Study of the structure of the energy spectrum of the system "atom+strong external electromagnetic field"

E. A. Volkova, A. M. Popov, and O. V. Tikhonova

D. V. Skobel'tsin Scientific-Research Institute of Nuclear Physics, M. V. Lomonosov State University, 119899 Moscow, Russia

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Direct numerical integration of the time-dependent Schrödinger equation is used to study the dynamics of a quantum system with a short-acting potential, subject to the action of a high-frequency electromagnetic field with superatomic field strength and a low-frequency field of moderate intensity. The results of the numerical experiments carried out in this work indicate the formation of a set of stationary states of the Kramers–Henneberger potential which describes the atom in a superstrong high-frequency field. © *1996 American Institute of Physics.*
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1. INTRODUCTION

The interaction of atomic systems with an intense electromagnetic field has aroused great interest in connection with a number of unexpected results obtained quite recently.[1] One of these is the prediction of a stabilization effect of a quantum system in the field of a high-intensity electromagnetic wave. The stabilization is characterized by a decrease in the ionization probability of the system with growth of the intensity of the laser radiation. The first papers indicating the possibility of such behavior of the system were Refs. 2 and 3.

An analysis of the reasons for stabilization leads to two main mechanisms. One is connected with destructive interference of the amplitudes of transitions to the continuum with nearby high-lying Rydberg states.[4–7] The second mechanism is responsible for stabilization of the low-lying states and even isolated atomic states in the high-frequency limit of the laser radiation. The physical essence of the relevant processes may be conveniently illustrated by replacing the laboratory coordinate system by the Kramers coordinate system oscillating together with the atomic electron. In this case the initial atomic potential and the external laser action are replaced by some effective potential—the Kramers–Henneberger potential, which describes the system "atom +field of the electromagnetic wave."[8–12] The emergent new quantum system, the Kramers–Henneberger atom, is characterized by its own system of stationary states and corresponding wave functions, which differ substantially from the stationary states of the initial atom. At superatomic intensities of the wave field the localization region of the electron in the Kramers–Henneberger potential turns out to be of the order of twice the amplitude of the oscillations of the free electron in the electromagnetic field and significantly exceeds the size of the initial atom.[13–16] This leads to greater stability of such states with respect to the ionization process.

However, from the point of view of a formal approach the description of the dynamics of the system in the basis of atomic wave functions and in the basis of Kramers–Henneberger states are equally valid. In this regard the question arises, which description corresponds to the real states of the system and, correspondingly, is preferable.[15,17,18]

The question of the reality of the structure of the levels of the Kramers–Henneberger potential was posed in the investigation of the phenomenon of stimulated photocapture to bound states in the process of electron scattering by an atom in the presence of an intense, high-frequency electromagnetic field.[17] The results of numerical calculations indicate a process of resonant population of the states of the Kramers–Henneberger potential depending on the energy of the incident electron. Thus, for superatomic field intensities photocapture proceeds not to a state of the initial atom[19,20] but to states of the Kramers–Henneberger potential.[17] An indirect proof of the formation in strong optical fields of stable states in the Kramers–Henneberger potential is the dichotomy of the electron wave function discovered and investigated in Refs. 10, 12, 14, 15, and 21 and enhancement of the stability of such a formation against ionization as the radiation intensity is increased.

The question of the physical validity of the choice of one or another basis of states of the quantum system can be most explicitly resolved by experimental means by examining the response of the system to some external action. As such an external action it is natural to choose electromagnetic dipole radiation of comparatively high intensity, allowing us to examine the structure of the energy spectrum of the atom in the presence of a strong electromagnetic field.

In the present paper we carry out direct numerical integration of the time-dependent Schrödinger equation to investigate the dynamics of a one-dimensional quantum system with a short-acting potential for simultaneous action of a strong, high-frequency field and a weak, low-frequency field. The results obtained are a direct proof of the real existence of the Kramers–Henneberger potential and corresponding stationary states for an atomic electron in the presence of an intense electromagnetic wave field. On the basis of the numerical experiments we have constructed a simple analytical model of an atomic system in a biharmonic laser field, allowing us a qualitative description of the physics of the processes taking place in the considered system.

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2. QUANTUM SYSTEM IN THE FIELD OF AN ELECTROMAGNETIC WAVE: BASIS OF STATES OF THE UNPERTURBED ATOM AND KRAMERS–HENNEBERGER BASIS

In the one-dimensional single-electron approximation the evolution of an atomic system in the field of an electromagnetic wave can be obtained by solving the following equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x,t) - exE_0 \cos \omega t \psi(x,t), \tag{1}$$

where ψ is the wave function of the system, $V(x)$ is the atomic potential, and E_0 is the amplitude of the electromagnetic wave field with frequency ω_0.

Equation (1) should be augmented by an initial condition of the form

$$\psi(x,t=0)=\varphi_0(x), \tag{2}$$

where $\varphi_0(x)$ is the wave function of the electron up to the start of laser action. It is usually assumed that $\varphi_0(x)$ is an eigenfunction of the atomic Hamiltonian

$$H_0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x), \tag{3}$$

i.e., that at the initial time the system is found in one of the stationary states, e.g., in the ground state.

Representing the general solution of Eq. (1) in the form of a superposition of stationary states of H_0, i.e.,

$$\psi(x,t) = \sum_n C_n(t)\varphi_n(x)\exp\left(-\frac{i}{\hbar}\varepsilon_n t\right) + \int C_\varepsilon(t)\varphi_\varepsilon(x)\exp\left(-\frac{i}{\hbar}\varepsilon t\right)d\tau \tag{4}$$

(here φ_n and φ_ε are the eigenfunctions of the atomic Hamiltonian of the discrete and continuous spectra, belonging to the eigenvalues ε_n and ε, respectively), it is easy to obtain a system of equations for the time-varying probability amplitudes of population of the atomic states of the discrete C_n and continuous C_ε spectra. From a physical point of view, such an approach is interpreted as transitions between various stationary states of the atom under the influence of the electromagnetic wave.

An alternative approach to the problem in the case of high-intensity electromagnetic fields consists in transforming to the Kramers coordinate system, oscillating with the free electron in the electromagnetic wave field. In this coordinate system the Schrödinger equation reduces to the form

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x + a_e \cos \omega t)\psi(x,t) \tag{5}$$

($a_e = eE_0/m\omega_2$ is the amplitude of vibrational motion of the electron in the wave field) and corresponds to motion of the electron in the field of the rapidly oscillating atomic shell. Expanding the oscillating potential $V(x+a_e \cos \omega t)$ in a Fourier series

$$V(x+a_e \cos \omega t) = V_{\mathrm{KH}}(x,a_e) + \sum_{n\neq 0} V_n(x,a_e)e^{in\omega t}, \tag{6}$$

we rewrite Eq. (5) in the form

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V_{\mathrm{KH}}(x,a_e)\psi(x,t) + \delta V(x,a_e,t)\psi(x,t). \tag{7}$$

Here $V_{\mathrm{KH}}(x,a_e)$ is the zeroth term of the Fourier series expansion—the Kramers–Henneberger potential—and $\delta V(x,a_e,t)$ is the sum of all remaining harmonics.

The solution of Eq. (7) can be represented in the form of transitions between the various stationary states in the Kramers–Henneberger potential, taking place under the action of the sum of harmonics $V_n(X,a_e)e^{in\omega t}$. In strong fields the quantity δV can be considered as a small perturbation, falling off with growth of the intensity of the electromagnetic wave,[16] which leads to an enhancement of the stability of the Kramers–Henneberger states and the phenomenon of stabilization.

From a mathematical point of view, both considered approaches are identically possible, and it becomes the question of convenience, which one of them is preferable. The results of direct numerical integration of the time-dependent Schrödinger equation are of course insensitive to the choice of basis. However, the physical interpretation turns out to be substantially different. In strong fields the localization regions of the atomic wave functions and wave functions of the stationary states of the Kramers–Henneberger potential differ substantially. Therefore, finding the system in one of the bound states of the atomic Hamiltonian means, with probability close to unity, an ionization state in the Kramers–Henneberger basis, and *vice versa*.

3. NUMERICAL MODEL

In the present paper, as our model atomic potential we will use the square-well potential

$$V(x) = \begin{cases} -V_0, & |x| \leq d/2, \\ 0, & |x| < d/2, \end{cases} \tag{8}$$

with parameters $V_0 = 3.33$ eV and $d = 3$ Å. In this well there is only one bound state, with energy $\varepsilon \cong -2.0$ eV. For these potential parameters the "atomic intensity" of radiation $P_a = 10^{14}$ W/cm^2. Switching on a strong field with $\hbar\omega = 5$ eV and superatomic intensity P_1 to a significant extent deforms (destroys) the original atomic potential. The Kramers–Henneberger formalism in this case leads to the characteristic Kramers–Henneberger double-well potential with width $2a_e$ (see Fig. 1) and a system of stationary states with energies $\varepsilon_1^{\mathrm{KH}}$ ($i=1,2,\ldots$). It is important here that the number of stationary states in the Kramers–Henneberger potential grows as the radiation intensity increases. Table I gives the energies of the three lowest states of the Kramers–Henneberger potential for different values of the radiation intensity, and Fig. 2 graphs the probability density distributions $|\varphi_i^{\mathrm{KH}}|^2$ for $P_1 = 10^{16}$ W/cm^2 for the three states existing in this case.

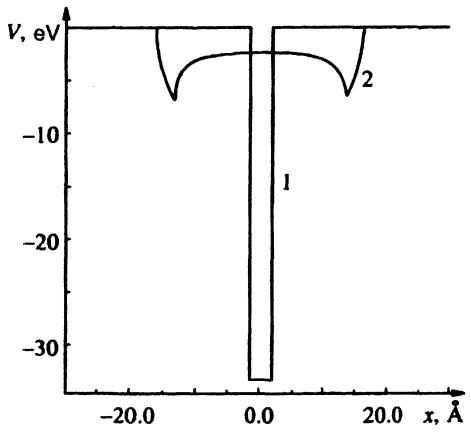

FIG. 1. Atomic potential (*1*) and Kramers–Henneberger potential (2) for radiation with intensity 10^{16} W/cm^2 and photon energy 5 eV.

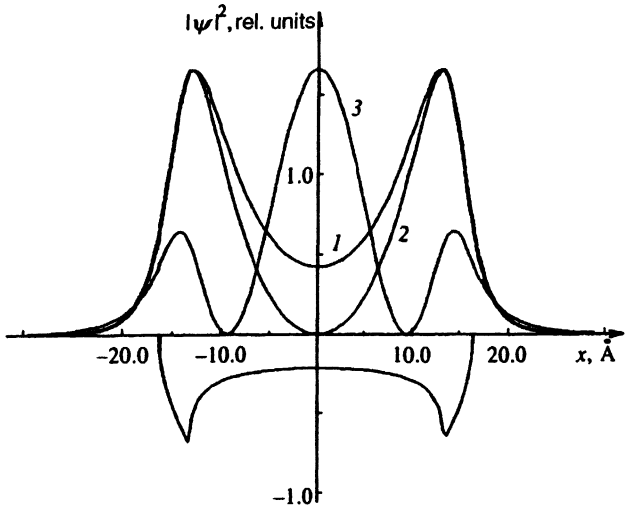

FIG. 2. Wave functions of the first (*1*), second (*2*), and third (*3*) stationary state in the Kramers–Henneberger potential corresponding to Fig. 1.

With the aim of revealing the real presence of such states of an atom in a strong field, we added a field with intensity $P_2=10^9-10^{10}$ W/cm^2 in the infrared frequency range $\hbar\omega_2=0.3-0.5$ eV. For a certain value of $\hbar\omega_2$ such an action can give rise to resonance transitions between the states of the discrete spectrum of the Kramers–Henneberger potential. From the physical point of view, it is precisely the presence of such resonances that allows us to determine the energy spectrum and potential parameters of the system "atom +high-frequency electromagnetic field."

In the coordinate system associated with the oscillating free electron in a strong field with frequency ω_1 the Schrödinger equation is written in the form

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+[V(x+a_e\cos\omega_1 t)-e(x+a_e\cos\omega_1 t)E_2\cos\omega_2 t]\psi(x,t), \tag{9}$$

$a_e=eE_1/m\omega_1^2$, and E_1 and E_2 are the amplitudes of the high-frequency and low-frequency fields.

By direct numerical integration of Eq. (9) we found an accurate exact solution for the wave function $\psi(x,t)$ for $P_1=10^{16}$ W/cm^2. Using this solution, we calculated the population dynamics of the various stationary states of the Kramers–Henneberger atom $W_i=|C_i(t)|^2$, $i=1,2,3$, and determined the probability of its ionization W as a function of the intensity and frequency of the probe pulse:

$$W_i(t)=|\langle\psi(x,t)|\varphi_i^{\mathrm{KH}}(x)\rangle|^2,$$

$$W(t)=1-\sum_{i=1}^{3}W_i(t). \tag{10}$$

Here $\varphi_i^{\mathrm{KH}}(x)$ is the wave function of the bound stationary state in the Kramers–Henneberger potential satisfying the equation

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V_{\mathrm{KH}}(x,a_e)\right]\varphi_i^{\mathrm{KH}}(x)=\varepsilon_i^{\mathrm{KH}}\varphi_i^{\mathrm{KH}}. \tag{11}$$

TABLE I. Energy of the lower three stationary states in the Kramers-Henneberger potential and their lifetimes for different values of the field intensity of the electromagnetic wave.

P_1, W/cm^2	$\varepsilon_i^{\mathrm{KH}}$, eV			τ_i^{KH}, fs		
3×10^{15}	−0.701	−0.379	–	16.9	27.3	–
1×10^{16}	−0.429	−0.343	−0.040	55.2	32.6	273
3×10^{16}	−0.276	−0.260	−0.114	373	235	983
1×10^{17}	−0.177	−0.176	−0.092	893	2390	3506

4. RESULTS AND DISCUSSION

4.1. Exponential decay of the Kramers–Henneberger states under the action of harmonics of the high-frequency electromagnetic field

First we separately analyzed the dynamics of the quantum system in only the high-frequency field with $\hbar\omega_1=5$ eV for various values of the radiation intensity. We assume that at the initial time the system is in one of the stationary states of the Kramers–Henneberger potential. Under this condition the probability of finding the system in this initial state as a function of time for $P_1=10^{16}$ W/cm^2 is shown in Fig. 3. As can be seen from the calculated results, the population of the states decays exponentially with characteristic time τ_i^{KH}, which is different for each of the different Kramers–Henneberger states. The lifetimes of the stationary Kramers–Henneberger states τ_i^{KH} are given in the table. The lifetimes and the corresponding widths of the states in the Kramers–Henneberger potential can be obtained in another way, e.g., by the Sturmian–Floquet method, as was done in Ref. 22. However, in our case the exponential nature of the decay follows directly from the numerical calculations without any additional assumptions. The rate of decay of these states $(\tau_i^{\mathrm{KH}})^{-1}$ is given approximately by the magnitude of the matrix element $\langle i|\delta V|e^{ikx}\rangle$ ($|i\rangle\equiv\varphi_i^{\mathrm{KH}}$, k is the wave number of the electron in the continuum) and falls off in the stabilization regime with increase of the radiation intensity. Note that at the radiation intensity

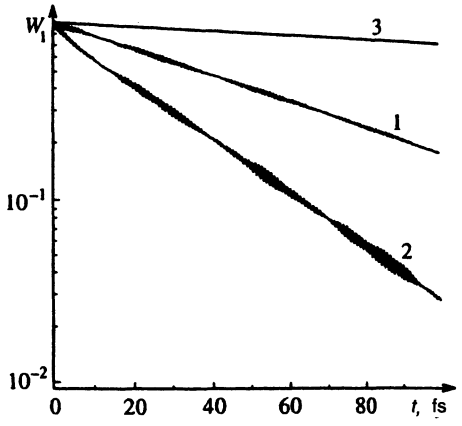

FIG. 3. Dynamics of the decay of the three lower stationary states in the Kramers–Henneberger potential for $P=10^{16}$ W/cm^2. The numbers labeling the curves correspond to Fig. 2.

$P_1=10^{16}$ W/cm^2 the decay times of the states in the Kramers–Henneberger potential satisfy the condition

$$\omega_{ji}\tau_i^{\mathrm{KH}}\gg 1, \tag{12}$$

where $\omega_{ji}=(\varepsilon_j^{\mathrm{KH}}-\varepsilon_i^{\mathrm{KH}})/\hbar$ are the frequencies of the transitions between the different states of the Kramers–Henneberger atom. This condition makes it possible to investigate the action of a low-frequency resonant field with frequency $\omega_2\cong\omega_j i$ without taking account in the zeroth approximation of the harmonics of the Kramers–Henneberger potential.

4.2. Analytical model

In the simplest case the action of a strong high-frequency field and a comparatively weak low-frequency field on the system under study can be treated in the approximation of a two-level system with outflow to the continuum.[23] The strong high-frequency field transforms the initial atomic potential into the Kramers–Henneberger potential with its set of states which decay under the action of the high-frequency harmonics of the strong field, each with its own characteristic decay time. The low-frequency field with frequency ω_2 singles out from the entire set of Kramers–Henneberger states two states, the frequency of the transition between which is close to the frequency of the low-frequency field (if such a field is present). Given this it is possible to limit the discussion to a two-level system in which the populations of the states vary due to possible transitions under the action of the low-frequency field and because of decay of the states under the action of the harmonics of the high-frequency field. In the case when the intensity of the high-frequency field $P_1=10^{16}$ W/cm^2 it turns out to be convenient to consider the transitions between the second and third states of the Kramers–Henneberger potential, the energy gap between which for the given value of P_1 is $\cong 0.30$ eV. In this case the frequency of the low-frequency field should be close to the indicated magnitude of the energy gap. Thus, the dynamics of the amplitudes of the populations of the second $C_2(t)$ and third $C_3(t)$ Kramers–Henneberger states after averaging over the rapidly oscillating terms is described by the system of equations

$$\frac{dC_2}{dt}=-\frac{C_2}{\tau_2}+C_3\frac{d_{32}E_0}{2i\hbar}e^{i\Delta\omega t},$$
$$\frac{dC_3}{dt}=-\frac{C_3}{\tau_3}+C_2\frac{d_{32}^*E_0}{2i\hbar}e^{-i\Delta\omega t}, \tag{13}$$

where τ_2 and τ_3 are the characteristic lifetimes of states $|2\rangle$ and $|3\rangle$ (see Table I), d_{32} is the dipole moment of the transition between these states, E_0 is the intensity of the low-frequency field, and $\Delta\omega$ is the detuning of the low-frequency field from the frequency of the transition between states $|2\rangle$ and $|3\rangle$. For $\hbar\omega_1=5$ eV and $P_1=10^{16}$ W/cm^2 the matrix element of the coordinate $x_{32}=d_{32}/e=5.55$ Å. Since the characteristic time $\tau_3\gg\tau_2$, we can neglect decay of the state $|3\rangle$ under the action of the high-frequency harmonics. The desired solutions for the population amplitudes $C_2(t)$ and $C_3(t)$ can be represented in the form

$$C_2(t)=(A_1\exp(\lambda_1 t)+B_1\exp(\lambda_2 t))e^{i\Delta\omega t},$$
$$C_3(t)=A_2\exp(\lambda_1 t)+B_2\exp(\lambda_2 t), \tag{14}$$

where

$$\lambda_{1,2}=-\frac{1}{2\tau_2}-\frac{i\Delta\omega}{2}\pm i\sqrt{\Omega^2-\frac{i\Delta\omega}{2\tau_2}}, \tag{15}$$

$$\Omega^2=\Omega_0^2+\left(\frac{\Delta\omega}{2}\right)^2-\left(\frac{1}{2\tau_2}\right)^2, \tag{16}$$

$$\Omega_0^2=\frac{|d_{32}|^2E_0^2}{4\hbar^2}, \tag{17}$$

and the coefficients $A_{1,2}$ and $B_{1,2}$ are found from the initial system of equations and the initial conditions for the populations of the Kramers–Henneberger states. Since under our conditions $\Delta\omega$ did not exceed 0.2 eV, we have $\Delta\omega/(2\tau_2\Omega^2)\ll 1$. In this case

$$\lambda_{1,2}\cong-\frac{1}{2\tau_2}-\frac{i\Delta\omega}{2}\pm i\Omega\left[1+\frac{1}{8}\left(\frac{\Delta\omega}{2\tau_2\Omega^2}\right)^2\right]\pm\frac{\Omega}{2}\frac{\Delta\omega}{2\tau_2\Omega^2}. \tag{18}$$

This expression leads to oscillations superimposed on the decay of the populations $W_2(t)$ and $W_3(t)$ of the two states under consideration with characteristic frequency 2Ω, where Ω is given by formula (16). Note that in the case of exact resonance ($\Delta\omega=0$) and neglecting the decay of state $|2\rangle$ under the action of the harmonics ($1/2\tau_2\ll\Omega_2$) the population oscillations take place with twice the frequency Ω_0, the expression for which coincides with the expression for the Rabi frequency in the two-level system in the presence of an electromagnetic field. For the parameter values used, the period of the population oscillations, calculated according to formula (17), is 105 fs for the intensity of the low-frequency field P_2 equal to 10^9 W/cm^2 and 33 fs for $P_2=10^{10}$ W/cm^2. Allowing for the finite lifetime τ_2 under resonance conditions leads to population oscillations with $\Omega=2\Omega_\tau$, where

$$\Omega_\tau^2=\Omega_0^2-(1/2\tau_2)^2. \tag{19}$$

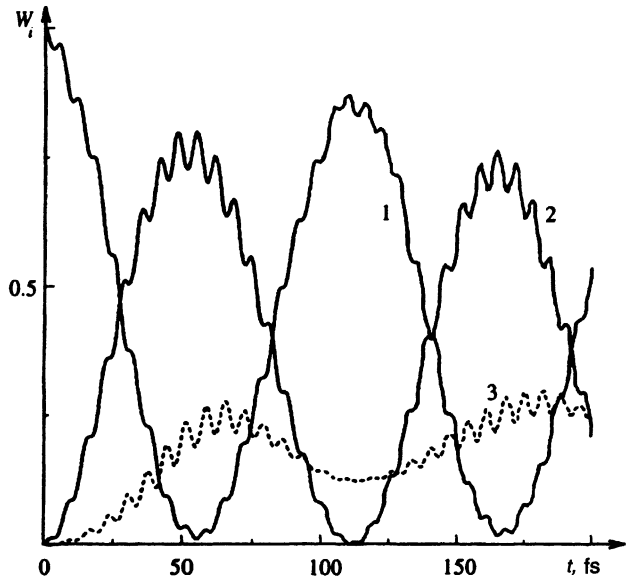

FIG. 4. Population dynamics of the second (*1*) and third (*2*) bound state in the Kramers–Henneberger potential, and also the continuum (*3*) in the presence of a low-frequency resonant field ($\hbar\omega=0.30$ eV) with intensity 10^9 W/cm^2. Harmonics of the the Kramers–Henneberger potential are not taken into account.

For $\tau_2=32.6$, fs, formula (19) leads to the value of the frequency of the population oscillations $\Omega_\tau=5.2\cdot 10^{13}$ s^{-1}, which corresponds to a period $T_\tau=2\pi/\Omega_\tau=120$ fs. If we also take account of decay of state $|2\rangle$ and the detuning from resonance, the population oscillations will take place with frequency 2Ω [formula (16)], which leads to values of the oscillation periods $T_1=40$ fs for $\Delta\omega=0.1$ eV and $T_2=20$ fs for $\Delta\omega=0.2$ eV.

Thus, based on the two-level model, we have obtained values of the periods of the oscillations of the populations of the first and second Kramers–Henneberger states for different intensities of the low-frequency field and different degrees of detuning of the frequency of the low-frequency field from resonance. It is of interest to compare the results of this analytical model with the results of numerical calculations of the dynamics of the populations of the investigated Kramers–Henneberger states.

4.3. Numerical calculations of the population dynamics of the Kramers–Henneberger states in the presence of a low-frequency probe field

The dynamics of the populations of the Kramers–Henneberger states $|2\rangle$ and $|3\rangle$ and the time-dependence of the probability of ionization of the system when the energy of a field quantum coincides with the distance between the levels for $P_2=10^9$ W/cm^2 without allowance for the high-frequency harmonics of the Kramers–Henneberger potential are shown in Fig. 4. It turns out that such an action indeed causes population oscillations in the system with probability of transition from state $|2\rangle$ to state $|3\rangle$, and *vice versa*, close to unity, and characteristic period $T=110$ fs. In this case, as a result of photo-ionization of state $|3\rangle$ under the action of the low-frequency field the total population of states $|2\rangle$ and $|3\rangle$ gradually decays. The observed picture is in good agreement with the analytical two-level model with Rabi oscillations appearing in it, including the value of the period of the oscillations calculated on the basis of this model.

The results of an exact solution of Eq. (9) for an atom in a biharmonic field, allowing for the simultaneous action of the low-frequency field with intensity 10^9 W/cm^2 and $\hbar\omega_2=0.30$ eV and the harmonics of the high-frequency field on the populations $W_2(t)$ and $W_3(t)$, are shown in Fig. 5a. These data were obtained for 100% population of state $|2\rangle$ at time $t=0$. Against the background of the decay of the states with decay times determined by the harmonics of the high-frequency field, the characteristic population oscillations are visible, of Rabi oscillation type with period $T=115$ fs, only a little longer in comparison with the case described above. Figure 5b plots the probability of ionization of the system as a function of time for the same conditions. The calculated curve has an undulatory character, which is a result of the different decay rates of states $|2\rangle$ and $|3\rangle$ (see Table I).

The results of our numerical experiment indicate that the system "atom+electromagnetic field" is characterized by a resonant transition at the frequency $\cong 0.30$ eV. This means that the Kramers–Henneberger states correspond precisely to the real physical states of the system. Figure 6a displays the dynamics of the populations of states $|2\rangle$ and $|3\rangle$ for the case in which the frequency of the probe field does not coincide

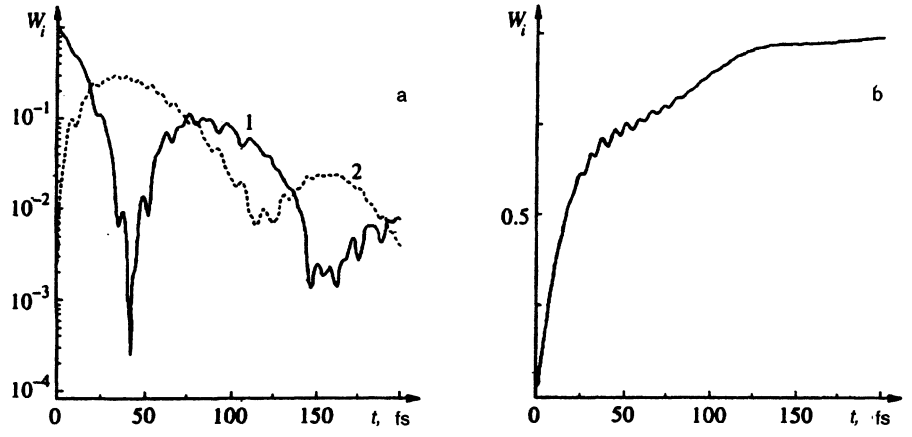

FIG. 5. Population dynamics of the second (*1*) and third (*2*) bound state in the Kramers–Henneberger potential (a), and the continuum (b) in the presence of a low-frequency resonant field ($\hbar\omega=0.30$ eV) with intensity 10^9 W/cm^2 with harmonics of the the Kramers–Henneberger potential taken into account.

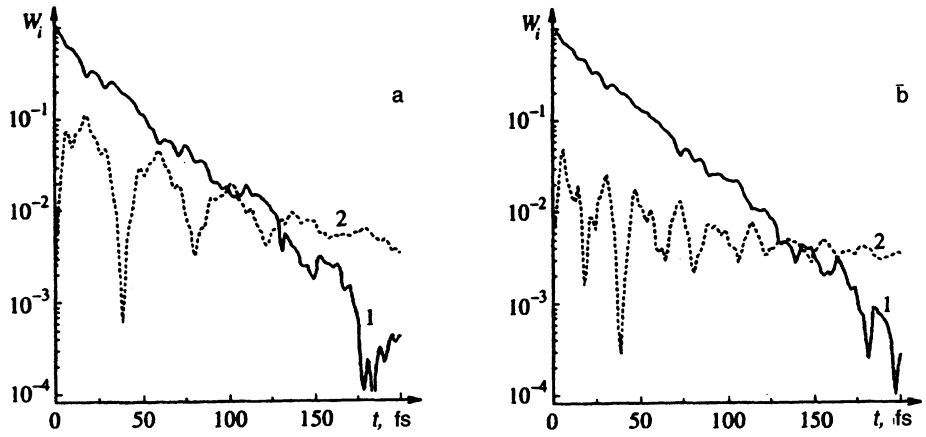

FIG. 6. Population dynamics of the second (*1*) and third (*2*) bound state in the Kramers–Henneberger potential (a), and the continuum (b) in the presence of a low-frequency resonant field with intensity 10^9 W/cm^2 and frequency $\hbar\omega = 0.40$ eV (a) and $\hbar\omega = 0.50$ eV (b).

with the frequency of the transition, ω_{32}. For the detuning $\Delta\omega = 0.1$ eV the oscillation period is 40–45 fs, which is in good agreement with the dependence of Ω on the detuning in formula (16). A decrease of the detuning $\Delta\omega$ to a value $\cong 0.2$ eV (Fig. 6b) leads to further decrease of the oscillation period and simultaneous decrease of the maximum value of the probability of finding the system in the state $|3\rangle$ down to a value on the order of $3 \cdot 10^{-2}$. As a result, the oscillations of the population of state $|2\rangle$ are practically invisible against the background of monotonic decay under the action of the high-frequency harmonics of the Kramers–Henneberger potential. In both cases with detuning from resonance, the period of the observed oscillations coincides with the value calculated from the two-level model.

In Fig. 7 one can observe similar oscillations of the populations $W_2(t)$ and $W_3(t)$ for the case of exact resonance, but a different intensity of the low-frequency field: $P_2 = 10^{10}$ W/cm^2. As could be expected, the oscillation period is decreased by roughly a factor of three in comparison with the results presented in Fig. 5a, which is in complete agreement with the calculational formula (17) for the Rabi frequency.

The above-mentioned results were obtained under the assumption that at the time the low-frequency field is switched on, the system is found in one of the stationary states of the Kramers–Henneberger potential. In the real situation the strong high-frequency field, which leads to the appearance of the Kramers–Henneberger potential, has a finite switching-on time, which causes all possible Kramers–Henneberger states to be populated, each with a definite probability. Therefore the question is of interest, what are the dynamics of an atomic system subject to the simultaneous action of a strong high-frequency field and a weak low-frequency field when the high-frequency field is switched on smoothly.

The question of the optimum duration of the front of the pulse to achieve the maximum probability of filling of the Kramers-Henneberger states has been discussed in a number of papers (see, e.g., Refs. 16 and 24). It appears possible to say that the optimum duration of the front τ_f is equal to roughly 2–6 periods of the wave field. Figure 8 plots the population dynamics of the states of the Kramers–Henneberger potential for the case where the high-frequency field is switched on over three periods ($\cong 2.5$ fs). The populations of the three states at the time the maximum of the amplitude of the field is reached turned out to be equal to

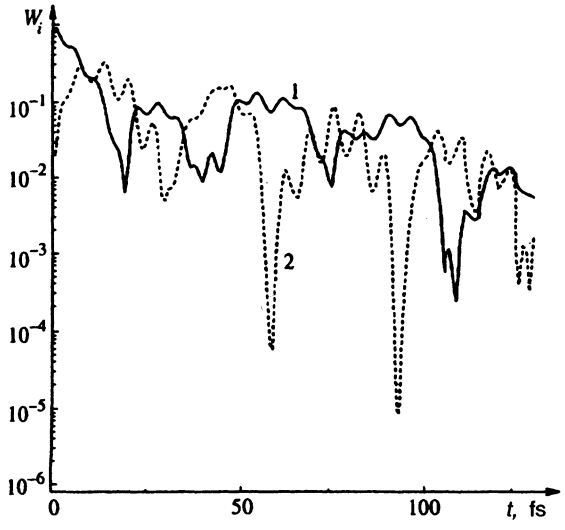

FIG. 7. The same as in Fig. 5, but for intensity of the low-frequency field equal to 10^{10} W/cm^2.

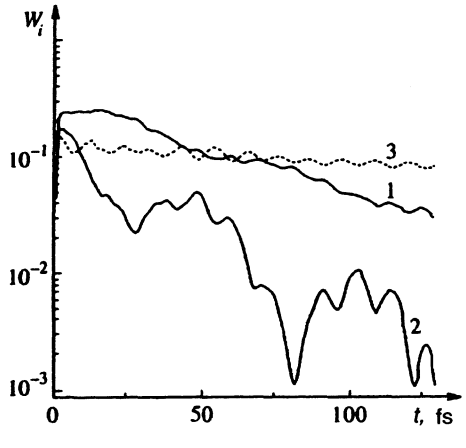

FIG. 8. Population dynamics of the first (*1*), second (*2*), and third (*3*) bound state in the Kramers–Henneberger potential after smoothly turning on the high-frequency field.

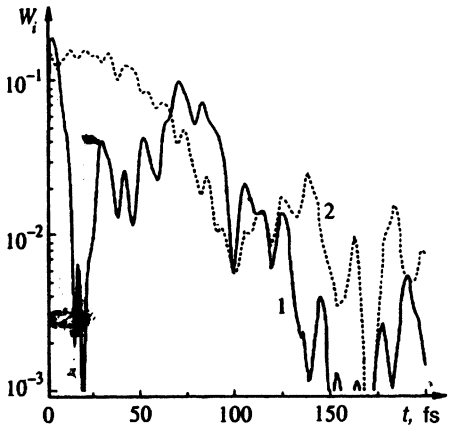

FIG. 9. The same as in Fig. 8, but in the presence of a low-frequency field with frequency $\hbar\omega = 0.30$ eV and intensity 10^9 W/cm^2.

$W_1 = 0.236$, $W_2 = 0.171$, and $W_3 = 0.145$. Further evolution of the states is governed by their decay under the action of the harmonics of the Kramers–Henneberger potential with characteristic decay times $\tau_1^{KH} \cong 55.5$ fs, $\tau_2^{KH} \cong 32.6$ fs, and $\tau_3^{KH} \cong 273$ fs. Figure 9 reflects the influence on the populations of states $|2\rangle$ and $|3\rangle$ of the Kramers–Henneberger potential of the resonance field for the case in which it is turned on instantaneously at the time $t = 0$. The high-frequency field, as before, takes a time $\tau_f \cong 2.5$ fs to reach its maximum amplitude. The characteristic oscillations of the populations, similar to those seen in Fig. 5a, are visible against the background of the decay of the states. Since the decay time of state $|3\rangle$ significantly exceeds the period of the oscillations, transitions between the states $|2\rangle$ and $|3\rangle$ are able to take place a considerable number of times before the population of state $|2\rangle$ becomes small due to its decay.

5. CONCLUSION

The numerical experiments which we have performed convincingly demonstrate that an atom in the presence of a strong high-frequency electromagnetic field is indeed described by the Kramers–Henneberger potential. A study of the energy levels of such a system with the help of a low-intensity electromagnetic probe field reveals the discrete structure of the levels corresponding to the Kramers–Henneberger approximation and differing substantially from the original atomic states. Since the system is characterized by a set of new stationary states over the duration of the strong laser pulse, the representation of the current solution of the time-dependent Schrödinger equation (9) in the form of a superposition of the original atomic states is not reasonable and does not lead to a satisfactory explanation of the obtained results. Thus, while the pulse acts, it is the Kramers–Henneberger atom that accurately represents the real physical system, which in the final analysis determines the set of basis wave functions.

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