# Metric description of hadronic interaction from Bose–Einstein correlation

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We discuss the problem of second-order correlation in pion production in high-energy processes (commonly known as Bose–Einstein correlation) by means of the concept of nonlocality and its mathematical realization via the isotopy of Hilbert and Minkowski spaces. Such a nonlocal approach allows one to describe the spatial shape of the source where pions are produced ("fireball"), and to account also for the correlation in phase. The correlation function obtained by this method does not contain free "*ad hoc*" parameters. Moreover, a test of this nonlocal correlation function performed on UAI experimental data is as good as that given by the conventional treatment. Such an approach suggests an interpretation of the pion production as a decay process of the fireball whose mean lifetime can be explicitly evaluated. Using the data of the UAI ramping run, we find an expression for the metric parameters as functions of the energy. They provide an effective dynamical description of the hadronic interaction in terms of a deformation of the Minkowski metric. The related parameters of the fireball admit of future experimental verification at DELPHI. The law of deformation of time in the presence of a hadronic field is derived. Its behavior with energy allows one to give an appealing picture of confinement and asymptotic freedom of hadronic constituents.
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## 1. INTRODUCTION

The phenomenon of second-order interference in pion production from high-energy collisions[1] was widely discussed in its formulation as a Bose–Einstein (BE) correlation among identical particles[2] by borrowing concepts from interferometry in radio astronomy.[3] Previously, it was merely considered as an unexpected and unforeseen correlation in the production of pions.[4]

The so-called BE correlation was later recognized as common to widely disparate processes, such as hadronic (also involving nuclei and heavy ions) and hadro-leptonic reactions, as well as pair annihilations and $\gamma\gamma$-reactions (for an experimental as well as theoretical review, see, e.g., Ref. 5).

From a phenomenological viewpoint, the effect is that pairs of identical bosons show a higher probability of emission at small opening angles—or, equivalently, at small relative momenta—than pairs of nonidentical particles. As already mentioned above, it was first interpreted, for equally charged pions, as a manifestation of their BE statistical properties.[4] The BE correlation picture of this phenomenon originates, of course, in the quantum-mechanical interference of the wave functions of the particles and the consequent requirement of a total wave function symmetric under particle exchange.[5]

Note that at the macroscopic level (for instance, in radio astronomical interferometry), the interference—and therefore the correlation—occurs in the space near the detector. This implies a coherent source. In contrast, at the microscopic level, we have to look for correlation, i.e., interference, in the spatial region near the source, which, *a priori*, is not expected to be coherent. This is the very reason for the unpredictability of the correlation effect first observed for pions: classically, the interference (and therefore the correlation) of the detected bosons is a strict consequence of source coherence. Although the principles at the very basis of the BE interpretation are, of course, of universal validity, we think that there is a difference in their application to the microscopic case as compared to the macroscopic one.

In our opinion, the inadequacy of BE correlation in the microscopic case is due to the model of source coherence used in deducing the correlation function.

To see this, let us critically review the main lines of the classical procedure of deriving the second-order correlation function $C_{(2)}$ defined by

$$C_{(2)}=\frac{P(p_1,p_2)}{P(p_1)P(p_2)}, \tag{1.1}$$

where $P(p_1,p_2)$ is the two-particle probability density subjected to BE symmetrization and $P(p_i)$ is the corresponding single-particle quantity for a particle with four-momentum $p_i$. In practice, one often uses the simplified expression

$$C_{(2)}=\frac{P(p_1,p_2)}{P_0(p_1,p_2)}, \tag{1.2}$$

where the reference probability density $P_0(p_1,p_2)$ is essentially the same as $P(p_1,p_2)$ apart from its lack of BE symmetrization.

At this point, in the standard treatment one introduces the model of "uncorrelated emission," which is considered to be a useful way to deal with an "uncorrelated source." This leads to an identification of the two concepts. However, in our opinion, these two concepts do not coincide in general, as we shall see in the following.

Incidentally, in the usual model of uncorrelated emission quantity $P(p_1,p_2)$ loses its character of probability density (in its usual, strict quantum, meaning) and rather takes the form of an "expectation value" of the space–time distribution of the boson sources, $\rho(r_i)$ (with $r_i$ being the position four-vector of the $i$th boson). For one has

$$P(p_1,p_2)=\int |\Psi_{12}^{\mathrm{BE}}(x_1,x_2;r_1,r_2)|^2\rho(r_1)\rho(r_2)d^4r_1d^4r_2, \tag{1.3}$$

where $\Psi_{12}^{\mathrm{BE}}$ is defined as the amplitude for a boson pair to be produced at $r_1$ and $r_2$ and registered in the detector at $x_1$ and $x_2$.

The next logical step in the standard treatment of the BE correlation is the identification of lack of correlation with casuality. In other words, the uncorrelated boson source is assumed to behave like a radioactive source, which *a priori* is not justifiable at all. Due to the identification of source and emission (cf. Eq. (1.3)), the uncorrelated emission is identified with casual emission. This leads to assuming a Gaussian space–time distribution of the source:

$$\rho(r)=\frac{1}{4\pi^2R^4}\exp\left(-\frac{r^2}{2R^2}\right). \tag{1.4}$$

The final result of these hypotheses is the well-known expression for $C_{(2)}$, i.e.,

$$C_{(2)}=1+\exp(-Q_{12}^2R^2), \tag{1.5}$$

where $Q_{12}=p_1-p_2$.

Equation (1.5) obtained by the standard approach describes the interference of bosons emitted by a totally incoherent source. However, as is well known, this formula is unable to account for the experimental findings. Indeed, in order to get satisfactory agreement with the experimental data, one is forced to introduce an "*ad hoc*" parameter $\lambda$ (totally alien to the model) physically interpreted as the fraction of pairs of identical particles that appear to interfere (and are therefore correlated):

$$C_{(2)}=1+\lambda\ \exp(-Q_{12}^2R^2). \tag{1.6}$$

Note that there is no way consistent with the standard approach to avoid the artificial introduction of the "chaotic" parameter $\lambda$ (although a justification of the parameter can be provided in the framework of very sophisticated—and somewhat involved—models; see, e.g., Ref. 6 and references therein).

Apart from the above quoted $\lambda$-problem, let us recall that some recent papers cite the (partial at least) inadequacy of the standard BE theory to account for the existing experimental data.[7]

In this paper we discuss a model of BE correlation that overcomes most of the critical remarks to the standard treatment we put forward above. The model relies in an essential way on the hypothesis of a nonlocal origin of the phenomenon.

In this connection, recall that in recent years there has been renewed interest in nonlocal gauge theories[8,9] (pioneered by Efimov[10] according to a very early suggestion by Wataghin in 1934[11]). In particular, it has been shown that nonlocality makes it possible to eliminate divergences[8,10] and to get a "hidden" Higgs boson.[9]

However, in the present paper we will treat nonlocal effects by exploiting some new mathematical tools explicitly constructed in the last decade in order to deal, among other things, with nonlocal effects, i.e., the so-called Lie-isotopic generalizations of special relativity theory and quantum mechanics.[12–16] (A comprehensive and detailed exposition of Lie-isotopic theories from both physical and mathematical points of view is given in Ref. 16.)

The main lines of the Lie-isotopic approach to BE correlation have been already given by Santilli.[17] However, our physical motivations are basically different, and the very derivation of the nonlocal correlation function departs, in some respects, from that in Ref. 17. As a matter of fact, we get a different final form of the physical correlation function for fitting the experimental data. Moreover, our basic aim is to extract (preliminary) information from BE correlation about the possible description of the hadronic interaction in terms of a deformed (isotopic) Minkowski metric.

The paper is organized as follows. In Sec. 2, we first discuss the physical model of the source when nonlocal effects are taken into account, and then carry out the explicit calculation of the correlation function $C_{(2)}$ in the nonlocal hypothesis. In Sec. 3, we discuss the main physical implications of the nonlocal model of BE correlation developed in the previous sections. Section 4 contains the fits to the experimental data. In Sec. 5 we derive explicit expressions for the parameters of the effective Minkowski metric describing the (nonlocal) hadronic interaction. The law of time deformation in a hadronic field is derived in Sec. 6. Finally, concluding remarks are presented in Sec. 7. Appendices A and B contain the mathematical rudiments of the Lie-isotopic generalization of quantum mechanics and classical analogs of an incoherent source producing correlated detection, respectively.

## 2. NONLOCAL DESCRIPTION OF BOSE–EINSTEIN CORRELATION

Let us summarize the fundamental concepts whereby we analyze second-order correlation in pion (boson)[1)] production in high-energy reactions. We have a source that is in general incoherent and a correlation among the mesons produced that occurs near the source (far from the detection place). We propose to separate the internal space of the source, where pions are produced, and the space external to the source, where they are detected and their correlation observed. In essence, the internal model of the source (discussed here) is introduced in order to account for the emergence of correlation from an incoherent source, while we explicitly calculate the correlation function in the external space where the detector is placed (as we shall see in the following).

We consider the pion (boson) source to be made up of a certain number of subsources, and we will refer to it henceforth as the total source.

The subsources are incoherent too, but they produce objects that are detected as correlated. Moreover, the subsources are indistinguishable; therefore, we do not know which subsource provides the contribution of incoherence (see Appendix B for a classical description of the source).

Although there may be other sources of nonlocality in the phenomenon,[2)] we claim that this indistinguishability can be well represented as a first contribution to nonlocality. Were we able to distinguish among the various subsources, our detector would be situated inside the total source and not outside as it actually is. Putting the detector inside the source would break this nonlocality, thus recovering the locality of the phenomenon.

A second contribution to nonlocality comes from the fact that, in general, the subsources are distributed inside the source in an anisotropic way. This has two different (but related) implications.

First, in general, we cannot assume that the source is spherical. Therefore, we have, in principle, to take account of its possible deformations, whose magnitude will be provided by the experimental data. Let us then assume hereinafter that the source is not spherical (or, otherwise speaking, it lacks global rotation symmetry, in its standard meaning, in the usual Minkowski space). From a physical viewpoint, we ascribe the loss of spherical symmetry of the source to the nonlocal origin of the correlation. In other words, the presence of nonlocal effects inside the source gives rise to an asymmetric pion production that appears externally as an effective nonspherical geometry of the source.

Second, we have to take into account the consequent anisotropy of the distribution function. The most general (but simplest) way to implement such an anisotropy is to assume that the source is still distributed according to a Gaussian function, but with different parameters for each direction of four-dimensional space–time (i.e., a four-vector distribution function):

$$\rho_\mu \propto e^{-r^2/2R_\mu^2}. \tag{2.1}$$

The exact meaning of the width parameters $R_\mu$ will be clarified later on.

In order to take account of these two nonlocal effects, we introduce an (external) Lie-isotopic Minkowski space defined as (see the mathematical appendix for the definition of the isotopic ⟨⟨star⟩⟩ product $a*b$)[13]

$$\hat{\mathscr{M}}(x,\hat{\eta},\hat{\mathbb{R}}):\ x^{\hat{2}}=x*x=x^\mu\hat{\eta}_{\mu\nu}x^\nu,\quad \hat{\mathbb{R}}=\mathbb{R}\hat{I},$$

$$\eta=\hat{T}g,\quad \hat{I}=\hat{T}^{-1}, \tag{2.2}$$

where $\mathbb{R}$ is the usual field of real numbers,[3)] $g=\mathrm{diag}(1,1,1,-1)$ is the usual metric tensor of the standard Minkowski space, and the Lie-isotopic element $\hat{T}$ is given by

$$\hat{T}=\mathrm{diag}(b_1^2,b_2^2,b_3^2,b_4^2),\quad b_\mu>0, \tag{2.3}$$

so that

$$\eta=(b_1^2,b_2^2,b_3^2,-b_4^2). \tag{2.4}$$

Moreover, we recall that in this framework, the $b$-parameters are not constant, but are to be regarded, in general, as functions of the physical quantities characterizing the system or the process considered. In particular, we assume in the following that the metric parameters do depend on the energy of the process (to be understood as the phenomenological energy measured by the detectors, far from the source, i.e., in full Minkowskian conditions). Therefore, in this respect, the parameters $b_\mu$ do play a dynamical role (cf. Sec. 5). We stress that the hypothesis of the dependence of the metric parameters on the energy is essential in order to get a comparison of the theoretical predictions with the experimental data, and is a basic point of our application of the isotopic formalism we put forward in the present paper.

The spatial part of $\hat{T}$ is given by

$$\hat{\delta}=\mathrm{diag}(b_1^2,b_2^2,b_3^2) \tag{2.5}$$

and describes the (possible) spatial deformation of the total source, whereas the fourth component $b_4$ replaces the chaoticity (which thus disappears from the model) and has the meaning of the temporal correlation (i.e., phase correlation) of the bosons.

We now explain why the source deformation is expressed by a deformation of the metric. Indeed, the deformations of a physical entity (i.e., its loss of global rotation symmetry) can be looked upon from either an active or a passive point of view. In the first case (active viewpoint), the physical entity is deformed, whereas the metric of the embedding space remains unchanged. From the passive point of view, the deformation of the physical entity is induced by the deformation of (the metric of) the space itself; in other words, both the entity and the space are deformed. As is easy to see, the deformation of the metric allows one to recover the symmetry lost by the physical entity. In more rigorous terms, the rotation symmetry, broken in the standard Minkowski space, is recovered as a Lie-isotopic rotation symmetry (isorotation symmetry) in the space endowed with the Lie-isotopic metric (Refs. 18 and 19)[4)].

In other words, the mathematical formalism of the isotopy of the Minkowski metric is introduced in order to recover the rotation symmetry (broken, for the deformation of the source, in the usual Minkowski space).

Let us now consider the modifications induced by nonlocal effects on the source distribution. In general, we have to assume a nonlocal distribution function $\rho(r_1,r_2)$ (where $r_i$ ($i=1,2$) are the position four-vectors of the subsources), which, besides being anisotropic, is not factorizable as a product of distribution functions of the two subsources. However, since we identify here in the nonlocality (indistinguishability) of the subsources the origin of casuality, we can assume for the function $\rho(r_1,r_2)$ the form

$$\rho(r_1,r_2)=\rho(r_1)\rho(r_2), \tag{2.6}$$

$$\rho(r_i)=\frac{1}{4\pi^2R^4}\exp\left(-\frac{r_i^2}{2R^2}\right), \tag{2.7}$$

where the $R$ is the Gaussian parameter.

Another possible point of view on the origin of the expression (2.6) and (2.7) for $\rho$ is that the interactions responsible for boson production are themselves partially nonlocal (and therefore nonpotential) in nature. These interactions, present inside the source previous to the emission, can be assumed to average to Gaussians, whence the form (2.6), (2.7) of the nonlocal function $\rho$.[17]

In different words, at the time of the boson production (inside the total source), we have, in general, interactions which are nonlocal and nonseparable (i.e., not factorizable into individual terms each one depending on the individual particle).[17]

After the completion of processes in the interior of the source and boson production, all interactions can be effectively approximated as being local and separable. However, the subsources cannot be regarded as pointlike, and this leads classically to Gaussian structure of the emission (already formally contained in the expression (2.6) and (2.7) for $\rho$).

In order to take into account the anisotropy of the metric, we have to assume, in general, that there are four distribution functions (of Gaussian type) different for each dimension. Equation (2.7) must be therefore replaced (cf. Eq. (2.1)) by

$$\rho_\mu(r)=\frac{1}{4\pi R_\mu^4}\exp\left(-\frac{r^{\hat{2}}}{2R_\mu^2}\right), \tag{2.8}$$

in which, for consistency, the usual square of the four-vector $r$ has been replaced by the Lie-isotopic square $r^{\hat{2}}=r*r$.

The second step to get a fully nonlocal distribution function is related to the change of meaning of the parameters $R_\mu$, which up to now have preserved the same "local" meaning as in Eq. (2.7), i.e., essentially the axes of a sphere. In the nonlocal case, the spatial parameters $R_k$ describe the sphere deformation, and, therefore, in general, must be regarded as the axes of an ellipsoid. This amounts to say that we have to put $R_k=a_k^{-1}$; generalizing this result to the time parameter, we have therefore to set

$$R_\mu=a_\mu^{-1}, \tag{2.9}$$

where the $a_\mu$ are the physical parameters describing the space region of the total source in which the subsources are distributed. It is easy to see that they are related to the parameters of the metric $\eta$ given by Eq. (2.4) of the isotopic Minkowski space by

$$a_k=\hbar c b_k, \tag{2.10a}$$

$$a_4=\hbar b_4. \tag{2.10b}$$

In the following, we use the standard convention $\hbar=c=1$, so that $a_\mu\equiv b_\mu$. The explicit distinction between the Gaussian parameters and the metric parameters will be exploited only in some special cases, in which we want to stress their different physical origin.

The above equations accomplish the link between the notion of nonlocality and its implementation by means of the metric deformation. Then, the vector distribution function becomes

$$\hat{\rho}_\mu(r)=\frac{1}{4\pi}a_\mu^4\exp\left(-\frac{r^{\hat{2}}a_\mu^2}{2}\right). \tag{2.11}$$

As a final step in the nonlocal description of BE correlation, we note that the standard expression of the two-particle probability density, Eq. (1.3), does possess naturally the structure of an isotopic scalar product. Indeed, putting

$$G=\hat{\rho}(r_1)\hat{\rho}(r_2) \tag{2.12}$$

(cf. Eq. (A9)), we can write the probability of getting two correlated pions with momenta $p_1$ and $p_2$, produced at $r_1$ and $r_2$ and detected at $x_1$ and $x_2$, as

$$\hat{P}(p_1,p_2)=\int d^4r_1d^4r_2\hat{\psi}^+{}_{12}(x_1,x_2;r_1,r_2),$$

$$\hat{\psi}_{12}(x_1,x_2;r_1,r_2)\hat{\rho}(r_1)\hat{\rho}(r_2)=(\hat{\psi}_{12}|G|\hat{\psi}_{12}). \tag{2.13}$$

Equation (2.13) represents the inner product of a Lie-isotopic Hilbert space $\hat{H}$. Henceforth, the wavefunctions with a hat, $\hat{\psi}$, are therefore to be regarded as vectors in $\hat{H}$. Moreover, $\hat{\rho}(r_i)$ is the anisotropic expression of the Gaussian source function.

Let us now introduce the Lie-isotopic correlation function

$$\hat{C}_{(2)}=\frac{\hat{P}(p_1,p_2)}{\hat{P}(p_1)\hat{P}(p_2)}, \tag{2.14}$$

where the hat means that the probabilities are to be evaluated in $\hat{H}$, according to Eq. (2.13).

The Lie-isotopic boson state (i.e., the symmetric Lie-isotopic wavefunction) is defined as[17]

$$\begin{aligned}\hat{\psi}_{12}(x_1,x_2;r_1,r_2)\equiv\frac{1}{\sqrt{2}}\{&\exp[ip_1*(x_1-r_1)]\\ &\times\exp[ip_2*(x_2-r_2)]\\ &+\exp[ip_1*(x_1-r_2)]\\ &\times\exp[ip_2*(x_2-r_1)]\},\end{aligned} \tag{2.15}$$

where the star product is to be considered in the isotopic Minkowski space (2.2). Replacing Eq. (2.8) in (2.13), on account of Eq. (2.15), we get

$$\hat{C}_{(2)}=1+|F_\mu|^2, \tag{2.16}$$

where

$$F_\mu=\int\exp(iQ*x)\hat{\rho}_\mu(x)d^4x. \tag{2.17}$$

If we want now to average over all subsources by also taking into account the anisotropy of their spatial distribution, we have to average over all space–time directions. In other words, we replace the average over the (unknown) distribution of subsources with an average over all dimensions that accounts also for their distribution in phase. This is easily accomplished by assuming that the squared modulus in Eq. (2.16) is an isotopic norm, i.e.,

$$\hat{C}_{(2)}=1+|F_\mu|^{\hat{2}}=1+F_\mu^*\eta^{\mu\nu}F_\nu. \tag{2.18}$$

Then, by repeating the various manipulations and integrations as in the conventional case,[5] we reach the following expression of the isotopic correlation function:[17]

$$\hat{C}_{(2)}=1+b_1^2\exp\left(-\frac{Q^{\hat{2}}}{a_1^2}\right)+b_2^2\exp\left(-\frac{Q^{\hat{2}}}{a_2^2}\right)+b_3^2\exp\left(-\frac{Q^{\hat{2}}}{a_3^2}\right)-b_4^2\exp\left(-\frac{Q^{\hat{2}}}{a_4^2}\right), \quad (2.19)$$

where

$$Q=p_1-p_2=(\mathbf{q},q_0). \quad (2.20)$$

Note that the terms in the exponents, given explicitly by

$$\frac{Q^{\hat{2}}}{a_\mu^2}=\frac{1}{a_\mu^2}(Q_1^2b_1^2+Q_2^2b_2^2+Q_3^2b_3^2-Q_4^2b_4^2), \quad (2.21)$$

are insensitive to any scalar renormalization of the metric; specifically they are invariant under the scale transformation

$$x^{\hat{2}}\Rightarrow Nx^{\hat{2}}=x^+\eta' x, \quad \eta'=N\eta, \quad N\in\mathrm{R}, \quad (2.22)$$

evidently because of the isotopic element $b_\mu^2$ in the denominator.

Equation (2.19) can be also rewritten in compact form, i.e.,

$$\hat{C}_{(2)}=1+\sum_{\mu=1}^{4}\eta_{\mu\mu}\exp\left(-\frac{Q^{\hat{2}}}{a_\mu^2}\right). \quad (2.23)$$

Notice that the above results are essentially exact with the sole approximation used in the calculation of the isotopic correlation function being that pertaining to the extended character of the sources of the individual particles and the selection of the Gaussian form for their spatial distribution. From now on, however, some approximations have to be made in order to reach an expression for $\hat{C}_{(2)}$ which is suitable for experimental verification.

## 3. NONLOCAL EXPRESSION OF THE CORRELATION FUNCTION

In order to compare the isotopic correlation function $\hat{C}_{(2)}$ derived from the nonlocal model with the experimental data, we need to express $Q^{\hat{2}}$ in terms of the quantities actually observed.

To this end, let us address some physical considerations aimed also to clarify some main points of the formalism we exploited.

It is easily seen from the above discussion that the spatial parameters $b_k$ of the metrics of the isotopic Minkowski space describe the possible spatial deformation of the source arising from interior nonlocal effects, whereas the time parameter $b_4$ represents the temporal deformation (i.e., the correlation in phase).

Let us assume that (as is indeed the case) spatial deformation of the source does actually occur. Then it is expected that the spherical geometry of the source (corresponding to an isotropic probability of pion production) is changed into an ellipsoid of rotation, in which the length of one of the semiaxes is much greater than the others.

Moreover, in general, the source volume is not preserved by the deformation process. Therefore, while before the deformation the source can be considered a spherical "fireball" of unit radius,

$$R^2=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1, \quad (3.1)$$

after the deformation we have[17]

$$\hat{R}^2=b_1^2+b_2^2+b_3^2=b^2\neq 1 \quad (3.2)$$

(we recall that $b_k=a_k/\hbar c$).

Obviously, the parameter $b$ provides a measure of the amount of deformation. Furthermore, since both the spatial deformation of the source and the appearance of the time parameter $b_4$ are to be ascribed to the same nonlocal effects, we expect on physical grounds that

$$b=\sqrt{b_1^2+b_2^2+b_3^2}\simeq b_4. \quad (3.3)$$

In particular, if one of the $b_k$, say, $b_2$, differs from $1/\sqrt{3}$ much more than the other two spatial parameters, it is clearly

$$b_4\simeq b_2\simeq b. \quad (3.4)$$

In other words, the $b$-number can be considered to be the "nonlocality parameter."

These relations can be interpreted as expressing a "towing effect"[5)] of the total deformation along one of the spatial dimensions. In general, we cannot say *a priori* which among the $b_k$ differs most from $1/\sqrt{3}$. Clearly, this depends on the phenomenon considered and, in this sense, our approach can be used for phenomena different from BE correlation. We can also define a "loss-of-spherical-symmetry" parameter $n_s$ as follows:

$$n_s\equiv\left|\frac{1}{\sqrt{3}}-b\right|. \quad (3.5)$$

We are now ready to express the correlation function $\hat{C}_{(2)}$ in terms of the observed variables. To this end, let us first of all adopt the Goldhaber convention, i.e.,

$$Q^2=q_t^2+q_l^2-q_0^2, \quad (3.6)$$

where $\mathbf{q}_t$ and $\mathbf{q}_l$ are, respectively, the components of $\mathbf{q}$ transverse and parallel to the vector $\mathbf{n}=\mathbf{P}/P$ $(\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2)$. Standard kinematic considerations yield $q_l\simeq q_0$, and therefore $Q^2\simeq q_t^2$.

Since the pion momentum is that observed by the detector, and therefore outside the nonlocality region (far from the total source), we can assume in the first approximation that the two-pion system remembers the nonlocal effects that produced it. Let us further assume that this memory is represented by a change in the modulus of the vector $Q^2$ proportional to the parameter $b$ that measures the amount of deformation (both spatial and temporal) of the metric (see Eqs. (3.2)–(3.4)), namely

$$Q^{\hat{2}}\simeq b^2Q^2=b^2(q_t^2+q_l^2-q_0^2). \quad (3.7)$$

Then we get

$$Q^{\hat{2}}\simeq b^2q_t^2\simeq b_4^2q_t^2, \quad (3.8)$$

where Eq. (4.4) has been taken into account in the last step.

The correlation function takes now the form

$$\hat{C}_{(2)}=1+b_4^2\left[b_1'^2 \exp\left(-\frac{q_t^2}{b_1'^2}\right)+b_2'^2 \exp\left(-\frac{q_t^2}{b_2'^2}\right)\right.$$
$$\left.+b_3'^2 \exp\left(-\frac{q_t^2}{b_3'^2}\right)-\exp(-q_t^2)\right], \tag{3.9}$$

where we put

$$b_k'=\frac{b_k}{b_4} \quad (k=1,2,3). \tag{3.10}$$

Let us explicitly stress that the above approximation leading from Eq. (2.19) to Eq. (3.9) does not represent a limitation of the model, but arises from the need for comparison with the experimental data on the basis of the experimental knowledge of the single kinematic parameter $q_t$.

Let us now discuss the limit of Eq. (3.9) in the conventional Minkowski space. In this case the values of the parameters $b_\mu$ are of the order of $1/\sqrt{3}$ (see Eq. (3.1)), and the metric $\eta_{\mu\nu}$ is replaced by the usual Minkowski metric $g_{\mu\nu}$, so that one gets[17]

$$C_{(2)}=1+\sum_{\mu=1}^{4} g_{\mu\mu}(1/3)\exp(-3q_t^2). \tag{3.11}$$

The above expression must be considered a local generalization of the standard correlation function obtained via an averaging process over all space–time directions. An interesting feature of the generalized formula (3.11) for $C_{(2)}$ is that it yields as a maximum value[17]

$$C_{(2)}^{\max}=1+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}-\frac{1}{3}=1.67, \tag{3.12}$$

which is verified by all available experimental data (see, e.g., those in Ref. 20). In this connection, we recall that the conventional expression for $C_{(2)}$, Eq. (1.5), gives as a maximum value $C_{(2)}^{\max}=2$ (which is lowered by hand by introducing the chaoticity parameter $\lambda$).

Let us explicitly stress, therefore, that taking the local limit of $\hat{C}_{(2)}$ for $q_t \to 0$ is not the mere operation of letting the variable go to zero: one first has to change the nonlocal function in a nonlocal metric into a local function in a local metric, and then let the variable going to zero.

From the above discussion, it follows, incidentally, that an *a posteriori* (but nevertheless valid) motivation for introducing nonlocality is that this is the only way which enables us to use different parameters for different dimensions in the Gaussian distributions. Otherwise, the correlation function we obtained would have no basis at all, and could be considered naively as a trivial attempt to fit the experimental data with the least possible number of different Gaussians (cf. Sec. 2).

Indeed, in the local limit of $\hat{C}_{(2)}$ (obtained by the recipe given above), all the Gaussians must be considered with identical parameters ($b_\mu^2=1/3$) (due to the space–time isotropy), and one cannot use different parameters for different dimensions in order to get a sum of different Gaussians. In

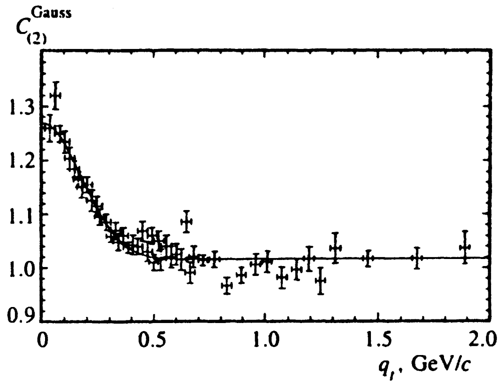


FIG. 1. Fit to the UAI experimental data obtained by the usual Gaussian correlation function $C_{(2)}^{\text{Gauss}}$ (Eq. (4.1)); $\Delta q_t=0.022$ GeV/$c$.

our opinion, this is perhaps the deepest reason that led us to consider a nonlocal model of BE correlation.

## 4. FIT OF THE UA1 DATA

The experimental data used to test the nonlocal correlation function $\hat{C}_{(2)}$ are those of the minimum bias of UA1 with energy $\sqrt{s}=630$ GeV, and $q_l$ and $q_t$ ranging over the intervals $0\leqslant q_l<0.2$ GeV/$c$, $0.02\leqslant q_t<2$ GeV/$c$.[14] The correlation function was derived by considering the ratio (like same-events/like mix-events).[20]

Figure 1 shows the fit of these data obtained by the usual Gaussian correlation function $C_{(2)}$ (see Eq. (1.6)) which we rewrite here for the reader's convenience:

$$C_{(2)}^{\text{Gauss}}=\gamma[1+\lambda e^{-\beta q_t^2}], \tag{4.1}$$

with $\gamma$ being a normalization factor, $\lambda$ the chaoticity parameter, and $R=\hbar c\sqrt{\beta}$, the fireball radius. The fit performed by Eq. (4.1) gives a reduced chi-square $\chi_n^2=1.39$, and the correlation coefficients show that the three fit parameters, $\gamma$, $\lambda$, and $\beta$, are not correlated (as expected).

In Fig. 2 we plot the fit obtained by the nonlocal correlation function $\hat{C}_{(2)}$ (see Eq. (3.9)) given by

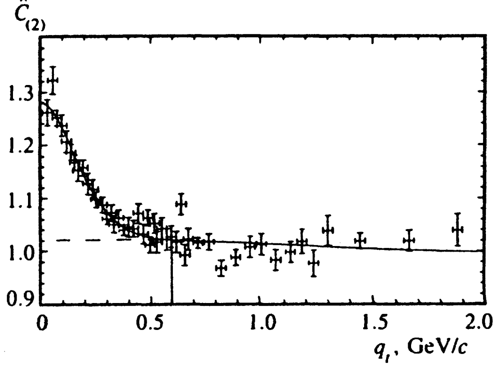


FIG. 2. Fit to the UAI experimental data obtained by the nonlocal correlation function $\hat{C}_{(2)}$ (Eq. (4.2)). The dashed horizontal line shows the estimated value of the "width" $\Gamma$ of the fireball (see Eq. (4.7) and the text); $\Delta q_t=0.022$ GeV/$c$.

TABLE I. Values of the metric parameters $b_\mu$ derived by the fit of $\hat{C}_{(2)}$ to the UAI data.

| | |
|---|---|
| $b_1$ | 0.267±0.054 |
| $b_2$ | 0.437±0.035 |
| $b_3$ | 1.661±0.013 |
| $b_4$ | 1.653±0.015 |

TABLE II. Values of the physical parameters $a_\mu$ of the fireball ($a_k=\hbar c b_k$, $a_4=\hbar b_4$).

| | |
|---|---|
| $a_1$ | $(0.053\pm0.011)\cdot10^{-13}$ cm |
| $a_2$ | $(0.086\pm0.007)\cdot10^{-13}$ cm |
| $a_3$ | $(0.328\pm0.003)\cdot10^{-13}$ cm |
| $a_4$ | $(1.09\ \pm0.01)\cdot10^{-24}$ s |

$$\hat{C}_{(2)}=\gamma\left\{1+b_4^2\left[b_1'^2\exp\left(-\frac{q_t^2}{b_1'^2}\right)+b_2'^2\exp\left(-\frac{q_t^2}{b_2'^2}\right)+b_3'^2\exp\left(-\frac{q_t^2}{b_3'^2}\right)-\exp(-q_t^2)\right]\right\}, \quad (4.2)$$

where $\gamma$ is again a normalization factor and $b_k'=b_k/b_4$ (cf. Eq. (3.9)).

The fit performed by Eq. (4.2) gives the reduced chi-square $\chi_n^2=1.22$, which is comparable that provided by Eq. (4.1).[6)]

The correlation coefficients among the fit parameters show that $\gamma$ is not correlated with any of the other parameters, as expected, and that $b_4$ is correlated with the $b_k'$, as expected.

The parameters of the nonlocal metric obtained by the fit are given in Table I. Using these values we get the following limit of $\hat{C}_{(2)}$ for $q_t\to0$:

$$\hat{C}_{(2)}(0)=1.28\pm0.15<\hat{C}_{(2)}^{\max}=1.67, \quad (4.3)$$

which does satisfy the constraint given by Eq. (3.12).[17]

Let us now check the consistency of the physical assumptions expressed by Eq. (3.3):

$$\sqrt{b_1^2+b_2^2+b_3^2}=1.738\pm0.029. \quad (4.4)$$

Because $b_4=1.653\pm0.015$, and on account of the $b_\mu$ values given in Table I, we can state that the hypothesis reported as an example in Eq. (3.4) is indeed satisfied by parameter $b_3=1.661\pm0.013$. Therefore, we have

$$b_3\simeq b_4\simeq\sqrt{b_1^2+b_2^2+b_3^2}. \quad (4.5)$$

Actually $b_3$ is also the spatial parameter with the greatest percentage difference from $1/\sqrt{3}$, so it controls the towing effect discussed in Sec. 3.

The values of the physical parameters $a_\mu$ of the fireball (total source) are given in Table II. Of course, the parameters $a_\mu$ yield the spatial shape of the fireball. A basic point to be stressed is that our analysis of the experimental data show a loss of spatial axial symmetry of the BE phenomenon.

It is worth noting that axial symmetry is usually assumed in any analysis of BE correlation simply because there is *a priori* no apparent physical reason which justifies the breakdown of axial symmetry in pion production. Therefore, if confirmed by further analyses, such a result could be a strict signature of the role of nonlocal effects in the phenomenon of BE correlation. Clearly, such an effect is related to the dependence of the metric parameters on the energy of the process (see Sec. 2 and next section).

Let us now discuss the physical meaning of the time parameter $a_4$. It represents the "extension" of the fireball along the time axis, and therefore it is easily seen that $a_4$ can be regarded as the mean lifetime of the fireball:

$$a_4=\tau=(1.09\pm0.01)\cdot10^{-24}\ \text{s}. \quad (4.6)$$

Following this interpretation, we can also define a kind of "width" $\Gamma$ of the fireball given by

$$\Gamma=\frac{\hbar}{a_4}=0.61\pm0.02\ \text{GeV}. \quad (4.7)$$

Moreover, since for particles with rest mass much lower than their energy, the energy can be identified with momentum, we may consider $\Gamma$ as that value of $q_t$ (i.e., our momentum variable, according to the adopted convention) for which we have $C_{(2)}>1$ if $q_t<\Gamma$ and $C_{(2)}\simeq1$ if $q_t>\Gamma$. As shown in Fig. 2 (see the dashed line), this interpretation of $\Gamma$ seems to be confirmed by the experimental data.

Eventually, the parameter $\tau$ may be regarded as the largest time interval separating the production of two correlated pions (bosons). In this sense, $c\tau$ is the maximum correlation length, i.e., the maximum distance at which two correlated pions (bosons) can be produced. In other words, $c\tau$ may be considered the maximum size of a subsource (or of a total source containing just a single subsource of size $c\tau$). Indeed, it is

$$c\tau=(0.330\pm0.045)\cdot10^{-13}\ \text{cm} \quad (4.8)$$

and from Table II we have $c\tau\simeq a_3$.

Let us end this section with some physical considerations. First, the presence of the parameters $b_\mu$ in the correlation function is a direct consequence of the nonlocal treatment. However, had the values of the $b_k$ (derived by the fit) been such as to point to an isotropic distribution of the subsources,[7)] it would be failed a necessary consequence of introducing the isotopic Hilbert space, which—as by now familiar—makes it possible to explain the correlation in an incoherent source. But we have checked by the fit that there is indeed anisotropy, and therefore this result constitutes a consistency test of our approach.

Second, we note that the hypothesis of subsources can also be introduced in the usual model. In this framework the shape of the correlation function for values larger than unity is interpreted as the envelope of a large number of different correlation functions (each unity corresponding to a single subsource), with different chaoticity parameters and different radii. However, this interpretation has found, up to now, no experimental evidence due essentially to the low resolution in the momentum variable $q_t$. Moreover, there is a basic objection to this procedure; indeed, the resulting correlation function is given by the sum of the correlation functions of the individual subsources, without any recipe to decide where the sum has to stop. Finally, this model implies a

TABLE III. Values of the metric parameters $b_\mu$ as energy functions.

| $E$, GeV | $b_1^2$ | $b_2^2$ | $b_3^2$ | $b_4^2$ |
|---|---|---|---|---|
| 200 | 0.054±0.02 | 0.142±0.04 | 1.150±0.195 | 0.083±0.168 |
| 260 | 0.052±0.02 | 0.0864±0.03 | 1.066±0.186 | 0.942±0.174 |
| 380 | 0.053±0.02 | 0.154±0.05 | 1.165±0.195 | 0.970±0.177 |
| 500 | 0.056±0.02 | 0.161±0.05 | 1.717±0.156 | 2.017±0.183 |
| 620 | 0.072±0.02 | 0.198±0.06 | 3.043±0.273 | 2.604±0.234 |
| 630 | 0.071±0.02 | 0.191±0.06 | 2.759±0.249 | 2.732±0.246 |

strictly isotropic distribution of the subsources, in order to be entitled to sum their contributions. On the contrary, the nonlocality requirement—which is independent of the fireball model—automatically considers the possible anisotropy of the subsources, which is accounted for by the average over space–time directions.

## 5. METRIC DESCRIPTION OF HADRONIC INTERACTION

We now show that it is possible, from the experimental data on BE correlation and the formalism discussed in the previous sections, to attempt an effective description of the hadronic interaction in terms of the parameters $b_\mu$ of the deformed Minkowski metric, Eq. (2.4).

To this end, we make the basic assumption (as already stressed in Sec. 2) that the parameters $b_\mu$ depend on the energy $E$ of the interaction process (i.e., the phenomenological energy measured by the detectors far from the source, namely in purely Minkowskian conditions):

$$b_\mu \equiv b_\mu(E). \tag{5.1}$$

Under such a hypothesis, we applied the results of Secs. 3 and 4 to the experimental data taken by the UA1 detector in the "ramping run" of 1984 and obtained for $b_\mu$ as functions of $E=\sqrt{s}$ the results given in Table III.

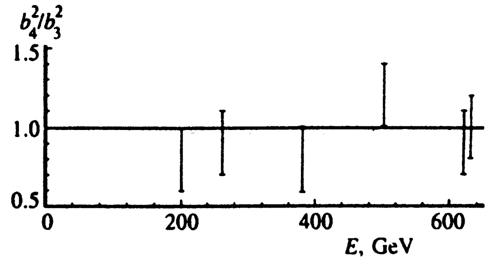


FIG. 4. Ratio of the hadronic metric parameters $b_4^2/b_3^2$. See text.

For each value of the energy, the parameters $b_\mu^2$ and the correlation function $\hat{C}_{(2)}$ satisfy the constraints expressed by Eqs. (3.3), (3.4), and (3.12). The behavior of the four parameters versus energy is plotted in Fig. 3.

The results of Table III clearly show that BE correlation is spatially anisotropic over the whole energy range 200–630 GeV.

Therefore, in spite of the phenomenological value of the nonlocal correlation function $\hat{C}_{(2)}$ given by Eq. (4.2) and the results of the previous section, our analysis yields strong evidence that hadronic interaction can be treated (according to the values of Table III) neither in terms of a metric with constant parameters nor even in terms of the (standard) Minkowski metric.

By interpolating the data of Table III, we obtain that the explicit form of the effective isotropic metric $\eta$ which describes the hadronic interaction as a function of $E=\sqrt{s}$ reads

$$\eta=\mathrm{diag}(b_1^2,b_2^2,b_3^2,-b_4^2), \quad b_1^2=(\sqrt{2}/5)^2, \quad b_2^2=(2/5)^2,$$

$$b_4^2=b_3^2=\begin{cases} 1, & 0\leq E\leq E_0=367.5\pm0.4\ \mathrm{GeV}, \\ (E/E_0)^2, & E_0\leq E. \end{cases} \tag{5.2}$$

Here $E_0$ must be understood as the energy value for which the metric becomes Minkowskian (i.e., $\eta(E_0)=g$).

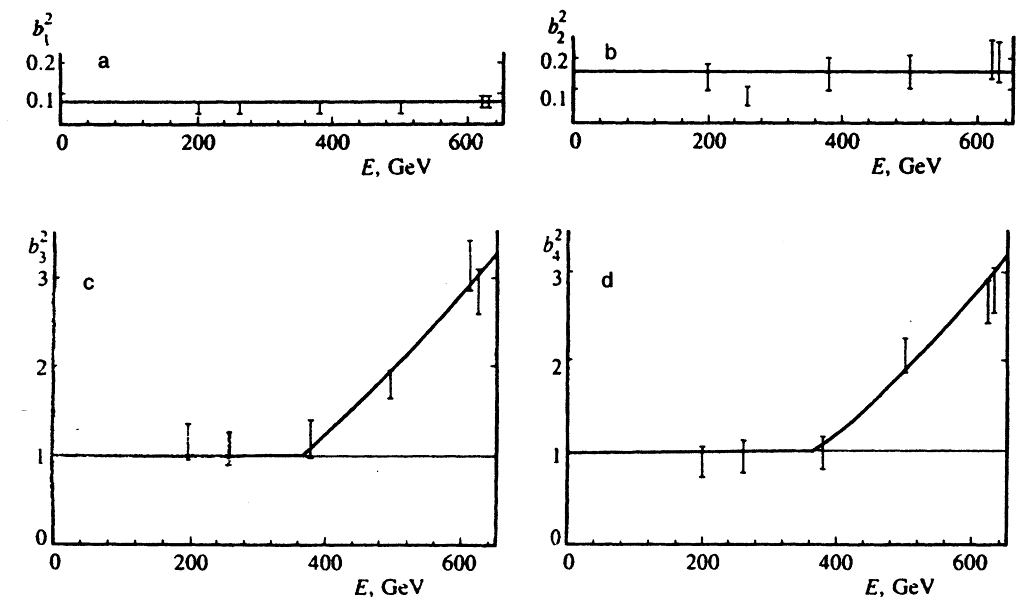


FIG. 3. Plots of the hadronic metric parameters versus energy: a) $b_1^2(E)=(\sqrt{2}/5)^2$; b) $b_2^2(E)=(2/5)^2$; c, d) $b_{3,4}^2(E)=(E/E_0)^2$, $E_0=367.5$ GeV and $b_{3,4}^2(E)=1$, $E<E_0$.

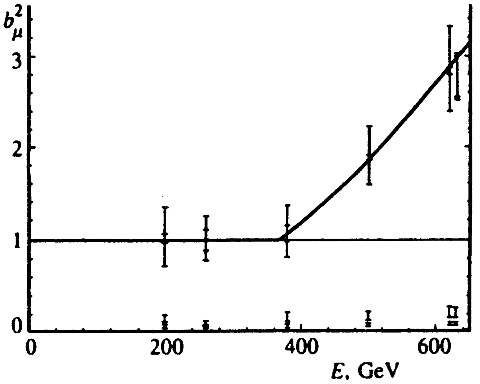


FIG. 5. Showing the behavior with energy of all four hadronic metric parameters $b_\mu^2$. See text and Eq. (5.2).

The ratio $b_4^2/b_3^2$ is plotted in Fig. 4 whence it immediately follows that we can put $b_3^2=b_4^2$ (see Eq. (5.2)). The behavior of all four parameters $b_\mu^2(E)$ of the hadronic metric is given, for comparison, in Fig. 5, which clearly illustrates the spatial anisotropy of the metric (and therefore of the fireball). Equation (5.2) allows one to rewrite Eq. (3.10) in the form

$$b_1'^2=\frac{b_1^2}{b_4^2}\begin{cases}(\sqrt{2}/5)^2, & 0\leq E_0\leq E,\\ (\sqrt{2}/5)^2(E_0/E)^2, & E_0\leq E,\end{cases}$$

$$b_2'^2=\frac{b_2^2}{b_4^2}\begin{cases}(2/5)^2, & 0\leq E\leq E_0,\\ (2/5)^2(E_0/E)^2, & E_0\leq E,\end{cases}$$

$$b_3'^2=b_3^2/b_4^2=1. \tag{5.3}$$

It follows from the previous relations that the nonlocal correlation function $\hat{C}_{(2)}$, Eq. (4.2), can be written as

$$\hat{C}_{(2)}=\gamma\left[1+b_1^2\exp\left(-\frac{q_t^2}{b_1'^2}\right)+b_2^2\exp\left(-\frac{q_t^2}{b_2'^2}\right)\right], \tag{5.4}$$

or

$$\hat{C}_{(2)}=\gamma\begin{cases}\left\{1+\left(\frac{\sqrt{2}}{5}\right)^2\exp\left[-q_t^2\left(\frac{5}{\sqrt{2}}\right)^2\right]+\left(\frac{2}{5}\right)^2\times\exp\left[-q_t^2\left(\frac{5}{2}\right)^2\right]\right\}, & 0\leq E\leq E_0, \quad (5.5)\\ \left\{1+\left(\frac{\sqrt{2}}{5}\right)^2\exp\left[-q_t^2\left(\frac{5}{\sqrt{2}}\right)\left(\frac{E}{E_0}\right)^2\right]+\left(\frac{2}{5}\right)^2\times\exp\left[-q_t^2\left(\frac{5}{2}\right)^2\left(\frac{E}{E_0}\right)^2\right]\right\}, & E_0\leq E, \quad (5.6)\end{cases}$$

TABLE IV. Values of the fireball physical parameters as energy functions.

| | |
|---|---|
| $a_1$ | $0.06\cdot 10^{-13}$ cm |
| $a_2$ | $0.08\cdot 10^{-13}$ cm |
| $a_3$ | $0.20\cdot 10^{-13}$ cm, $0\leq E\leq E_0$ |
| | $0.20\cdot 10^{-13}(E/E_0)$ cm, $E_0\leq E$ |
| $a_4$ | $0.67\cdot 10^{-24}$ s, $0\leq E\leq E_0$ |
| | $0.67\cdot 10^{-24}(E/E_0)$ s, $E_0\leq E$ |
| $\Gamma$ | $0.93\pm 0.17$ GeV, $0\leq E\leq E_0$ |
| | $E/E_0$ GeV, $E_0\leq E$ |

where $E=\sqrt{s}$. Relation (5.5) admits of experimental verification by future measurements on BE correlation with DELPHI when the LEP 2 phase starts in 1995/96. Indeed, the DELPHI collaboration aims to take data at four or five different energies in the range $\sqrt{s}=80-180$ GeV.[8] But the same measurements, it will be also possible to check the physical parameters of the fireball given by $a_k=\hbar c b_k$ and $a_4=\hbar b_4$ (see Eqs. (2.10)), and its width $\Gamma=\hbar/a_4$. The values of $a_\mu$ and $\Gamma$ are summarized as functions of energy in Table IV.

Obviously, in the energy range explored by LEP 2, the values of the fireball physical parameters which admit of experimental verification are those given in Table V.

## 6. HADRONIC TIME DEFORMATION

The metric given by Eqs. (5.2), derived in the previous section from the UAI experimental data, is not always isochronous with the usual Minkowski metric ($b_4^2=1$) (which, of course, characterizes the electromagnetic interaction). Actually, it follows from Eqs. (5.2) that it is $b_4^2\neq 1$ for $E_0<E$.

Such a case is not new; indeed, as is well known, the same happens for the gravitational interaction, as shown, e.g., by the various measurements of red or blue shifts of electromagnetic radiation in a gravitational field, or by the relative delays of atomic clocks put at different heights in the presence of gravity.

Let us investigate the possible implications of such an anisochronism of the hadronic metric. We denote by $dt$ the time interval taken by a certain hadronic process for a particle at rest ("hadronic clock"). The same process, when referred to a Minkowskian electromagnetic metric, will take a time $\Delta t_{\rm el}$ to happen. Adopting methods and notation from the general theory of relativity, we can state that for a particle at rest,

$$\frac{dt_{\rm had}}{\Delta t_{\rm el}}=(-g_{00})^{1/2}. \tag{6.1}$$

Since for the isotopic metric given by Eqs. (5.2) it is $g_{00}=-b_4^2(E)$, we get

$$\frac{dt_{\rm had}}{\Delta t_{\rm el}}=\begin{cases}1 & 0\leq E\leq E_0=367.5\pm 0.4\ \text{GeV},\\ E_0/E, & E_0\pm E.\end{cases} \tag{6.2}$$

Equation (6.2) provides the law of time deformation in a hadronic field. Figure 6 shows the behavior of law in Eq. (6.2), i.e., the plot of $dt_{\rm had}$ versus the energy in units of $\Delta t_{\rm el}$. It is easily seen that there is isochronism at low energies (i.e., physical processes have the same rate whether referred to a hadronic metric or an electromagnetic one), whereas there is

TABLE V. Predicted values of the fireball physical parameters in the range $\sqrt{s}=80-180$ GeV.

| | |
|---|---|
| $a_1$ | $0.06\cdot 10^{-13}$ cm |
| $a_2$ | $0.08\cdot 10^{-13}$ cm |
| $a_3$ | $0.20\cdot 10^{-13}$ cm |
| $a_4$ | $0.67\cdot 10^{-24}$ s |
| $\Gamma$ | $0.93\pm 0.17$ GeV |

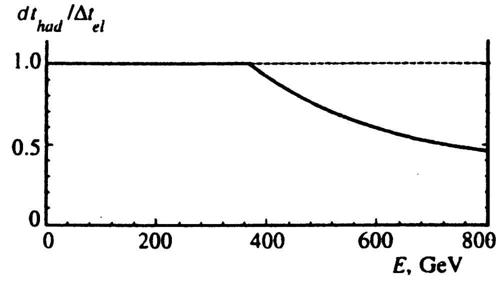


FIG. 6. Time deformation law in a hadronic field versus the energy. See text and Eq. (6.2).

a time contraction at high energies. In other words, hadronic processes are faster when observed with respect to an electromagnetic metric.

Such results provide an interesting representation of two fundamental features of strong interactions, i.e., asymptotic freedom and confinement. We recall that in deep inelastic scattering, where such properties are observed, the probe particles interact electromagnetically with the hadron constituents. When low-energy probe particles are involved—i.e., energy exchange occurs between the probe leptons (which undergo scattering) and the hadronic constituents (scattering centers) at energy values low with respect to the energy scale of hadronic interactions—hadronic constituents behave essentially as free particles, that is, they are "asymptotically free". On the basis of Eq. (6.2), we can interpret such a fact in terms of equal time intervals for both electromagnetic and hadronic interactions, during which the same amount of energy is exchanged. In other words, both electromagnetic and hadronic processes require the same time interval in order that particles exchange the same amount of energy. Therefore, in exchanging energy at such "low" values, hadron constituents behave exactly like the electromagnetic probes which, when scattered, do not keep any trace of the "bond" due to the hadronic interaction.

In contrast, with increasing energy exchange between electromagnetic probes and hadronic constituents, Eq. (6.2) shows that different time intervals are required for electromagnetic and hadronic interactions at the same energy levels in a given process. Specifically, it is seen from Eq. (6.2) that, energies being equal, hadronic processes require a shorter time interval to occur than do electromagnetic ones. When the energy of the process increases, Eq. (6.2) shows that the time interval needed for hadronic interactions falls off according to a hyperbolic law with respect to the time required for electromagnetic interactions at the same energy. Indeed, in energy exchange at "high" values, hadronic constituents are seen as bound particles by the electromagnetic probes, which show a "bond" with intensity greater than that produced by an electromagnetic interaction. Therefore, at increasing energy exchange, the hadronic constituents appear more and more bound—that is, "confined". Thus the confinement of hadronic constituents finds a natural (qualitative, at least) interpretation on the basis of Eq. (6.2).

Due to the time deformation law, Eq. (6.2), therefore, a system built up by hadronic bonds requires, in order to exchange energy among its constituents, a time interval which, with increasing energy, is still smaller than the time needed to supply energy to the system via the electromagnetic interaction. Thus, the greater the supplied energy, the faster the bond responds to the solicitation.

Finally, we want to spend some words on the problem of isolating hadronic constituents, still in light of Eq. (6.2).

In our opinion, it is wrong to look at such a problem from the point of view of finding the energy value at which the system becomes unstable, so that its constituents might be isolated as particles "free" from the hadron system.

Apart from the question of whether such an energy could be supplied to the system by means of electromagnetic or hadronic probe particles, the true problem is that we are just trying to get the hadron constituents to move in a "space" whose (Minkowskian) metric is not their own (hadronic) metric.

Thus, we deem that the proper way to put correctly the problem of the isolation of hadronic constituents is as follows: getting an object to move in a space whose metric is not its own, and studying its motion. A possible analogy is provided by the motion of a real (electromagnetic) photon in a space endowed with a gravitational metric, or by the motion of a virtual (electromagnetic) photon in a space endowed with a hadronic metric.

## 7. CONCLUSIONS

In this paper we have discussed the problem of second-order BE correlation on the hypothesis that nonlocal effects do play a role in the phenomenon, and that they admit of an effective description in terms of a deformation (isotopy) of the Minkowski and Hilbert spaces.

The correlation function obtained by this approach (first considered in Ref. 17) does not contain free "*ad hoc*" parameters.

Indeed, the four parameters $b_\mu$ entering into its expression are nothing but the coefficients of the deformed Minkowski metric, and, as such, are a strict consequence of the theoretical formalism. Moreover, they admit of a clear physical interpretation. The spatial parameters are related to the shape of the source (i.e., the interaction region); the time parameter describes the correlation in phase, and is related to the mean lifetime of the source. Therefore, such a nonlocal approach suggests an interpretation of the pion production as a decay process of the fireball.

We carried out a fit to the UAI data, whose reduced $\chi$-square is comparable to that obtained by a correlation function with Gaussian source. The values of the physical parameters $a_k$ (cf. Eq. (2.10a)) obtained by the fit yield, for the first time, detailed information about the actual shape of the interaction region. The "width" $\Gamma$ of the fireball, which is related to the time parameter $a_4$ (lifetime of the source; cf. Eq. (4.7)) is in striking agreement with the length of the $q_t$ interval over which $C_{(2)}>1$ (see Fig. 2).

We used also the data of the UAI ramping run to obtain explicit expressions for the four parameters $b_\mu$ as functions of energy. Both such behavior with energy and the related parameters $a_\mu$ of the fireball admit of possible experimental verification (e.g., in future DELPHI experiments).

In this connection, we must emphasize the loss of symmetry of the BE phenomenon, which shows up in the asymmetrical values of the spatial parameters of the fireball (see Table II). This is a quite unforeseen result, since one would expect on physical grounds an ellipsoid of rotation for the spatial shape of the source. If confirmed by further analyses of other experimental data, such asymmetry could be regarded as a clear signature of the presence and effectiveness of nonlocal effects in BE correlation.

We also emphasize that such an approach provides an (effective) phenomenological description of the hadronic interaction in terms of a deformation of the Minkowski metric. Indeed, the specification of the behavior of the four parameters with energy (the energy measured by a detector outside the source, i.e., in fully Minkowskian conditions) yields a "dynamical map" of the hadronic interaction, which can be regarded as alternative (or complementary) to a potential description of the interaction. Clearly, in cases in which a description in terms of a potential does not exist (such as that of hadronic interaction), the description in terms of a (local) deformation of the Minkowski space–time might turn out to be a useful tool, as we hope to have shown in our analysis of the BE phenomenon.

Note that Lorentz invariance—although broken in its conventional sense (at least locally)—is preserved in a generalized (isotopic) sense[13,16] by the nonlocal formalism, just as rotation symmetry (broken due to the spatial deformation of the source) is recovered when considering invariance under the Lie-isotopic rotation group.[18,19] The phenomenological breakdown of gauge theories, and its effective representation in terms of a deformed Minkowski metric, was considered some years ago by Nielsen and Picek.[21]

Needless to say, we are very far from a full geometrization of the hadronic interaction at the degree attained, e.g., by the general relativity theory in the geometrization of gravitation. This can be only regarded as a preliminary step toward such a very far-reaching goal. However, in our opinion, such a "geometrical" approach can provide quite interesting insights in some physical aspects of strong interactions, as we have tried to show by deriving the deformation law of time intervals in a hadronic field. Indeed, the "hadronic law of time deformation" derived in the previous section on a purely geometrical basic yields a qualitative but appealing picture of the well-known phenomena of asymptotic freedom and confinement of hadronic constituents.

We have the pleasure of thanking Prof. G. Salvini for communicating in 1990 the following which lies thought, at the very foundation of this work: "In the phenomenon of correlation in pion production. Nature is giving us some information I deem fundamental, but we are not yet able to fully understand it." Moreover, we are greatly indebted to M. Gaspero for invaluable discussions on the theoretical foundations and experimental aspects of BE correlation. Useful discussions with U. Amaldi, L. Chiatti, A. De Rujula, and F. Francaviglia are also gratefully acknowledged. Last but not least, special thanks are due to G. Caricato and E. Ferrari for their constructive criticism, continuous encouragement, and sincere appreciation of our work.

## APPENDIX A

### Elements of the Lie-isotopic generalization of quantum mechanics

The purpose of this appendix is to acquaint the reader with the basic rudiments of that branch of the Lie-isotopic formalism of interest to us, i.e., essentially its operator counterpart that allows one to build a generalized quantum mechanics able to account for nonlocal, non-Hamiltonian interactions (Refs. 15 and 16)[9].

First of all, we stress that the term "isotopy" must be understood here in its mathematical meaning (first introduced in the context of set theory[22]), i.e., in general, as a change in a mathematical structure that leaves the axioms of the structure invariant.

The term "Lie-isotopic" is due to the fact that this concept was rediscovered by Santilli[12] and first applied to the isotopy of Lie algebras.

A simple example of isotopy is provided by an isotopic algebra. Consider a standard linear algebra $\mathscr{A}$ with elements $A$ (e.g., the algebra of linear operators acting on a vector space) equipped with the usual sum and multiplication. Select a given operator $T$ such that $T^{-1}$ exists. The isotopy $\hat{\mathscr{A}}$ of the algebra $\mathscr{A}$ is obtained by considering the same set of elements $\{A\}$ with the same definition of sum, but multiplication given by

$$A*B=ATB \quad \forall A,B\in\hat{\mathscr{A}}. \tag{A1}$$

It is trivially checked that $\hat{\mathscr{A}}$ is still a linear algebra, whose identity is just given by

$$\hat{I}=T^{-1}. \tag{A2}$$

Note that for a given isotopy $\hat{\mathscr{A}}$ of $\mathscr{A}$, the isotopic element $T$ is fixed once and for all; in general, there is no need that $T$ be an element of $\mathscr{A}$—it is only required that $T$ be invertible (in the multiplicative group to which it belongs) and that the product $TA$ be well-defined for any $A$.

It is also clear that there exists a whole class $\{\hat{\mathscr{A}}\}$ of infinite possible isotopies of a given algebra $\mathscr{A}$ corresponding to the infinitely many choices of the isotopic element $T$.

We now immediately see the connection with the isotopy of a Lie algebra. Indeed, suppose that $\mathscr{A}$ is the enveloping associative algebra of a Lie algebra $\mathscr{A}_{\mathscr{A}}$ with product given by the usual bracket

$$[A,B]=AB-BA. \tag{A3}$$

Then, to the isotopic algebra $\hat{\mathscr{A}}$ there corresponds the isotopic Lie algebra $\hat{\mathscr{L}}_{\hat{\mathscr{A}}}$ with product

$$[A,B]_*=A*B-B*A=ATB-BTA. \tag{A4}$$

As is well known, the mathematical structure of quantum mechanics is represented by the Hilbert space of states $\mathscr{H}$ and the algebra $\mathscr{A}$ of operators acting on it. Thus, it is easily seen that the nonlocal Lie-isotopic generalization of quantum mechanics is obtained by the following isotopies[15,16]:

i) the isotopy of $\mathcal{A}$ according to the rules (A1), (A2), i.e.,

$$\mathcal{A}:AB \rightarrow \hat{\mathcal{A}}:A*B=ATB, \quad (A5)$$

$$I:AI=IA=A \rightarrow \hat{I}=T^{-1}:\hat{I}*A=A*\hat{I}=A, \quad (A6)$$

where $T$ is assumed, in this case, to be also (conventionally) Hermitian ($T^+=T$);

ii) the isotopy of the field $\mathscr{C}$ of complex numbers:

$$\mathscr{C} \rightarrow \hat{\mathscr{C}}=\{\hat{c}|\hat{c}=c\hat{I}, c \in \mathscr{C}\}. \quad (A7)$$

The isotopic field $\hat{\mathscr{C}}$ is obviously a field with ordinary sum and multiplication

$$\hat{a}*\hat{b}=\hat{a}T\hat{b}=(ab)\hat{I}, \quad (A8)$$

and its elements are called isoscalars (or isonumbers).

We stress that as a consequence of (A8), the isotopic ("star") product of isonumbers gives essentially the usual product of the corresponding numbers. Moreover, the isotopic product of an isonumber $\hat{n}$ by a quantity $Q$ coincides with conventional multiplication by a scalar ($\hat{n}*Q = nQ$). However, introducing the isofield $\hat{\mathscr{C}}$ is necessary to preserve linearity in the isotopic sense in the action of operators (see below);

iii) the isotopy of the inner product defined on $\mathscr{H}$:

$$\mathscr{H}:\langle\varphi|\psi\rangle \in \mathscr{C} \rightarrow \hat{\mathscr{H}}:\langle\hat{\varphi}|\hat{o}|\hat{\psi}\rangle=\hat{I}\langle\hat{\varphi}|G|\hat{\psi}\rangle \in \hat{\mathscr{C}}, \quad (A9)$$

where the operator $G$ (with, in general $G \neq T$) is nonsingular (conventionally) Hermitian and positive definite, the elements $|\hat{\psi}\rangle \in \hat{\mathscr{H}}$ are isokets (essentially coinciding with $|\psi\rangle \in \mathscr{H}$, except for a possible different normalization), and the isobras $\langle\hat{\varphi}|$ are the conventional Hermitian conjugates of isokets and therefore belong to the dual space $\hat{\mathscr{H}}_D$ of $\hat{\mathscr{H}}$:

$$\hat{\mathscr{H}}_D:\langle\hat{\varphi}|=(|\hat{\varphi}\rangle)^*, \quad |\hat{\varphi}\rangle \in \hat{\mathscr{H}}. \quad (A10)$$

It is easy to see that due to the positive-definiteness of $G$, the isotopic space $\hat{\mathscr{H}}$ is still a Hilbert space in its strict sense. Note also that the isotopic inner product $\langle|\hat{o}|\rangle$ is not new in the physical literature: it can be traced back to Pauli[23] (who followed a preceding, unpublished study by Dirac) and was used by Gupta and Bleuler in their indefinite-metric approach to electrodynamics.[24] In essence, it is easily seen that this "lifting" of the inner product leads to a deformation of the metric structure of the original Hilbert space $\mathscr{H}$;

iv) the isotopy of the action of the operators of $\mathcal{A}$ on $\mathscr{H}$:

$$A|\psi\rangle \rightarrow A*|\hat{\psi}\rangle=AT|\hat{\psi}\rangle. \quad (A11)$$

Of course, this implies the following isotopic generalization of the eigenvalue equations:

$$A|\psi\rangle=a|\psi\rangle \rightarrow A*|\hat{\psi}\rangle=\hat{a}*|\hat{\psi}\rangle=a|\psi\rangle, \quad (A12)$$

where the last step underlines the fact that in the isotopic case the eigenvalues are ordinary numbers too.

A number of implications of the above generalization of quantum mechanics follow, both on the physical and mathematical side. We merely recall that, e.g., the definitions of hermiticity and unitarity are to be suitably generalized. However, it can be shown that, in the case when the two isotopic elements $T$ and $G$ coincide, the definition of Hermitian operator is still the usual one. The case $T=G$ is just that considered in this paper. For further details, the reader is referred to Refs. 10 and 11.

Finally, we give the definition of an isotopic metric (or pseudometric) space (an example is provided, of course, by the isotopic Minkowski space considered in this paper).[13,18,19] Given an $n$-dimensional metric or pseudometric space $M(x,g,F)$, where $x$ are the elements of $M$ (local coordinates), $g$ is the metric tensor, and $F$ is the field over which $M$ is defined, its isotopy $\hat{M}(x,\hat{g},\hat{F})$ is a (metric or pseudometric) space with the same dimension $n$ and the same elements $x$ of $M$, with isotopic metric tensor given by

$$\hat{g}=Tg, \quad \det T \neq 0, \quad T^+=T \quad (A13)$$

and defined on the isotopic field $\hat{F}$:

$$\hat{F}=F\hat{I}, \quad \hat{I}=T^{-1}. \quad (A14)$$

Further mathematical details on the application of the Lie-isotopic formalism to BE correlation can be found in Ref. 17.

## APPENDIX B

### Classical model of an incoherent source with correlated detection

We now illustrate by a classical model, involving plane waves incident on a double slit, how it is possible, at least in principle, to build an incoherent source that nevertheless gives rise to correlated detection.

Let us consider the source to be formed by the two slits, and state the following detector-source duality: a source consisting of a double slit is detected by a screen (the detector), and this is analogous to two detectors measuring the emission of a source of unknown structure.

The analogy is carried out in four steps, corresponding to the four cases pictured in Fig. 7.

**Case 1.** The source consists of two slits in an opaque screen, on which a single monochromatic wave is incident (Fig. 7a). The detecting screen shows a correlated detection (spatial correlation), i.e. (on the basis of the detector–source duality) we have a second-order correlation.[1] In this case, we have to symmetrize the wavefunction (here regarded as a collection of bosons) both inside the source and outside the source (in the space between source and detector).

**Case 2.** The source consists of two slits in a screen, which are now separated by a diaphragm. The different monochromatic waves are now incident on each slit (see Fig. 7b). The detecting screen exhibits uncorrelated detection. In this case, the two waves are different, and therefore speaking about symmetry of the source does not make sense.

**Case 3.** The source consists of two slits in a screen, still separated by a diaphragm, but a monochromatic wave is incident on only one of the slits (Fig. 7c). The detecting screen shows correlated detection (first-order correlation).

**Case 4.** This case is obtained by "summing" cases 1 and 3 (or, what is the same, cases 1 and 2). The source consists of two slits separated by a diaphragm. A monochromatic wave common to both slits is incident on the slit screen, while a second, different, monochromatic wave is

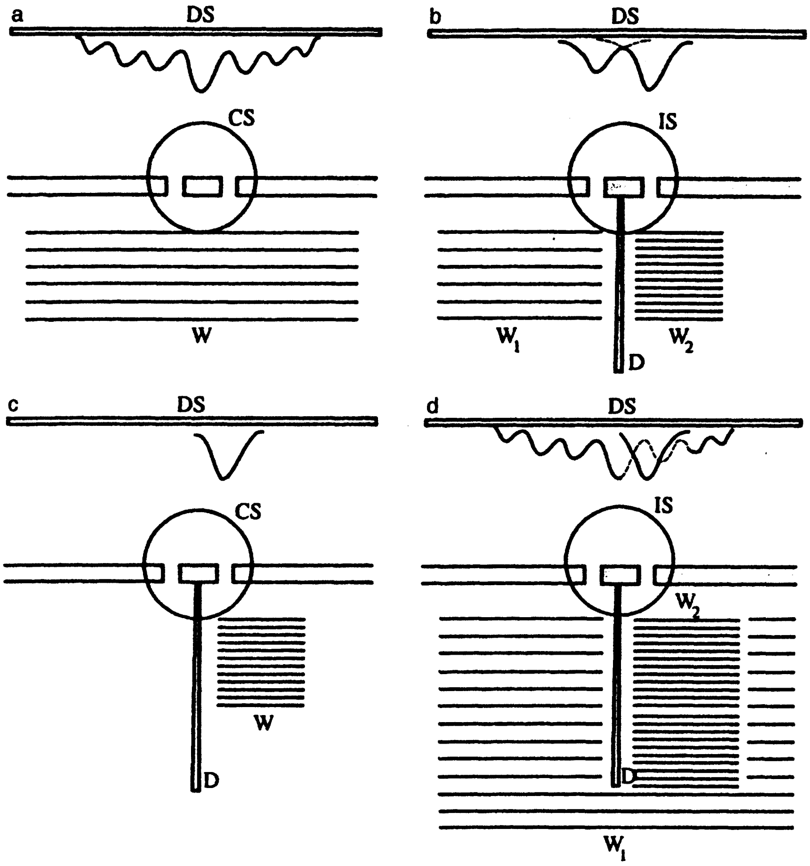


FIG. 7. Illustration of the classical model corresponding to: a) case 1—a coherent source (CS) and correlated detection (second-order correlation); b) case 2—incoherent source (IS) and uncorrelated detection; c) case 3—a coherent source and correlated detection (first-order correlation); d) case 4—incoherent source and correlated (but perturbed) detection (second-order correlation). Here DS is a detecting screen, W is a monochromatic wave ($W_{1,2}$ are the first (common for Fig. 7d) and second monochromatic waves); D is a diaphragm.

incident on only one of the slits (see Fig. 7d). The detecting screen shows a correlated but perturbed detection. In this case there is no need to symmetrize the wavefunction outside the source, in the space between source and detector.

In our opinion, this simple classical model can provide an intuitive picture of the logical process we followed in our analysis of the birth of correlation at the detectors for an incoherent source.

However it is impossible to establish a full analogy between this model and the interpretation of BE correlation, because we can "look" neither inside the fireball (total source) nor inside the subsources (or, in other words, it is impossible to know on which of the two slits a wave other than the common one incident on both slits falls).

1)Hereafter, we shall consider, to be concrete, that the particles involved in correlation are pions; however, it is easy to see that the whole treatment holds true for any particles obeying BE statistics.

2)Other contributions to nonlocality may come, for instance, from possible nonlocal terms of standard (electroweak) interactions at high energies, and from the fact that for hadrons, the de Broglie wavelength is, at the high energies we are going to deal with, of the order of the range of strong interactions.

3)We recall that for practical purposes it is possible to consider the product of numbers in an isotopic Minkowski space (i.e., the elements of the isotopic field $\hat{R}$) to be the usual product of real numbers. See Appendix A for further details.

4)From a mathematical point of view, this is due to the local isomorphism between the standard rotation group in three dimensions, $O(3)$, and the Lie-isotopic group $\hat{O}(3)$ (which, roughly speaking, is the group of isometries of an ellipsoid). See Ref. 19.

5)We acknowledge that the term "towing effect" is not ideal, and it would be more appropriate to use "leading effect." However, this last name has been already used to denote another effect in high-energy collisions among particles, and we wish to avoid confusion.

6)However, strictly speaking, one ought to compare this value with that provided by the fit of the usual correlation function without the parameter $\lambda$ (see Eq. (1.5)), which, as stressed in Sec. 1, is actually introduced by hand. Indeed, the fit of the data by the correlation function $C_{(2)}^{\text{Gauss}}=\gamma[1+\exp(-\beta q_t^2)]$ gives a reduced chi-square $\chi_n^2=16.72$. Moreover, the correlation coefficients of this fit show that $\beta$ is correlated with the normalization factor $\gamma$!

7)Let us notice that the four-vector $\rho_\mu$ accounts for possible anisotropy; in case of actual isotropy, $\rho_\mu$ collapses into $\rho g$ ($g$ being the usual Minkowski metric), and one then recovers the conventional case (except for the time part that as already said, takes account of the phase correlation).

8)U. Amaldi and A. De Angelis: private communication (1993).

9)The Lie-isotopic generalization of quantum mechanics is a branch of the so-called "hadronic mechanics" so named by Santilli[12] because of its possible role in describing strong interactions (supposed to be basically of nonlocal, nonpotential nature).

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