Quasiclassical description of resonant Raman scattering in quantum wells in the presence of a magnetic field

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We discuss the intensity and polarization characteristics of Raman scattering in a quantum well in the presence of a magnetic field, taking into account the real structure of the valence band. From a quasiclassical point of view an electron and hole are created and recombine where their orbits intersect. The only orbits that contribute to resonant Raman scattering are those for which the time required to move from one intersection point to another is the same for electrons and holes, so that after creation they encounter each other again. In the interval between their creation and re-encounter, the carriers interact with phonons. We show that in weak fields oscillations in the scattering intensity and polarization are possible, connected with interference between the results of encounters of various types on wiggly orbits.
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1. INTRODUCTION

A giant increase of the intensity of light scattering with participation of acoustic phonons (so-called geminate recombination) in GaAs/GaAlAs quantum-well structures in a magnetic field was observed by Mirlin *et al.* [1,2] (see also Refs. 3 and 4). An analogous effect was observed previously for Raman scattering with participation of optical phonons.[5]

This effect can be explained by starting with a quasiclassical picture of the scattering.[2,6] According to this picture, the scattering event consists of three stages. (1) Creation of an electron-hole pair by the excitation photon. The electron and hole are created at the same point in space, with momenta that are equal in magnitude and opposite in direction. (2) The interaction of an electron or hole with phonons. (3) Recombination of the pair with emission of the scattered photon. In this case the electron and the hole should also be at the same point in space with oppositely directed momenta.

Without a magnetic field, the processes of creation and recombination should occur almost simultaneously. The time delay $\hbar/\varepsilon$ between them (where ε is the kinetic energy with which the particles fly apart) arises only from quantum indeterminacy of the coordinates and momenta of the particle.

In a magnetic field, for the simplest case of circular orbits the particles move along the same circle in opposite directions after creation. Because of this they have the possibility of recombining after encountering each other again, once more within a time interval $t_H=2\pi/\omega_H$, where ω_H is the sum of the cyclotron frequencies of the electron and hole. Between the instants of creation and recombination, the electron-hole pair can undergo interactions with phonons. This interaction period lasts considerably longer in a magnetic field than for zero field if we have $t_H \gg \hbar/\varepsilon$ (i.e., $\varepsilon \gg \hbar\omega$).

We can expect that in sufficiently weak fields the primary magnetic field dependence will be given by the exponential $\exp[-t_H/\tau]$, where τ is the time it takes the pair to leave its initial state due to any scattering process. This exponential determines the probability that an electron-hole pair will survive in its initial state until the first encounter. For sufficiently strong fields satisfying $t_H<\tau$, many encounters can take place and the interference between them will give resonances at the Landau levels.

This picture was formulated and confirmed by quantum calculations in Refs. 2 and 6 under the assumption of simple bands and circular orbits. However, the real isoenergetic surfaces of holes are rippled, and the orbit of a hole in a magnetic field is not circular. Under these conditions the electron and hole do not necessarily encounter one another after creation. It is also necessary to include the complex structure of the valence band to explain the polarization characteristics of the scattering.

The task of the present paper is to investigate scattering when the real structure of the valence band is included. In this paper we show that in a magnetic field there exist orbits for which the electron and hole really can encounter one another. It is these orbits that give the primary contribution to the scattering intensity. When the valence band is rippled, the total kinetic energy $\varepsilon_c+\varepsilon_v$ of the electron and hole which is determined by the energy of the excitation photon, no longer fixes the energies of the particles ε_c and ε_v individually (in contrast to the case of a circular orbit). The individual orbits on which the encounter takes place correspond to a definite choice of ε_c and ε_v. The scattered light is a result of interference of the radiation from encounters on different orbits.

In the calculations we will use the selection rules and quasiclassical expressions for the matrix elements of optical transitions between Landau levels of an electron and hole in a quantum well obtained in Ref. 7. We will discuss a symmetric well, neglecting the absence of a center of inversion in the bulk material. We assume that the magnetic field and excitation light beam are perpendicular to the plane of the well (the Faraday geometry). The Coulomb interaction between the electron and hole is not taken into account. We will discuss only one-phonon processes. In this geometry of the problem, the momentum of a phonon radiated by an electron or hole is directed along the z axis.

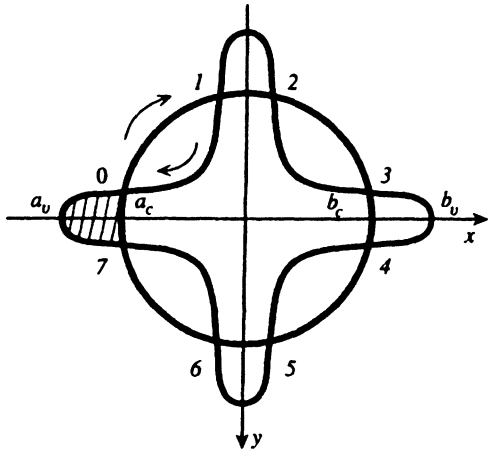

FIG. 1. Trajectories of an electron in the valence band and in the conduction band. The centers of the orbits are located at the point $x=y=0$. The intersection points of the orbits are labeled. The arrows indicate the direction of motion of particles along the orbits.

2. CONDITIONS FOR ENCOUNTER

Figure 1 shows the paths for an electron and hole created by the absorption of a photon in a magnetic field. The centers of the orbits coincide, because at the instant of creation the coordinates of the particles are the same, and their momenta are opposite. Pair creation takes place at one of the intersection points of the orbits. Recombination should also take place at such an intersection point. However, the electron and hole need not encounter one another at an intersection point, since the times required for them to move from the point of creation to this intersection point are, in general, different. The conditions for equality of these times can be satisfied only for special relations between the energies of the electron and hole. Their total energy, given by the excitation photon, does not fix their energies individually because the energy of the hole depends not only on the value of the wave vector but also on its direction. Because of this, we can find those orbits for which the condition that the two times be equal is satisfied for a given photon energy $\hbar\omega$. In this case, for each pair of orbits there will be only one encounter in general.

The processes of creation and encounter are conveniently classified as follows. Type-I processes are those in which a pair is created at an even-labeled intersection point of the orbits (see Fig. 1) and recombines at an odd-labeled intersection point. In type-II processes the pair is created at an odd point and recombines at an even point. In type-III processes the pair is created and recombines at points with the same parity.

We denote by t_c and t_v the times it takes the electron and hole to move from the point of creation to some point of intersection of the orbits. Then the values of the energies ε_c and ε_v for which an encounter occurs should be found from two conditions—the condition that these times be equal, and conservation of energy:

$$t_c=t_v, \quad \varepsilon_c+\varepsilon_v=E, \tag{1}$$

where $E=\varepsilon_0$ (entrance resonance) or $E=\varepsilon_0-\hbar\Omega$ (exit resonance). Here $\varepsilon_0=\hbar\omega-E_g$ is the total kinetic energy of an electron and hole at creation, and E_g is the width of the forbidden gap, including the size-quantization energy.

The times t_c and t_v for processes of various kinds are determined by the expressions

$$t_{c\mathrm{I}}=T_c-\Delta t_c-\frac{K}{4}T_c+N_cT_c,$$

$$t_{v\mathrm{I}}=\Delta t_v+\frac{K}{4}T_v+N_vT_v, \tag{2}$$

$$t_{c\mathrm{II}}=T_c-\left(\frac{T_c}{4}-\Delta t_c\right)-\frac{K}{4}T_c+N_cT_c,$$

$$t_{v\mathrm{II}}=\left(\frac{T_v}{4}-\Delta t_v\right)+\frac{K}{4}T_v+N_vT_v, \tag{3}$$

$$t_{c\mathrm{III}}=T_c-\frac{K}{4}T_c+N_cT_c, \quad t_{v\mathrm{III}}=\frac{K}{4}T_v+N_vT_v. \tag{4}$$

Here $T_c=2\pi/\omega_c$ and $T_v=2\pi/\omega_v$ are the cyclotron periods of the electron and hole; ω_c and ω_v are the cyclotron frequencies; N_c and N_v are non-negative whole numbers; K runs over the values 0, 1, 2, 3; Δt_c is the time it takes an electron to move from an odd point to the nearest even point (for example, from point 7 to point 0); and Δt_v is the time it takes a hole to move from an even point to the nearest odd point (for example, from point 0 to point 7).

Equations (2)–(4) have a simple meaning. For a fixed point of creation the four values of the number K correspond to four possible encounter points of a given type; N_c and N_v denote the number of complete turns an electron or hole makes from creation to the encounter.

Note that the condition that the times for processes of type III be equal requires that the cyclotron periods of the electron and hole be commensurate, and can be satisfied only in exceptional cases. These cases correspond to an accidental coincidence of the frequencies of several transitions that satisfy the selection rules (7) (see below).

Let us introduce the notation $t_R\equiv t_{cR}=t_{vR}$ for the time interval from creation to recombination of a pair in a process of type R. If scattering with a characteristic time τ occurs, then the amplitude of the radiation that appears at the encounter is proportional to the exponential $\exp[-t_R/2\tau]$, since the square of the amplitude is proportional to the probability that the pair survives for a time t_R without scattering. For small magnetic fields, for which $t_R>\tau$, the primary contribution to the radiation comes from the encounter with the smallest time t_R, i.e., the principal encounter.

Assume, for example, that a pair is created at the point 0. Then if the mass of the hole is much larger than the mass of the electron, the principal type-I encounter takes place at point 7. In this case, the hole will necessarily have traversed a very small path, whereas the electron traverses almost a complete orbit. As the relative positions of the orbits change from that shown in Fig. 2a to those shown in Fig. 2b, the time for motion of the hole changes from 0 to $T_v/4$, while the time for motion of the electron changes from T_c to $3T_c/4$. Therefore, if $T_v>3T_c$ holds, we can always find that arrangement of the orbits (i.e., those energies ε_c and ε_v) for

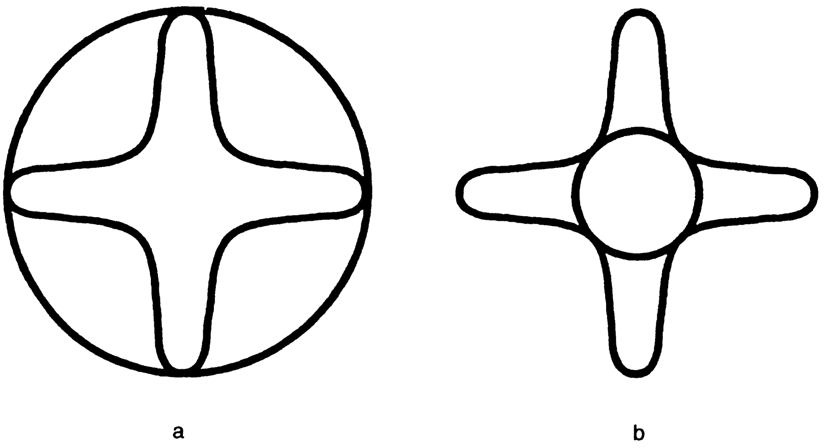

FIG. 2. Limiting cases of the mutual positions of electron and hole orbits.

which an encounter at point 7 will be possible. The competing process will be a process of type II: for example, creation of a pair at point 1 with an encounter at point 0. The interference between these two types of processes leads to an additional dependence of the intensity and polarization of the scattered light on magnetic field (see below).

As an illustration, using the results of Ref. 8, we numerically solved Eqs. (1) for GaAs-based quantum wells with infinite barriers, Luttinger parameters $\gamma_1 = 6.8$, $\gamma_2 = 1.9$, $\gamma_3 = 2.7$, and an electron mass $m_c = 0.067 m_0$. In Fig. 3 we show the energy ε_v, and also the times t_{I} and t_{II} corresponding to principal encounters of types I and II, as functions of the pair kinetic energy ε_0.

The points of intersection of the horizontal straight line $3T_c/4$ with the curves t_{I} and t_{II} correspond to the condition $T_v = 3T_c$. The situation between these points (where encounters of type III are possible) and to the left of them requires a special discussion. However, in this region the hole orbits become practically circular, so that the cylindrical approximation can be used and classification of the encounters based on type loses its meaning.

From Fig. 3 it is clear that the times t_{I} and t_{II} are quite close to one another over the entire range of energies ε_0, so that competition between principal encounters of types I and II is actually possible.

3. QUASICLASSICAL CALCULATIONS

The scattering amplitude for Raman scattering with emission of a phonon with frequency Ω and wave vector Q directed along the z axis is determined by the expression

$$\mathscr{A} = \frac{1}{2\pi\lambda^2} \times \sum_a \frac{\langle 0|\mathbf{e}\hat{\mathbf{p}}|a\rangle\langle a|\hat{V}_{e-ph}(Q)|aQ\rangle\langle aQ|\mathbf{e}_l\hat{\mathbf{p}}|0Q\rangle}{(\hbar\omega - E_a + i\Gamma_a/2)(\hbar\omega - E_a - \hbar\Omega + i\Gamma_a/2)}, \quad (5)$$

where $\hat{\mathbf{p}}$ is the momentum operator, the vectors $\mathbf{e}$ and $\mathbf{e}_l$ characterize the polarization of the excited and scattered light, and λ is the magnetic length. The state $|a\rangle$ is a state of the electron-hole pair that is virtually excited by the light, $|aQ\rangle$ is a state of a pair plus phonon, and $|0Q\rangle$ is the ground

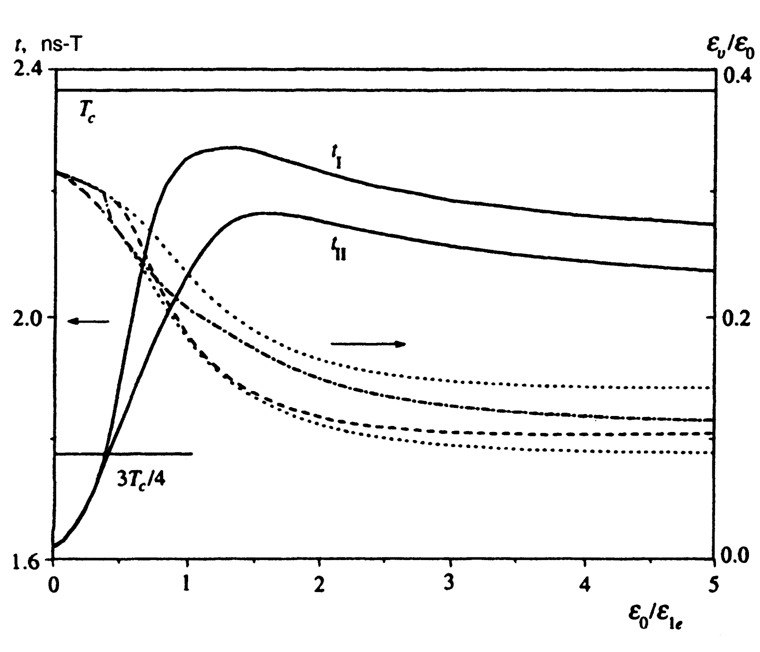

FIG. 3. Times for motion of particles and kinetic energies of holes as a function of the kinetic energy of a pair ε_0 (here $\varepsilon_{1e} = (\hbar^2/2m_c)(\pi/L)^2$, where L is the width of the quantum well). T_e is the cyclotron period of an electron. The dotted curves are the minimum and maximum kinetic energies of holes ε_v for given ε_0. The dashed and chain curves represent ε_v for primary encounters of types I and II respectively.

state of the crystal + phonon. The state $|a\rangle$ is characterized by labels for the size-quantized subbands, labels for Landau levels of the electron n_c and hole n_v, and also the spin labels m_c and m_v ($m_c=\pm 1/2$ for electrons in a subband of the conduction band, $m_v=\pm 3/2$ for electrons in the subbands of heavy holes, $m_v=\pm 1/2$ for electrons in the subbands of light holes). The classification of the states using m_c, m_v is connected with their symmetry under the reflection $\hat{\Pi}_z$, and is explained in Ref. 7. We will assume that the electron-phonon interaction $\hat{V}_{e-ph}$ does not change the state of the pair $|a\rangle$.

The matrix element $\langle 0|\mathbf{e}\hat{\mathbf{p}}|a\rangle$ can be written in the form [7]

$$\langle 0|\mathbf{e}\hat{\mathbf{p}}|a\rangle=e_+F_-+e_-F_+,\quad e_\pm=e_x\pm ie_y. \tag{6}$$

The quantities $F_\pm$ differ from zero when the following selection rule is satisfied:

$$(\pm 1+n_v+m_v)-(n_c+m_c)=4N, \tag{7}$$

where N is a whole number. Due to the difference in selection rules for F_+ and F_-, Eq. (5) does not contain terms with the products $e_1e^*_{1-}$ or $e_1e^*_{1+}$ This implies that the circular polarization of the light is preserved under scattering. This conclusion (which is in approximate agreement with experiment [2,3]) is of course connected with the assumption that the electron-phonon interaction does not change the state of the electron-hole pair in the scattering process. Thus, we can rewrite Eq. (5) in the form

$$\mathscr{A}=\mathscr{A}^+e_-e^*_{1-}+\mathscr{A}^-e_+e^*_{1+}. \tag{8}$$

The quantities $\mathcal{A}^+$ and $\mathcal{A}^-$ are transition amplitudes for (σ^+,σ^+)- and (σ^-,σ^-)-processes respectively. In calculating them we first sum over n_c and n_v. It is therefore convenient to write the expression for $\mathcal{A}^\pm$ in the form

$$\mathscr{A}^\pm=\sum_b \mathscr{A}^\pm_b;$$

$$\mathscr{A}^\pm_b=\frac{1}{2\pi\lambda^2}\sum_{n_c\,n_v}\frac{|F_\pm|^2V_{e-ph}}{(\varepsilon_0-\varepsilon+i\Gamma/2)(\varepsilon_0-\varepsilon-\hbar\Omega+i\Gamma/2)}. \tag{9}$$

Here $\varepsilon\equiv\varepsilon_c+\varepsilon_v$ is the total kinetic energy of the electron and hole, $\Gamma\equiv\hbar/\tau$ is the width of the level, and $V_{e-ph}\equiv\langle a|\hat{V}_{e-ph}(Q)|aQ\rangle$. We use the label b to denote all the quantum numbers that characterize the state $|a\rangle$ except for n_c and n_v. All the quantities under the summation sign depend on b, n_c, n_v; however, they are omitted for brevity.

According to Eq. (24) of Ref. 7, the quantity $|F_\pm|^2$ can be written in the form of a sum of three terms:

$$|F_\pm|^2=g^\pm_{\rm I}\exp[2i\Delta s_0]+g^\pm_{\rm II}\exp[-2i\Delta s_0]+g^\pm_{\rm III}, \tag{10}$$

where

$$\Delta s_0=\int_{a_v}^{x_0}k_{vx}dx-\int_{a_c}^{x_0}k_{cx}dx,$$

$$\Delta m_0=m_v(x_0)-m_c(x_0),$$

$$g^\pm_{\rm I}=\{4Bf_\pm(\mathbf{k}_0)\exp[i\Delta m_0-i(\Delta_\pm+1)\pi/4]\}^2, \tag{11}$$

$$g^\pm_{\rm II}=(g^\pm_{\rm I})^*,\quad g^\pm_{\rm III}=2|g^\pm_{\rm I}|,\quad \Delta_\pm=m_v-m_c\pm 1.$$

The quantities $f_\pm(\mathbf{k})$ are expressed in terms of the envelopes of the wave functions of electrons and holes in the well without a magnetic field (Eq. (20) of Ref. 7), and $m(x)$ is the phase set of the wave function connected with the magnetic moment (Eq. (10) from Ref. 7). The quantity B is defined by Eq. (20) from Ref. 7. The vectors $\mathbf{k}_v$ and $\mathbf{k}_c$ are wave vectors of electrons in the valence and conduction bands, which depend on the position of the electron in the orbit. In the expressions for Δs_0, the coordinate x_0 refers to the 0 point of intersection of the orbits (see Fig. 1). The quantity $2\Delta s_0$ is the hatched area shown in Fig. 1, divided by λ^2.

Substitution of (10) into (9) gives the transition amplitude in the form of three terms, which as we will show below correspond to the three types of encounters of an electron and hole described in the previous section. Let us calculate the first of these terms:

$$\mathscr{A}^\pm_{b\rm I}=\frac{1}{2\pi\lambda^2}\sum_{N,\,n_c}g^\pm_{\rm I}V_{e-ph}\times\frac{\exp[2i\Delta s_0]}{(\varepsilon_0-\varepsilon+i\Gamma/2)(\varepsilon_0-\varepsilon-\hbar\Omega+i\Gamma/2)}. \tag{12}$$

Here we have taken into account the selection rule (7), so that $n_v=n_c+4N-\Delta_\pm$.

In order to show that the primary contribution to the transition amplitude comes from orbits on which an encounter takes place, let us transform the sum in (12) according to the Poisson formula:

$$\sum_{N,n_c}\cdots\to\sum_{\tilde N\,\tilde n_c}\int dNdn_c\exp[2i\pi(N\tilde N+n_c\tilde n_c)]\cdots$$

In these integrals we replace the variables N and n_c by ε_v and $\varepsilon\equiv\varepsilon_c+\varepsilon_v$, and in the sums we change from the summation variables $\tilde N$ and $\tilde n_c$ to N_c, N_v and K, where $\tilde N=4N_v+K$, $\tilde n_c=N_c+N_v+1$, and K runs over the values 0, 1, 2, 3. Then (12) takes the form

$$\mathscr{A}^\pm_{b\rm I}=\frac{1}{2\pi\lambda^2}\sum_{N_c,N_v,K}\int d\varepsilon\times\frac{\exp[iK\Delta_\pm\pi/2]}{(\varepsilon_0-\varepsilon+i\Gamma/2)(\varepsilon_0-\varepsilon-\hbar\Omega+i\Gamma/2)}\times\int d\varepsilon_v\frac{g^\pm_{\rm I}V_{e-ph}}{4\hbar\omega_c\hbar\omega_v}\exp[i\alpha_{\rm I}], \tag{13}$$

$$\alpha_{\rm I}=2\Delta s_0+2\pi(N_c+1-K/4)n_c+2\pi(N_v+K/4)n_v. \tag{14}$$

It should be understood that n_v and n_c (like the other quantities under the integral) depend on the hole energy ε_v and electron energy $\varepsilon_c=\varepsilon-\varepsilon_v$.

Let us evaluate the integral over ε_v in (13) by the saddle-point method, assuming that only $\exp[i\alpha_{\rm I}]$ is a rapidly oscillating function of ε_v. In this case we use the relations

$$\frac{dn_v}{d\varepsilon_v}=\frac{1}{\hbar\omega_v},\quad \frac{dn_c}{d\varepsilon_v}=-\frac{dn_c}{d\varepsilon_c}=-\frac{1}{\hbar\omega_c},$$

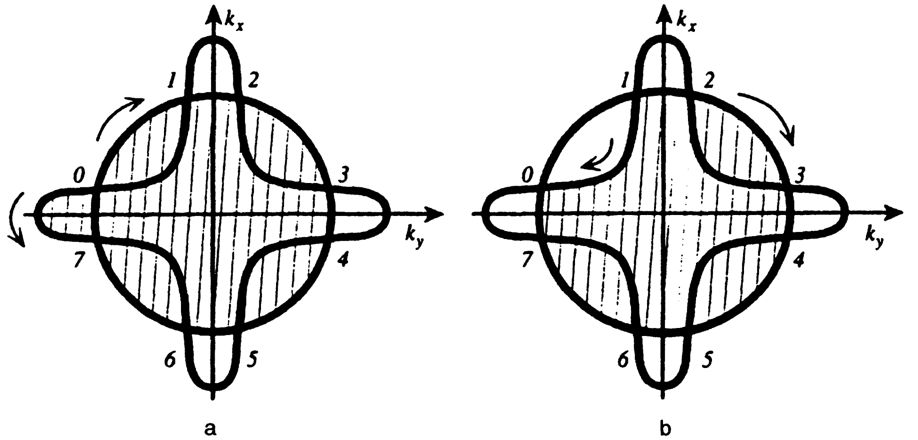

FIG. 4. Areas $S_{\rm I}$ (a) and $S_{\rm II}$ (b) that determine the phase set from creation to primary encounters of types I and II.

$$\frac{d\Delta s_0}{d\varepsilon_v}=\frac{\Delta t_v+\Delta t_c}{2\hbar},$$

where Δt_c and Δt_v are exactly those time intervals that enter into Eq. (2). It is easy to verify that the saddle-point condition $d\alpha_{\rm I}/d\varepsilon_v = 0$ reduces to the condition that the times for the type-I encounters are equal. We evaluate the remaining integral over ε in (13) by residues, extending the integration from $-\infty$ to $+\infty$. We first expand the phase α_I with respect to ε near the points ε_0, $\varepsilon_0-\hbar\Omega$ for those values of ε_v that satisfy the saddle-point condition:

$$\alpha_{\rm I}(\varepsilon)=\alpha_{\rm I}(E)+(\varepsilon-E)t_{\rm I}/\hbar,$$

where $t_{\rm I}$ is the time interval from creation of an electron-hole pair to a type-I encounter when the pair energy equals E. Then Eq. (13) has the form

$$\mathscr{A}_{b\rm I}^{\pm}=\frac{1}{\hbar\Omega}\{\mathscr{A}_{b\rm I}^{\pm}(\varepsilon_0)-\mathscr{A}_{b\rm I}^{\pm}(\varepsilon_0-\hbar\Omega)\}, \tag{15}$$

where

$$\mathscr{A}_{b\rm I}^{\pm}=\sum_{N_c,N_v,K}\exp\left[iK\Delta\pm\frac{\pi}{2}+i\,\mathrm{sign}(\xi_{\rm I})\frac{\pi}{4}\right]\times\sqrt{\frac{2\pi}{|\xi_{\rm I}|}}\frac{ig_{\rm I}^{\pm}V_{e-ph}}{4\lambda^2\hbar\omega_c\hbar\omega_v}\exp\left[i\alpha_{\rm I}(E)-\frac{t_{\rm I}}{2\tau}\right],$$

$$\xi_{\rm I}=\left.\frac{d^2\alpha_{\rm I}}{d\varepsilon_v^2}\right|_{\varepsilon=E}. \tag{16}$$

The computation by residues gives a nonzero answer for $t_{\rm I}>0$ (i.e., N_c, $N_v>0$), which is completely natural from the point of view of the encounter picture.

In Eq. (16) all the quantities depend on the electron and hole energies ε_c and ε_v. These energies are found from the two Eqs. (1). Thus, the contribution to the transition amplitude (9) connected with the first term in (10) actually corresponds to encounters of type I.

In exactly the same way we can show the contributions associated with the second and third terms in (10) correspond to encounters of types II and III. The expressions for the amplitudes $\mathcal{A}_{b\rm II}^{\pm}$ differ from (16) by replacement of the label I by II and K by $K+1$. The expression for $\alpha_{\rm II}$ differs from (14) by replacement of Δs_0 by $(-\Delta s_0)$ and K by $K+1$. The expression for $\alpha_{\rm III}$ is obtained from (14) by eliminating from (14) the term $2\Delta s_0$.

4. WEAK MAGNETIC FIELDS

Let us discuss the range of magnetic fields that are so weak that resonances connected with Landau levels do not enter in ($\omega_c\tau<1$). In this case, as we have already mentioned above, the primary contribution to the scattering intensity is given by the encounter with the smallest time t_R (the principal encounter). For definiteness, let us consider scattering by acoustic phonons (geminate recombination), so that $\hbar\Omega\ll\varepsilon_0$. We will assume that "first heavy-hole subband-first electron subband" transitions play the primary role. For each polarization there are two types of transitions of this kind: an electron-hole pair is excited either in a state with $m_v=3/2$, $m_c=1/2$ or in a state with $m_v=-3/2$, $m_c=-1/2$. If we take into account these assumptions and Eq. (16), the transition amplitude for the principal encounter of type I is conveniently written in the form

$$\mathscr{A}_{\rm I}^{\mp}=\lambda C_{\rm I}\exp[i\lambda^2S_{\rm I}-t_{\rm I}/2\tau\mp i\psi_{\rm I}], \tag{17}$$

where $C_{\rm I}$ and $\psi_{\rm I}$ depend on the energy ε_0 and do not depend on magnetic field. The quantity $C_{\rm I}\exp[\mp i\psi_{\rm I}]$ contains, in particular, the product of amplitudes $f_\pm$ for processes of creation and recombination of pairs, calculated without the magnetic field. The quantity $S_{\rm I}$ is the area in **k**-space bounded by the trajectories of the particles from creation at the point $\mathbf{k}_0$ up to encounter at the point $\mathbf{k}_7$ (the hatched area in Fig. 4a). We note that λ^2S is the phase set from creation to recombination caused by the magnetic flux passing through the area bounded by the particle trajectories in real space.

In deriving (17), we used the quantization conditions, and also the properties of the amplitudes $f_\pm$ (Eqs. (13), (21), (25) of Ref. 7).

A competitor with this process is radiation arising from a primary encounter of type II. The amplitude of this process $\mathscr{A}_{\rm II}^{\mp}$ is described by an expression analogous to (17). The area $S_{\rm II}$ is hatched-in in Fig. 4b.

It is clear from (17) that the scattering intensity is enhanced as the magnetic field increases, due to the exponential $\exp[-t_R/\tau]$ and the fact that the time t_R from creation of the pair to recombination decreases with field (just as in the spherical approximation): $t_R \sim 1/H$.

Let us now discuss the problem of polarization characteristics of the scattered light. It is clear from Eq. (8) that if the transition amplitude is written in the form $\mathscr{A}^{\pm}=\sqrt{I^{\pm}}\exp[-i\psi_{\pm}]$, then I^+ and I^- are proportional to scattering intensities with excitation of right- and left-handed polarized light respectively. (We emphasize once more that the scattered light has the same circular polarization as the excitation light.)

When the excitation light is linearly polarized, the total scattered intensity is $I=(I^+ + I^-)/2$. The half-difference $(\psi_+ - \psi_-)/2$ gives the angle of rotation ψ_l of the plane of linear polarization of the scattered light with respect to the excitation light. In this case, the maximum degree of linear polarization ρ_l (i.e., the degree of polarization, referenced to axes connected with the plane of polarization of the scattered light) is $\rho_l=2\sqrt{I^+I^-}/(I^+ + I^-)$.

From Eq. (17) it follows that if the overwhelming contribution to the scattering comes from only a single process, i.e., one encounter, then $I^+=I^-$, $\rho_l=1$, and the angle of rotation $\psi_l=\psi_{\mathrm{I}}$ for fixed energy ε_0 does not depend on magnetic field. We can show that in the cylindrical approximation (i.e., if we neglect warping of the hole orbits) the same results are obtained: an angle of rotation of the plane of polarization that is independent of magnetic field and 100% linear polarization.

However, in experiment [1] the angle of rotation and degree of linear polarization of the scattered light depend on magnetic field. This implies that the contributions to the transition amplitude should come from at least two competing processes. These processes can be primary encounters of types I and II, as we have shown already above.

If we assume that $\mathscr{A}=\mathscr{A}_{\mathrm{I}}^{\pm}+\mathscr{A}_{\mathrm{II}}^{\pm}$ and use Eq. (17), then the following relations are found. The scattering intensity for circularly polarized excitation is:

$$I^{\pm}=I_{\mathrm{I}}+I_{\mathrm{II}}+2\sqrt{I_{\mathrm{I}}I_{\mathrm{II}}}\cos[\lambda^2\Delta S\pm\Delta\psi]. \tag{18}$$

For linearly polarized excitation light we have

$$\rho_l^2=1-\left\{\frac{\beta\sin(\Delta\psi)\sin(\lambda^2\Delta S)}{1+\beta\cos(\Delta\psi)\cos(\lambda^2\Delta S)}\right\}^2,$$

$$\psi_l=\frac{\psi_{\mathrm{I}}+\psi_{\mathrm{II}}}{2}+\arctan\left\{\frac{\kappa\sin(\Delta\psi)}{\cos(\Delta\psi)+\beta\cos(\lambda^2\Delta S)}\right\}. \tag{19}$$

Here, according to Eq. (17), we have

$$I_R=\lambda^2|C_R|^2\exp[-t_R/\tau]\quad(R=\mathrm{I},\ \mathrm{II}),$$

$$\Delta\psi=\psi_{\mathrm{II}}-\psi_{\mathrm{I}},\quad \Delta S=S_{\mathrm{II}}-S_{\mathrm{I}},$$

$$\kappa=(I_{\mathrm{II}}-I_{\mathrm{I}})/(I_{\mathrm{II}}+I_{\mathrm{I}}),\quad \beta=\sqrt{1-\kappa^2}.$$

From these expressions it is clear that except for the exponential dependence of the scattered intensity on magnetic field (due to the fact that $t_R \propto 1/H$) the competition between principal encounters of types I and II (connected with wiggling of the hole orbits) should lead to oscillations in all the scattering characteristics ($I^{\pm}$, ρ_l, $\psi^{\pm}$). These oscillations are caused by the phase difference $\lambda^2 S$ between amplitudes for processes of types I and II. We emphasize once more that these oscillations are not related to the ordinary oscillations connected with resonances in the energy of the excitation photon with scattering between Landau levels of the electron and hole. These anomalous oscillations should be observed under just those conditions where the distance between Landau levels is smaller than the width of the levels.

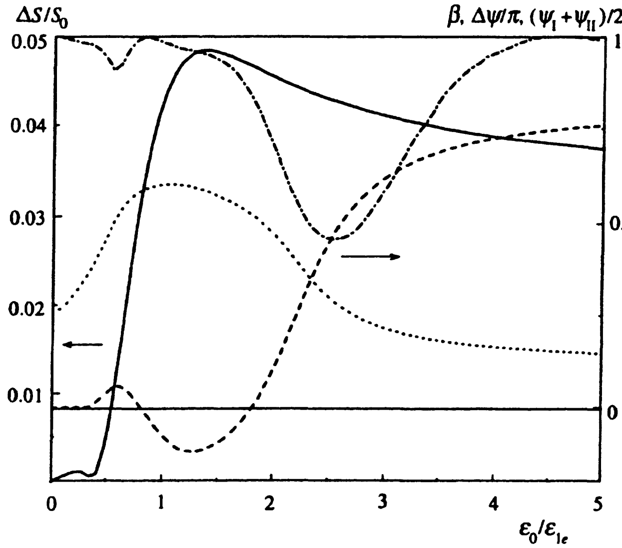

FIG. 5. Dependences of parameters entering into (18) and (19) on the kinetic energy ε_0 of a pair. The solid curve represents ΔS, the dashed curve $\Delta\psi$, the dotted curve $\psi_{\mathrm{I}}+\psi_{\mathrm{II}}$, and the dotted-dashed curve β.

Figure 5 shows the dependence of the parameters entering into Eqs. (18) and (19) calculated within the same model as the encounter times given in Fig. 3. In the calculations of the quantity β we have neglected the difference in times t_{I} and t_{II}, so that the computed β does not depend on the magnetic field or on the scattering time τ. It is clear that β is not small, which confirms the possibility of a competition between processes I and II. Note that $S_0 \equiv 2\pi m_c\varepsilon_0/\hbar^2$ is approximately the area of an electron orbit. Therefore, it is clear from the plot for ΔS that the period of the anomalous oscillations versus inverse magnetic field is approximately 20 times longer than the period that would be connected with Landau levels.

Thus, in this paper we have proposed a quasiclassical description of geminate recombination that takes into account the complex rippled structure of the valence band. This description shows that the primary contribution to the scattering comes from isolated orbits on which electrons and holes encounter one another after creation by an excitation photon. Anomalous oscillations are predicted in the intensity and polarization characteristics of the scattering that are not due to the presence of Landau levels and are connected with interference between two types of encounters on the rippled orbits.

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[1]P. S. Kop'ev, D. N. Mirlin, V. F. Sapega, and A. A. Sirenko, JETP Lett. **51**, 708 (1990).
[2]D. N. Mirlin, I. A. Merkulov, V. I. Perel' *et al.*, Solid State Commun. **82**, 305 (1992).
[3]V. F. Sapega, V. I. Belitsky, A. J. Shields *et al.*, Solid State Commun. **84**, 1039 (1992).
[4]V. F. Sapega, V. I. Belitsky, T. Ruf *et al.*, Phys. Rev. B **46**, 16005 (1992).
[5]D. Gammon, R. Merlin, and H. Morcoç, Phys. Rev. B **35**, 2552 (1987).
[6]I. A. Merkulov and V. I. Perel', J. Luminescence **60-61**, 293 (1994).
[7]A. Yu. Dobin and V. I. Perel', Zh. Éksp. Teor. Fiz. [JETP]. (to be published)].
[8]I. A. Merkulov, V. I. Perel', and M. E. Portnoĭ, Zh. Éksp. Teor. Fiz. **99** 1202 (1991) [Sov. Phys. JETP **72**, 669 (1991)].

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