Electromagnetic excitation of transverse ultrasound under inhomogeneous electromagnetic-to-acoustic conversion in a tangential magnetic field

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We demonstrate by theoretical means that inhomogeneous electromagnetic-to-acoustic conversion in isotropic ferromagnetic metals in a constant tangential magnetic field can lead to effective generation of transverse ultrasound in addition to longitudinal. We establish the conditions in which only transverse elastic waves are generated in such geometry. Finally, we show that inhomogeneous electromagnetic-to-acoustic conversion can lead to resonant amplification of the excited elastic vibrations. © *1996 American Institute of Physics.* [S1063-7761(96)02303-6]

It is known that homogeneous electromagnetic-to-acoustic conversion in a constant tangential magnetic field in isotropic and polycrystalline ferromagnets can lead to intense excitation of only longitudinal elastic vibrations.[1–3] Generally, in real experiments the electromagnetic field is inhomogeneous.[4] According to Ref. 4, such a field can be represented by a continuous sum of plane waves (spatial harmonics) propagating along an axis with an infinite spectrum of wave numbers q and a set of amplitudes $\mathbf{h}_q$. Here the projection of the wave vector $\mathbf{q}$ on the axis of wave propagation can take on arbitrary values in the interval from 0 to q. In this case transverse vibrations are excited in the medium in addition to longitudinal elastic vibrations.

In this paper we show that under certain conditions transverse ultrasound can be intensively excited in isotropic ferromagnets placed in a tangential magnetic field by inhomogeneous electromagnetic-to-acoustic conversion. Longitudinal ultrasound may be entirely absent in this case.

We study a semi-infinite isotropic ferromagnetic metal occupying the half-space $z<0$. The ferromagnet's surface coincides with the xy plane. The magnetic field lies in the zy plane: $\mathbf{H}=(0, H_{0y}, -H_{0z})$.

To solve the problem we employed a system of Maxwell, Landau–Lifshitz, and elasticity equations in an approximation in which the coupling between the electromagnetic, spin, and elastic vibrations is small:

$$\xi = H_0^2/\rho c^2 \ll 1, \quad \zeta = b^2 M_0^2 \chi^2/\rho c^2 \ll 1, \tag{1}$$

where ξ and ζ are, respectively, the dimensionless parameters of electromagnetic-acoustic and magnetoelastic coupling, ρ is the metal's density, c is the speed of sound, b is the magnetoelastic constant, and M_0 and χ are the ferromagnet's saturation magnetization and susceptibility. The second equation excludes the range of magnetic fields and frequencies close to various resonant values,[2] where the dynamic susceptibility χ may be high. Another approximation is that we ignore spatial and temporal dispersion in the ferromagnet's susceptibility. In this case the Landau–Lifshitz equations establish a simple relationship between the ferromagnet's magnetization and the magnetic field strength:[2,4]

$$M_i = \chi_{ik} H_k, \tag{2}$$

where only the diagonal components of the tensor $\hat{\chi}$ are nonzero. This relationship is valid in the following range of magnetic field strengths:

$$H_0 \gg \alpha M_0 (\omega/c)^2, \ \ \omega/g, \tag{3}$$

where α is the inhomogeneous exchange constant, ω is the frequency of the electromagnetic wave, and g is the gyromagnetic ratio.

Within these approximations we can ignore the electromagnetic field that emerges as a result of excitation of elastic vibrations in comparison to the source field. The problem of ultrasound excitation in a ferromagnet breaks down into two problems: first the Maxwell equations are used to solve the electrodynamic problem of determining the electromagnetic field strength in the metal from the known external field, and then the elasticity equations are used with the established values of the field to find the amplitudes of the generated elastic waves. Both problems are solved with standard boundary conditions at the surface $z=0$ of the ferromagnet for the electromagnetic field and the stress tensor.[2,4]

For an inhomogeneous electromagnetic field in the chosen geometry, the solution of the first problem yields[4]

$$\mathbf{e},\mathbf{h}(z,y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathbf{e}_q, \mathbf{h}_q(z,q) e^{iqy} dy, \tag{4}$$

where $\mathbf{e}_q(z,q)$ and $\mathbf{h}_q(z,q)$ are the Fourier transforms of the electromagnetic field in the metal,

$$\begin{aligned} h_{qy} &= 2h_{0y}\exp(-i^{3/2}kz)/(1-\eta), \\ h_{qz} &= (\mu_y q/\mu_z i^{1/2} k) h_{qy}/(1-\eta), \\ e_{qx} &= -(v k_y^2/4\pi\sigma i^{1/2} k) h_{qy}/(1-\eta), \end{aligned} \tag{5}$$

with h_{0y} the amplitude of the variable magnetic field along the y axis outside the metal at the metal's surface,[4]

σ the metal's conductivity, $k^2=k_y^2-iq^2\mu_y/\mu_z$, $k_y^2=4\pi\mu_y\omega\sigma/v^2$, $\eta=i^{3/2}(k^2-q^2)^{1/2}\mu_y/k$, μ_y and μ_z the components of the ferromagnet's permeability tensor, and v the speed of light in vacuum.

If we allow for (5), we can write the elasticity equations, which determine the components of the displacement vector u, as follows:

$$\frac{d^4u_{y,z}}{dz^4}+(q_{0l}^2+q_{0t}^2)\frac{d^2u_{y,z}}{dz^2}+q_{0l}^2q_{0t}^2u_{y,z}=F_{y,z}(z), \quad (6)$$

where $q_{0t,l}=(q_{t,l}^2-q^2)^{1/2}\equiv q_{t,l}\cos\gamma_{t,l}$ are the projections of the wave vectors of the shear ($q_t=\omega/c_t$) and longitudinal ($q_l=\omega/c_l$) waves on the z axis, $F_{z.y}$ are functions proportional to the projections of coordinate axes of the total bulk force, which allows for the induction and magnetoelastic mechanisms (the corresponding expressions are given below).

Equation (6) is solved by variation of constants. After using this method we employ the boundary conditions imposed on the stress tensor at $z=0$ to find the amplitudes of the excited ultrasonic vibrations. As a result, for the components of the displacement vector that propagate in the metal we have

$$u_{y,z}=u_{y,zt}\exp(-iq_{0t}z)+u_{y,zl}\exp(-iq_{0l}z), \quad (7)$$

where

$$u_{zt}=q[s^2(q_{0t}^2-q^2)(F_{1s}+F_{1v})+2qq_{0l}(F_{2s}+F_{2v})]/s^2\Delta,$$

$$u_{zl}=q_{0l}[(q_{0t}^2-q^2)(F_{2s}+F_{2v})-2qq_{0t}s^2(F_{1s}+F_{1v})]/s^2\Delta, \quad (8)$$

$$u_{yt}=q_{0t}u_{zt}/q, \quad u_{yl}=-qu_{zl}/q_{0l},$$

$$\Delta=4q^2q_{0t}q_{0l}+(q_{0t}^2-q^2)^2, \quad s=c_t/c_l.$$

Here by F_{is} and F_{iv} we denote the dimensionless total surface and bulk forces, respectively:

$$F_{1s}+F_{1v}=\frac{h_{0y}}{\rho c_t^2}\left\{-i^{1/2}M_{0y}\frac{q}{k}\left[b_0\kappa_y(1+s^2)+b_1\left(2\kappa_y-\kappa_z+\frac{\mu_y\kappa_z}{\mu_z}\right)\right]+\frac{iB_{0z}}{4\pi}-i^{1/2}\frac{q}{k}\times\left[\frac{(1-2s^2)B_{0y}}{4\pi}+(1+s^2)M_{0y}\right]\right\}, \quad (9)$$

$$F_{2s}+F_{2v}=-\frac{ih_{0y}}{\rho c_t^2}\left[\frac{s^2H_{0y}}{4\pi}-i^{1/2}\frac{q}{k}s^2\left(\mu_yM_{0z}-\frac{B_{0z}}{4\pi}\right)\right], \quad (10)$$

where b_0 and b_1 are, respectively, the bulk and relativistic magnetostriction constants, $\mathbf{B}=\mathbf{H}_0+4\pi\mathbf{M}_0$ is the magnetic field induction in the ferromagnet, and $\hat{\varkappa}$ is the magnetic susceptibility tensor. Equations (9) and (10) are written to frst power of the ratio q/k_y, which is assumed small. Indeed, for bulk waves the wave number q is bounded from above: $0<q<q_t$, and the ratio q_t/k_y, as demonstrated in Ref. 4, is much smaller than unity in most ferromagnetic metals at the frequencies usually used in experiments and practical applications ($\omega\leqslant10^8\,s^{-1}$).

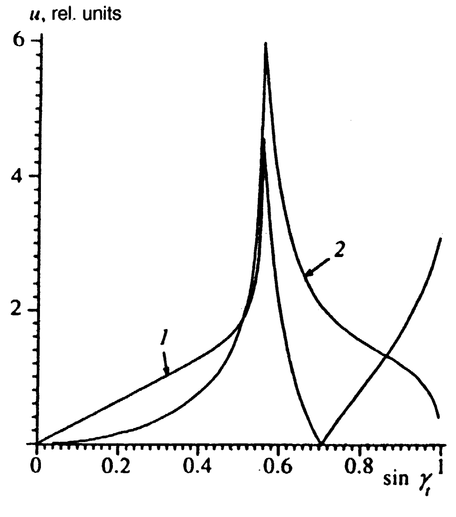

FIG. 1. The absolute value of the relative amplitude of elastic displacements excited by the magnetoelastic mechanism in a constant tangential magnetic field, as a function of the acceptance angle of vibrations: curve 1—longitudinal waves; curve 2—transverse waves.

Equations (8)–(10) imply that when the variable electromagnetic field is inhomogeneous, usually both longitudinal and transverse ultrasonic vibrations are excited in a ferromagnetic metal.

Comparison of u_{yt} with u_{zl} shows that for small values of q the efficiency of excitation by the magnetoelastic mechanism (the terms proportional to b_0 and b_1 in Eqs. (9) and (10)) of transverse ultrasound exceeds the efficiency of excitation of longitudinal ultrasound, since $u_{yt}\propto q$ and $u_{zl}\propto q^2$. In the first approximation in the small parameter q/k, the magnetoelastic mechanism of ultrasound excitation is attributable solely to the tangential component M_{0y} of magnetization.

With the induction mechanism and a constant tangential magnetic field ($H_0=H_{0y}$) only longitudinal waves are efficiently excited, while in a normal magnetic field ($H_0=H_{0z}$) the excited waves are transverse. The other vibrations are weakened by a factor of k/q.

At $q=q_l$ ($\gamma_l=90°$) a longitudinal wave propagates along the metal's boundary. In the wave-number range $q_l<q<q_t$ longitudinal vibrations are damped, since q_{0l} is complex-valued within this range. As q_{0l} grows, the damping of the longitudinal waves increases. However, the transverse waves in this range remain undamped. Thus, if the vibrational spectrum of the electromagnetic field contains mainly harmonics with wave numbers from the range $q_l<q<q_t$, only transverse elastic vibrations are excited. This is true for both the magnetoelastic excitation mechanism and the induction mechanism.

Let us illustrate these points by numerical calculations, say for $s=0.5$. The results are depicted in Figs. 1 and 2, which show the dependence of the relative values of the amplitudes of the elastic waves, $u_l\propto|u_{zl}/\cos\gamma_l|$ and $u_t\propto|u_{yt}/\cos\gamma_l|$, excited by the magnetoelastic (Fig. 1) and induction (Fig. 2) mechanisms on the acceptance angle of vibrations.

Figures 1 and 2 show that for the magnetoelastic mecha-

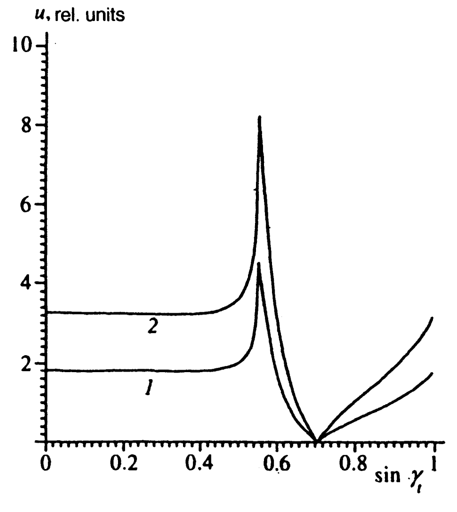

FIG. 2. The absolute value of the relative amplitude of elastic displacements excited by the induction mechanism in a constant tangential magnetic field, as a function of the acceptance angle of vibrations: curve *1*—longitudinal waves at $\mathbf{H}_0 \| \mathbf{y}$, and curve *2*—transverse waves at $\mathbf{H}_0 \| \mathbf{z}$.

nism and small angles γ_t, the amplitude of the transverse sound exceeds that of the longitudinal, as noted earlier. For both mechanisms at $\gamma_l \simeq 90°$ the amplitudes of both longitudinal and transverse sound resonantly grow. Since for $\gamma_l \geqslant 90°$ longitudinal vibrations are damped, only transverse ultrasound is intensively excited in this case. We note also that for both mechanisms there are angles at which no vibrations are excited. For the induction mechanism and small γ_t, the amplitude of elastic vibrations is essentially independent of γ_t (or of q). This is the main difference between the induction mechanism and the magnetoelastic, in which u_l and u_t are proportional to q^2 and q, respectively.

Thus, our analysis shows that in contrast to the homogeneous case, in inhomogeneous electromagnetic-to-acoustic conversion in a constant tangential magnetic field, transverse elastic vibrations can be intensively excited in addition to longitudinal. The dominant mechanism here is the magnetoelastic. Transverse vibrations are dominant, first, when the electromagnetic spectrum contains mostly vibrations with small wave numbers and, second, when the wave numbers lie within the range $q_l < q < q_t$, where the longitudinal waves decay. The strongest (in a resonant manner) transverse vibrations are excited at acceptance angles $\gamma_l \geqslant 90°$ $(q \geqslant q_l)$. In the $q \leqslant q_l$ range there is also a longitudinal vibration resonance. There are likewise angles at which no vibrations are excited.

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