Photon emission by an electron in a collision with a short focused laser pulse

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The spectral–angular distributions and the probability spectra of photon emission by an electron in a collision with a short focused laser pulse are obtained. The feasibility of using a monochromatic plane wave model to describe a short laser pulse is discussed. © *1996 American Institute of Physics.* [S1063-7761(96)00307-1]

1. INTRODUCTION

The interaction of electrons and photons with a strong laser field has been under intensive study since the mid-1960s. As a rule, the laser field is modeled by a monochromatic electromagnetic plane wave. A detailed review of the most important results in this field can be found in Ref. 1.

The probabilities of quantum processes induced by one particle in the field of a plane wave are regulated by two invariant parameters whose orders of magnitude are determined by the relations

$$\eta \sim \frac{eF}{m\omega}, \quad \chi \sim \frac{eF\varepsilon}{m^3} \sim \frac{\omega\varepsilon}{m^2}\eta, \tag{1}$$

where F is the amplitude of the wave field, ω is the frequency of the wave, and ε is the particle energy.[1)] The range of these parameters of greatest interest is that in which they are both at least of order unity. The processes occurring in the field of a wave are then of a multiphoton character and their probabilities become strongly nonlinear functions of the intensity of the field. The parameter η does not depend on the Planck constant and is a classical parameter of nonlinearity, while χ is responsible for the nonlinear quantum effects.

The development of compact optical-frequency lasers with pulse power $W \sim 10^{18}-10^{19}$ W/cm^2 has made it possible, comparatively recently, to check the equations of strong-field nonlinear quantum electrodynamics experimentally. This power level corresponds to a field intensity at the focal point $F \sim 10^{11}$ V/cm or $\eta \sim 1$. The parameter χ is also of the order of unity for electrons with energy $\varepsilon \sim 50$ GeV. A series of experiments checking quantum electrodynamics in strong fields is now underway in McDonald's group at SLAC.[2,3] The feasibility of such experiments has also been discussed in Ref. 4.

The use of ultrashort (pico- or even femtosecond) and sharply focused (the spot size at the focus is of the order of several wavelengths) pulses has made it possible to achieve laser intensities corresponding to $\eta \sim 1$. The amplitude of the field in such a pulse varies strongly in both space and time, and for this reason the justification for using a monochromatic plane wave model to describe a laser pulse in experiments in which the pulse collides with electrons or photons becomes somewhat dubious.

For a classical electron, the spatial and temporal nonuniformity of the amplitude of the laser field result, correspondingly, in ponderomotive scattering and surfing effects (see, for example, Ref. 5). In the former effect, the trajectory of an electron averaged over rapid oscillations is curved or the electron is even reflected from the pulse. In the second effect, the energy of an electron passing through the pulse changes. Both effects have now found direct experimental confirmation.[6] Moreover, the spectrum radiated by a classical electron is substantially different from that of a monochromatic plane wave.[7,8]

In the present paper, we present a quantum electrodynamic calculation of the angular and spectral distributions of the radiation from an electron in a collision with a laser pulse under conditions close to those of McDonald's experiment.

It is assumed that before the collision the laser and electron beams propagate toward one another. It is also assumed that the electrons are ultrarelativistic and the pulse is so short that during the interaction there is not enough time for the electron to be deflected appreciably from the initial direction of motion as a result of the ponderomotive effect. In this case, the spatial nonuniformity of the beam in the transverse direction can be neglected and hence the laser field can be modeled by a plane but, of course, not monochromatic wave.

As is well known, the Dirac equation possesses exact solutions for a plane wave field of arbitrary spectral composition—the so-called Volkov solutions, which we shall employ to describe the initial and final states of the electron. This makes it possible to advance far in the analytic calculations.

Admittedly, this formulation of the problem presupposes that the transverse size of the electron beam is small compared to the laser spot diameter, which is quite difficult to achieve experimentally. However, the results obtained in our approach can be interpreted as formulas for electrons colliding with a laser pulse with a definite value of the impact parameter. For this, the transverse size of the wave packet corresponding to the incident electron must be assumed to be large or at least of the order of the formation length of the radiation process on the one hand, and much smaller than the size of the laser focus on the other. Then, to describe a realistic situation, for which the transverse size of the electron beam is greater than the size of the focus, in our formulas the intensity of the field must be regarded as a given function of the impact parameter and averaging over the impact parameter must be performed.

As shown in the present work, in order that the condition formulated above be valid for $\eta \gtrsim 1$, the size R of the focal spot must be much greater than the characteristic wavelength λ of the laser radiation. We shall assume that the condition

$$R/\lambda \gg 1 \quad \text{or} \quad kR \gg 1 \tag{2}$$

(where $k=\omega=2\pi/\lambda$ is the characteristic wave number) is satisfied. Moreover, the pulse must be sufficiently short so as to be able to neglect the spreading of the packet. As will be evident from what follows, this condition and the condition for neglecting ponderomotive scattering are quite mild and they make it possible, specifically, to assume that for pulse duration τ we have

$$\omega\tau \gg 1, \tag{3}$$

which we shall assume to be the case in our calculations.

2. PROBABILITY OF PHOTON EMISSION BY AN ELECTRON IN A NONMONOCHROMATIC PLANE WAVE

We shall describe the field of a plane-wave laser pulse with the aid of the 4-potential

$$A^{\mu}=g\left(\frac{\varphi}{\omega\tau}\right)\{a_1^{\mu}\cos\varphi+a_2^{\mu}\sin\varphi\}, \tag{4}$$

where $\varphi=kx$, $k^{\mu}=(\omega, \mathbf{k})$ is the 4-vector of the wave, $a_{1\mu}$ and $a_{2\mu}$ are the amplitudes of the potential, which satisfy the conditions

$$k^2=0, \quad ka_1=ka_2=a_1a_2=0,$$

and $g(\varphi/\omega\tau)$ is the envelope of the potential, which we require to equal 1 at the center of the pulse, $g(0)=1$, and to decrease exponentially for $|\varphi|\gg\omega\tau$. Then the quantity τ can be regarded as the pulse duration.

In the present work, we shall assume for simplicity that the laser pulse is circularly polarized, meaning that $a_1^2=a_2^2$.

The Volkov solution in the field of a circularly polarized wave with the potential (4) has the form

$$\Psi_{\mathrm{p}r}=\left\{1+\frac{e(\gamma k)(\gamma A)}{2pk}\right\}\frac{u_{\mathrm{p}r}}{\sqrt{2p_0}}\exp(iS_{\mathrm{p}}), \tag{5}$$

where

$$S_{\mathrm{p}}=-px-\frac{\eta^2m^2}{2pk}\int_{-\infty}^{\varphi}d\varphi g^2\left(\frac{\varphi}{\omega\tau}\right)$$
$$-\frac{epa_1}{pk}\int_{-\infty}^{\varphi}d\varphi g\left(\frac{\varphi}{\omega\tau}\right)\cos\varphi$$
$$-\frac{epa_2}{pk}\int_{-\infty}^{\varphi}d\varphi g\left(\frac{\varphi}{\omega\tau}\right)\sin\varphi \tag{6}$$

is the classical action of an electron in the field with the potential (4), p^{μ} is the 4-momentum of an electron outside the pulse, $u_{\mathrm{p}r}$ is the free Dirac bispinor, normalized by the condition $\bar{u}_{\mathrm{p}r}u_{\mathrm{p}r}=2m$, and

$$\eta^2=-\frac{e^2a_1^2}{m^2}=-\frac{e^2a_2^2}{m^2}=\frac{e^2F^2}{m^2\omega^2} \tag{7}$$

(F is the amplitude of the field intensity at the center of the pulse). The solutions of Eqs. (5) form a complete orthonormal set.[1]

For laser pulses of optical frequency and duration of several picoseconds (or even tens of femtoseconds), the condition (3) holds conservatively. Then, neglecting terms $\sim 1/\omega\tau$, the expression (6) can be rewritten in the form

$$S_{\mathrm{p}}\simeq-\int dx_{\mu}q^{\mu}-g\left(\frac{\varphi}{\omega\tau}\right)\left(\frac{eqa_1}{qk}\sin\varphi-\frac{eqa_2}{qk}\cos\varphi\right), \tag{8}$$

where we have introduced the notation

$$q^{\mu}=p^{\mu}+g^2\left(\frac{\varphi}{\omega\tau}\right)\frac{m^2\eta^2}{2pk}k^{\mu}. \tag{9}$$

Therefore the classical action of an electron in the field (4) in the approximation $\omega\tau\gg1$ is identical to the corresponding expression for a monochromatic wave[1,9] with the difference that the 4-vector q_{μ} and the amplitudes of the potential in our case depend slowly on the variable φ. This structure of the classical action corresponds to separation of the motion of a classical electron for $\omega\tau\gg1$ into a systematic motion along a continuous trajectory and rapid oscillations with frequency ω around this trajectory (compare Sec. 30 in Ref. 10).

The dependence of q_{μ} on the variable φ is the so-called "ponderomotive" scattering effect.[5] As a result of this dependence, the 4-vector q_{μ}, in contrast to the case of a monochromatic wave, can no longer be regarded as the quasimomentum of the particle. However, once again, we shall call the quantity m_*, whose square equals the squared average kinetic momentum q_{μ}, the φ-dependent effective mass of an electron in the field (4):

$$m_*^2(\varphi)=q^2=m^2\left(1+\eta^2g^2\left(\frac{\varphi}{\omega\tau}\right)\right). \tag{10}$$

The element of the S matrix for a transition of an electron from the state $\Psi_{\mathrm{p}r}$ into the state $\Psi_{\mathrm{p}'r'}$ with the emission of a photon with momentum l and polarization e' has the form

$$S_{i\to f}^{(1)}=-ie\int d^4x\bar{\Psi}_{\mathrm{p}'r'}(\gamma e'^*)\Psi_{\mathrm{p}r}\frac{e^{ilx}}{\sqrt{2l_0}}, \tag{11}$$

where $\Psi_{\mathrm{p}r}$ and $\Psi_{\mathrm{p}'r'}$ are the wave functions (5) with S_{p} in the form (8).

The integrand in Eq. (11) is a linear combination of the quantities

$$\{1,\cos\varphi,\sin\varphi\}\exp\left[-i\alpha_1\left(\frac{\varphi}{\omega\tau}\right)\sin\varphi+i\alpha_2\left(\frac{\varphi}{\omega\tau}\right)\cos\varphi\right], \tag{12}$$

where

$$\alpha_i\left(\frac{\varphi}{\omega\tau}\right)=eg\left(\frac{\varphi}{\omega\tau}\right)\left(\frac{qa_i}{qk}-\frac{q'a_i}{q'k}\right), \quad i=1,2, \tag{13}$$

and equals, to within a slow dependence of the quantities α_i on φ, the corresponding expression in the field of a monochromatic wave.[9]

Strictly speaking, expressions of the type (12) are not periodic functions of φ, and in contrast to the case of a monochromatic wave they cannot be expanded in a Fourier series over the entire range of values of φ. However, any continuously differentiable function $h(\varphi, g(\varphi,\omega\tau))$ can be expanded in a Fourier series over the interval $[\varphi,\varphi+2\pi]$:

$$h\left(\varphi,g\left(\frac{\varphi}{\omega\tau}\right)\right)=\sum_{s=-\infty}^{\infty} h_s(\varphi)\exp(-is\varphi),$$

$$h_s(\varphi)=\frac{1}{2\pi}\int_{\varphi}^{\varphi+2\pi} d\varphi' h\left(\varphi',g\left(\frac{\varphi'}{\omega\tau}\right)\right)\exp(is\varphi').$$

Under the condition $2\pi/\omega\tau \ll 1$ the function g varies very slowly over the interval $[\varphi,\varphi+2\pi]$, so that taking account of the property

$$h\left(\varphi+2\pi,g\left(\frac{\varphi}{\omega\tau}\right)\right)=h\left(\varphi,g\left(\frac{\varphi}{\omega\tau}\right)\right),$$

we obtain for the coefficients

$$h_s(\varphi)\simeq\frac{1}{2\pi}\int_0^{2\pi} d\varphi' h\left(\varphi',g\left(\frac{\varphi}{\omega\tau}\right)\right)\exp(is\varphi')$$
$$+\frac{1}{\omega\tau}\frac{g'(\varphi/\omega\tau)}{2\pi}\int_{\varphi}^{\varphi+2\pi} d\varphi'(\varphi'-\varphi)$$
$$\times\frac{\partial h}{\partial g}\left(\varphi',g\left(\frac{\varphi}{\omega\tau}\right)\right)\exp(is\varphi').$$

Therefore, in the zeroth approximation in the parameter $1/\omega\tau$, the Fourier coefficients h_s depend on the initial point of the interval $[\varphi,\varphi+2\pi]$ only via the slowly varying function $g(\varphi/\omega\tau)$. This dependence is universal for the entire range of the variable φ, and therefore the formula

$$\{1,\ \cos\varphi,\ \sin\varphi\}\exp(-i\alpha_1\sin\varphi+i\alpha_2\cos\varphi\}$$
$$=\sum_{s=-\infty}^{\infty}\{B_{0s}B_{1s}B_{2s}\}\exp(-is\varphi), \tag{14}$$

holds up to terms $\sim 1/\omega\tau$ and, just as in the field of a monochromatic wave, the coefficients B_{0s}, B_{1s}, and B_{2s} can be expressed in terms of Bessel functions

$$B_{0s}=J_s(z)\exp(is\varphi_0),$$
$$B_{1s}=\frac{1}{2}\{J_{s+1}(z)\exp[i(s+1)\varphi_0]$$
$$+J_{s-1}(z)\exp[i(s-1)\varphi_0]\}, \tag{15}$$
$$B_{2s}=\frac{1}{2i}\{J_{s+1}(z)\exp[i(s+1)\varphi_0]$$
$$-J_{s-1}(z)\exp[i(s-1)\varphi_0]\},$$

whose argument is now a slow function of φ

$$z=\sqrt{\alpha_1^2\left(\frac{\varphi}{\omega\tau}\right)+\alpha_2^2\left(\frac{\varphi}{\omega\tau}\right)},\quad \cos\varphi_0=\frac{\alpha_1}{z},\quad \sin\varphi_0=\frac{\alpha_2}{z}. \tag{16}$$

For an element of the S matrix (11) we have, therefore,

$$S_{i\to f}^{(1)}=\frac{-ie}{\sqrt{8l_0p_0p_0'}}\sum_{s=-\infty}^{\infty}\int d^4x\bar{u}_{\mathbf{p}'r'}O_s\left(\frac{\varphi}{\omega\tau}\right)u_{\mathbf{p}r}$$
$$\times\exp\left\{i(l-sk)x+i\int dx_\mu(q'^\mu-q^\mu)\right\}, \tag{17}$$

$$O_s\left(\frac{\varphi}{\omega\tau}\right)=\left[(\gamma e'^*)+\frac{\eta^2m^2ke'^*}{2(qk)(q'k)}(\gamma k)g^2\left(\frac{\varphi}{\omega\tau}\right)\right]B_{0s}\left(\frac{\varphi}{\omega\tau}\right)$$
$$+\sum_{i=1}^{2}eg\left(\frac{\varphi}{\omega\tau}\right)\left[\frac{(\gamma a_i)(\gamma k)(\gamma e'^*)}{2q'k}\right.$$
$$\left.+\frac{(\gamma e'^*)(\gamma k)(\gamma a_i)}{2qk}\right]B_{is}\left(\frac{\varphi}{\omega\tau}\right). \tag{18}$$

We carry out subsequent calculations in a special coordinate system, in which the 3 axis is oriented along the wave vector $\mathbf{k}$ and the 1 and 2 axes are oriented along the corresponding polarization vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. Therefore in the special coordinate system

$$k^\mu=(\omega,0,0,\omega),\quad a_1^\mu=(0,a,0,0),\quad a_2^\mu=(0,0,a,0). \tag{19}$$

Since in our formulation of the problem the initial electron moves toward the laser pulse, we have in this system

$$p^\mu=(\varepsilon,0,0,-\sqrt{\varepsilon^2-m^2}). \tag{20}$$

Switching in Eq. (17) to integration over the variables $x_\pm=x^0\pm x^3$, which are the natural variables for a plane-wave field, we obtain

$$S_{i\to f}^{(1)}=\frac{-ie(2\pi)^3\tau}{\sqrt{8l_0p_0p_0'}}\delta(l^1+p'^1)\delta(l^2+p'^2)$$
$$\times\delta(l_-+p'_--p_-)\sum_{s=-\infty}^{\infty}M_s, \tag{21}$$

$$M_s=\int_{-\infty}^{\infty}d\xi\bar{u}_{\mathbf{p}'r'}O_s(\xi)u_{\mathbf{p}r}\exp[i\omega\tau G_s(\xi)], \tag{22}$$

$$G_s(\xi)=\left(\frac{l_++p'_+-p_+}{2\omega}-s\right)\xi+\frac{m^2\eta^2}{2}\left(\frac{1}{p'k}\right.$$
$$\left.-\frac{1}{pk}\right)\int_{-\infty}^{\xi}d\xi' g^2(\xi'). \tag{23}$$

Here the indices $\pm$ indicate the corresponding combination of the components of the momentum, for example, $l_\pm=l^0\pm l^3$, and the integration variable ξ equals

$$\xi=\frac{\varphi}{\omega\tau}=\frac{x_-}{\tau}. \tag{24}$$

For $\omega\tau\gg 1$, the partial amplitude M_s can be calculated by the stationary-phase method. The equation for the points ξ_* of stationary phase is

$$G_s'(\xi_*)=\frac{l_++p'_+-p_+}{2\omega}-s+g^2(\xi_*)\frac{m^2\eta^2}{2}\left(\frac{1}{p'k}-\frac{1}{pk}\right)=0, \tag{25}$$

or

$$\frac{l_+ + q'_+(\xi_*) - q_+(\xi_*)}{2\omega} - s = 0. \tag{26}$$

The equation (25) (or (26)) possesses real solutions only for positive values of s. Indeed, it follows from Eqs. (21) and (26) that at the points of stationary phase the law of conservation holds for all four components of the average kinetic momentum q_μ:

$$l^\mu + q'^\mu(\xi_*) = sk^\mu + q^\mu(\xi_*). \tag{27}$$

Hence

$$s = \frac{lq'(\xi_*)}{kq(\xi_*)} > 0. \tag{28}$$

For nonpositive values of s, Eq. (25) possesses only complex solutions. Then the partial amplitudes M_s can be calculated by the saddle-point method and are exponentially small.

For simplicity, we shall assume that $g(\xi)$ is an even function of its argument. Then Eq. (25) for $s \geqslant 1$ has two solutions which are located symmetrically relative to the center of the pulse:

$$\xi_*^{1,2} = \pm \xi_0^s, \quad \xi_0^s > 0, \tag{29}$$

and if these points are sufficiently far from one another, then the integral for M_s (22) can be represented as a sum of two terms, each of which corresponds to the contribution of one of the stationary-phase points

$$M_s = \sqrt{\frac{2\pi}{\omega\tau|G_s''(\xi_0^s)|}}\bar{u}_{\mathbf{p}'r'}\{\exp[i\omega\tau G_s(-\xi_0^s) + i\pi/4]O_s(-\xi_0^s) + \exp[i\omega\tau G_s(\xi_0^s) - i\pi/4]O_s(\xi_0^s)\}u_{\mathbf{p}r}. \tag{30}$$

In the next section we make more precise the meaning of the expression "sufficiently far from one another," but it is obvious at the outset that this condition will be violated if the points of stationary phase lie near $\xi = 0$, where the envelope $g(\xi)$ possesses a maximum and $G_s''(\xi)$ is close to zero. In this situation, the stationary-phase method for the integral (22) no longer works, in its pure form, and in order to calculate the integral the phase $G_s(\xi)$ must be expanded near $\xi = 0$ up to the term with the third derivative. As a result, M_s assumes the form

$$M_s = 2\left[\omega\tau|g''(0)|\frac{m^2\eta^2 lk}{2(pk)(p'k)}\right]^{-1/3} \times \exp[i\omega\tau G_s(0)]\Phi(y_s)\bar{u}_{\mathbf{p}'r'}O_s(0)u_{\mathbf{p}r}, \tag{31}$$

where

$$\Phi(y) = \int_0^\infty dx \cos\left(yx + \frac{x^3}{3}\right)$$

is the Airy function with argument

$$y_s = (\omega\tau)^{2/3}\left[|g''(0)|\frac{m^2\eta^2 lk}{2(pk)(p'k)}\right]^{-1/3} \times \left(s - \frac{l_+ + p'_+ - p_+}{2\omega} - \frac{m^2\eta^2 lk}{2(pk)(p'k)}\right). \tag{32}$$

We note that for $s \leqslant 0$ the argument of the Airy function is large and positive, i.e., the partial amplitudes with $s \leqslant 0$ are exponentially small. Therefore, taking into consideration the remark immediately following Eq. (28), we claim that only terms with $s \geqslant 1$ need be taken into account in the matrix element (21).

If now the normalization volume V, which we previously set equal to 1, is restored in our formulas and the relation[11]

$$[\delta(l^1 + p'^1)\delta(l^2 + p'^2)\delta(l_- + p'_- - p_-)]^2 = \frac{V}{(2\pi)^3}\frac{p_0}{p_-}\delta(l^1 + p'^1)\delta(l^2 + p'^2)\delta(l_- + p'_- - p_-),$$

is used, then we obtain for the probability of emission over the entire observation time

$$dW = \frac{e^2\tau^2}{8(2\pi)^3 p_-}\delta(l^1 + p'^1)\delta(l^2 + p'^2)\delta(l_- + p'_- - p_-)\sum_{s,s'=1}^{\infty} M_s M_{s'}^* \frac{d^3p'}{p_0'}\frac{d^3l}{l_0}. \tag{33}$$

Only terms with $s = s'$ need be retained in the double sum in Eq. (33). The point is that for $\omega\tau \gg 1$ the matrix element M_s is a rapidly oscillating function of the frequency. For matrix elements with different values of s, these oscillations are incoherent. Since any spectrometer is characterized by a finite resolution, the differential probability measured experimentally must be obtained from the expression (33) by averaging over a frequency interval determined by the resolution of the detector. Under such averaging, on account of the incoherent oscillations, terms with $s \neq s'$ in the double sum in Eq. (33) make a small contribution compared to terms in the sum with the same values of s, and they can therefore be dropped.

Switching next in the formula (33) from the variable p_3' to p_-' and integrating over the momenta of the recoil electron with the aid of a δ-function, we obtain

$$\frac{dW}{d\omega' d\Omega} = \frac{e^2\eta^2(\omega\tau)^2\omega'}{64\pi^3 m^4\chi^2}(1+u)\sum_{s=1}^{\infty}|M_s|^2, \tag{34}$$

where

$$\chi = \frac{\eta(pk)}{m^2}, \quad u = \frac{kl}{kp - kl},$$

and $\omega' = l_0$ is the frequency of the emitted photon.

This expression is the probability of emission of a polarized photon by a polarized electron. If the polarizations are not of interest, then this expression must be summed over the polarizations of the photon and recoil electron and averaged over the polarizations of the initial electron. The result is

$$\frac{dW}{d\omega' d\Omega} = \frac{e^2\eta^2(\omega\tau)^2\omega'}{32\pi^3 m^2\chi^2}\sum_{s=1}^{\infty}\left\{\frac{\eta^2}{2}(2 + 2u + u^2)(|F_{s,1}^{(1)}|^2 + |F_{s,-1}^{(1)}|^2) - \left(2 + 2u - u^2 + 2su\frac{\chi}{\eta}\right.\right.$$

$$-z^2(0)\frac{\chi^2}{\eta^4}\bigg)|F_{s,0}^{(0)}|^2-2\eta^2(1$$

$$+u)\mathrm{Re}(F_{s,0}^{(2)}F_{s,0}^{(0)*})\bigg\}, \quad (35)$$

where the integrals $F_{s,i}^{(k)}$ are defined as

$$F_{s,i}^{(k)}=\int_{-\infty}^{\infty}d\xi g^k(\xi)J_{s+i}[z(\xi)]\exp[i\omega\tau G_s(\xi)] \quad (36)$$

with

$$z(\xi)=(1+u)\frac{\eta g(\xi)}{m^2\chi}[e^2(a_1l)^2+e^2(a_2l)^2]^{1/2} \quad (37)$$

and, depending on the distance between the two solutions, the Eqs. (25) must be calculated either by the standard or a modified stationary-phase method.

3. FORMATION REGION OF THE PROCESS

We shall now study the radiation formation region or the coherence interval. By coherence interval we mean the region of space-time where the functions determining the matrix element (21), i.e., the functions B_{is} (15), the ξ-integral in Eq. (22), and the δ-function in Eq. (21), are formed.

The functions B_{is} are determined by the integrals over the phase φ from 0 to 2π, where the amplitude of the field can be regarded as a constant since $\omega\tau\gg 1$. Therefore their formation region is the same as in the case of the field of a monochromatic plane wave,[1] with the stipulation that in a pulsed field the coherence interval $\Delta\varphi$ is generally position-dependent: $\Delta\varphi\sim 2\pi$ for $\eta g(\varphi/\omega\tau)\lesssim 1$ and $\Delta\varphi\sim[\eta g(\varphi/\omega\tau)]^{-1}$ for $\eta g(\varphi/\omega\tau)\gg 1$. Since we are interested in the case $\eta\sim 1$, we assume that

$$\Delta\varphi\sim 2\pi \quad (38)$$

over the entire range of the phase.

In the stationary-phase method, every term in Eq. (30) is formed near the corresponding stationary-phase point in the interval

$$\Delta\xi\sim[\omega\tau|G_s''(\xi_0^s)|]^{-1/2} \text{ or } \Delta\varphi\sim[\omega\tau/|G_s''(\xi_0^s)|]^{1/2}.$$

In the special coordinate system, $|G_s''(\xi_0^s)|$has the form

$$|G_s''(\xi_0^s)|=g(\xi_0^s)|g'(\xi_0^s)|\frac{m^2\eta^2}{\omega}\frac{l_-}{p_-p_-'}$$

$$=g(\xi_0^s)|g'(\xi_0^s)|\frac{m^2\eta^2}{\omega}$$

$$\times\frac{\omega'(1+\cos\theta)}{2m\gamma[2m\gamma-\omega'(1+\cos\theta)]}, \quad (39)$$

where θ is the angle of emergence of the radiated photon with respect to the direction of the momentum of the initial electron, and frequency ω' of the photon is determined from the formula (25), which can be rewritten in the form

$$\omega'=\frac{4s\gamma^2\omega}{1+2\gamma^2(1-\cos\theta)+\left(\frac{2s\gamma\omega}{m}+\frac{1}{2}g^2(\xi_0^s)\eta^2\right)(1+\cos\theta)}. \quad (40)$$

In the formulas (39) and (40), we employed the fact that for an ultrarelativistic particle $p_-\simeq 2m\gamma$, $p\simeq m\gamma$, and $\gamma=\varepsilon/m$.

Since we are interested in the range of parameters $\eta\sim 1$, $\omega\gamma\sim m$, the first few harmonics, i.e., $s\sim 1$, will make a large contribution to the emission probability, just as in the case of a monochromatic wave. Then for photons emitted in a narrow cone with angle $\theta\sim 1/\gamma$ characteristic of the ultrarelativistic case, ω' is of the order of $\omega\gamma^2$. The points of stationary phase are located, by assumption, far from the maximum of the envelope g. Therefore it can be assumed that $g(\xi_0^s)|g'(\xi_0^s)|\sim 1$. As a result, we obtain the estimate

$$\Delta\varphi\sim\sqrt{\omega\tau}. \quad (41)$$

It follows immediately from this result that the representation (30) will be valid only when the distance between the points of stationary phase, equal to $2\xi_0^s$, is greater than $(\omega\tau)^{-1/2}$. In the opposite case, the coherence intervals overlap and the interference of radiation formed at the points $\pm\xi_0^s$ must be taken into account. This is achieved by using the representation (31) for M_s.

It the radiation is produced near the center of the pulse, the coherence interval is determined by the phase interval in which the Airy function in Eq. (31) is produced: $\Delta\varphi\sim[2(\omega\tau)^2/|G'''(0)|]^{1/3}$. Using arguments similar to those leading to the estimate (41), it is easily shown that for $\eta\sim 1$, $\omega\gamma\sim m$, and $\theta\sim 1/\gamma$

$$\Delta\varphi\sim(\omega\tau)^{2/3}. \quad (42)$$

This result determines more accurately the region of applicability of the representation (30). It follows from Eq. (42) that the representation (30) is valid if the distance $2\xi_0^s$ between the stationary-phase points is greater than $(\omega\tau)^{-1/3}$.

Therefore we can see that the process is characterized by two coherence intervals. The first one (38) characterizes the size of the region where the functions determining the nonlinear dependence of the emission probability on the intensity of the field are formed. The second one (41) (or (42)) characterizes the size of the region that dominates the probability of emission of a photon with given frequency and in a given direction is formed.

The same situation also occurs in the case of the field of a monochromatic wave.[1] For $\eta\sim 1$, as we have already noted, the first coherence interval is determined by the relation (38) and the second corresponds to the region of formation of the δ-function determining the law of conservation for the "+" component of the 4-vector q^μ, i.e. it equals infinity. In practice, it is sufficient to take N periods of the wave as the second coherence interval; this will ensure that the conservation law holds to within $\sim\omega/N$. This means that if the spectrometer is turned on for N periods of the wave, the angular distribution will transform from a δ-function distribution into a set of narrow lines of width $\sim\omega/N$. Therefore the second coherence interval in the field of the monochromatic wave is determined by the measurement accuracy required in a specific experiment. It is important, however, that this accuracy can in principle be arbitrary, since no matter how far away they are from one another, the first coherence intervals make the same contribution to the probability.

For the formulas for the emission probability in the field of a monochromatic wave to be applicable in our case of a pulsed field, the spectrometer must be turned on for a time $\Delta t \lesssim \sqrt{\tau/\omega}$ at the time when the electron is located at the periphery of the pulse. The width of the sth spectral line or harmonic is then determined not by the accuracy of the instruments employed in the experiment but by the duration of the pulse, and in principle cannot be less than $\sim \sqrt{\omega/\tau}$. Obviously, a necessary condition for the applicability of the monochromatic-wave formulas is $\sqrt{\omega\tau} \gtrsim 2\pi$.

This formulation of the experiment is hardly possible, however, in the case of a collision of a short laser pulse with an electron beam from an accelerator. In a real experiment, spectrometers record photons for time intervals $\gtrsim \tau$. Then the measured probability will be a sum of contributions from various second coherence intervals. These intervals are the maximum phase intervals where the amplitude of the field intensity can be assumed to be constant. Therefore the frequencies of the photons emitted in a given direction from neighboring coherence intervals differ by a quantity at least of order $\sqrt{\omega/\tau}$. As a result, the sth harmonic acquires a width that no longer depends on the pulse duration but is determined only by the maximum intensity of the field in the pulse, i.e., by the parameter η. For this reason, our formulas for the emission probability (35) do not turn into the corresponding formulas for the case of a monochromatic wave for any values of $\omega\tau$.

Another fundamental difference of a pulsed field from the field of a monochromatic wave is that a situation in which the distance between the centers of the two second coherence intervals, located on different sides of the center of the pulse, is at most of order $(\omega\tau)^{2/3}$ is possible. In this case, interference of the radiation produced near these points occurs, and this produces fine structure in the spectral lines.

All arguments in this section pertain to photon emission at angles $\theta \lesssim 1/\gamma$. Although this angular interval is of greatest interest, we note that for angles $\gamma\theta > 1$ the second coherence interval is substantially different from the values (41) or (42) found above. It is easy to see from Eqs. (39) and (40) that if a coherence interval is centered at the periphery of the pulse, then it is of order $\Delta\varphi \sim \sqrt{\omega\tau}|\gamma \tan(\theta/2)|$. It can also be shown that for coherence intervals centered near the center of the pulse $\Delta\varphi \sim (\omega\tau)^{2/3}|\gamma \tan(\theta/2)|^{2/3}$. In both cases the coherence interval increases with θ. This in turn means that as θ increases, the width of the harmonic decreases and the form of the angular distribution approaches the form characteristic of a monochromatic wave. The intensity of the lines, of course, decreases rapidly.

In closing this section, we determine the transverse coherence interval, i.e., the size of the region where the δ-functions in Eq. (21), which determine the conservation law for the transverse components of the momentum, are formed. The transverse momentum of a photon emitted at an angle $\theta \sim 1/\gamma$ is of order

$$l_\perp \sim \omega'/\gamma \sim \omega\gamma \sim m.$$

Therefore it is completely sufficient for the conservation law to hold with accuracy $\lesssim \omega$. The formation length $\gtrsim \lambda$ corresponds to this accuracy, and we shall adopt it for the transverse coherence interval.

4. SPECTRAL–ANGULAR DISTRIBUTION AND THE EMISSION SPECTRUM

In this section, we shall present computational results for the spectral–angular distribution (35) and the emission spectrum of an electron. An envelope of the form

$$g = \left(\cosh \frac{\varphi}{\omega\tau} \right)^{-1}, \tag{43}$$

where the laser frequency $\omega = 1.17$ eV, was used in the calculations.

As already noted, the spectral–angular distribution of the radiation in a pulsed field consists of a superposition of broadened lines or harmonics, each of which corresponds to a definite value of s in Eq. (35). The right-hand limit of the sth harmonic ω_c^s is determined from Eq. (40) with $g=0$:

$$\omega_c^s(\theta) = \frac{4s\gamma^2\omega}{1+2\gamma^2(1-\cos\theta) + \dfrac{2s\gamma\omega}{m}(1+\cos\theta)}. \tag{44}$$

This frequency corresponds to radiation produced at the periphery of the pulse and is identical to the frequency of a Compton photon emitted by an electron which has absorbed s photons of the wave. The left-hand limit of the harmonic ω_η^s corresponds to the frequency at which the argument of the Airy function (32) vanishes. It is determined from Eq. (40) with $g=1$ and is identical to the frequency of the sth harmonic emitted by an electron in the field of a monochromatic wave of intensity η

$$\omega_\eta^s(\theta) = \frac{4s\gamma^2\omega}{1+2\gamma^2(1-\cos\theta) + \left(\dfrac{2s\gamma\omega}{m} + \dfrac{\eta^2}{2} \right)(1+\cos\theta)}. \tag{45}$$

The width of the sth harmonic is thus determined by the formula

$$\Delta\omega'(\theta) = \frac{\omega_c^s \omega_\eta^s}{8s\gamma^2\omega} \eta^2(1+\cos\theta). \tag{46}$$

For $\eta \ll 1$, $\Delta\omega' \sim \eta^2$, for $\eta \sim \gamma\omega/m \sim 1$, $\theta \sim 1/\gamma$, and $\Delta\omega' \sim \omega'$ and for $\eta \gg 1$ $\Delta\omega'$ does not depend on η at all. The last remark is, admittedly, purely formal, since for $\eta \gg 1$ in Eq. (35), just as in the case of a monochromatic wave,[1] $s \sim \eta^3$ are effective and the concept of a harmonic becomes meaningless.

Figures 1a and b display the first two harmonics for a fixed angle of emission $\theta = 10^{-5}$ and various values of η. As one can see from the figures, the spectral lines possess fine structure, which results from interference of the radiation produced at two close points symmetrically disposed about the center of the pulse. This fine structure is described by oscillations of the Airy function in Eq. (31). Strictly speaking, the oscillations occur over the entire width of the line. However, their frequency increases as the right-hand limit of the harmonic is approached, and on averaging over a small

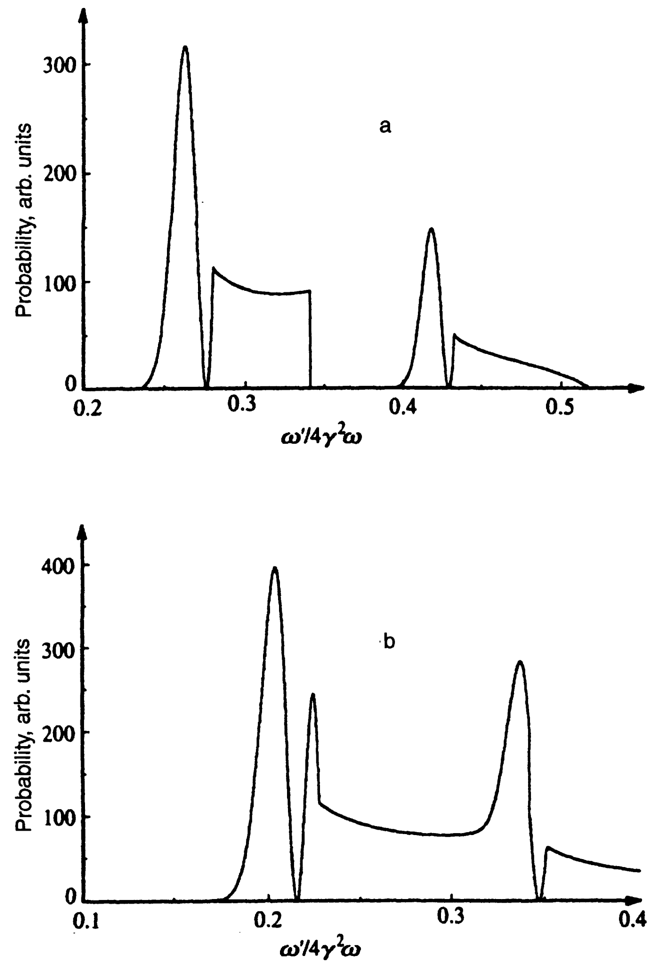

FIG. 1. First and second harmonics in the spectral–angular distribution of the probability, $\gamma=10^5$, $\omega\tau=50$, $\theta=10^{-5}$; $\eta=1$ (a), 1.5 (b).

range of frequencies corresponding to the resolution of the spectrometer, the contribution of the oscillating terms becomes vanishingly small. The magnitude of the frequency range over which the harmonics shown in Fig. 1 are averaged was set equal to 5% of the frequency ω', which corresponds to the resolution of spectrometers operating over an energy range of order 50 GeV.

As the parameter $\omega\tau$ increases, the frequency of the oscillations increases, and at some value of $\omega\tau$ the fine structure of the lines becomes indistinguishable for a spectrometer with fixed resolution (Fig. 2).

As η increases, the harmonics start to overlap at the same time that the line widths increase. The spectrum becomes continuous with a characteristic spiked structure, which is clearly seen in Figs. 1a and b and especially in Fig. 3, where three overlapping harmonics are presented. The position of the spikes is determined by the frequencies ω_η^s.

The condition for overlap of the sth and $(s+1)$-th harmonics has the form

$$\omega_\eta^{s+1}<\omega_c^s$$

or

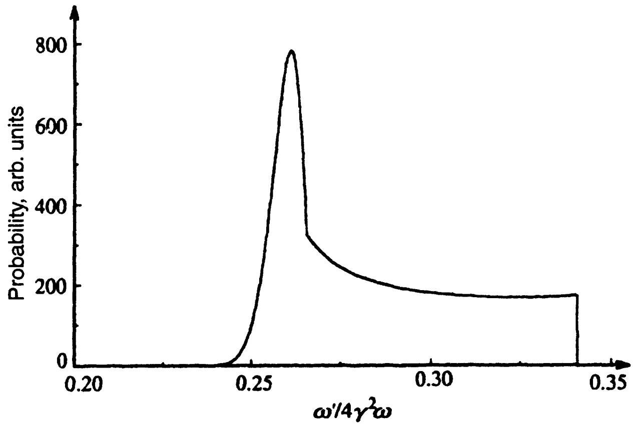

FIG. 2. First harmonic in the spectral–angular distribution of the probability, $\gamma=10^5$, $\omega\tau=200$, $\theta=10^{-5}$; $\eta=1$.

$$\frac{\omega_c^s}{4\gamma^2\omega}\left[\frac{2\gamma\omega}{m}+\frac{\eta^2}{2}\right](1+\cos\theta)>1. \tag{47}$$

Plots of the first harmonic with different values of the angle of emergence θ of the emitted photon are displayed in Fig. 4. It is clearly seen that the width of the harmonic decreases rapidly as θ increases, in complete agreement with the arguments presented in the preceding section.

The radiative spectrum is obtained by integrating the expression (35) over angles. In contrast to the case of a monochromatic wave, the integration can only be done numerically. The results are displayed in Fig. 6. The spectrum in the field of a monochromatic wave is presented for comparison in Fig. 5.

The spectrum in a monochromatic wave corresponding to the sth harmonic has a sharp limit at the frequency $\omega_\eta^s(0)$, i.e., the frequency of radiation in the direction of the momentum of the initial electron (Fig. 5).[2)]

The spectrum corresponding to the first harmonic in a pulsed field is displayed in Fig. 6a. In contrast to the spectrum in a monochromatic wave, it possesses an asymmetric peak at the point $\omega_\eta^1(0)$ and its right-hand limit is shifted to $\omega_c^1(0)$. We note that the distance between the maximum and

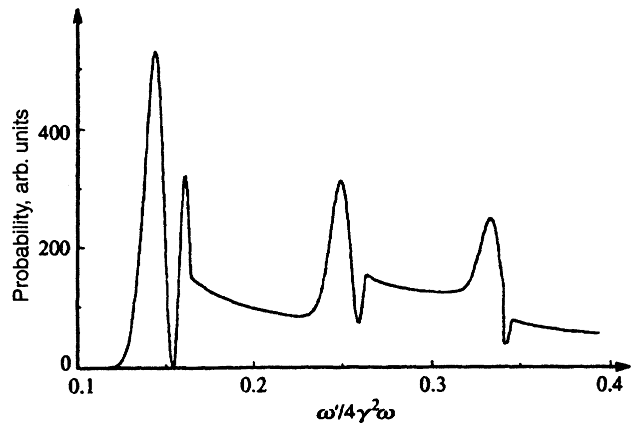

FIG. 3. First three harmonics in the spectral–angular distribution of the probability, $\gamma=10^5$, $\omega\tau=50$, $\theta=10^{-5}$; $\eta=2$.

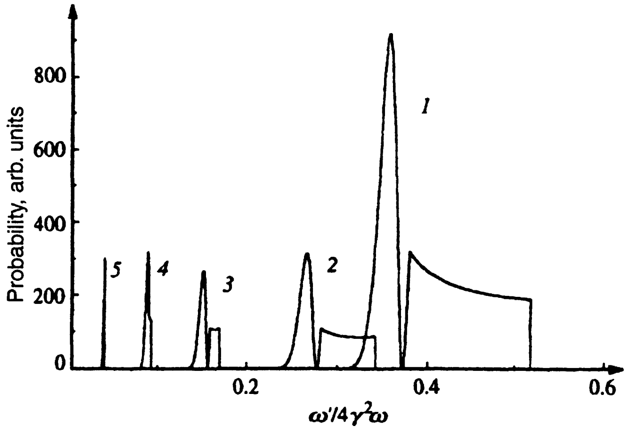

FIG. 4. First harmonic in the spectral–angular distribution of the probability for different values of θ: $\gamma=10^5$, $\eta=1$, $\omega\tau=50$, $\theta=0$ *(1)*, $1\cdot10^{-5}$ *(2)*, $2\cdot10^{-5}$ *(3)*, $3\cdot10^{-5}$ *(4)*, $5\cdot10^{-5}$ *(5)*.

its right-hand limit equals, as it should, the width of the spectral line emitted in the forward direction:

$$\Delta\omega'(0)=\frac{\omega_c^1(0)\,\omega_\eta^1(0)}{4\gamma^2\omega}\,\eta^2. \tag{48}$$

The spectrum with the contribution of the second harmonic taken into account is displayed in Fig. 6b. It is clear that the criterion for the observation of the second harmonic is the detection of photons with frequency greater than $\omega_c^1(0)$ and not $\omega_\eta^1(0)$, as would be the case in the field of a monochromatic wave.

We now discuss the applicability of our model of the field (4) to a focused laser pulse actually used in an experiment. Clearly, the nonuniformity of the field transverse to the direction of propagation can be neglected if the transverse size b of the packet describing the electron incident on the pulse and the deflection of the trajectory of the center of the packet on account of the ponderomotive effect are small compared to the size R of the laser focus. Moreover, in order for our results to be correct, the size b must be at least of the order of the transverse coherence interval, which in our problem we take to be of the order of the wavelength (see Sec. 3):

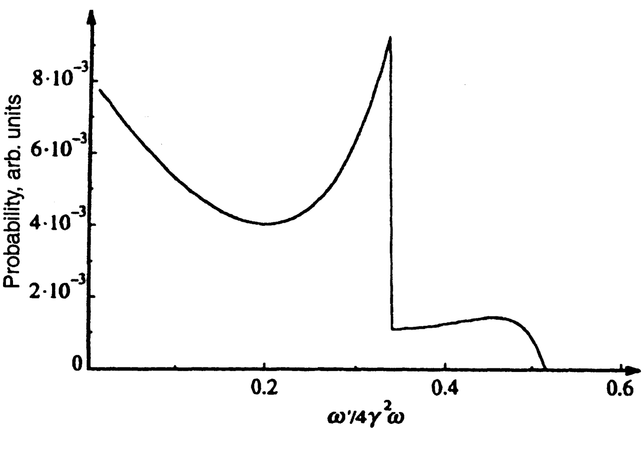

FIG. 5. Contribution to the spectral distribution of the probability from the first two harmonics in the monochromatic plane wave, $\gamma=10^5$, $\eta=1$.

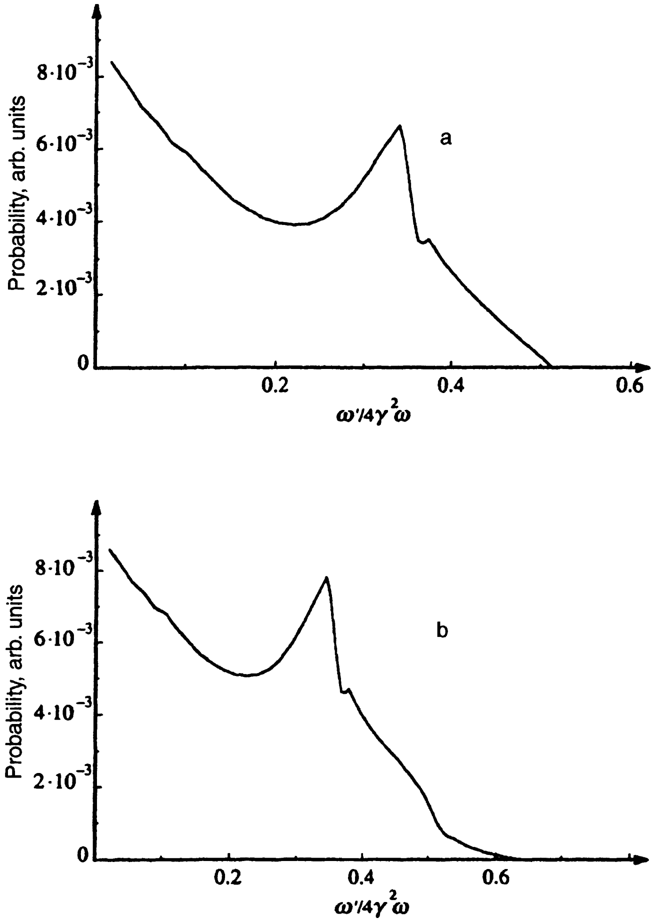

FIG. 6. Contributions from the first harmonic (a) and the first two harmonics (b) to the spectral distribution of the probability in the case of passage through a short laser pulse, $\gamma=10^5$, $\omega\tau=50$, $\eta=1$.

$$\lambda\lesssim b\ll R.$$

These conditions require that $\lambda\ll R$ or $kR\gg1$.

The motion of the center of the electron packet is described by the classical equations of motion. Specifically,[5]

$$\frac{dp_\perp}{d\varphi}=-\frac{m}{\omega p_-}\frac{\partial U}{\partial r_\perp},$$

where U is the ponderomotive potential, equal to

$$U=\frac{e^2\langle E^2\rangle}{2m\omega^2},$$

and averaging extends over the rapid oscillations of the field. In our case of a pulsed field, U depends on both the transverse coordinates and the phase φ, and it reaches a maximum at the center of the focus at the moment $\varphi=0$. Therefore

$$\left|\frac{\partial U}{\partial r_\perp}\right|<\frac{m\eta^2}{R},$$

$\Delta p_\perp$ is the increment in $p_\perp$ over the time τ, and it does not exceed a value of the order of $(m\eta^2/\gamma R)\tau$. For an ultrarelativistic electron $\Delta p_\perp\simeq m\gamma\Delta v_\perp$, and hence the upper limit of the transverse displacement of an electron over a time τ is of the order of

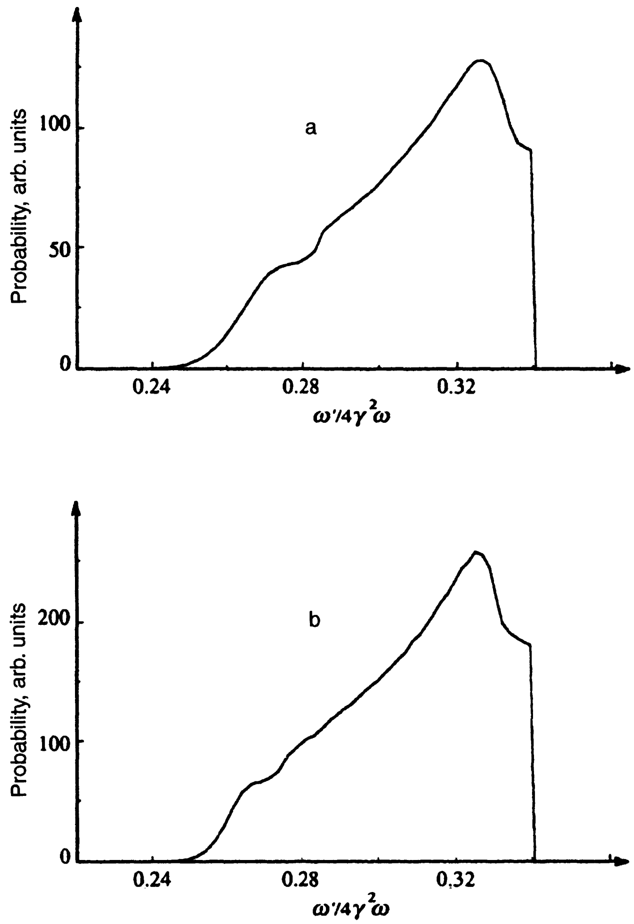

FIG. 7. First harmonic in the spectral–angular distribution of the probability for different values of $\omega\tau$ in the focused laser pulse, $\gamma=10^5$, $\theta=10^{-5}$, $\eta=1$, $\omega\tau=50$ (a) and 100 (b).

$$\Delta r_\perp \sim \Delta v_\perp \tau \sim \frac{\eta^2}{\gamma^2}\frac{\tau^2}{R}.$$

The relation $\Delta r_\perp \ll R$ must be satisfied, whence we obtain the condition

$$(kR)^2 \gg (\omega\tau)^2 \frac{\eta^2}{\gamma^2}. \tag{49}$$

Since in our problem η/γ is a small parameter $\eta/\gamma \sim 10^{-5}$, the condition (49) is quite mild and easily admits a situation in which $kR \ll \omega\tau$.

One more condition must be satisfied in order for our results to be applicable to a focused pulse. The spreading of the electron packet over the time during which it interacts with the laser pulse must be negligible. As is well known (see, for example, Ref. 13), the packet spreads most rapidly in the transverse direction, and it can be neglected at time t if $t/m\gamma b^2 \ll 1$. This imposes an upper limit on the possible pulse duration, which, taking account of the fact that $b \sim \lambda$, can be written in the form

$$\omega\tau \ll \frac{m\gamma}{\omega}, \tag{50}$$

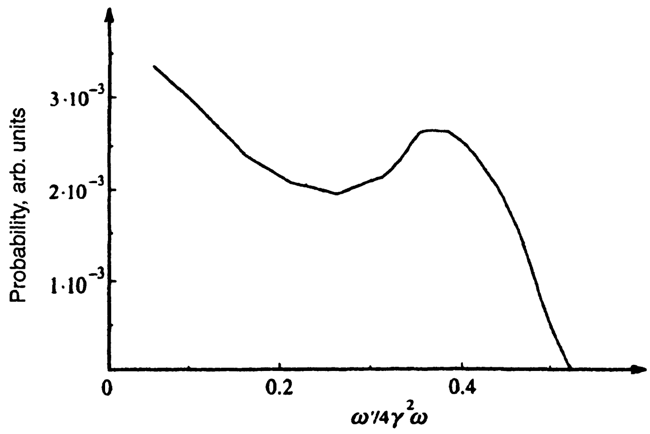

FIG. 8. Spectral distribution of the probability in a focused laser pulse, first harmonic, $\gamma=10^5$, $\omega\tau=50$, $\eta=1$.

where ω is an optical frequency and in our problem γ is very large, so that, for example, for $\gamma \sim 10^5$ the right-hand side of Eq. (50) is of the order of 10^{11} and therefore this condition does not interfere at all with the inequality $\omega\tau \gg 1$.

As a result, the results obtained in this work under the conditions (2), (49), and (50) can be used to describe the emission of a photon by an ultrarelativistic electron in a collision with a focused laser pulse at impact parameter ρ, if in our formulas η is interpreted as the maximum intensity of the field at a distance ρ from the center of the focus.

To obtain formulas for the emission probability which are valid in an experiment where a laser pulse collides with an electron beam whose radius is, as a rule, much larger than the laser focus, it is important to average the expression (35) over impact parameters in accordance with the formula

$$\left\langle \frac{dW}{d\omega' d\Omega} \right\rangle_\rho = \frac{1}{\pi R_e^2} \int_0^{R_e} 2\pi\rho d\rho \frac{dW}{d\omega' d\Omega}\left[\eta\left(\frac{\rho}{R}\right)\right]. \tag{51}$$

Here R_e is the effective radius of the electron beam, equal to the maximum impact parameter at which the influence of the radiation can be recorded in a given experiment.

Averaging over the impact parameters can radically change the form of the spectral–angular distribution of the radiation. Figures 7a and b display the first harmonics of the radiation for different values of the parameter $\omega\tau$. The function

$$\eta\left(\frac{\rho}{R}\right) = \eta \exp\left(-\frac{\rho^2}{R^2}\right)$$

was chosen for $\eta(\rho/R)$. One can see that the line profile has changed substantially. Its maximum has shifted appreciably to the right. This means that photons were more likely to be emitted from the periphery than from the center of the focus, which is simply related to an increase of $2\pi p$ in the weighting factor over the integral (51). Moreover, the fine structure of the line is smoothed out. Fine structure is manifested only in the nonmonotonic increase of the probability to the left of the peak for moderate values of $\omega\tau$; see Fig. 7b.

The changes in the spectral distribution of the probability are not so dramatic. Figure 8 displays the spectrum corresponding to the first harmonic averaged over the impact

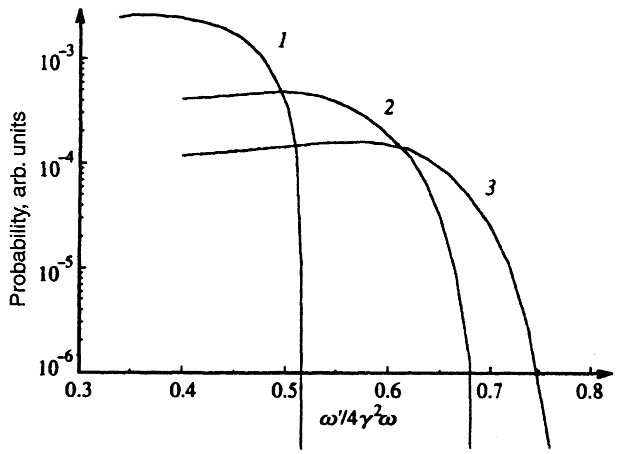

FIG. 9. Contribution to the spectral distribution of the probability in a short focused laser pulse from the first (1), second (2), and third (3) harmonics, $\gamma=10^5, \omega\tau=50$, $\eta=1$.

parameters. One can see that the averaging procedure broadens the sharp peak characterizing the spectrum of an individual electron (Fig. 6a). Moreover, its position shifts slightly toward the blue. The right-hand limit of the spectrum, of course, remains unchanged.

Figure 9 displays the impact-parameter-averaged spectra corresponding to the first three harmonics on a logarithmic scale. These plots agree qualitatively and the position of the right-hand limits agrees quantitatively with the experimental results of Refs. 2 and 3.[3)]

In conclusion, we note that although our calculations were all performed with a potential envelope $g(\varphi/\omega\tau)$ of the specific form (43), the results are qualitatively essentially independent of the form of the envelope.

Figure 10 displays the computational result for the first harmonic of the spectral–angular distribution of the emission probability for a Gaussian envelope

$$g\left(\frac{\varphi}{\omega\tau}\right)=\exp\left[-\frac{\varphi^2}{(\omega\tau)^2}\right]. \tag{52}$$

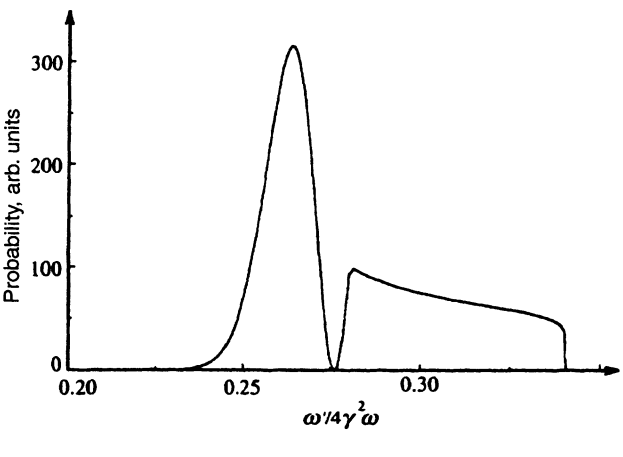

FIG. 10. First harmonic in the spectral-angular distribution of the probability for an envelope g of the form (52). The parameters are the same as in Fig. 1a.

The values of the parameters η, $\omega\tau$, and θ are taken to be the same as in the case displayed in Fig. 1a. It is clearly seen that the form of the first harmonic for the envelope (52) differs from the corresponding curve in Fig. 1a only by a small decrease in the probability in the violet part; this is explained by the fact that the field decreases more rapidly at the periphery of the pulse.

Our assertion that the results are virtually independent of the form of the envelope pertains, however, only to one-parameter curves, which make it possible to describe short pulses. In principle, one can conceive of an envelope that is characterized by the pulse rise and fall times τ_s and by the pulse duration τ_i during which the amplitude of the field remains constant. Of course, even in this case our result that the width of the spectral line is independent of the pulse duration remains valid for an experimental scheme in which the rise time of the spectrometer is greater than τ_i. Nonetheless, the shape of the line will be strongly dependent on the ratio of τ_i and τ_s. Specifically, in the case $\tau_i \gg \tau_s$ the contribution of the periphery will be negligible compared to the contribution of the region corresponding to the plateau of the envelope, and the larger the ratio τ_i/τ_s, the better the spectral–angular and spectral distributions will correspond to the monochromatic-wave model.

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[1)]We employ a system of units $\hbar=c=1$ and metric such that the scalar product of two 4-vectors is $AB=A^{\mu}B_{\mu}=A^0B^0-\mathbf{A}\cdot\mathbf{B}$.

[2)]The first numerical calculation of the emission spectrum in a monochromatic wave was performed in Ref. 2. See also Ref. 12.

[3)]Reported by Professor D. D. Meierhofer at the SILAP IV international conference in Moscow in 1995.

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