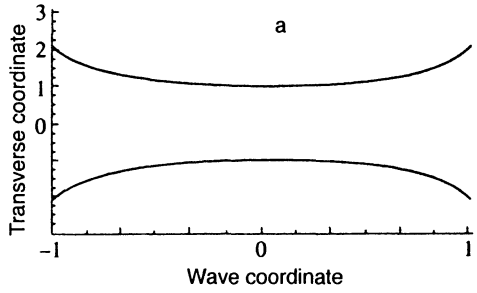
If the interaction of the field with the matter were linear, the wave equation could be solved by the method of separation of variables and the expression (12) would not contain a field dependence. But, we are dealing with a strongly nonlinear interaction, and the transverse coordinate enters in the solution for the field (3) to keep the pulse area constant and equal to 2π over the entire cross section of the beam. Therefore the correction to the refractive index contains explicitly a dependence on the time and the field intensity. Substituting the solution (3) into Eq. (12) and choosing for the amplitude of the field a Gaussian function of the transverse coordinate

$$A_0(\rho)=A_0(\rho=0)\exp\left(\frac{-\rho^2}{2}\right),$$

we obtain finally the following expression for the refractive index:

$$\eta=\eta_0+\delta\eta=\eta_0-\frac{1}{2(kr_0)^2}\left\{2u\tanh u+\rho^2\times\left[u^2\left(\frac{2}{\cosh u}-1\right)+3u\tanh u\right]\right\}. \tag{13}$$

One can see from Eq. (13) that the refractive index increases as $u\to 0$, i.e. away from the leading edge of the pulse toward the center of the pulse, where the amplitude of the field of the pulse is maximum. Just as in Kerr media, the increase in the refractive index with increasing intensity re-



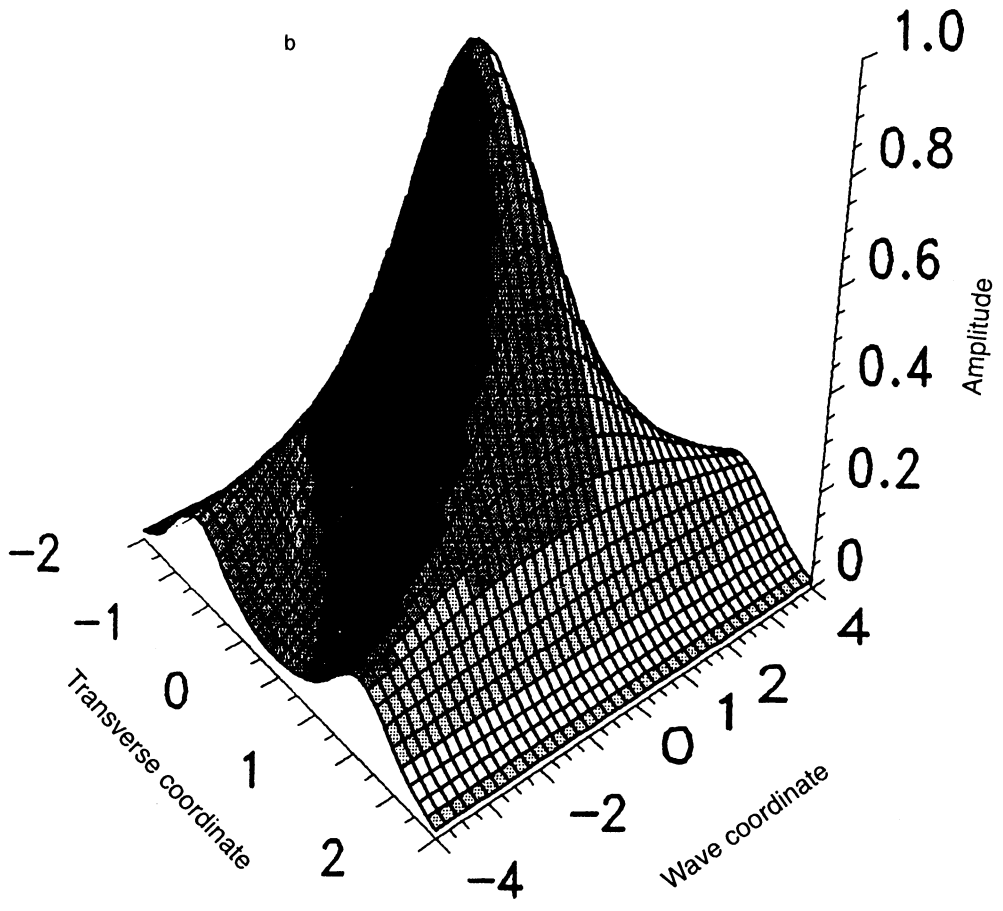


FIG. 1. a — Schematic diagram of the beam cross section in the energy carrying part of the pulse ($u<1$) along the wave coordinate. It is seen that the radius tends to decrease monotonically as the amplitude increases; the minimum is reached at $u=0$, i.e., at the peak of the pulse. b — Form of the three-dimensional self-induced transparency soliton in a moving coordinate system. The distribution of the density of the absorbing atoms is taken in the form $n(\rho) \propto \exp[-\rho^2]$.

sults in beam self-focusing, which in the case of stationary propagation of the pulse is expressed in an oscillatory dependence of the beam radius on the time with a maximum at the peak of the pulse (see Fig. 1a). Figure 2 displays the relation $\delta\eta_{nls} \propto |\mathcal{E}|^2$, which is characteristic for Kerr media, as compared with the relation found above for self-induced transparency: $\delta\eta(u, \rho=0)$. The curves are obviously similar.

The analogy drawn above is based on an extension of the concept of a nonlinear refractive index to the case of coherent interaction of a pulse with a resonantly absorbing medium. A detailed derivation shows that the complicated function $\delta\eta(u,\rho)$ can be replaced by the simpler function δn_{nls}, which holds near the maximum of the field ($u<1$, $\rho<1$). The reduction to a simpler function is dictated by the fact that it is convenient to interpret the pulse dynamics in terms of a refractive index which is quadratic in the field, since the effect of such an index on the pulse has been well studied.

It is well known that a positive correction to the refractive index results not only in beam self-focusing but also, which is no less important for the pulse, to a shift of the "red" frequencies in the spectrum of the field on the leading edge of the pulse and a shift of the "blue" frequencies to the trailing edge of the pulse (i.e. phase self-modulation).[39] In

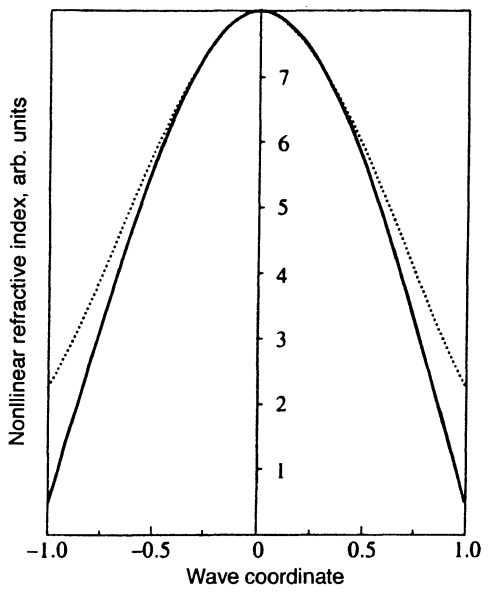


FIG. 2. Plot of the nonlinear correction to the refractive index calculated according to Eq. (12) (solid curve) compared with the nonlinear correction to the refractive index in the case of a Kerr nonlinearity (dotted curve).

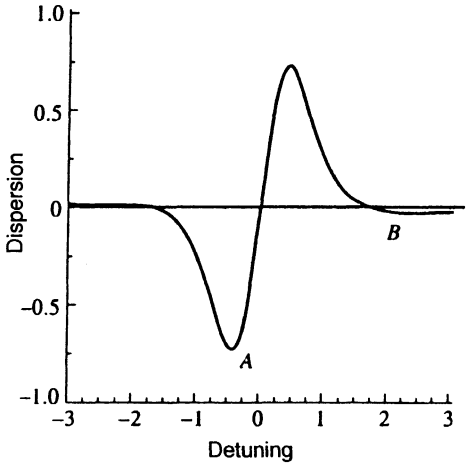


FIG. 3. Plot of the nonlinear resonant dispersion of the group velocity for the case of a homogeneously broadened absorption line. The letters A and B designate local minima of the dispersion. The dimensionless detuning normalized to the pulse duration τ is plotted along the abscissa.

the process, the spectrum becomes deformed. This deformation is expressed in a symmetric broadening of the pulse on both sides of the central frequency. Cancellation of the frequency spreading, which ensures a stationary shape, is possible only in the region of anomalous dispersion of the group velocity: $\partial^2 k/\partial\omega^2 < 0$.[40]

To determine the nature of the resonant dispersion, we shall find an expresson for the polarization induced in the medium in the case when a self-induced transparency soliton propagates in it (in the plane-wave approximation). This problem with arbitrary detuning is solved, for example, in Refs. 5, 6, and 12. The dispersion of the medium is described by the polarization component which fluctuates in phase with the field:

$$\mathrm{Re}\,\mathscr{P}=\frac{\Delta\omega\tau}{1+(\Delta\omega\tau)^2}|\mathscr{E}|. \tag{14}$$

The dispersion relation for the field interacting coherently with resonantly absorbing atoms follows immediately from the expression (14):

$$k=\frac{\omega_0}{c}+\frac{1}{L_{nl}}\frac{\Delta\omega\tau}{1+(\Delta\omega\tau)^2}. \tag{15}$$

In the context of the present exposition, we are interested not in the wave number k itself, but rather its second frequency derivative, $\partial^2 k/\partial\omega^2$, i.e. the dispersion of the group velocity. It is seen from Fig. 3 that the dispersion of the medium near resonance has a complicated frequency dependence. Even though it is completely similar to the curve of linear dispersion of the resonant matter, the half-width of the curve being determined by the homogeneous lifetime T_2, here we are dealing with a manifestation of nonlinearity and the half-width of the curve is determined by the pulse duration τ. The curve of the nonlinear resonant dispersion of the group velocity determines two regions of anomalous dispersion: a deep well at low frequencies (near the point A) and a weak anomalous dispersion at high frequencies (near the point B).

The most favorable conditions for compensation of phase self-modulation, i.e. large anomalous dispersion, occur to the left of the resonance. Therefore, as diffraction increases (the parameter ε increases), the nonlinear correction $\delta\eta$ to the refractive index will increase, and in order to compensate for the additional phase self-modulation the pulse carrier frequency must shift into the region with large values of the anomalous dispersion of the group velocity, i.e., in the red direction from resonance. This monotonic dependence on the magnitude of the phase self-modulation is correct for small frequency shifts. The expression (8), which likewise determines the monotonic increase in the red shift with increasing parameter ε and serves as a confirmation of the qualitative description of the shift, was obtained in the same approximation.

Therefore the dynamics of a three-dimensional self-induced transparency pulse admits a simple and clear interpretation in terms of phase self-modulation and dispersion of the group velocity. The interaction of these two effects is responsible for the conservation of pulse shape and explains all characteristic features of the phase dynamics of the field.

5. ANALYSIS OF SUPERTRANSPARENCY FROM THE STANDPOINT OF THE DISPERSION–DIFFRACTION MECHANISM OF STABILIZATION

The main experimental result is the construction of a curve of the shift of the pulse carrier frequency versus the pulse intensity and the optical density of the resonant matter. In Ref. 11 the semi-empirical formula

$$\Delta\nu=\frac{\nu_0}{(kr_0)^2}\frac{S}{1+S^2}, \tag{16}$$

was proposed for describing the frequency shift as a function of the parameter S introduced above. The formula (16) was obtained as an extension of the theoretical result (8) to the

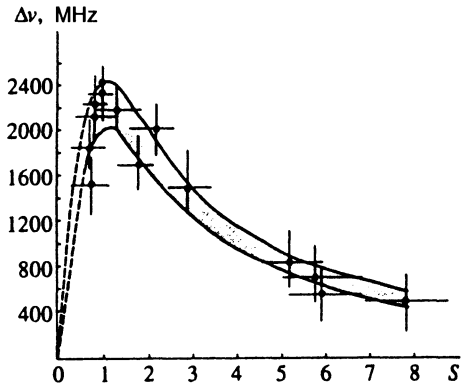


FIG. 4. Comparison of the experimental results with the magnitude of the shift calculated according to the semi-empirical formula (16).

case of large values of S, where the energy of the field exceeds the energy that the medium can absorb. The quantitative and qualitative agreement with the experimental results (designated by dots with an indication of the error) serves as a confirmation of the correctness of the semi-empirical formula (16); see Fig. 4. The solid lines in the figure represent the dependence of the spectral shift $\Delta\nu$ on the parameter S, as calculated from the formula (16) taking account of the experimentally measured parameters, which we have at our disposal, of the source and the absorbing medium. The beam diameter at the surface of the lens is $D_l=2.0\pm0.1$ mm, the focal length of the lens is $F_l=20$ cm, the peak power of the pulse is $P_{\max}=0.2-0.7$ W, and the radius at the waist of the beam is $r_0=\lambda F_l/2D_l=3\cdot10^{-3}$ cm. The density of metastable atoms $n=10^{12}$ cm^{-3} corresponds to the optical density $L/L_{ab}\approx60$ with a 30 cm long absorbing cell. The region of the computed values within the limits of error is shaded in the figure. All experimental points fall within the theoretically predicted region. The formula (16) reflects the fundamental role of the reradiation processes in the supertransparency effect. The shift depends not on the intensity and density separately but rather on the combined parameter S, which incorporates the characteristics of both the medium and the field.

The absence of experimental points on the initial section of the curve (for $S<0.6$) is due to the existence of a threshold for observing self-induced transparency — the pulse area at the entrance must be greater than π. From this condition it is possible to calculate the threshold light-flux density $I_{cr}=6.12$ W/cm^2, which in turn results in the cutoff of small values of the parameter S, so that the values of the frequency shifts corresponding to $S<S_{cr}$ are in principle unobservable.

The condition $\theta\geqslant\pi$ at the surface of the lens with beam diameter $D_l=2.0\pm0.1$ mm corresponds to radiation power at the entrance $P_{\max}\approx0.17-0.21$ W. We obtain finally $S_{cr}=0.60-0.86$ and $\Delta\nu_{cr}=1700-2400$ MHz with the absorbing-atom density $n=10^{12}$ cm^{-3} ($L/L_{ab}=60$). In reality, the experimental values of $\Delta\nu_{\rm cr}$ should be somewhat lower, since it was assumed that the threshold condition holds on the surface of the lens and not at the entrance into the medium.

It is evident from Fig. 4 that large shifts are observed in a quite narrow range of variation of the parameter S and are largest near $S=1$. The intensity $\mathscr{E}_0=\sqrt{4\pi\hbar\omega n}$ of the field of the light wave at which the frequency shift reaches its largest value corresponds to the situation when half the energy is stored in the matter and the other half is stored in the field of the pulse. For $S\gg1$, which corresponds to the situation when the number of photons in the light beam is much larger than the number of absorbing particles, the spectral shift decreases rapidly with increasing S. This corresponds to the experimental situation realized for optical densities $L/L_{ab}<17$ with the same power at the entrance. To achieve, once again, large spectral shifts for the indicated small values of the density it is necessary to decrease the initial intensity of the light beam, i.e., the parameter S must be decreased.

According to Eq. (16), the frequency shift is inversely proportional to the beam radius squared. The convergent light-beam geometry employed in the supertransparency experiments gave a radius which decreased gradually to a comparatively low value and thereby resulted in the appearance of large spectral shifts.

Analysis of the experimental data on the basis of the formula (16) shows that the magnitude of the shift of the pulse spectrum is a sensitive function of three parameters — the peak intensity of the field, the optical density of the absorber, and the beam radius. Therefore the absence of large spectral shifts in the earlier classical self-induced transparency experiments is apparently due to the fact that in those experiments a geometry close to a planar wave beam was used and the value of the parameter S was not optimal.

We can conclude on the basis of the quantitative and qualitative agreement between the experimental data and the theory for small S and the simplicity of extending the formula without invoking additional physical parameters that supertransparency is of the same nature as a three-dimensional self-induced transparency pulse. A unified description of both phenomena is possible on the basis of the dispersion–diffraction stabilization mechanism, which, specifically, will make it possible to construct a complete picture of supertransparency and to describe the mechanism by which energy is transferred into the red region of the spectrum.

In the preceding section we noted that diffraction is the source of phase self-modulation and gives rise to symmetric spreading of the frequencies with respect to the central frequency. Part of the broadened spectrum falls into the region of violet detuning, where the dispersion of the group velocity is positive and therefore these spectral components disperse in time. The dispersive spreading process results in rapid absorption of the violet components of the field. The part of the spectrum falling within the region of red detuning from the resonance frequency is subject to anomalous dispersion of the group velocity, and the spectral components of the field run together in time. The mutual compensation of the phase self-modulation and anomalous dispersion of the group velocity produces favorable conditions for the formation of a stationary pulse shape. This is the process responsible for the transfer of the energy stored in the field into the red region of the spectrum, as observed in supertransparency experiments.

The magnitude of the shift depends on the ratio of the magnitudes of the phase self-modulation and dispersion of the group velocity. Let us consider first the case of small parameters S, when phase self-modulation is not too large. Then the initial section of the curve in Fig. 4, characterized by a direct proportionality between the magnitude of the shift and the intensity and an inverse proportionality between the magnitude of the shift and the density, describes well the dependence of the frequency shift on the parameter S. Consider now the curve of the nonlinear dispersion of the group velocity (see Fig. 3). As the intensity increases, the pulse duration decreases in accordance with the ability of a self-induced transparency soliton to keep the envelope area equal to 2π, and so the characteristic width of the dispersion curve increases. In the process, the pulse spectrum shifts to lower frequencies, since now the same value of the dispersion corresponds to a frequency farther from resonance. As the absorbing-particle density increases, the half-width of the dispersion curve does not change, but rather the curve extends along the vertical axis. Now the anomalous dispersion of the group velocity required to balance phase self-modulation shifts closer to resonance. Ultimately, the red shift decreases.

The dynamics of the shift of the pulse frequency as a function of the position of the pulse in the medium can be easily traced systematically on the basis of the mechanism described above. For a fixed magnitude of the diffraction (i.e., for fixed beam radius) the pulse carrier frequency self-tunes to the frequency near which the anomalous dispersion makes possible complete balancing of the phase-self-modulation and therefore prevents the pulse from breaking up. In supertransparency experiments, the radius and therefore the magnitude of the diffraction changed with distance. Adiabatic adjustment of the pulse parameters to the instantaneous beam radius results in a continuous increase in the shift of the pulse carrier frequency with decreasing beam radius. This process continues until the pulse spectrum is completely pushed out from under the contour of the absorption line.

We shall show that the dispersion–diffraction stabilization mechanism which we have proposed gives a satisfactory quantitative estimate for the maximum frequency shift.

The theory developed is restricted by the assumption $\varepsilon \ll 1$ and does not contain an explanation of the basic feature of the curve presented in Fig. 4 — the saturation of the shift. However, this feature can be easily understood on the basis of a more complete analysis of the dispersion curve. In the process of focusing of the beam by the lens, the diffraction parameter ε and the phase-self-modulation increase. The self-modulation is balanced by a gradual shifting of the frequency into the region of large anomalous dispersion of the group velocity, i.e., in the direction of low frequencies. This happens until the frequency is located near the minimum — near the point A. A further shift of the carrier frequency is disadvantageous — beyond the point of the minimum the anomalous dispersion of the group velocity starts to decrease as the detuning increases. Therefore the region near the minimum of the dispersion curve is the natural limit for the further motion of the frequency.

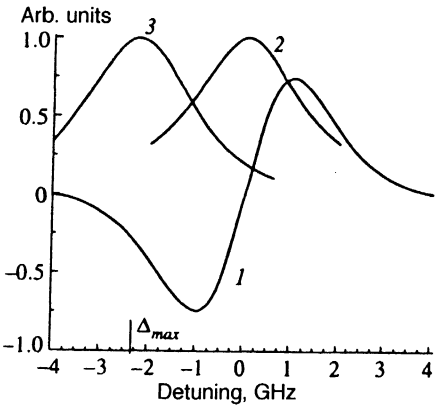


FIG. 5. Plot of the nonlinear resonant dispersion of the group velocity calculated for the case of an inhomogeneously broadened absorption line according to Eq. (17) with substitution of the experimentally measured values of the parameters (curve *1*). Profile of an inhomogeneously broadened line with width $\delta\nu_D = 1400$ MHz (curve *2*). Pulse spectrum of width $\delta\nu = 1000$ MHz (curve *3*). The maximum shift of the carrier frequency is $\Delta_{max} = 2400$ MHz.

The experimentally observed independence of the frequency shift from the initial detuning can be explained from the same standpoint of the dispersion–diffraction stabilization mechanism. For prescribed parameters of the field and the medium, the frequency corresponding to exact balancing of the phase self-modulation and dispersion of the group velocity is determined naturally according to the dispersion curve, and in order for the form of the curve to remain the same the pulse spectrum inevitably shifts toward this frequency, regardless of where the pulse is located (within reasonable limits).

To get an idea of the maximum shift occurring under the experimental conditions, having extended the theory for an inhomogeneously broadened absorption line, we shall present the relevant quantitative estimates. From the theory of self-induced transparency it is known that in this case the dispersion relation assumes the form

$$k(\Delta\omega) = \frac{\omega_0}{c} + \frac{1}{L_{nl}} \frac{T_2^*}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp[-(\Delta\Omega T_2^*)^2] \times \frac{\Delta\omega\tau + \Delta\Omega\tau}{1 + (\Delta\omega\tau + \Delta\Omega\tau)^2}\, d\Delta\Omega, \qquad (17)$$

where the averaging extends over the inhomogeneously broadened contour of width T_2^*. Differentiating the expression (17) twice with respect to frequency, we obtain the desired frequency dependence of the dispersion of the group velocity.

We now possess all information required to make a quantitative comparison of the theory with the experimental data. The computed dispersion curve of the group velocity for the case of an inhomogeneously broadened resonance transition with half-width $\Delta\nu_D = 1400$ MHz and for a pulse with a spectrum of width 1000 MHz is displayed in Fig. 5. It is easy to see that the inhomogeneously broadened character of the transition causes the dispersion curve to stretch out along the frequency axis. Figure 5 also shows schematically the pulse spectrum shifted in the red direction away from

resonance by the amount of the maximum experimentally recorded shift. It is easy to see that the minimum of the dispersion curve lies closer to resonance than the pulse carrier frequency with the maximum recorded shift. This small discrepancy is easily explained by the asymmetry of the dispersion well and the finite width of the spectrum of the pulse. The maximum anomalous dispersion of the group velocity for the pulse as a whole is reached not in the well itself but somewhat to the left of the well. After this correction, the agreement between the theoretical results and the experimental data becomes better.

It should be noted that the nature of the supertransparency effect is of a fundamental character and is independent of the specific geometry of the experiment. The effect always occurs during the coherent propagation of transversely bounded light pulses through resonantly absorbing media. In our case the experimental detection of frequency shifts and their interpretation are facilitated by using lenses, though in principle the same result can also be obtained in a lens-free geometry on account of the coherent resonant self-focusing of the beam.[14] The effect of the transverse dynamics on the temporal dynamics and vice versa is manifested as time-dependent diffraction of the beam. The interplay is strongest when the scale $\approx L_{nl}$ of the longitudinal changes in the pulse is comparable to the scale $\approx L_d$ of the transverse changes. This was recorded in our experiments as a strong red shift of the frequency.

Returning to the question of the nature of the anomalously long pulse propagation paths, two basic points can be distinguished. The first one, which is not associated directly with the characteristic features of the nonlinear dynamics, is that the lens plays a key role in supertransparency. The results of the experiments performed in a lens-free geometry serve as proof of this assertion. The medium completely absorbed the pulse energy and the useful signal at the exit from the cell with an absorber with optical thickness $L \approx 3L_{ab}$ became comparable to the noise level. When the lens was introduced, beam focusing occurred and the energy became concentrated near the axis. The dissipation due to polarization relaxation was thereby effectively balanced by a constant inflow of energy from the periphery of the beam. This can serve as an explanation of the fact that the width of the pulse spectrum is approximately the same at the entrance and exit of the medium. In the experiments with a lens under optimal conditions, the total energy losses from the field were at most 70% of the input energy.

The second reason for effective stabilization of the beam is of a nonlinear nature and does not depend on the coherent geometry of the experiment. Nonetheless, lenses make it possible to create favorable conditions for the dispersion–diffraction mechanism of stabilization. As the beam becomes narrower, the frequency shift increases monotonically, displacing the pulse spectrum from under the absorption curve. Therefore the effect of the dissipative processes in the medium on the pulse decreases (approximately as the square of the detuning) and the growth rate of the disturbances decreases (also approximately as the square of the detuning; see Ref. 41).

6. CONCLUSIONS

In the forgoing discussion we gave a detailed description of the experimentally discovered supertransparency effect, which is observed when a pulse interacts coherently with a two-level absorbing medium under the conditions of a convergent light-beam geometry. We showed in detail that the two-level approximation is applicable and that the requirements for self-induced transparency are satisfied. We showed that existing theories cannot give an adequate explanation of all experimental results.

Having found that the difference between the classical self-induced transparency experiments and supertransparency experiments is due mainly to the use of light beams which are bounded in the transverse direction, we constructed a theory of a three-dimensional self-induced transparency pulse. The condition that the transverse density profile match the transverse profile of the field, as required for stationary propagation of a pulse, is equivalent to the use of a focusing lens in the experiments. Analysis of the transverse dynamics of a three-dimensional self-induced transparency pulse uncovered the reasons why a stable spatiotemporal structure is formed, the entire collection of factors being called the dispersion–diffraction mechanism of stabilization. The heart of the new mechanism is that phase self-modulation, caused by diffraction mixing of rays with different phases, and nonlinear resonant dispersion induced in the group velocity in the medium during the propagation of the self-induced transparency pulse in it balance one another.

All the basic features of supertransparency were explained. The experimentally measured curves of the magnitude of the shift as a function of the energy stored in the field and as a function of the optical density of the absorber can be explained on the basis of the dispersion–diffraction mechanism of stabilization and merge in a natural manner into a single curve as a function of the combined parameter S. The theoretical formula for the frequency shift, containing the radius of the light beam and the parameter S, requires a slight generalization in order to produce complete agreement with the experimentally measured curves. In so doing, we remain completely within the model employed and do not invoke any new parameters or new physical mechanisms.

The asymmetry of the shift of the pulse carrier frequency is explained by the asymmetry of the nonlinear dispersion curve, and the most favorable conditions for stabilization of the temporal shape of the pulse are obtained only on the low-frequency side of the absorption line. The shift recorded in the experiment may saturate because of a minimum in the frequency dependence of the dispersion. After the minimum of the dispersion is reached, a further motion of the spectrum into the low-frequency region becomes disadvantageous, since the conditions for balancing phase self-modulation become worse. For a fixed beam radius, diffraction is determined by the corresponding degree of phase self-modulation, to compensate which a definite value of the dispersion is required, and so the pulse self-tunes to the frequency at which this value is reached. Therefore, within wide limits, the experimentally recorded shift is independent of the magnitude of the initial detuning.

The additional stabilization of the pulse as a result of the

pulse carrier frequency being pushed out from under the absorption contour made it possible to increase the propagation path of the pulse up to anomalously large values ($L > 60L_{ab}$), since then the dissipative processes in the medium became less efficient and development of transverse instability was suppressed.

In summary, both distinguishing features of supertransparency — pulse stabilization over large distances and large spectral shifts — are found to be inseparably interrelated and they are based on the same mechanism — dispersion-diffraction stabilization.

We thank V. V. Samartsev and E. I. Yakubovich for encouragement. The work of one of us (K. V. V.) was made possible by a stipend from the Swedish Royal Academy of Sciences and was performed as part of the research program of the International Center of Fundamental Physics in Moscow. K. V. V. is also grateful to the government of St. Petersburg and Heidelberg University for financial support (Rotary Club stipend).

[1)]If a medium with an inhomogeneously broadened absorption line is studied, then the nonlinear length L_{nl} must be replaced by the absorption length $L_{ab} = 1/k_0$, which differs in that the width T^* of the absorption line replaces the pulse duration τ.

[2]The medium is considered to be dense if $L > L_{ab}$ and $N/n \leqslant 1$ hold, where N is the number of photons per unit volume and n is the same for atoms.

[3]A value close to $\varepsilon = 1$ is reached only at the exit from the absorbing medium.

[4]The fact that adequate concepts for describing all coherent spatial effects which we are discussing are not available in the literature justifies the introduction of a new term.

[5]Suppression of disturbances is possible only for sufficiently small beam radii ($\varepsilon \approx 1$), when a narrow resonant waveguide is chosen.

[6]A detailed description of the experiment is given in Ref. 11. We present below only the results that are required in our exposition.

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Translated by M. E. Alferieff