On the circular polarization and spin dependence of surface positron radiation

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It is shown that the radiation accompanying surface reflection of positrons from a crystal plane for a small angle of incidence onto the crystallographic axis is circularly polarized, but its probability of emission depends on the degree of longitudinal polarization of the positrons. This opens up new opportunities for measuring the degree of longitudinal polarization of positrons and obtaining circularly polarized photon and longitudinally polarized positron beams by using the surface radiation of internal, induced, and secondary beams of unpolarized positrons in the nondipole as well as dipole regime.
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1. INTRODUCTION

As is well known, polarization phenomena in crystals have opened up wide opportunities for obtaining high-energy polarized electron–photon beams.[1,2] These opportunities are based on the fact that under certain conditions the interaction of the electrons and positrons with the crystals is analogous to their interaction with an extremely intense plane-wave[1] or quasidipole[2] field. The most obvious advantage of these methods[1,2] over the widely discussed method of inverse Compton scattering of high-power laser radiation[3–5] is that because the crystal field is stationary, methods which use it to obtain polarized electron–photon beams are still applicable for particle pulses of considerable duration, typical of the secondary electron–photon beams of proton accelerators,[6] whose energies substantially exceed the energies of the electron accelerators on which it is proposed to make use of inverse Compton scattering.[3–5]

A quite complicated scheme has recently been proposed for using crystals to form circularly polarized photon beams based on the idea of transforming unpolarized photons first into linearly polarized, and then into circularly polarized photons by utilizing properties of the dichroism and birefringence of the crystal associated with either coherent bremsstrahlung[1] or, at higher energies, magnetic bremsstrahlung related to electron–positron pair production.[2] Only recently[7,8] was a method proposed of obtaining high-energy circularly-polarized photon beams by collimating the axial electron radiation in the nondipole regime. This method rests on the fact that when a fast particle moves in an atomic potential, the projection of its angular velocity onto its direction of motion results in circular polarization of its radiation. Developing this idea, we show in Sec. 2 that when the particle moves parallel to the crystal plane the effective field of the family of crystal axes lying in this plane possesses a degree of polarization in excess of 90%.

This allows us to predict in Sec. 3 that the radiation accompanying the reflection of positrons from a surface of the crystal parallel to the crystal plane should possess a high degree of circular polarization. And, since all the positrons reflecting from the crystal plane interact only with the effective crystal field, which possesses a distinct direction of polarization, in contrast to the situation considered in Refs. 7 and 8, the entire angular radiation spectrum of all the positrons reflected from the crystal will possess significant circular polarization of the same sign. Thanks to this, the necessity of using collimated radiation in the nondipole regime to obtain polarized photons falls away, which makes it possible to obtain polarized photons in the dipole regime in the axis field at markedly lower positron energies than proposed in Refs. 7 and 8.

Section 3 also introduces the approximation of constant amplitudes of the harmonics of the effective crystal potential and the approximation of independent formation of soft radiation in the field of the crystal plane and hard radiation in the field of the crystal axes. These approximations make it possible to substantially simplify the description of the surface radiation and the considered polarization phenomena in the most interesting region of positron energies $\varepsilon \ll 1$ TeV.

The interaction of the positrons with an effective crystal field possessing definite circular polarization also causes the spectral (and total) probability of emission of surface radiation to depend on the longitudinal component of the positron spin, analyzed in Sec. 4. This dependence opens up the possibility of obtaining longitudinally polarized positron beams and measuring their polarization.

This same section compares the proposed methods with methods based on the used of inverse Compton scattered laser radiation, and also discusses some technical aspects of realizing surface reflection of positrons from the crystal surface and the accompanying polarization phenomena.

2. CIRCULAR POLARIZATION OF THE EFFECTIVE CRYSTAL FIELD

All of the predicted phenomena owe their existence to circular polarization of the harmonics of the Fourier expansion of the crystal potential $V(\mathbf{r})=\Sigma_{\mathbf{q}}V(\mathbf{q})\exp(i\mathbf{q}\cdot\mathbf{r})$ over the inverse lattice vectors $\mathbf{q}$. We note at once that this polarization is oppositely directed on the two sides of the crystal plane and therefore is not manifested, for example, in coherent bremsstrahlung in the interior of the crystal. The present paper, like Refs. 7 and 8, is based in essence on using different means of realizing the interaction of positrons and electrons, respectively, with an effective crystal field of specifically circular polarization.

Thus, let the positron be incident upon a family of crystal axes parallel to the z axis and lying in the yz plane normal to the x axis. We assume for simplicity that the distance between any two neighboring axes in this plane is identical and equal to d_{iax}. Although the polarization effects in question are manifested over a considerably wider energy range, we concentrate our attention on the region 1 GeV$\leqslant\varepsilon\leqslant$1 TeV both because of its accessibility and its present importance and because, as will be shown in detail below, the theoretical description of these effects in that range can be substantially simplified.

The requirements of a reasonable probability of emission and a hard coherent bremsstrahlung spectrum in the positron energy range under consideration are satisfied at incidence angles $\psi\leqslant 10$ mrad onto the crystal axes (see Secs. 3 and 4). Under these conditions, we can at once set $q_z=0$, describe the evolution of the x coordinate by the averaged plane potential $V(x)=\Sigma_{q_x}V(q_x,0,0)\exp(iq_x x)$, and, what is most important, neglect its variation during the scattering of the positron by a large number of axes, ensuring the formation of the desired hard radiation (see below). In this case, we can use the effective field of the crystal to describe the motion of the positrons and their radiation, where this effective field is the sum of the mean field of the crystal planes $-(1/e)dV(x)/dx$ (e is the positron charge) and the harmonics

$$\mathbf{E}(x,y)=\sum_{q_y\neq 0}[\mathbf{n}_x E_x(q_y,x)\cos(q_y y)+\mathbf{n}_y E_y(q_y,x)\sin(q_y y)] \qquad (1)$$

with wave vectors $q_y=2\pi n_y/d_{iax}$, $n_y=\pm1,\pm2,\ldots$ and amplitudes

$$\begin{Bmatrix} E_x(q_y,x) \\ E_y(q_y,x) \end{Bmatrix}=\frac{4}{e}\sum_{q_x}\frac{V(q_x,q_y)}{1+\delta_{q_x,0}}\begin{Bmatrix} q_x\sin(q_x x) \\ q_y\cos(q_x x) \end{Bmatrix} \qquad (2)$$

(we assume $V(q_x,q_y)=V(q_x,-q_y)$). This representation of the effective crystal potential (1), (2) allows us to explicitly demonstrate the circular polarization of its harmonics $\lambda_2=2E_xE_y/E^2$ and its change of sign from one side of the plane to the other (with change of sign of x) ruling out, in particular, circular polarization of the effective crystal field (1) by rectilinear propagation of particles in the interior of the crystal. The x-dependence of λ_2 and $E=\sqrt{E_x^2+E_y^2}$ for the first three harmonics of the potential of the $\langle 100\rangle$ axes of silicon is shown in Fig. 1. In the calculation we neglected the difference between the behavior of the amplitudes (2) and potential $V(x)$ inside the crystal and on the surface of the crystal at $x\sim d/2$, where d is the interplanar distance, since the contribution of this region to the emission probability is small. Note that for $d=d_{iax}$ it is possible to obtain values of λ_2 close to 100%.

The interaction of ultrarelativistic particles with the circularly polarized harmonics of the expansion of the crystal field (1) differs in essence only slightly from inverse Compton scattering[3,4] of circularly polarized laser radiation which has been widely discussed recently in connection with the development of photon–photon colliders.[5] The equivalent frequencies of the latter are close to the frequencies of excimer lasers or even several times higher (see below). Without disputing the advantages of inverse Compton scattering, we note that its realization in the extracted or secondary electron beams of proton accelerators, which can persist for tens of seconds, or even in accumulator beams, which are characterized by a very high pulse repetition rate, is extremely difficult since it requires a minimum of a gigawatt of average laser power. At the same time, the amplitude $p_\perp\simeq eEd_{iax}/\psi$ of the oscillations of the transverse positron momentum in the harmonics of the effective crystal potential (1) for $E\sim 10$ GV/cm (see Fig. 1) and $\psi\sim 1-10$ mrad reaches $\sim(0.1-1)mc$, where m is the electron mass, and corresponds to a power level of the laser wave of $I\sim 10^{17}-10^{18}$ W/cm^2 and higher, which is realistically achievable only for subpicosecond pulses[9] whereas if one uses a crystal target such conditions exists continuously.

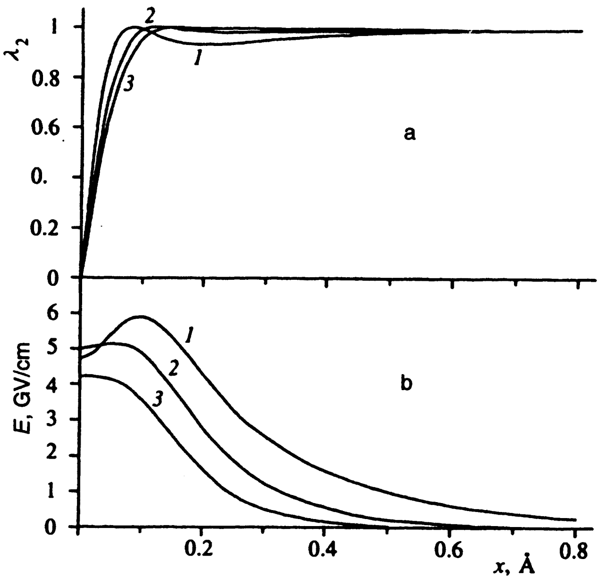

FIG. 1. Degree of circular polarization (a) and field strength (b) of the first three harmonics of the potential of the $\langle 100\rangle$ axes of silicon plotted versus distance from the (110) plane. The curves are labeled by the number of the corresponding harmonic.

3. CIRCULAR POLARIZATION OF THE RADIATION

The analogy with inverse Compton scattering of polarized photons allows us to predict the degree of circular polarization of surface positron radiation. To describe this degree of polarization we use the quasiclassical expression for the probability of emission[10 1)]

$$dW^{(j)}=\frac{i\alpha}{2\pi}d\omega\int dt\int\frac{d\tau}{\tau-i0}T^{(j)}\exp\left\{-\frac{i\omega\varepsilon\tau}{2\varepsilon'}\left[\frac{1}{\gamma^2}+\int_{t_1}^{t_2}[\Delta\mathbf{v}(t')]^2dt'\frac{1}{\tau}-\left(\int_{t_1}^{t_2}\Delta\mathbf{v}(t')dt'\frac{1}{\tau}\right)^2\right]\right\}, \qquad (3)$$

where $\alpha=1/137$, $\gamma=\varepsilon/m$, and $\varepsilon'=\varepsilon-\omega$. For $j=0$, $T^{(0)}=\gamma^{-2}+\varphi(\varepsilon)[\Delta\mathbf{v}_2-\Delta\mathbf{v}_1]^2/4$, and $\varphi=\varepsilon/\varepsilon'+\varepsilon'/\varepsilon$, this expression describes the probability of emission, and for $j=2$ and

$$T^{(2)}=(i/2)\varphi(\varepsilon)\mathbf{v}_0\cdot\left\{\mathbf{v}_2\times\mathbf{v}_1-\left[(\mathbf{v}_2-\mathbf{v}_1)\times\int_{t_1}^{t_2}\mathbf{v}(t')dt'/\tau\right]\right\} \quad (4)$$

the degree of circular polarization of the emitted radiation is $\lambda_2=dW^{(2)}/dW^{(0)}$. The subscripts 1 and 2 refer to the times $t_{1,2}=t\pm\tau/2$. In the situation under consideration the integral over t follows the trajectory of the positron in the potential $V(x)$ defining the dependences $x(t)$ and $\mathbf{v}(t)$.

Expression (3) is commonly used in conjunction with the approximation of rectilinear trajectories for the velocity of the radiating electrons (positrons) to describe coherent bremsstrahlung.[10] Implementing this approximation when calculating the velocity perturbation due to the crystal potential, we write the radius vector of the particle in the form $\mathbf{r}=\mathbf{r}_0+\mathbf{v}_0t$, corresponding to rectilinear motion, in the expression for the effective crystal field analogous to expression (1). Subsequent averaging over trajectories (over $\mathbf{r}_0$) removes the multiple sum over the three components of the inverse lattice vector $\mathbf{q}$.

In the case of surface radiation under consideration the rectilinear trajectory approximation does not even allow us to describe the very process of reflection from the crystal surface. However, in this situation it is possible to introduce two other approximations allowing us to simplify expression (3) as much as the rectilinear trajectory approximation. The first of these approximations is based on the requirement of smallness of the variations of the x coordinate and, as a consequence, the amplitudes (2) of the harmonics of the effective potential (1) and the field strength of the plane field during the formation time of the radiation. This approximation is nothing but a generalization of the uniform plane field approximation to the amplitudes of the oscillating component of the crystal field (1) and allows us locally to reduce the description of the surface radiation to a treatment of the radiation from the positrons moving in the field of the transverse electrostatic harmonics of constant amplitude in the presence of the quasi-uniform field of the crystal planes. Describing the radiation process in this way at an arbitrary fixed value of the x coordinate, we need only perform the integration over time t, allowing for the deviation from a linear dependence of $x(t)$ corresponding to the process of reflection of a positron from the surface of the crystal.

However, in the more interesting energy range $\varepsilon\ll 1$ TeV it is possible to go somewhat further in the simplification of the description of the radiative process and also neglect the influence of the plane field on the formation of hard radiation, which is equivalent to going over to the rectilinear trajectory approximation. Here, in contrast to the theory of coherent bremsstrahlung,[1] under the present conditions this approximation is employed only in place of the approximation of constant amplitudes (2). Note that at higher positron energies $\varepsilon\geqslant 1$ TeV the quasi-uniform plane field starts to have an effect on the entire radiation spectrum,[2] rendering the second of the approximations introduced inadequate, in contrast to the constant-amplitude approximation (2), which remains valid at all positron energies, no matter how high.

The perturbation of the transverse component of the velocity of the ultrarelativistic positron leading to emission is found from the equation of motion in the effective field (1) (with accuracy $\sim\psi^2/2\leqslant 10^{-4}$ the latter can be taken to be transverse). Here, in accordance with the constant-amplitude approximation (2) the value of the x coordinate everywhere in the integration over t can be taken to be constant. Taking the next step and neglecting the influence of the plane field on the formation of the hard radiation, we should therefore use for the y coordinate the expression $y=y(t)+v_y(x)\tau$, based on neglecting the variation in the photon formation region of the corresponding component of the velocity under the influence of the averaged plane potential. In addition, for simplicity we may also neglect the velocity variation due to oscillations in the field of the axes, setting $v_y(x)=\psi$, although the use of this approximation is not essential and in the calculation of the characteristics of the radiation in the constant amplitude approximation (2) it is possible to use as $v_y(x)$ the y coordinate of the velocity averaged over the oscillations in the effective potential (1) (cf. Ref. 11). Thus, setting $v_y(x)=\psi$, assuming that the x coordinate and the amplitudes (2) that depend on it are constant, we obtain for the τ-dependent part of the velocity

$$\Delta\mathbf{v}(t+\tau)=-\frac{e}{\varepsilon\psi}\sum_{q_y\neq 0}\frac{1}{q_y}[i\mathbf{n}_xE_x(q_y,x)+\mathbf{n}_yE_y(q_y,x)]\exp\{iq_y[y(t)+\psi\tau]\}. \quad (5)$$

The frequencies of the oscillations of the transverse components of the velocity (5) in the harmonics of the effective crystal potential coincide with the frequencies of the oscillations in the field of the counterpropagating electromagnetic waves with effective frequencies $\omega_{\mathrm{eff}}=q_y\psi/2$ and wavelengths $\lambda_{\mathrm{eff}}=0.2d_{iax}(\text{Å})/[n_y\psi(\mu\text{rad})]$. Although the possibility of adjusting the latter is in practice limited by the rapid decrease of the emission probability with increase of the angle of incidence of the positrons, on the axis, this effective wavelength can actually be, as a minimum, an order of magnitude shorter than the wavelengths of existing high-power laser systems.

After substituting the velocity perturbation (5) into expressions (3) and (4) and expanding the exponential in powers of this perturbation, for example, in the circular-polarization dependent contribution to the probability of emission, we obtain

$$dW^{(2)}=\frac{i\alpha^2\omega d\omega}{4\pi\varepsilon^2}\varphi(\varepsilon)\int dt\int\frac{d\tau}{\tau-i0}\exp\left\{-\frac{\omega m^2\tau}{\varepsilon\varepsilon'}\right\}\sum_{q_y,q_y'\neq 0}\frac{E_x(q_y,x)E_y(q_y,x)}{q_\parallel q_\parallel'}\times\left\{\sin(q_\parallel-q_\parallel')\tau+\sin q_\parallel'\tau\frac{\sin q_\parallel\tau}{q_\parallel\tau}-\sin q_\parallel\tau\frac{\sin q_\parallel'\tau}{q_\parallel'\tau}\right\}\exp[-i(q_y+q_y')y], \quad (6)$$

where, following Ref. 10, in order to simplify the form of the resulting expression the variable τ has been replaced by 2τ. Note that in the present approximation q_x does not contribute to $q_\parallel = \mathbf{qv} \simeq q_y \psi$ (see below). The y coordinate of the positron at the time t entering into the argument of the last exponential can be represented in the form

$$y(t) = y_0 + \int_{t_0}^{t} v_y(t')dt',$$

where y_0 is its value at some arbitrary time t_0 chosen, for example, as the time at which the positron is found a certain distance from the plane. Averaging of the points where the positrons are incident on the plane over this coordinate leads to the appearance of the Kronecker symbol δ_{q_y, q'_y}, allowing us to drop the sum over q'_y in expression (6) and avoid having to consider the integral contribution to $y(t)$. If we do not at once set $q_z = 0$, it is possible in a similar way to simplify the sum over the z component of the inverse lattice vector.

At the same time, the absence of translational symmetry in the direction of the x axis (the axis normal to the crystal plane from which reflection takes place) means that there is no analogous conservation law and yields a procedure for integrating over the time variable t equivalent to integrating over the x coordinate. As a result, we arrive at the expression

$$dW^{(0)} = \frac{\alpha^2 d\omega}{4\varepsilon^2} \int dt \sum_{q_y} \frac{E^2(q_y, x)}{q_\parallel^2} \left\{ \varphi(\varepsilon) - \frac{2\omega m^2}{\varepsilon\varepsilon' q_\parallel^2} \times \left(\left| q_\parallel \right| - \frac{\omega m^2}{2\varepsilon\varepsilon'} \right) \right\} \theta\left(\left| q_\parallel \right| - \frac{\omega m^2}{2\varepsilon\varepsilon'} \right),$$

$$dW^{(2)} = \frac{\alpha^2 d\omega}{\varepsilon^2} \varphi(\varepsilon) \int dt \sum_{q_y} \frac{E_x(q_y, x) E_y(q_y, x)}{q_\parallel^2} \times \left(\frac{\omega m^2}{\varepsilon\varepsilon' |q_\parallel|} - 1 \right) \theta\left(\left| q_\parallel \right| - \frac{\omega m^2}{2\varepsilon\varepsilon'} \right), \qquad (7)$$

where $\theta(a)$ is the Heaviside unit step function, i.e., $\theta(a) = 1$ for $a \geqslant 0$ and $\theta(a) = 0$ for $a < 0$.

Before presenting the results of using these expressions, let us discuss in more detail their applicability. As was already noted, the constant-amplitude approximation generalizes the magnetic-bremsstrahlung approximation[2] used to describe the radiation in the field of the crystal planes at energies $\varepsilon > \tilde{\varepsilon}$, where the characteristic energy

$$\tilde{\varepsilon} = m^2/2V_0 \sim 125 \text{ GeV}/V_0(\text{eV}) \qquad (8)$$

is equal to several GeV. However, in the description of the hard radiation of interest, this approximation, as will be shown below, also applies at lower energies. Since we have neglected distortions of the positron trajectories in the field of the crystal planes in the calculation of the local emission probability in the integrand of Eq. (7), these expressions do not describe the soft radiation, whose formation is governed mainly by the plane potential $V(x)$, and not by the sum over the harmonics (1) which makes the main contribution to (7). The characteristic frequencies of this radiation in the regions of applicability of the dipole ($\varepsilon < \tilde{\varepsilon}$) and magnetic bremsstrahlung ($\varepsilon > \tilde{\varepsilon}$) approximations can be estimated by the formulas $\omega(\text{MeV}) \leqslant \varepsilon^{3/2}(\text{GeV}) V_0^{1/2}(\text{eV})/d(\text{Å})$ and $\omega(\text{MeV}) \leqslant 0.1\varepsilon^2(\text{GeV}) V_0(\text{eV})/d(\text{Å})$, the latter of which allows us to show that for $\varepsilon \ll 1$ TeV the characteristic energy of the photons emitted in the field of the principal planes of silicon ($V_0 \leqslant 20$ eV) and germanium ($V_0 \leqslant 40$ eV) indeed satisfy the condition $\omega \ll \varepsilon$, confirming the validity of neglecting the mutual influence of the emission of soft photons in the field of the crystal planes and the emission of substantially harder (see below) photons in the field of the crystal axes forming these planes. Note that the presence of types of radiation so different in range can be used to predict the characteristics of one of them from results of observing the other (see Sec. 4).

At photon energies ω_{n_y} corresponding to peaks of the coherent bremsstrahlung spectrum with indices $n_y = 1, 2, \ldots$, the coherence length $l_{\text{coh}} = 2\varepsilon\varepsilon'/m^2\omega$ satisfies the condition $l/l_{\text{coh}} = q_\parallel = 2\pi n_y \psi / d_{iax}$, fitting n_y times on a line segment of length d_{iax}/ψ, equal to the distance between collisions of the positrons with the axes. Here even a few such collisions are sufficient for the formation of distinct interference peaks in the spectrum.

So that we can analyze the possibility of using the constant amplitude approximation (2) under these conditions, we will estimate the number of axes with which the positrons collide during the time it takes for their x coordinate to change by, say, $\Delta x \leqslant 0.1$ Å. Since all of the possible integrands in Eq. (3) are bilinear in the squares of the amplitudes (2), the main contribution to the emission probability comes from the vicinity of the turning point of the transverse motion of the positron in the field of the plane. Using the harmonic-potential approximation for this field, we obtain for the desired number of axes $N_{ax} = \sqrt{\Delta x/d}(\psi/\theta_c)$, where d is as before the interplanar distance and $\theta_c = \sqrt{2V_0/\varepsilon}$ is the characteristic turning angle of the positrons in the field of the crystal plane, which places an upper bound on the angle of incidence onto the plane θ at which reflection takes place (V_0 is the amplitude of variation of the plane potential $V(x)$).

Rewriting the condition $N_{ax} \gg 1$ in the form $\psi(\text{mrad})\varepsilon^{1/2}(\text{MeV}) \gg 1$, one can convince oneself that it does not restrict the energy of the positrons to the value given by relation (8) which is independent of their angle of incidence on the axis. It is also not hard to see that this condition is fulfilled for the parameter ranges $\psi \geqslant 10$ mrad, $\varepsilon \geqslant 1$ GeV and $\psi \geqslant 10$ mrad, $\varepsilon \geqslant 1$ GeV which are actually used in practice to obtain coherent bremsstrahlung, and also in the case $\varepsilon \sim 100$ GeV and $\psi \sim 1$ mrad, considered below by way of an example. Figure 2 displays the spectral dependence of the polarization of the positron radiation at $\varepsilon = 3$ GeV and angles of incidence $\psi = 15$ mrad on the $\langle 100 \rangle$ axis and $\theta \simeq 0.1$ mrad on the (110) plane of silicon ($\theta_c \simeq 0.13$ mrad). The directions of polarization in Figs. 1 and 2, similar to the case of inverse Compton scattering, are oppositely directed. In the calculations following formulas (3) and (4) here and below, the probability of incoherent emission, calculated according to the Bethe–Heitler formula for the local density of nuclei, has been added to $dW^{(0)}$.

Since under the conditions of surface reflection the angle θ of deflection of the direction of motion of the positrons from the crystal plane is bounded from above by the charac-

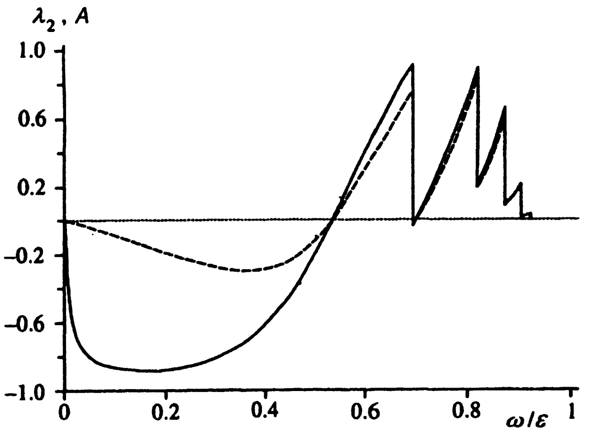

FIG. 2. Spectral distribution of the polarization of the radiation, λ_2 (solid curve), and spin asymmetry of scattering, A (dashed curve), of positrons with energy $\varepsilon = 3$ GeV incident at an angle of $\theta \simeq 0.1$ mrad with respect to the (110) plane and $\psi = 15$ mrad relative to the $\langle 100 \rangle$ axis of a silicon crystal.

teristic channeling angle θ_c, satisfaction of the condition

$$N_{ax} = \sqrt{\Delta x/d}(\psi/\theta_c) \gg 1$$

also allows us to neglect the contribution of the x component of the inverse lattice vector $q_x = 2\pi n_x/d$, $n_x = \pm 1, \pm 2, \ldots$, equal to $\theta q_x < \theta_c q_x$, to the longitudinal component $q_\parallel$ of this vector in comparison with the contribution of the y component $q_y = 2\pi n_y/d_{iax}$, $n_y = \pm 1, \pm 2, \ldots$. The fact that $q_\parallel$ is independent of q_x allows us to obtain expressions (6) and (7) by an alternate route from the corresponding expressions of the theory of coherent bremsstrahlung, thanks to the independent summation over the x components of the inverse lattice vector which becomes possible in this case and which distinguishes the combinations (2) in expressions (6) and (7).

Although expressions (7) are written in a form more appropriate for the theory of coherent bremsstrahlung,[1,10] it is not hard to make out their close connection with the theory of inverse Compton scattering.[3–5] Here it should be borne in mind that we are considering scattering by positrons not of a monochromatic counterpropagating laser wave, but of harmonics of an electrostatic wave, which are characterized by the 4-momentum vectors $q = (0, q_y \mathbf{n}_y)$ and the parameter

$$\bar{x} = 2pq/m^2 = 2\varepsilon q_\parallel/m^2, \tag{9}$$

proportional to their product with the 4-momentum of the positron $p = \varepsilon(1, \mathbf{v})$, which defines the probability of inverse Compton scattering and the maximum frequency of the scattering (emitted) photons. Thus, nonrelativistic oscillations of the positrons in the n_y-th harmonic of their transverse motion lead to emission with maximum frequency $\omega_{n_y} = \bar{x}\varepsilon/(1+\bar{x})$. Here the condition for $\omega_{n_y} \sim \varepsilon$ for hard radiation turns out to be equivalent to $\bar{x} \sim 1$. After the definition of the parameter (9) is rewritten as the estimate

$$\psi(\text{mrad}) \simeq 10\bar{x} d_{iax}(\text{Å})/[\varepsilon(\text{GeV}) n_y], \tag{10}$$

it is convenient to use this condition to choose the magnitude of the angle of incidence of the positrons on the crystal axis. Here it should be borne in mind that only a few of the first harmonics of the effective crystal potential have comparable amplitudes (i.e., $n_y \sim 1-5$), and also that the falling off of the angle (10) with increase of the positron energy can lead to a loss of applicability of expressions (7), which were obtained in the limit of smallness of the turning angle $\delta\vartheta$ of the positron in the axis field in comparison with the characteristic angle of emission $1/\gamma$. This circumstance constrains the magnitude of the angle of incidence of the positrons on the axis.

Indeed, at not too small angles ψ (the angles of interest) it is more convenient to estimate the inclination angle of the positron in the axis field from the magnitude of the field strength of the plane field, obtained by averaging the axis potential in the direction lying in the plane and normal to the axes. The maximum value of the inclination angle of the positron to the axis $\delta\vartheta_{\max} = eE_{\max} d_{iax}/\psi\varepsilon$ is reached when it moves at that distance from the axis and the plane at which the field strength of the plane field also reaches its maximum value $E_{\max}$. Using this estimate, it can be shown that the interesting situation $\delta\vartheta \leqslant \delta\vartheta_{\max} < 1/\gamma$ is realized at incidence angles of the positrons on the axis $\psi > eE_{\max} d_{iax}/m \sim 300-500$ μrad (our estimates, as before, are for silicon and germanium crystals, the most likely crystals to be used in the experiments, for whose principal planes $E_{\max} \sim 10^{10}$V/cm). Thus, the simplified theoretical approach based on formulas (7) to the description of the hard radiation accompanying surface reflection of positrons remains valid for incidence on the axes at angles $\psi \gg 1$ mrad, which must be taken into account when using estimate (10).

4. SPIN DEPENDENCE OF THE PROBABILITY OF EMISSION

Circular polarization of the harmonics of the crystal potential, in complete analogy with Compton scattering of photons by ultrarelativistic electrons, leads to a substantial spin dependence of the spectral and total probabilities of emission, which makes it possible to obtain polarized positron beams and measure their polarization. That part of the emission probability $dW_\zeta^{(2)}$ depending on the spin vector of the incident positron $\boldsymbol{\zeta}$ is obtained from $dW^{(2)}$ by replacing $T^{(2)}$ by

$$T^{(\zeta)} = \frac{i\omega}{2\varepsilon}\left(1 + \frac{\varepsilon}{\varepsilon'}\right)\boldsymbol{\zeta}\cdot\{[\mathbf{v}_2 \cdot \mathbf{v}_1] - [(\mathbf{v}_2 - \mathbf{v}_1),$$

$$\int_{t_1}^{t_2} \mathbf{v}(t')dt'/\tau]\}. \tag{11}$$

The projection of the angular velocity of the positrons onto their direction of motion, arising in the effective potential (1), leads to the contribution $dW_\zeta^{(2)}$ depending on the longitudinal component of the spin $\zeta_\parallel = \boldsymbol{\zeta}\mathbf{v}$. The spectral dependence of the scattering asymmetry, $A(\omega) = |dW^{(2)}_{\zeta_\parallel = \pm 1}|/dW^{(0)}$, which is equal to the longitudinal polarization acquired by the positrons upon the surface emission of a photon with frequency ω, is plotted in Fig. 2, confirming the possibility of obtaining highly polarized positrons in the usual tagging scheme.[6]

The substantial spin dependence of the probability of surface emission also makes it possible to measure the polarization of the incident positrons. Usually in such situations one compares the emission probabilities for different mutual orientations of the spin of the incident particles. In the case under consideration this can be done, for example, by placing reflective crystals on opposite sides of the beam and comparing the probabilities of emission of polarized positrons in the field of the harmonics of the effective potential (1) having opposite directions of circular polarization at the surface of these crystals.

However, there exists another, more effective way of determining the degree of polarization, namely the way discussed in Sec. 3. We are talking here about the possibility of predicting the spin-independent part of the emission probability $dW^{(0)}$ of a hard photon on the basis of measurements of the closely related probability of emission of soft radiation in the plane field. Not pretending to a general analysis of the possibility of using this connection, we note its substantial simplification in the limit of low divergence of the positron beam $\Delta\theta\ll\theta\sim\theta_c$, when the difference between the various positron trajectories can be neglected, by virtue of which the differential and total probabilities of emission of the hard radiation, as well as its polarization characteristics, are uniquely related to the total probability of emission of soft radiation in the plane field. Here by virtue of the significant magnitude of the latter the independent data on the polarization of the hard photons and the positrons that have emitted them, obtained on the basis of measurements of the probability of emission of soft photons, possess low statistical uncertainty.

The probability of emission of hard radiation in the field of the crystal axes during the interaction of the photon with the crystal surface can be easily estimated using the parabolic approximation of the potential of the crystal planes, for which

$$\int dtE^2(q_y,x)\simeq E^2(q_y,x_{\min})T/4,$$

where $x_{\min}$ is the x coordinate of the turning point of the positron and $T=\pi d_{ipl}/\theta_c$ is the period of transverse motion of the positrons in the plane field. Next integrating over the photon spectrum, we obtain the following estimate for the total probability of surface positron emission:

$$W\simeq\sum_{q_y}\sigma_c(\tilde{x})\frac{E^2(q_y,x_{\min})}{4\pi(q_\parallel/2)}\frac{T}{8}, \tag{12}$$

demonstrating explicitly its close analogy with inverse Compton scattering. Here $\sigma_C(\tilde{x})$ is the Compton scattering cross section, defined by the magnitude of the invariant parameter $\tilde{x}$ [Eq. (9)].

According to the above, surface positron radiation is equivalent to inverse Compton scattering of electromagnetic waves with field strengths $E(q_y,x_{\min})\sim10$ GV, lengths $\lambda_{\text{eff}}=0.2d_{iax}(\text{Å})/[n_y\psi(\mu\text{rad})]$, $n_y=\pm1,\pm2,\ldots$ and pulse duration $t_i=T/8\simeq\pi d_{iax}/8\theta_c\sim1$ μm. The smallness of the latter in comparison with the typical length of the laser pulses for inverse Compton scattering[5] does not allow one to achieve as high a probability of emission when positrons in the energy range under consideration are scattered by the crystal surface. Choosing the angle of incidence of the positrons onto the axis as prescribed by Eq. (10) and employing formula (12) after first setting $x_{\min}\sim0.1$ Å, we readily ascertain that for one harmonic the probability (12) can vary from 10^{-5} at $\varepsilon\sim1$ GeV to 10^{-2} at $\varepsilon\sim100$ GeV. As was already noted, only the first few harmonics of the crystal potential can give a comparable contribution to the emission probability, depending on the atomic number of the material of the crystal and its temperature.

With the aim of making more effective use of the phenomena under discussion, it is not hard to imagine schemes that allow one to realize multiple scattering of the positrons by the crystal surface, based, for example, on the bending the surface or introducing a transverse magnetic field deflecting the scattered positrons in the direction of the crystal surface. However, in the light of the lack of study of the possibility of realizing single scattering of high-energy positrons by a crystal surface, a discussion of these schemes seems premature.

Multiple scattering of positrons by a crystal surface can be attempted also in the inner beam of an accumulator in which, besides multiple passes of a high-quality beam, there exist efficient mechanisms for suppressing transverse surging and for compensation of small energy losses of the positrons. The most important circumstance restricting the possibility of using multiple collisions of the positrons with the surface is the intense soft radiation they emit as they are deflected in the field of a crystal plane. The estimates of the frequency of this radiation given in Sec. 3 show that for $\varepsilon\leqslant10$ GeV the photon energies of this radiation are less than one percent of the energy of the positrons, which makes it quite easy, once the positrons have emitted, to keep them from leaving the inner beam of the accumulator. The emission of such soft photons also has practically no effect on the conventional tagging scheme in the extracted beam[6] for selecting polarized hard photons and also the positrons which have emitted them and become polarized themselves in the process.

However, the substantial intensity of the radiation in the plane field becomes a major obstacle to the use of multiple reflection of positrons from the crystal surface at energies significantly higher than 10 GeV. The point is that at these energies this radiation becomes so hard that the typical energy of its photons, growing in proportion to ε^2, will exceed, at a minimum, several percent of the energy of the positrons (see the corresponding estimate in Sec. 3). At such energy losses, with the emission of just one photon great difficulties arise in keeping the post-emission positron contained within the inner beam of the accumulator. At the same time, the total probability for emission of soft photons, estimated in the magnetic-bremsstrahlung approximation, for a single collision with the crystal surface, $5\alpha\sqrt{2\varepsilon V_0/3m^2}\simeq1.8\cdot10^{-3}\sqrt{\varepsilon(\text{GeV})V_0(\text{eV})}$ can reach $3-10\%$, which exceeds the probability of hard emission in the axis field by an order of magnitude or more. For such a probability of escaping from the accumulator beam, the average positron "survives" only on the order of ten reflections. If one uses a tagging scheme in the extracted or secondary beam, a total emission probability of a few percent will be not far from optimal.[6]

Let us dwell briefly on some technical aspects associated with achieving positron reflection from a crystal surface. We start from the fact that with the creation of a surface precisely parallel to a crystal plane, the appearance of defects having the form of fragments of successive atomic planes is unavoidable. Such defects will lead to undesirable trapping in the positron channeling regime at their front edges. An effective means of preventing this phenomenon from happening is to polish the crystal surface at a small angle Θ to the crystal plane, ensuring a favorable transition to the deeper lying planes along the direction of motion of the positrons. In order that "breakoff" of the planes in this case only weakly affect the emission process, the typical distance between the boundaries of neighboring atomic layers, equal to d/Θ, should substantially exceed the displacement of the positron during the time $t \simeq T/2 + d/\theta_c \simeq (\pi/2+1)d/\theta_c$ it resides at distances $x \leqslant d$ from the reflecting surface. Thus, the polishing angle of the crystal surface relative to the direction of the crystal plane should satisfy the quite stringent condition $\Theta \ll \theta_c/(\pi/2+1)$.

Another difficulty has to do with the fact that even for transverse beam dimensions $2r_b \sim 100\,\mu$m and $\theta_c \sim 100\,\mu$rad the reflection length of the entire beam reaches the value $L = 2r_b/\theta_c \simeq 1$ m.[2)] Naturally, the formation of a reflecting surface from crystal blocks processed as described above, say, of length $L_b \leqslant 1$ cm would be preferable to the fabrication of such an extended single crystal. It would be necessary to maintain the parallelism of the implemented family of crystal planes of all such $N_b \simeq L/L_b$ blocks with an accuracy of $\Delta\Theta < \theta_c$ at a minimum, or better yet $\Delta\Theta \ll \theta_c$ (this would make it possible to satisfy the condition $\Delta\theta \ll \theta_c$, see above), and the allowable normal displacement of the crystals would have to satisfy the condition $\Delta x_b \ll L_b\theta_c \sim 1\,\mu$m. Although these requirements are quite stringent, they are realizable in practice if germanium or silicon crystals are used.

We also point out a more compact, albeit less simple, method of achieving positron reflection from an atomic surface and the accompanying polarization effects. Here we are talking about the possibility of replacing a single extended reflecting surface by a stack of parallel crystal plates of thickness a, spaced a distance b apart. If the condition $a \ll b$ is fulfilled and the beam is incident at an angle $\theta < \theta_c$ on the side of such a structure, the majority of the positrons will experience surface reflection and only a small fraction $a/b \ll 1$ of the beam will be incident on the sides of the crystals. In order to have all the positrons incident on a crystal surface, the length of the crystal plates L should be b/θ. Besides the possibility of a substantial decrease in this length (see below) an important advantage of this structure is the absence of fundamental constraints on its transverse dimensions and on the cross section of the positron beams. In keeping with the requirement to conserve the phase volume of the beam during focusing (more accurately, defocusing) at the expense of increasing its cross section, the angular divergence can be decreased to a level $\Delta\theta \ll \theta_c$, thereby allowing us not only to choose the optimum value of the angle of incidence of all the positrons onto the plane, but also to make more efficient use of the method described above for collected data on the soft positron radiation in the plane field in order to obtain information about the hard positron radiation in the axis field and its polarization characteristics.

The answer to the question, what are the possible parameters of the above-described crystal structure, requires an analysis of specific technologies. Without formulating this problem here, we note that for layered growth the thickness a will be bounded from below only by the thickness of an atomic layer. Thus it is possible to set $a \ll b \simeq 0.01\,\mu$m, which corresponds to the length $L \simeq b/\theta$ of the crystal plates along the beam, not exceeding a fraction of a millimeter even for $\varepsilon \sim 100$ GeV. Note that filling the space separating the crystals from whose surfaces the positrons are to be reflected with a sufficiently light material under these conditions and also for significantly greater values of the parameter b does not introduce any fundamental complications.

The approximations on which the simple calculational method used in this paper is based—namely the approximation of constant amplitude of the harmonics of the effective potential (1) and the approximation of independent formation of the soft photons in the plane field and hard photons in the axis field—provide a sufficiently accurate description of the hard surface positron radiation and the accompanying polarization phenomena in the interesting energy region $\varepsilon \sim 1-100$ GeV at angles of incidence on the axis $\psi \sim 1-10$ mrad.

However, it should be noted that the effects under consideration are manifested over a significantly wider range of the parameters in which these approximations break down. Thus, at intermediate energies $\varepsilon \sim 10-100$ MeV and angles of incidence small enough to ensure substantial radiation intensity, surface positron reflection will take place for scattering from just a few or even one crystal axis and the variation of the impact parameter of the positrons with respect to the axes as well as the variation of the amplitudes (2) over the distance within which the photons are emitted can become quite substantial.

In the hundred-GeV energy region and higher, the approximation of constant amplitudes, on the other hand, possesses very good accuracy. However, the small bending of the trajectories in the plane potential in this region of positron energies, without contradicting this approximation will, nevertheless, have a substantial affect on the entire radiation spectrum, including the part that interests us, the hard part. An indication of this is given by the fact that in this region the energy of the magnetic bremsstrahlung photons emitted by the positrons in the plane potential[2] becomes comparable with the energy of the latter, indicating the inadequacy of treating the magnetic-bremsstrahlung mechanism and the coherent-bremsstrahlung mechanisms independently in this case. The emission process and the accompanying polarization phenomena are described in this limit in a way analogous to the situation of Compton scattering in a uniform external field, considered in Ref. 11 in application to the case of above-barrier motion. Thus, for high as well as low positron energies the description of the effects considered here requires that we depart from the approximation of constant amplitude of the harmonics of the effective potential and independent formation of the hard and soft radiation in the field of planes formed by the atomic axes, which requires separate treatment.

5. CONCLUSION

Thus, circular polarization of the harmonics of the effective crystal potential and the resultant circular polarization of the surface positron radiation lead to new opportunities for obtaining circularly polarized photon beams and longitudinally polarized positron beams as well as new ways of analyzing the polarization of the latter. These opportunities can be realized using internal as well as extracted and secondary positron beams.

The circular polarization and spin dependence of the surface radiation, which lie at the basis of these opportunities, are manifested over a wide range of positron energies, essentially without an upper bound. The theoretical description of these effects simplifies substantially in the energy range that is currently of greatest interest, from one to a several hundred GeV thanks to the fact that the approximations of constant amplitude of the harmonics of the effective crystal potential and independent formation of soft and hard radiation in the field of the planes formed by the atomic axes apply in that energy range.

The predicted polarization phenomena are for the most part analogous to the polarization phenomena associated with inverse Compton scattering of laser radiation. A significant advantage of the first of these is the time independence of the crystal field, which allows us to realize these effects in beams of arbitrary long duration, including the secondary beams of proton accelerators in which it has been proposed to produce inverse Compton scattering of laser radiation. Another of its advantages is the possibility of having the positrons interact with the harmonics of the crystal potential, the effective wavelengths of which can be, at least, an order of magnitude shorter than the wavelengths of high-power short-pulsed lasers.

The drawbacks of the predicted possibilities include, first, the fact that the probability of emission of the hard photons by the positrons reflected from a crystal plane is not too high. Thus, at positron energies of the order of 100 GeV this probability is at most a few percent. Note that this shortcoming disappears at positron energies above a few hundred GeV, where the probability reaches 10% and more. Here, however, the theoretical description of the predicted effects must be modified somewhat.

Although the soft radiation of the positrons in the plane field does not complicate the theoretical description of the interesting hard radiation in the field of the crystal axes, it is necessary to take it into account in any practical realization of these effects in the inner positron beams of accumulators. Its negative effect shows up most strongly when the energies of the latter exceed a few tens of GeV and the soft photon emission in the plane field leads to a substantially faster escape of post-emission positrons from the beam than the hard photon emission in the axis field. However, for positron energies less than 10 GeV, their soft emission in the plane field apparently does not complicate the realization of the considered effects in accumulators as badly. By virtue of its small total probability of emission, the soft radiation from positrons in extracted and secondary beams in the energy ranges considered should not introduce any fundamental difficulties in the selection of events accompanied by emission of hard photons. Note that the intense positron radiation in the field of the crystal planes can play a positive role. Specifically, when using positron beams of low angular divergence it opens up the possibility of independently determining the characteristics of the hard radiation with high statistical accuracy.

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[1)]Here we use the system of units $\hbar = c = 1$.

[2)]Note that to measure the polarization of a positron beam it is sufficient to have scattering in only a small part of the crystal. This can be provided by simpler designs than are discussed below.

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