Surface effects in a nonlocal critical-state model

V. S. Gorbachev and S. E. Savel'ev

Moscow Engineering-Physics Institute, 115409 Moscow, Russia
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A nonlocal critical-state model, which takes into account in a unified manner the effects associated with the nonlocal relationship between the magnetic induction and the vortex density, as well as the Bean–Livingston surface barrier, is presented. When the surface effects are considered in a nonlocal critical-state model, the presence of vortex-free regions near the boundary, whose thickness depends on the magnitude of the bulk critical current and the history of the sample, must be taken into account. It is shown that vortex-free regions, whose dimensions are comparable to the London penetration depth, naturally appear in the nonlocal critical-state model not only near the boundary, but also at sites of abrupt variation of the critical current density (for example, in layered superconductors) and should thus produce features on plots of the magnetization, susceptibility, etc. Experimental measurements of the remanent magnetization in superconducting particles having diameters comparable to the effective penetration depth are discussed. The experimental results cannot be described within the local critical-state model. The theoretical plots of the remanent magnetization calculated within the nonlocal model reproduce all the characteristic features of the experimental curves. Physically this is because a considerable part of the magnetic moment in small superconducting particles is created by Meissner magnetic fields, which do not contribute to the remanent magnetization. © *1996 American Institute of Physics.* [S1063-7761(96)02404-3]

1. INTRODUCTION

The electrodynamics of hard superconductors in external magnetic fields are usually described using the critical-state model (see, for example, Refs. 1–3). In this model it is assumed that shielding currents with a density equal to the critical current $\pm J_c$ are induced in a superconductor near the boundary in response to any variation of the external field. This is because the magnetic field penetrates a hard superconductor in the form of Abrikosov vortices, which are generated on the surface of the sample and move into it until the force created by the vortex gradient is equilibrated by the pinning force.

The traditional formulation of the critical-state model[1–3] does not take into account several phenomena. First, this model is local, i.e., it neglects the nonlocal relationship between the magnetic induction B and the density of the Abrikosov vortices n. This is justifiable only when spatial scales of the order of the London penetration depth λ are not considered. Thus, this model is unsuitable for describing superconducting samples with dimensions of the order of λ. In addition, it is not applicable to the description of the surface effects, which require consideration of the interaction of the vortices with their images and the Meissner currents, whose spatial scale is also of the order of λ. Second, the critical-state model usually does not consider the nonlocal effects associated with "reversible vortex displacement" in the bulk pinning potential, i.e., it does not take into account the fact that a vortex is displaced from its equilibrium position when there is a small change in the field. In this case the vortex moves reversibly in the pinning potential, and the current density is less than the critical value. When the displacement of a vortex becomes equal to the interaction distance d_0, the vortex is depinned.

The effects associated with the finite value of λ were first taken into account self-consistently in the nonlocal critical-state model introduced in Ref. 4. This model was described in detail in Ref. 5, in which reversible vortex motion in a pinning potential was also considered. The nonlocal critical-state model has not only a single equation relating the field to the bulk current density, but also a system of equations which relates the field induction to the vortex density and to the reversible vortex displacements. It was found that consideration of these effects leads to several features in the distributions of the magnetic field and the vortex density, which are not related to one another by the simple linear relation $B=n\Phi_0$ in the nonlocal model. When the external field is varied, a superconductor splits into two regions in the general case: a region where irreversible dissipative displacement of the vortices (detachment of the vortices from the pinning centers) takes place and a region where the displacement of the vortices has a reversible character. We shall refer to the former region (as in Ref. 5) as the critical region and the latter region as the subcritical region. The vortex density has a discontinuity on the boundary between these regions.[5] If the external field H_0 reaches a certain value $H_{\max}$ and then begins to decrease, no irreversible vortex displacement occurs in the range of fields $H_{\max}-\Delta H<H<H_{\max}$. If the amplitude of the variable field is less than ΔH, the hysteretic losses in the superconductor are equal to zero. In addition, if the superconductor has the form of a plate, there is always a region in the center of the plate where the vortex density satisfies $n=0$ for any value of $H_{\max}$. (Of course, the vortex density can be nonzero in the center of the plate, if, for

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example, the magnetic field is frozen in the plate when the transition to the superconducting state occurs.) This result disagrees qualitatively with the predictions of the traditional critical-state model, in which a region with $n=0$ exists only in a field weaker than the penetration field $H_p=(2\pi/c)J_c d$. We stress that all these features of the nonlocal model vanish when we go to the local limit, in which the model becomes the familiar Bean critical-state model.[1]

We also note that some of the features enumerated (particularly, the field interval ΔH) are shared by the model in Ref. 6, but it is a totally different model from the physical standpoint. The model in Ref. 6, which is local, takes into account the granular structure of a high-T_c superconducting ceramic and the finite value of the granule-depairing field. In the present work (as in Refs. 4 and 5) a homogeneous hard superconductor is considered.

The nonlocal critical-state model in Refs. 4 and 5 neglects the presence of the Bean–Livingston surface barrier,[7] which is associated with the interaction of vortices with an ideally flat surface of a superconductor. Therefore, the results in Refs. 4 and 5 are rigorously valid only for superconductors in which the Bean–Livingston barrier is suppressed (for example, due to the roughness of the surface or in strong magnetic fields).

The effects due to the existence of the Bean–Livingston barrier have been thoroughly studied for soft superconductors, in which bulk pinning can be neglected.[8,9] A microscopic description which considers individual Abrikosov vortices[8] is generally needed. However, a simple macroscopic model for describing soft superconductors in the mixed state was proposed in Ref. 9. In this model (for the case of an increasing external field) there is a vortex-free boundary region with a thickness l, which is specified by the relation

$$l=\lambda\ \text{arccosh}\left(\frac{H_0}{B}\right), \tag{1}$$

where H_0 is the external field and B is the magnetic induction. Here the magnetic induction is given by the expression $B=\sqrt{H_0^2-H_s^2}$, where $H_s=H_c=(\Phi_0/4\pi)\lambda\xi$ is the critical thermodynamic field, ξ is the coherence length, and Φ_0 is the magnetic flux quantum. We note that H_s is equal to the critical thermodynamic field H_c only in the case of an ideally smooth surface. The presence of roughness results in suppression of the Bean–Livingston barrier; therefore, in the general case $H_s\leqslant H_c$.

In recent years there has been a lively discussion of the appearance of the Bean–Livingston barrier in high-T_c superconductors (see, for example, Refs. 10–18). The relationship between the Bean–Livingston barrier and the features of magnetization curves was discussed in Ref. 10–12. In Ref. 13 it was shown that the presence of the Bean–Livingston barrier results in hysteresis of the critical current of a high-T_c superconducting ceramic. The features of the magnetic flux creep and relaxation through a surface barrier were investigated in Refs. 14 and 15. The surface impedance of a soft superconductor with a Bean–Livingston barrier in parallel and perpendicular magnetic fields was studied in Ref. 16.

The critical state in a superconducting plate with a Bean–Livingston barrier in a perpendicular magnetic field was studied in Refs. 17 and 18. In this case the surface barrier is renormalized due to the geometric factor. In addition, the interaction of the vortices has a power-function, rather than an exponential character. The latter creates a situation in which the vortex-free region near the surface is of the order of the thickness of the plate and not of the order of λ, as in the parallel geometry. Such a vortex-free region can be observed experimentally.[19]

In this paper we consider a superconductor in a parallel magnetic field. In this case the thickness of the vortex-free region is $l\leqslant\lambda$; therefore, correct consideration of the Bean–Livingston barrier for a hard superconductor requires consideration of the nonlocal effects, i.e., these effects cannot be taken into account self-consistently within the local Bean model in the general case. A model which self-consistently takes into account the nonlocal effects and the Bean–Livingston surface barrier is introduced below. Vortex-free regions appear in a natural manner in this model both near the surface and at sites of sharp variation of the critical current density. The thickness of these regions depends on the value of the bulk critical current and the history of the sample. It is shown that the magnitude of the interval ΔH, in which no irreversible vortex displacements occur as the external field varies, depends on the thickness of the vortex-free region.

The measurements of the remanent magnetization in ceramic particles having diameters comparable to the effective penetration depth in Ref. 20 are discussed next. These experimental results cannot be described within the traditional critical-state model. An anomalous decrease in the remanent magnetization is observed for superconducting particles with small diameters. The possibility of the presence of a vortex-free region, which appears when the external field is removed, was taken into account in Ref. 20 to explain this anomaly. Since the thickness of this region is $x_f<\lambda$, a rigorously correct treatment must consider the nonlocal effects. The theoretical plots of the remanent magnetization calculated within the nonlocal critical-state model reproduce all the characteristic features of the experimental curves. The principal physical cause of this anomaly is that in small superconducting particles a considerable part of the magnetic moment is created by Meissner magnetic fields, which do not contribute to the remanent magnetization.

2. MODEL

We use the nonlocal critical-state model[4,5] to investigate the surface effects. We consider a hard type-II superconductor in the form of an arbitrary cylinder in an external field H_0 parallel to its generatrix (the z axis). We assume

$$H_{c1} \ll H_0 \ll H_{c2}, \tag{2}$$

where H_{c1} and H_{c2} are the lower and upper critical fields.

For a macroscopic description the characteristic scale d, on which the averaging of the microscopic equations is performed, must be much greater than the characteristic distance between vortices a. Another characteristic scale is the London penetration depth λ. When $d \geqslant \lambda$ holds, we obtain the local model, for which the relation $B=n\Phi_0$ is valid. If the microscopic equations are averaged on scales $d \ll \lambda$, we arrive at the nonlocal critical-state model. As was noted in the Introduction, an additional spatial scale, viz., the thickness of the vortex-free region l, appears when the surface effects are investigated; therefore, the condition $l \gg a$ must also hold.

Hence we obtain the following conditions for applicability of a macroscopic description of the surface effects in the nonlocal model:

$$\lambda \gg a, \tag{3}$$

$$l \gg a. \tag{4}$$

If these conditions do not hold, a microscopic approach must be used in the general case. We shall show that a macroscopic approach is applicable when (2) holds. In fact, both relations (3) and (4) are valid in this case. The validity of (3) is obvious, since in the fields (2) the mean distance between vortices a satisfies the relation $\xi \ll a \ll \lambda$. Let us prove the validity of (4). In fields $H_0 \sim H_s$ from (1) we have $l \sim \lambda$, i.e., the condition (4) is satisfied. In fields $H_0 \gg H_s$ we obtain $l \sim \lambda H_s / H_0 \sim a\sqrt{H_{c2}/H_0} \gg a$. Thus, the condition (4) breaks down only in fields $H_0 \sim H_{c2}$.

The derivation of the equations of the nonlocal critical-state model was given in Ref. 5. The magnetic induction B, which reflects the averaged microscopic field, is determined by the equation

$$B\mathbf{e}_z + \lambda^2 \operatorname{curl}\operatorname{curl} B\mathbf{e}_z = \Phi_0 n \mathbf{e}_z \tag{5}$$

with the boundary condition $B=H_0$, where $\mathbf{e}_z$ is a unit vector parallel to the z axis. The solution of this equation can be represented in the form of the sum of the vortex (B_v) and Meissner (B_m) components, which satisfy the equations

$$B_v\mathbf{e}_z + \lambda^2 \operatorname{curl}\operatorname{curl} B_v\mathbf{e}_z = \Phi_0 n \mathbf{e}_z, \tag{6}$$

$$B_m\mathbf{e}_z + \lambda^2 \operatorname{curl}\operatorname{curl} B_m\mathbf{e}_z = 0 \tag{7}$$

with the boundary conditions $B_m = H_0$ and $B_v = 0$. The physical meaning of the latter boundary condition is that the magnetic flux of a vortex tends to zero as it approaches the superconductor boundary. The solution (6) can be represented in the form

$$B_v = \int d^2\boldsymbol{\rho}'\, n(\boldsymbol{\rho}') h_1(\boldsymbol{\rho},\boldsymbol{\rho}'), \tag{8}$$

where h_1 is the magnetic field of a single vortex located at the point $\boldsymbol{\rho}'$. The function h_1 satisfies the equation

$$h_1\mathbf{e}_z + \lambda^2 \operatorname{curl}\operatorname{curl} h_1\mathbf{e}_z = \Phi_0 \mathbf{e}_z(\boldsymbol{\rho}-\boldsymbol{\rho}') \tag{9}$$

with the boundary condition $h_1(\boldsymbol{\rho}_{\text{surf}},\boldsymbol{\rho}')=0$, where $\boldsymbol{\rho}_{\text{surf}}$ is the coordinate of the superconductor surface. The magnetic flux of a single vortex located at the point $\boldsymbol{\rho}$ is defined by the expression

$$\Phi(\boldsymbol{\rho}) = \int d^2\boldsymbol{\rho}'\, h_1(\boldsymbol{\rho}',\boldsymbol{\rho}).$$

Near the superconductor surface $\Phi(\boldsymbol{\rho})$ is related to the Meissner component B_m of the field by the expression[21]

$$\Phi(\boldsymbol{\rho}) = \Phi_0(1 - B_m(\boldsymbol{\rho})/H_0), \tag{10}$$

which takes the form $\Phi = \Phi_0(1-\exp(-x/\lambda))$ for a half-space.

The second equation of the model is obtained from the variation of the Gibbs free energy G, which can be represented as the sum of the electromagnetic energy $G_{e/m}$ and the work of the pinning forces G_p^{crit}. In this paper, in contrast to Ref. 5, the effects associated with reversible vortex displacement in the pinning potential are disregarded for simplicity. All the results in Ref. 5 pertaining to these effects remain valid even when the Bean–Livingston barrier is taken into account.

After averaging the free energy on scales $d \gg a$, we obtain (the averaging procedure was presented in Ref. 5)

$$G_{e/m} = \frac{1}{8\pi}\int d^2\boldsymbol{\rho}(\Phi_0 n B_v - 2H_0 B_v). \tag{11}$$

The latter expression is not exact. As was shown in Ref. 5, it is calculated with a relative accuracy $\sim H_{c1}/H \ll 1$, which corresponds in fields $H \gg H_{c1}$ to neglect of the difference between the magnetic induction B and the field strength $H_{\text{eq}}(B)$. All the calculations below are performed with this accuracy.

The work of the pinning forces G_p^{crit} has the form

$$G_p^{\text{crit}} = \int d^2\boldsymbol{\rho}\, n(\boldsymbol{\rho})\mu_p. \tag{12}$$

Here the integration is carried out over the critical region. The notation for the pinning potential μ_p was introduced in Eq. (12). It is defined so that the variation of μ_p associated with a vortex displacement $\mathbf{u}$ is equal to the force f_p acting on the vortex. In the critical region $f_p = \pm P_c$, where P_c is the maximum pinning force, and in the subcritical region the inequality $-P_c < f_p < P_c$ holds.

To obtain the material equation describing the balance between the forces acting on a vortex, the free energy $G=G_{e/m}+G_p$ must be varied with respect to vortex displacements $\mathbf{u}$. (We note that in Ref. 5 the free energy was varied with respect to the vortex density n. As will be seen from the following, variation with respect to $\mathbf{u}$ is more convenient.) The variation of the vortex density has the form

$$\delta n=-\operatorname{div}(n\,\delta\mathbf{u}).$$

Varying the electromagnetic part of the free energy $G_{e/m}$ (see Appendix 2 in Ref. 5), we obtain

$$\delta G_{e/m}=-\frac{\Phi_0}{4\pi}\int d^2\boldsymbol{\rho}\,\operatorname{div}(n\,\delta\mathbf{u})\left(B_v-H_0\frac{\Phi(\boldsymbol{\rho})}{\Phi_0}\right). \quad (13)$$

Adding the contribution from the variation of G_p to (13) and using the expression (10) for the flux of a single vortex Φ, for the variation of the total free energy we obtain

$$\delta G=\frac{\Phi_0}{4\pi}\int d^2\boldsymbol{\rho}\,\delta\,\mathbf{u}\;n(\boldsymbol{\rho})\left(\nabla B-\frac{4\pi}{\Phi_0}\frac{\partial\mu_p}{\partial\mathbf{u}}\right). \quad (14)$$

Setting δG equal to zero, we find that wherever the vortex density n is nonzero, the following balance equation of the forces must be satisfied:

$$\nabla B=\frac{4\pi}{\Phi_0}f_p. \quad (15)$$

The magnetic induction need not obey this equation in regions where $n=0$, since the condition $\delta G=0$ is satisfied identically in that case.

The expression (15) is the usual expression of the critical state (this becomes obvious, if it is taken into account that $B=(\mathbf{B})_z$ and $|\nabla B|=|\operatorname{curl}\mathbf{B}|$). We stress, however, that in deriving it (as in Ref. 5) we explicitly took into account the Meissner currents and the boundary conditions for the magnetic field; therefore, (15) is valid in the immediate vicinity of the boundary.

Let us consider the boundary conditions when the Bean–Livingston barrier is taken into account and the external magnetic field H_0 increases. Two alternatives are possible. First, the barrier can prevent the entry of vortices into the superconductor. In this case the total number of vortices in the superconductor does not vary:

$$\int d^2\boldsymbol{\rho}\;n(\boldsymbol{\rho})=\text{const.} \quad (16)$$

Second, vortices can penetrate into the superconductor. The condition for the penetration of a vortex into a superconductor is equality of the total electromagnetic force acting on it to zero. Ultimately, in analogy to Ref. 9, we obtain

$$\frac{\partial B}{\partial l}=-\frac{H_s}{\lambda}, \quad (17)$$

where $\partial/\partial l$ is the derivative in the direction of the internal normal to the superconductor surface. In deriving (17) we disregarded the bulk pinning, since we assumed that the bulk critical current density satisfies $J_c \ll cH_s/4\pi\lambda$. This is justified, since $cH_s/4\pi\lambda$ coincides in order of magnitude with the depairing current, and all real critical currents are usually much smaller.

It follows from the conditions (16) and (17) that a vortex-free region appears near the surface as the external field increases. In fact, if there is no vortex-free region, the following condition holds on the boundary:

$$|dB/dx|=(4\pi/c)J_c<H_s/\lambda.$$

As H_0 increases, the vortices move deeper into the superconductor, but new vortices cannot enter the superconductor in this case, since the condition (17) does not hold. A vortex-free region appears near the surface. At a certain value of the external field the condition (17) begins to hold, and new vortices enter the superconductor. The subsequent entry of vortices does not result in filling of the vortex-free region, since the current near the surface exceeds the critical value, and vortices cannot remain in this region. The new vortices are "blown" deeper into the superconductor by the large Meissner surface currents. From the mathematical standpoint, such a region must exist, because the solution of the balance equation (15) of the forces (which is a first-order equation) cannot simultaneously satisfy the two boundary conditions $B=H_0$ and (17). Equation (5) with $n=0$, which defines the field in the vortex-free region, is a second-order equation, and its solution can satisfy the two boundary conditions.

When the external field decreases, the boundary conditions change. Strictly speaking, to derive them we must investigate the stability of the vortices nearest the surface with respect to small displacements from their equilibrium positions. However, in the macroscopic approach these conditions can be derived from simple arguments. When the external field decreases, the vortex-free region near the surface begins to shrink. As long as the thickness of the vortex-free region is nonzero, vortices do not exit the superconductor, and the condition (16) holds. In a certain external field H_{exit} the thickness of the vortex-free region becomes equal to zero. When the field diminishes further, there is no vortex-free region, vortices exit the superconductor, and the balance equation (15) of the forces is valid right on the boundary. In this case the boundary condition $B=H_0$ alone is sufficient.

Now we can formulate the basic assumptions of the nonlocal critical-state model with consideration of the Bean–Livingston surface barrier. For simplicity we restrict ourselves to the one-dimensional case, which alone will be considered below.

The basic equations of the model can be written in the form

$$\frac{dB}{dx}=\frac{4\pi}{c}J, \quad (18)$$

$$B-\lambda^2\frac{d^2B}{dx^2}=\Phi_0 n. \quad (19)$$

To construct the solutions of the nonlocal model, we must also formulate an algorithm to take into account the magnetic history. Let the distributions of the magnetic induction B_{old} and the vortex density n_{old} corresponding to a certain value of the external field H_0 be known to us. Let the external field vary. Then a critical region appears near the surface in the general case. In this region the vortex density n differs from n_{old}, but the current density J, whose absolute value equals the critical current density $J_c=(4\pi/\Phi_0)P_c$, is a known quantity. This enables us to find the magnetic induction in this region, using Eq. (18) with $J=\pm J_c$. Now there can be different boundary conditions, depending on whether or not the vortex-free boundary region exists. Knowing the magnetic induction, we can find the new vortex density in the critical region from (19). In the subcritical region the variation of the electromagnetic forces is insufficient for depinning of the vortices, and with neglect of the reversible displacements the vortex density remains the same as before the external field was varied, i.e., $n=n_{old}$. The distribution of the magnetic induction is found from (19), and the current density must be determined from (18). We only know *a priori* that $|J|\leq J_c$.

The matching conditions on the boundary between the critical and subcritical regions were discussed in Ref. 5. They are continuity of the magnetic induction and the current density. Moreover, these conditions are valid, even if the vortex density n has a discontinuity at the matching point. This is not so in the case of a δ-function feature on n (an example in which this possibility is realized is presented below).

3. SURFACE EFFECTS

In this paper we shall not discuss the features of the distribution of the magnetic induction B and the vortex density n in the interior of a homogeneous superconductor. They were discussed in detail in Refs. 4 and 5. Here we shall focus our main attention on the boundary effects. We shall examine these effects in the case of a semi-infinite superconducting space $x>0$.

Let the external field H_0 be equal to zero initially, and let there be no frozen magnetic flux within the superconductor. As H_0 increases, there is only a Meissner state at first. Vortices do not penetrate the superconductor until the condition (17) is satisfied. It is satisfied only when $H_0=H_s$. As the field increases further, there appears a vortex-free region of thickness l, beyond which (for $l<x<b$) there is a critical region, where $dB/dx=-(4\pi/c)J_c$. For $x>b$ the density n equals zero (as in Ref. 5, we shall call the point $x=b$ the penetration depth). The expressions for the field $B(x)$ and the density $n(x)$ have the form (Fig. 1a)

$$B(x)=\begin{cases} H_0\cosh\dfrac{x}{\lambda}-H_s\sinh\dfrac{x}{\lambda} \quad \text{for } 0<x<l,\\ H_0\cosh\dfrac{l}{\lambda}-H_s\sinh\dfrac{l}{\lambda}-\dfrac{4\pi}{c}J_c(x-l)\\ \quad\text{for } l<x<b,\\ \dfrac{4\pi}{c}J_c\lambda\exp\left(-\dfrac{x-b}{\lambda}\right) \quad \text{for } x>b, \end{cases} \tag{20}$$

$$n(x)=\begin{cases} 0, \quad \text{for } 0<x<l,\\ \dfrac{1}{\Phi_0}\left(H_0\cosh\dfrac{l}{\lambda}-H_s\sinh\dfrac{l}{\lambda}-\dfrac{4\pi}{c}J_c(x-l)\right)\\ \quad\text{for } l<x<b,\\ 0, \quad \text{for } x>b, \end{cases} \tag{21}$$

where l and b are defined implicitly by the equations

$$H_0\sinh\frac{l}{\lambda}-H_s\cosh\frac{l}{\lambda}=-\frac{4\pi}{c}J_c\lambda, \tag{22}$$

$$H_0\cosh\frac{l}{\lambda}-H_s\sinh\frac{l}{\lambda}-\frac{4\pi}{c}J_c\lambda=\frac{4\pi}{c}J_c(b-l). \tag{23}$$

Let us discuss the features of the solution obtained. When we go to the soft-superconductor limit $J_c\to 0$, it coincides with the model in Ref. 9. In this case $b\to\infty$, Eq. (22) transforms into (1), the vortex density is constant ($n=\text{const}$), and the magnetic induction equals

$$B=\Phi_0 n=\sqrt{H_0^2-H_s^2}. \tag{24}$$

This expression gives the magnetic induction appearing in a soft superconductor when the field increases.[8,9]

In the case of a hard superconductor, the density n depends on the coordinates, and instead of (24) we have

$$\Phi_0 n(l)=\sqrt{H_0^2-H_s^2+\left(\frac{4\pi}{c}J_c\lambda\right)^2}. \tag{25}$$

If the external field H_0 increases to the value H_{max} and then begins to decrease, a critical region fails to appear in the superconductor in a certain range of fields $H_{max}-\Delta H<H_0<H_{max}$. In this range of fields the vortex density remains constant everywhere in the superconductor, and the pinning force acting on the vortices near the boundary varies from $+P_c$ to $-P_c$. The magnetic induction also varies, since the Meissner component of the field varies. In this range of fields the hysteretic losses in the superconductor are equal to zero. The interval ΔH is proportional to $J_c\lambda$ and depends on the geometry of the system (it was calculated for a semi-infinite space and a plate without consideration of the Bean–Livingston barrier in Refs. 4 and 5). When the surface barrier is taken into account, the expression for ΔH has the form

$$\Delta H=\frac{8\pi}{c}=J_c\lambda\exp\frac{l}{\lambda}. \tag{26}$$

It is seen that the presence of the vortex-free region near the surface leads to an increase in the barrier ΔH.

When the external field decreases further, the change in the electromagnetic force can no longer be balanced by a corresponding change in the pinning force. A critical region appears, and the vortices are irreversibly displaced toward the surface; however, no vortices exit the superconductor, and the thickness of the vortex-free region l_d simply decreases. The distribution of the magnetic induction and the vortex density in this case has the form (Fig. 1b)

$$
B(x)=\begin{cases} H_0 \cosh \frac{x}{\lambda}-\frac{\sinh(x/\lambda)}{\cosh(l_d/\lambda)}\left(H_0 \sinh \frac{l_d}{\lambda}-\frac{4\pi}{c}J_c\lambda\right) & \text{for } 0<x<l_d, \\ H_0 \cosh \frac{l_d}{\lambda}-\frac{\sinh(l_d/\lambda)}{\cosh(l_d/\lambda)}\left(H_0 \sinh \frac{l_d}{\lambda}-\frac{4\pi}{c}J_c\lambda\right)+\frac{4\pi}{c}J_c(x-l_d) & \text{for } l_d<x<x_0, \\ H_{\max} \cosh\frac{l}{\lambda}-H_s \sinh \frac{l}{\lambda}-\frac{4\pi}{c}J(x-l)-\frac{8\pi}{c}J_c\lambda \exp\left(-\frac{x-x_0}{\lambda}\right) & \text{for } x_0<x<b, \\ \frac{4\pi}{c}J_c\lambda \exp\left(-\frac{x-b}{\lambda}\right)-\frac{8\pi}{c}J_c\lambda \exp\left(-\frac{x-x_0}{\lambda}\right) & \text{for } x>b, \end{cases} \quad (27)
$$

$$
\Phi_0 n(x)=\begin{cases} 0 & \text{for } 0<x<l_d, \\ H_0 \cosh \frac{l_d}{\lambda}-\frac{\sinh(l_d/\lambda)}{\cosh(l_d/\lambda)}\left(H_0 \sinh \frac{l_d}{\lambda}-\frac{4\pi}{c}J_c\lambda\right)+\frac{4\pi}{c}J_c(x-l_d) & \text{for } l_d<x<x_0, \\ H_{\max} \cosh \frac{l}{\lambda}-H_s \sinh \frac{l}{\lambda}-\frac{4\pi}{c}J_c(x-l) & \text{for } x_0<x<b, \\ 0 & \text{for } x>b, \end{cases} \quad (28)
$$

where l and b are determined from Eqs. (22) and (23) when $H=H_{\max}$. The position of the boundary between the critical and subcritical regions x_0 is determined from the continuity of the magnetic induction and the current density, and the thickness of the vortex-free region l_d is determined from conservation of the total number of particles (16). (The transcendental equations specifying l_d and x_0 are not presented, because they are quite involved). When the external field decreases further, the vortex-free region vanishes, and vortices exit the superconductor (Fig. 1c).

Therefore, two barriers should be distinguished. The first is the barrier in Ref. 5, which prevents irreversible vortex displacement in the superconductor. It is determined from the bulk critical current density, and this barrier, of course, is not present in a soft superconductor. The second barrier prevents the entry of vortices into the superconductor and their exit from it. This barrier was investigated for a soft superconductor in Refs. 8 and 9. In the case of a hard superconductor, the field for the entry of vortices is specified by (25), and the exit field H_{exit} can be found from the condition $l_d=0$. Figures 2 and 3 present plots of l_d and H_{exit} as functions of the external field for various values of the critical current density. We stress that both effects can be taken into account systematically and self-consistently within the nonlocal critical-state model.

We also note another special feature of the solutions obtained. The value of ΔH in the presence of the Bean–Livingston surface barrier depends on the thickness of the vortex-free region near the surface [see (26)]. At the same time, for a given value of the external field H_0 the thickness of the vortex-free region depends on the history of the sample. For this reason ΔH depends on the history.

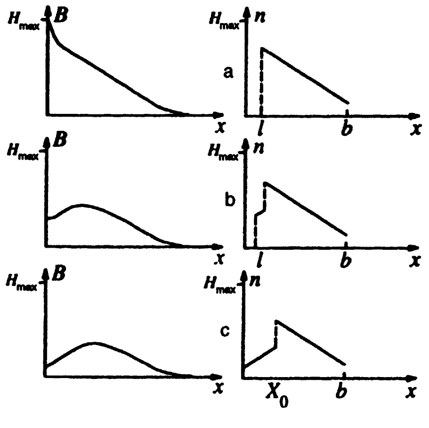

FIG. 1. Distribution of the magnetic induction (left-hand column) and the vortex density (right-hand column) in the half-space $x>0$ in the presence of the Bean–Livingston barrier: a) initial increase in the external field; b) decrease in the external field, during which vortices do not exit the superconductor and only the thickness of the vortex-free region decreases; c) further decrease in the external field, after which there is no vortex-free region and vortices exit the superconductor.

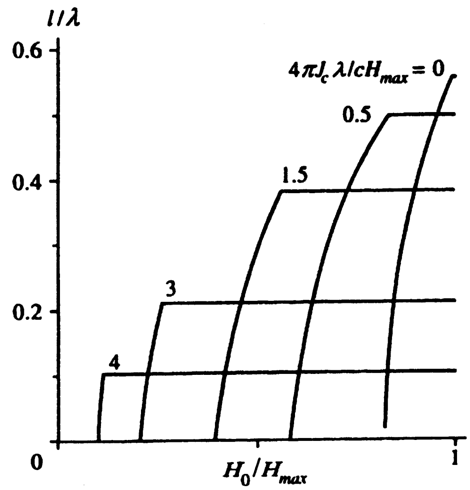

FIG. 2. Thickness of the vortex-free region l as a function of the magnetic field H_0 as it decreases from the maximum value $H_{\max}$ for various values of the critical current density J_c.

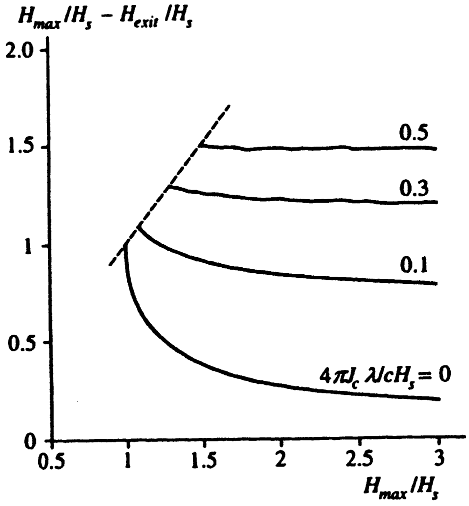

FIG. 3. Exit field H_{exit} as the external field decreases from the maximum value H_{max} for various values of the critical current density.

4. BOUNDARY BETWEEN TWO REGIONS WITH DIFFERENT CRITICAL CURRENTS

In the preceding section we considered the nonlocal effects on the surface of a superconductor. It was shown that a vortex-free region appears near the surface when the external field increases. In this section it will be shown that similar effects appear on the boundary between two superconductors with different critical current densities.

Let the boundary between the superconductors be the $x=x_b$ plane. Let the critical current density at $0<x<x_b$ be J_{c1}, and let the critical current density at $x>x_b$ be J_{c2}. We also assume that there is no additional surface pinning at $x=x_b$. We first consider the case in which $J_{c1}=J_{c2}$. In addition, let the external field first increase, and let there be no frozen vortices. At first the field penetrates precisely as in the case of a homogeneous half-space with the critical current J_{c1}. At a certain value of the external field the penetration depth b becomes equal to x_b. When the field increases further, the vortices found on the distribution front are now in a region of weaker pinning. On the other hand, if the vortex density n does not have δ-function features, the Lorentz force is continuous and has a characteristic spatial scale $\sim\lambda$. Because the Lorentz force to the left of the point x_b equals $\Phi_0 J_{c1}/c$, it is also close to $\Phi_0 J_{c1}/c$ immediately to the right of x_b at a distance much smaller than λ. Here the Lorentz force cannot be balanced by the pinning force $P_{c2}=\Phi_0 J_{c2}/c$ because of the condition $J_{c1}>J_{c2}$. The nearest point at which an equilibrium position is possible is located at a finite ($\sim\lambda$) distance from x_b. Mathematically this is manifested in the fact that it is impossible to construct a solution of Eqs. (15) and (5) which would satisfy the field and current continuity conditions without introducing a vortex-free region. The thickness of the vortex-free region l_b is implicitly assigned by the expression

$$B(x_b)\sinh\frac{l_b}{\lambda}-\frac{4\pi}{c}J_{c1}\lambda\cosh\frac{l_b}{\lambda}=-\frac{4\pi}{c}J_{c2}\lambda, \qquad (29)$$

where $B(x_b)$ is the value of the magnetic induction at x_b. The maximum value of l_b is observed in the first moment when the vortex front reaches x_b. It equals

$$l_b=\lambda\ln\left(\frac{J_{c1}}{J_{c2}}\right). \qquad (30)$$

We stress that the physical reason for the appearance of a vortex-free region near the surface when the Bean–Livingston barrier is taken into account is the same, since the Bean–Livingston barrier is associated with strong vortex pinning on an ideal flat surface.

Let us now consider the case $J_{c1}<J_{c2}$. Similarly, when $b<x_b$ holds, penetration takes place precisely as in the case of a homogeneous half-space with a critical current J_{c1}. When the external field increases further, the penetration of the field occurs in a totally different manner than in the case of $J_{c1}>J_{c2}$. Vortices do not penetrate the region $x>x_b$ until the magnetic force exceeds the pinning force $P_{c2}=\Phi_0 J_{c2}/c$. A macroscopically large number of vortices accumulates near x_b. Mathematically this means that it is impossible to construct a solution of Eqs. (15) and (5) without a current-density discontinuity, which results in the appearance of a δ-function feature on the vortex density at $x=x_b$. The expression for the vortex density in the vicinity of the point x_b has the form

$$n(x)=n_{reg}(x)+\frac{4\pi}{c\Phi_0}(J_{c1}-J_{c2})\lambda^2\delta(x-x_b), \qquad (31)$$

where $n_{reg}(x)$ is a term which does not have a singularity at x_b. The presence of such singularity means that vortices accumulate in a region whose thickness is smaller than the averaging scale α. This does not contradict the microscopic picture. In fact, when the jump in the critical current density $J_{c2}-J_{c1}$ is smaller than the depairing current $cH_s/2\pi\lambda$, the vortex density in this region is less than the maximum permissible value H_{c2}/Φ_0, at which superconductivity vanishes.

5. CRITICAL STATE IN SMALL SUPERCONDUCTING PARTICLES

Here we would like to discuss the experiment in Ref. 20, which, in our opinion, can be described only when the nonlocal effects are taken into account.

In that experiment the isothermal remanent magnetization M_{IR} was studied as a function of the magnetic field H_0, which was first applied to the superconductor and then removed. Figure 4 presents experimental plots of $M_{IR}(H_0)$ for small superconducting particles of various diameters d. The data are taken from Ref. 20, where the details of the experiment were described. Crushed samples of a superconducting Y–Ba–Cu–O ceramic were divided into the following groups by size: 10–60 μm, 60–75 μm, 100–130 μm, 130–260 μm, and >260 μm. Here we wish to restrict ourselves to consideration of the region of weak magnetic fields below the field for penetration into the granules. In this case the ceramic can be described as an effective Josephson medium. From the mathematical standpoint such a medium is identical (if we apply the continuous limit) to a homogenous

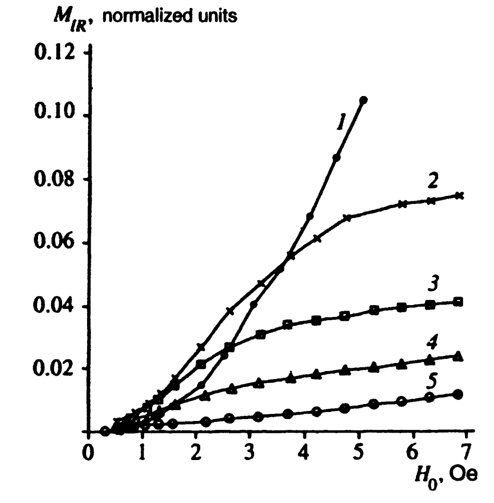

FIG. 4. Experimental field dependence of $M_{\rm IR}$ on H_0 in the weak-field range (0–7 Oe) for samples *1–5*, which consist of ceramic particles measuring >260 μm, 130–260 μm, 100–130 μm, 60–75 μm, and 10–60 μm, respectively. The data were taken from Ref. 20.

type-II superconductor (Ref. 22; see also Ref. 23, where a theory which is valid for arbitrary temperatures was devised). The main difference is that the effective penetration depth λ_J is considerably greater than the London depth λ in the granules. For this reason, the lower critical field H_{c1}^J is small, and the magnetic field penetrates the ceramic in the form of hypervortices, whose diameter is much greater than the diameter of the granules. Bearing in mind everything stated above, we shall henceforth speak in terms of homogeneous hard superconductors in this section, and we shall use δ to denote the effective penetration depth λ_J, H_{c1} to denote the field H_{c1}^J, etc. Thus, the notations adopted coincide with those used in Ref. 20.

As was noted in Ref. 20, the experimental plots presented in Fig. 4 cannot be explained with the traditional critical-state model. For example, in the Bean model[1] (the local model with $J_c=\text{const}$) the expression for the remanent magnetization of a plate of thickness d

$$M_{\rm IR}=\frac{1}{4\pi d}\int_{-d/2}^{d/2}B(x)dx \tag{32}$$

has the form

$$4\pi M_{\rm IR}^{\rm Bean}(H_0)=\begin{cases}\dfrac{H_0^2}{4H_p} & \text{for } H_0<H_p,\\[2mm] H_0-\dfrac{H_p}{2}-\dfrac{H_0^2}{4H_p} & \text{for } H_p<H_0<2H_p,\\[2mm] \dfrac{H_p}{2} & \text{for } H_0>2H_p,\end{cases} \tag{33}$$

where $H_p=(2\pi/c)J_cd$ is the penetration field. The value of the remanent magnetization $M_{\rm IR}$ is inversely proportional to d in weak fields $H_0<H_p$ and is directly proportional to d in strong fields $H_0>2H_p$. This results in crossing of the plots of $M_{\rm IR}$ for different values of d. In the experiment in Ref. 20 this is true only for sufficiently large superconducting particles with diameters $d>260$ μm (see Fig. 4). For smaller particles $M_{\rm IR}$ increases with increasing d over the entire range of fields.

To account for the anomaly pointed out, Blinov *et al.*[20] advanced an hypothesis that there is a vortex-free region near the surface, which forms when the external field decreases to zero. As was discussed above, a vortex-free region appears, if there is a difference between the external field H_0 and the magnetic induction B_0 within the superconductor near the boundary. For example, when the Bean–Livingston barrier is considered, this situation is realized when the external field (24) increases. When the external field decreases, we have $B_0=H_0$, which corresponds to the absence of a vortex-free region in fields $H \gg H_{c1}$. It was postulated in Ref. 20 that a vortex-free region appears for $H_0 \sim H_{c1}$ due to the difference between the field strength H and the magnetic induction B. The thickness of the vortex-free region satisfies $x_f<\delta$; therefore, a nonlocal treatment is necessary. The local critical-state model was used in Ref. 20. For this reason, the vortex-free region in Ref. 20 was introduced as the region in which the magnetic induction B equals zero and the field strength H satisfies the London equation. These two conditions contradict the fact that H is the thermodynamic field and $H(B)$ is a known function. This contradiction does not arise in the nonlocal model, since the vortex-free region is now determined by the condition $n=0$, and B satisfies the averaged London equation in the vortex-free region. The field strength $H(B)$ can now be found from thermodynamic relations.

In order for the presence of the vortex-free region to have an effect on the remanent magnetization, its thickness x_f must be comparable to the diameter of the superconducting particle d. The estimate $x_f \sim 30$ μm was given in Ref. 20. The thickness of the vortex-free region is $x_f<\delta$. Therefore, the effective lower critical field can be estimated: $H_{c1}<10^{-2}$ Oe. Thus, the interpretation given in Ref. 20 is fairly sensitive to the accuracy with which the external field is zeroed.

From our point of view, another interpretation of the experiment just described can be given. The only circumstance which, in our opinion, must be taken into account systematically is the fact that the penetration depth δ is comparable to the diameter of the superconducting particles d. This parameter does not appear in the traditional formulation of the critical-state model; therefore, within it there cannot be any singularities of the magnetic moment for small superconducting particles. On the other hand, for $d \leqslant \delta$, it is obvious that a significant role must be played by the Meissner fields, which are likewise not taken into account in the Bean critical-state model.

We use the nonlocal critical-state model consisting of (15) and (5) to calculate the remanent magnetic moment of a plate of thickness d $(-d/2<x<d/2)$. Here we shall neglect the Bean–Livingston barrier and the difference between B and H, i.e., we shall take into account the possible presence of vortex-free regions near the surface. This is done to demonstrate the main cause of the anomaly under discussion, i.e., the comparability of d and δ.

Let the external field H first increase from zero to H_0.

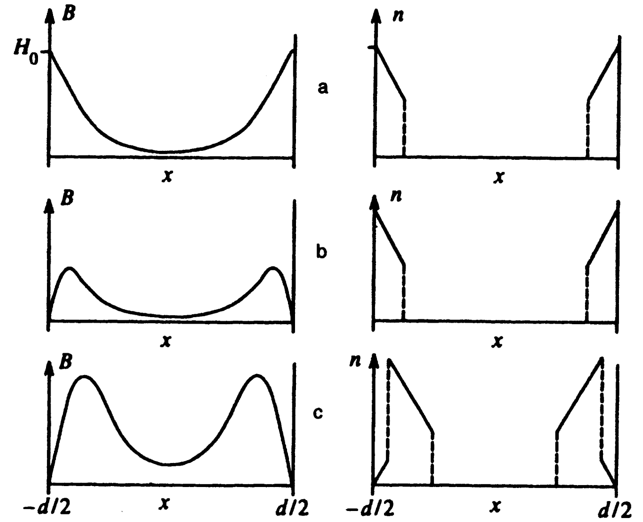

FIG. 5. Spatial distributions of the magnetic induction $B(x)$ (left-hand column) and the vortex density $n(x)$ (right-hand column): a) initial application of the external field $H=H_0$; b) $H=0$, $H_\delta<H_0<2H_\delta$; c) $H=0$, $H_0>2H_\delta$.

For $H_0<H_\delta = (4\pi/c)j_c\delta\cosh(d/2\delta)$, vortices do not penetrate the plate. In this case the Meissner current density j_m is less than j_c everywhere in the plate. Only for $H_0>H_\delta$ do we have $j_m>j_c$ near the surface. Now the force of the Meissner currents acting on the first vortex generated at the surface exceeds P_c, and the vortex moves deeper into the superconductor. An example of the distributions of $B(x)$ and $n(x)$ for this case is presented in Fig. 5a. In the center of the plate there is a region $(-x_0<x<x_0)$, where $n=0$. The magnetic field is nonzero everywhere.

Let the external field next decrease from H_0 to zero. Two alternatives are possible here. For $H_\delta<H_0<2H_\delta$ the distribution of the vortices in the plate remains unchanged when the external field is removed. The pinning force acting on the vortices near the boundary changes direction, but its amplitude does not yet reach the maximum value P_c. The magnetic induction varies here, since the Meissner component B_m varies. The distributions of $B(x)$ and $n(x)$ for this case are presented in Fig. 5b. When $H_0>2H_\delta$ holds, a surface region $(x_1<|x|<d/2)$ appears, in which the vortex density is lower than in the case presented in Fig. 5a (see Fig. 5c). The values of x_0 and x_1 are determined by the following equations:

$$d_h=\frac{d}{2}-x_0+\delta\cosh\frac{x_0}{\delta},\qquad \frac{d_h}{2}=\frac{d}{2}-x_1+\delta\cosh\frac{x_0}{\delta}, \tag{34}$$

where $d_h=cH_0/4\pi j_c$.

For the remanent magnetization $M_{\rm IR}$ we have the following expressions:

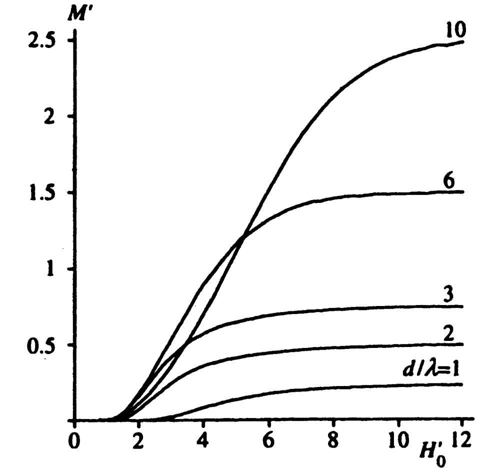

FIG. 6. Theoretical plots of the dependence of $M_{\rm IR}$ of a superconducting plate on H_0 for various values of d/δ. The dimensionless quantities $M'=cM_{\rm IR}/16\pi^2j_c\delta$ and $H_0'=cH_0/4\pi j_c\delta$ are plotted along the axes.

$$M_{\rm IR}=\begin{cases} 0 \quad \text{for } H_0<H_\delta, \\ \dfrac{H_0}{2\pi}\left(\dfrac{y_0}{d}-\dfrac{\delta}{d}\tanh\left(\dfrac{d}{2\delta}\right)\right)+\dfrac{H_p}{\pi}\left(\dfrac{\delta^2}{d^2}-\dfrac{y_0^2}{2d^2}\right) \\ \quad \text{for } H_\delta<H_0<2H_\delta, \\ \dfrac{H_0}{2\pi}\left(\dfrac{y_0}{d}-\dfrac{y_1}{d}\right)+\dfrac{H_p}{\pi}\left(\dfrac{y_1^2}{d^2}-\dfrac{\delta^2}{d^2}-\dfrac{y_0^2}{2d^2}\right) \\ \quad \text{for } H_0>2H_\delta. \end{cases} \tag{35}$$

Here we have introduced the notations $y_0=d/2-x_0$ and $y_1=d/2-x_1$. It is easy to see that in the limit $\delta\to 0$ these expressions coincide with the expressions obtained in the Bean model (33). An explicit analytical expression can also be obtained in the opposite limit $d/\delta\ll 1$. We have

$$4\pi M_{\rm IR}=\begin{cases} 0 H_0<H_\delta, \\ H_0\left(\dfrac{\delta^4}{3dd_h^3}-\dfrac{d}{4d_h}+\dfrac{d^2}{12\delta^2}\right) \\ \quad \text{for } H_\delta<H_0<2H_\delta, \\ \dfrac{1}{2}H_p\left(1-\dfrac{28\delta^4}{3d^2d_h^2}\right) \quad \text{for } H_0>2H_\delta. \end{cases} \tag{36}$$

Plots of the theoretical field dependence of $M_{\rm IR}(H_0)$ are presented in Fig. 6. In strong fields the dependence of the remanent magnetization on the diameter of the superconductor d is identical: $M_{\rm IR}=H_p/8\pi$. The situation changes radically at small H_0. At a certain fixed value of H_0 the remanent magnetization increases with decreasing d, if $d\gg\delta$, and it decreases, if $d\ll\delta$. A feature of just this kind is seen on the experimental curves presented in Fig. 4. Physically this is because the Meissner magnetic fields, which do not contribute to the remanent magnetization, create a considerable part of the magnetic moment when $d\ll\delta$.

Thus, the experiment in Ref. 20 has a natural qualitative interpretation within the nonlocal critical-state model. A quantitative comparison with experiment requires consideration of the factors which were discussed in Ref. 20, as well as the geometric shape of the particles etc. The following critical experiment can be proposed to distinguish between the effects associated with the vortex-free region and the comparability of d and δ. When the remanent magnetization is measured, the external field must be lowered to a certain small finite value $H^* \sim 1\,\mathrm{Oe} \gg H_{c1}$, rather than removed completely. In this case our proposed interpretation remains practically unchanged, and there should be no vortex-free region.

6. CONCLUSIONS

The critical-state model is a fairly rough approximation. In particular, it is unsuitable for describing numerous phenomena in systems whose characteristic dimensions are commensurate with the London penetration depth λ. In this case the nonlocal effects must be taken into account.

The behavior of the distributions of the magnetic induction B and the vortex density n in the bulk of a homogeneous superconductor was discussed in detail in Ref. 5. Here we focused our main attention on the boundary effects. The nonlocal critical-state model must then be used, since a new characteristic scale, which is comparable to λ, i.e., the thickness of the vortex-free region, appears. Correct consideration of the vortex-free region is possible only in the nonlocal critical-state model, since the vortex density n and the magnetic induction B, which are related in this case not by the simple local relation $B = n\Phi_0$, but by the more general nonlocal equation (5), must now be treated separately.

It was shown in this work that a vortex-free region appears when the Bean–Livingston barrier is taken into account or, in a more general case, when there is a jump in the critical current density. Physically the vortex-free region arises because a vortex which has been pinned at a certain point x_b on the boundary of a region with a strong critical current and then enters a region with a weak current has its nearest equilibrium point at a finite distance ($\sim\lambda$) from x_b. Mathematically this is shown by the impossibility of constructing a solution of Eqs. (15) and (5) which would satisfy the field and current continuity conditions without introducing a vortex-free region.

Experimental measurements[20] of the remanent magnetization in ceramic particles whose dimensions are comparable to the effective penetration depth were discussed in this paper. These experimental results cannot be described within the traditional critical-state model. It was shown that this is because the finite value of the London penetration depth is usually not taken into account in the critical-state model. When this factor is taken into account, it becomes necessary to use the nonlocal critical-state model, which self-consistently describes the experimental results. The calculated theoretical plots of the remanent magnetization reproduce all the features of the experimental plots. The anomaly discussed arises physically because in small superconducting particles a considerable part of the magnetic moment is created by the Meissner magnetic fields, which do not contribute to the remanent magnetization.

The model presented is valid in fields $H_{c1} \ll H_0 \ll H_{c2}$. Here the effects due to the difference between the thermodynamic field H_{eq} and the magnetic induction B were disregarded, which is justified in fields $H \gg H_{c1}$. A quantitative comparison with experimental also requires consideration of the factors that are significant for $H \sim H_{c1}$, which were discussed in Ref. 20, as well as the geometric shape of the superconducting particles, etc.

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