Ion transport in gases and weakly ionized nonequilibrium plasmas in a variable electric field

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A study is made of ion transport in gases and weakly ionized plasmas under the influence of a strong electric field under conditions when the velocity distribution of the ions is non-Maxwellian and anisotropic. An expression is obtained for the ion flux due to variation in space and/or time of the reduced electric field, on which the distribution function depends. The corresponding ion transport coefficients are determined for some model collision integrals.
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1. INTRODUCTION

Study of the processes of collisional ion transport in weakly ionized gases and plasmas has both scientific and practical interest.[1–5] The transport of charged particles determines many properties of laboratory and ionospheric plasmas. Analysis of experimental data on transport coefficients makes it possible to establish the ion–atom (ion–molecule) interaction potential.[2,3,5] Data on these coefficients are needed to analyze the results of investigations of ion-molecular reactions in a plasma. Ion transport processes are taken into account in the modeling of electric discharges in gases and in some atmospheric effects.

Ion transport in gases and plasmas in an electric field has been the subject of numerous experimental and theoretical studies,[2,6] which have treated both equilibrium conditions (weak fields) and strongly nonequilibrium ones (strong fields), when the ion energy distribution is non-Maxwellian. Most of the investigations have been devoted to ion drift and diffusion. In a plasma, gradients of the charged-particle densities are always associated with gradients of the electric field, the gas density, etc. Therefore, the transport equations must also take into account the fluxes due to the inhomogeneity of these parameters and behave like thermal diffusion. Ion thermal diffusion may also be important in the analysis of the error in the measurement of the ionic mobility in gases by the drift-tube method.[2,3]

These ion transport processes in gases and plasmas have been quite fully studied; on the other hand, all studies have been made for equilibrium conditions (see, for example, Refs. 7–9), However, it is known[9–12] that the description of the analogous transport processes for electrons in a strong electric field, when the energy distribution is non-Maxwellian, differs appreciably from the one adopted for equilibrium conditions. In the absence of equilibrium, the system of hydrodynamic equations is changed, and unusual fluxes appear in them. Mechanical extension of these results to ions is not justified, since the description of the electron processes uses the fact that the electron mass is small relative to the mass of the neutral particles. This last makes it possible to employ the well-known two-term approximation,[13] which is associated with the weak anisotropy of the electron energy distribution. For ions, this approximation is not adequate except in rare cases. On the other hand, the description of ion transport in gases and plasmas can be somewhat simplified because of the comparatively small (in contrast to the electron case) number of inelastic processes that affect the energy distribution.[2,3] Thus, interest attaches to the study of ion transport in gases and in nonequilibrium weakly ionized plasmas when the electric field and of the other parameters on which the ion energy distribution depends vary in space and time. Although ions become nonequilibrium under more stringent conditions than electrons, these conditions are nevertheless realized in a gas-discharge plasma[14] and in drift-tube experiments.[2,3] Therefore, this problem warrants a detailed investigation. Only the first steps have been taken in this direction. For example, there have been studies of the relaxation of the ion drift velocity in a gas when a strong electric field is suddenly switched on[15] and of the change of the ion distribution function in a time-dependent field.[16]

In the present paper, the approach used earlier to describe electrons[11,12] is generalized to ions, i.e., we make a study not restricted to the two-term approximation of ion transport in weakly ionized gases and plasmas under the influence of a strong electric field when the ion velocity distribution is nonequilibrium and anisotropic. We obtain expressions for the coefficients that describe the ion transport when the electric field and of the other parameters on which the ion velocity distribution depends vary in space and time. We determine these transport coefficients for some model collision integrals for ions interacting with neutral particles.

2. BASIC EQUATIONS

We consider a weakly ionized gas or plasma in an electric field that is sufficiently strong that the ions satisfy the conditions

$$|q|E\lambda > T, \quad \nu \gg \nu_{\mathrm{Coul}}.$$

Here q is the ion charge, E is the electric field strength, λ is the relaxation length of the ion energy, T is the temperature of the gas, ν is the relaxation frequency of the ion energy in collisions, and $\nu_{\rm Coul}$ is the frequency of Coulomb collisions. If the first inequality is satisfied, the ion temperature is strongly "decoupled" from the gas temperature, and when the second inequality is also satisfied the ion velocity distribution function becomes non-Maxwellian.[2,3] For the most typical case $T=300$ K and ions and neutral particles having nearly the same masses, these conditions reduce to

$$E/N>2\cdot 10^{-16}-10^{-15}\ \mathrm{V\cdot cm^2},\quad \alpha\ll 10^{-3}-10^{-2},$$

where N is the density of the neutral particles, and α is the ionization cross section. It is more difficult to satisfy the first condition in the case of ions in their own gas; this is due to the process of resonant charge exchange, which is characterized by a large scattering cross section and, accordingly, a small value of λ. It is harder to satisfy the first inequality for ions, which almost always exchange energy with the neutral particles more efficiently, and easier to satisfy the second condition then for electrons. The given range of parameters of the considered system is encountered in drift-tube experiments[1–5] and in gas discharges.[14]

Under these conditions, as was shown earlier[10,11] (see also Refs. 9 and 12) for the example of electrons, the system of hydrodynamic equations describing the motion of the charged particles simplifies and reduces to a single equation for their density. On the other hand, it is necessary to take into account in it the fluxes due to the spatial and temporal variation of the parameters (E/N, etc.) on which the energy distribution of the charged particles depends. All this is equally true for ions. However, the derivation of the expressions for the electron transport coefficients[9–12] uses the well-known two-term approximation,[13] which is based on the small value of the electron mass. It is of interest to develop an approach analogous to that of Refs. 10–12 for ions too, using the Boltzmann equation for an arbitrary mass ratio of the charged and neutral particles. This is the main aim of the present paper.

Under these conditions, the ion velocity distribution function $f(\mathbf{v})$ is described by the linear Boltzmann equation

$$\frac{\partial(nf)}{\partial t}+\mathbf{v}\,\frac{\partial(nf)}{\partial \mathbf{r}}+\frac{q\mathbf{E}n}{m}\,\frac{\partial f}{\partial \mathbf{v}}=S(f)=nN\tilde{S}(f),\qquad (1)$$

where n and m are the ion density and mass, and S is the collision integral for interaction of the ions with the neutral particles. Here the distribution function is normalized by the condition

$$\int f(\mathbf{v})d\mathbf{v}=1.\qquad (2)$$

When Eq. (1) is integrated over the velocities, we obtain a balance equation for the ion density:

$$\frac{\partial n}{\partial t}+\mathrm{div}(n\mathbf{w})=Q,\qquad (3)$$

where we have the mean ion velocity

$$\mathbf{w}=\int \mathbf{v}f(\mathbf{v})d\mathbf{v},\qquad (4)$$

and Q describes bulk processes of creation and annihilation of ions. Substituting (3) in (1), we obtain

$$\frac{\partial f}{\partial t}-\frac{f}{n}\,\mathrm{div}(n\mathbf{w})+\frac{\mathbf{v}}{n}\,\frac{\partial(nf)}{\partial \mathbf{r}}+\frac{q\mathbf{E}}{m}\,\frac{\partial f}{\partial \mathbf{v}}=N\tilde{S}(f)-\frac{fQ}{n}.\qquad (5)$$

Following the approach developed in Refs. 11 and 12, we assume that the distribution function $f(\mathbf{v})$ depends on a certain scalar parameter γ, which varies slowly in space and time:

$$\lambda\ll L,\qquad (6)$$

$$\omega\ll\nu,\qquad (7)$$

where

$$L\sim\left(\frac{1}{\gamma}\,\frac{\partial\gamma}{\partial x}\right)^{-1},\quad \omega\sim\frac{1}{\gamma}\,\frac{\partial\gamma}{\partial t}.$$

For simplicity, we also set $Q=0$ and $n=$const, i.e., we shall not consider the well-studied processes of ion diffusion. Then the function $f(\mathbf{v})$ in the inhomogeneous nonstationary case can be found by means of perturbation theory with respect to the small parameters λ/L and ω/ν:

$$f=f_0(1-\varphi).\qquad (8)$$

The zeroth approximation is found from Eq. (5), neglecting all the terms that describe the spatial and temporal variation,

$$J(f_0)\equiv\frac{q\mathbf{E}}{m}\,\frac{\partial f_0}{\partial \mathbf{v}}-N\tilde{S}(f_0)=0,\qquad (9)$$

and from the normalization condition (2), where f is replaced by f_0.

The equation for the first-order correction has the form

$$\frac{\partial f_0}{\partial t}-f_0\ \mathrm{div}\ \mathbf{w}_0+\mathbf{v}\,\frac{\partial f_0}{\partial \mathbf{r}}=J(f_0\varphi)\qquad (10)$$

with normalization condition

$$\int f_0\varphi\ d\mathbf{v}=0,\qquad (11)$$

which follows from the analogous conditions for f and f_0.

In a space having only one distinguished direction—the direction of the electric field—the arguments of the function f_0 are the ion velocities parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the field. As independent variables, we choose $v_\parallel$ and $v=(v_\parallel^2+v_\perp^2)^{1/2}$. In accordance with (9), f_0 also depends on the parameter $\gamma=E/N$, which varies in space and time.

Under these conditions, Eq. (10) can be rewritten in the form

$$\frac{\partial f_0}{\partial\ln\gamma}\,\frac{\partial\ln\gamma}{\partial t}-f_0\,\frac{\partial w_0}{\partial\ln\gamma}\,\mathbf{e}\nabla\ln\gamma+\frac{\partial f_0}{\partial\ln\gamma}\,\mathbf{v}\nabla\ln\gamma$$
$$-f_0w_0\ \mathrm{div}\ \mathbf{e}+\frac{\partial f_0}{\partial v_\parallel}\,\mathbf{v}\,\frac{\partial\mathbf{e}}{\partial t}+\frac{\partial f_0}{\partial v_\parallel}\,\mathbf{v}\otimes\mathbf{v}:\nabla\otimes\mathbf{e}=J(f_0\varphi),\qquad (12)$$

where $\mathbf{e}=\mathbf{E}/E$ is the unit vector along the electric field, $\mathbf{a}\otimes\mathbf{b}$ is the tensor with components $a_i b_j$, and the colon denotes contraction. Because the functional J is linear, the solution of Eq. (12) must be sought in the form

$$\varphi(\mathbf{v})=A(\mathbf{v})\,\frac{\partial\ln\gamma}{w_0\partial t}+\mathbf{B}(\mathbf{v})\nabla\ln\gamma+\mathbf{C}(\mathbf{v})\,\frac{\partial\mathbf{e}}{w_0\partial t}+\hat{F}(\mathbf{v}):\nabla\otimes\mathbf{e}, \tag{13}$$

where A, $\mathbf{B}$, $\mathbf{C}$, $\hat{F}$ are, respectively, scalar, vector, and tensor (second-rank) functions. To ensure that they all have the same dimensions, the time derivatives have been divided by w_0, which is the ion drift velocity in the homogeneous stationary case and is determined from (4) by substituting f_0 in place of f.

From the fact that the different terms in (12) are independent, we obtain equations for A, $\mathbf{B}$, $\mathbf{C}$, and $\hat{F}$:

$$w_0\,\frac{\partial f_0}{\partial\ln\gamma}=J(f_0A), \tag{14}$$

$$-\mathbf{e}\,\frac{\partial w_0}{\partial\ln\gamma}f_0+\mathbf{v}\,\frac{\partial f_0}{\partial\ln\gamma}=J(f_0\mathbf{B}), \tag{15}$$

$$\mathbf{v}_\perp w_0\,\frac{\partial f_0}{\partial v_\parallel}=J(f_0\mathbf{C}), \tag{16}$$

$$-(\hat{I}-\mathbf{e}\otimes\mathbf{e})w_0f_0+\mathbf{v}\otimes\mathbf{v}_\perp\,\frac{\partial f_0}{\partial v_\parallel}=J(f_0\hat{F}), \tag{17}$$

where $\hat{I}$ is the second-rank unit tensor. [In the derivation of (16) and (17), we have taken into account the constancy of the magnitude of $\mathbf{e}$.] It should be borne in mind that here and in what follows we understand by $\partial f_0/\partial v_\parallel$ the derivative $(\partial f_0/\partial v_\parallel)_v$. The normalization condition (11) remains as before with φ replaced by A, $\mathbf{B}$, $\mathbf{C}$, and $\hat{F}$, respectively. The scalar function A depends on the same arguments and parameters as f_0, i.e., cylindrical symmetry is preserved for it. Similarly, the vector and tensor functions must be expressed in terms of linear combinations of the vectors and tensors that occur on the left-hand sides of the corresponding equations. Thus, it follows from (15) that

$$\mathbf{B}=\mathbf{v}_\parallel B_\parallel+\mathbf{v}_\perp B_\perp,$$

where the scalar functions $B_\parallel$ and $B_\perp$ satisfy the equations

$$-f_0\,\frac{\partial w_0}{\partial\ln\gamma}+v_\parallel\,\frac{\partial f_0}{\partial\ln\gamma}=J(f_0v_\parallel B_\parallel) \tag{18}$$

and

$$v_\perp\,\frac{\partial f_0}{\partial\ln\gamma}=J(f_0v_\perp B_\perp). \tag{19}$$

Similarly, from (16) we obtain

$$\mathbf{C}=\mathbf{v}_\perp C,$$

$$v_\perp w_0\,\frac{\partial f_0}{\partial v_\parallel}=J(f_0v_\perp C). \tag{20}$$

For the tensor $\hat{F}$, it follows from (17) that

$$\hat{F}=\mathbf{e}\otimes\mathbf{v}_\perp v_\parallel F_{\parallel\perp}+(\mathbf{i}_x\otimes\mathbf{i}_xv_{\perp x}^2+\mathbf{i}_y\otimes\mathbf{i}_yv_{\perp y}^2)F_\perp+(\mathbf{i}_x\otimes\mathbf{i}_y+\mathbf{i}_y\otimes\mathbf{i}_x)v_{\perp x}v_{\perp y}F_{\perp xy},$$

where $\mathbf{i}_x$ and $\mathbf{i}_y$ are unit vectors along the x and y axes of a rectangular coordinate system with z axis along $\mathbf{e}$; $v_{\perp x}$ and $v_{\perp y}$ are the corresponding components of the vector $\mathbf{v}_\perp$. The equations for the cylindrically symmetric scalar functions $F_{\parallel\perp}$, $F_\perp$, and $F_{\perp xy}$ have the form

$$v_\perp v_\parallel\,\frac{\partial f_0}{\partial v_\parallel}=J(f_0v_\perp v_\parallel F_{\parallel\perp}), \tag{21}$$

$$v_{\perp x}v_{\perp y}\,\frac{\partial f_0}{\partial v_\parallel}=J(f_0v_{\perp x}v_{\perp y}F_{\perp xy}), \tag{22}$$

$$-w_0f_0+v_{\perp x}^2\,\frac{\partial f_0}{\partial v_\parallel}=J(f_0v_{\perp x}^2F_\perp). \tag{23}$$

Substitution of (8) and (13) in (4) with allowance for the above gives for the mean ion velocity in the time-dependent inhomogeneous field the expression

$$\mathbf{w}=\mathbf{w}_0-D_t\mathbf{e}\,\frac{\partial\ln\gamma}{w_0\partial t}-\hat{D}_\gamma\nabla\times\ln\gamma-\hat{D}_{et}\,\frac{\partial\mathbf{e}}{w_0\partial t}-\hat{\hat{D}}_e:\nabla\otimes\mathbf{e}, \tag{24}$$

where

$$D_t=\int f_0v_\parallel A\,d\mathbf{v},\quad \hat{D}_\gamma=\int f_0\mathbf{v}\otimes\mathbf{B}\,d\mathbf{v},$$

$$\hat{D}_{et}=\int f_0\mathbf{v}\otimes\mathbf{C}\,d\mathbf{v},\quad \hat{\hat{D}}_e=\int f_0\mathbf{v}\otimes\hat{F}\,d\mathbf{v}.$$

Here D_t is a scalar, $\hat{D}_\gamma$ and $\hat{D}_{et}$ are second-rank tensors, and $\hat{\hat{D}}_e$ is a third-rank tensor.

The expression (24) can be significantly simplified by taking into account the cylindrical symmetry of many functions and the following relations for an arbitrary function $g(v,v_\parallel)$:

$$\int g(v,v_\parallel)v_\perp d\mathbf{v}=\int g(v,v_\parallel)v_{\perp x}v_{\perp y}d\mathbf{v}=\int g(v,v_\parallel)v_\perp^3d\mathbf{v}=0$$

and

$$\int g(v,v_\parallel)v_{\perp x}^2d\mathbf{v}=\int g(v,v_\parallel)v_{\perp y}^2d\mathbf{v}=\frac{1}{2}\int g(v,v_\parallel)(v^2-v_\parallel^2)d\mathbf{v}.$$

Using them, we obtain

$$\hat{D}_\gamma=\int f_0(\mathbf{v}_\perp+v_\parallel\mathbf{e})\otimes(B_\parallel v_\parallel\mathbf{e}+B_\perp\mathbf{v}_\perp)d\mathbf{v}=D_{\gamma\parallel}\mathbf{e}\otimes\mathbf{e}$$
$$+D_{\gamma\perp}(\hat{I}-\mathbf{e}\otimes\mathbf{e}),$$

where

$$D_{\gamma\parallel}=\int f_0B_\parallel v_\parallel^2d\mathbf{v},\quad D_{\gamma\perp}=\frac{1}{2}\int f_0B_\perp(v^2-v_\parallel^2)d\mathbf{v}.$$

Thus, in a system of coordinates for which one of the axes is parallel to the vector **E** the tensor $\hat{D}_\gamma$ has diagonal form and is characterized by two scalar quantities–the coefficients of the corresponding transport parallel and perpendicular to the field. Here the situation is analogous to the diffusion transport of ions described by coefficients of longitudinal and transverse diffusion.[1-3]

Similarly, for $\hat{D}_{et}$ we obtain

$$\hat{D}_{et}=\int f_0(\mathbf{v}_\perp+v_\parallel\mathbf{e})\otimes\mathbf{v}_\perp C d\mathbf{v}=D_{et}(\hat{I}-\mathbf{e}\otimes\mathbf{e}),$$

where

$$D_{et}=\frac{1}{2}\int f_0C(v^2-v_\parallel^2)d\mathbf{v}.$$

It follows from this that

$$\hat{D}_{et}\frac{\partial\mathbf{e}}{w_0\partial t}=D_{et}\frac{\partial\mathbf{e}}{w_0\partial t},$$

i.e., the ion flux due to the time dependence of the direction of the electric field is due to just a single coefficient, like the flux associated with the variation of γ. However, in contrast to the latter, the flux is directed, not along the electric field, but along the vector $\partial\mathbf{e}/\partial t$.

Taking into account the expression for the tensor $\hat{F}$, we find for $\hat{\hat{D}}_e$

$$\hat{\hat{D}}_e=\int f_0(\mathbf{v}_\perp+v_\parallel\mathbf{e})\otimes\hat{F}d\mathbf{v}=\mathbf{e}\otimes(\hat{I}-\mathbf{e}\otimes\mathbf{e})D_e+(\mathbf{i}_x\otimes\mathbf{e}$$
$$\otimes\mathbf{i}_x+\mathbf{i}_y\otimes\mathbf{e}\otimes\mathbf{i}_y)D_{e\nabla},$$

where

$$D_e=\frac{1}{2}\int f_0v_\parallel(v^2-v_\parallel^2)F_\perp d\mathbf{v},$$

$$D_{e\nabla}=\frac{1}{2}\int f_0v_\parallel(v^2-v_\parallel^2)F_{\perp\parallel}d\mathbf{v}.$$

As a result, the final term in (24) reduces to

$$\hat{\hat{D}}_e:\nabla\otimes\mathbf{e}=D_e\mathbf{e}\ \mathrm{div}\ \mathbf{e}+D_{e\nabla}(\mathbf{e}\nabla)\mathbf{e},$$

i.e., the ion flux due to the variation in space of the direction of the electric field is described by two scalar coefficients.

With allowance for the results given above, the expression (24) reduces to

$$\mathbf{w}=\mathbf{w}_0-D_t\mathbf{e}\frac{\partial\ \ln\gamma}{w_0\partial t}-D_{\gamma\perp}\nabla_\perp\ln\gamma-D_{\gamma\parallel}\mathbf{e}(\mathbf{e}\nabla)\ln\gamma$$
$$-D_{et}\frac{\partial\mathbf{e}}{w_0\partial t}-D_e\mathbf{e}\ \mathrm{div}\ \mathbf{e}-D_{e\nabla}(\mathbf{e}\nabla)\mathbf{e}. \tag{25}$$

All the coefficients we have introduced have the dimensions of a diffusion coefficient and describe nonlocal (to first order in λ/L) and inertial (to first order in ω/ν) effects for the ion velocity distribution function, i.e., (25) contains six scalar coefficients.

The coefficients $D_{\gamma\perp}$ and $D_{\gamma\parallel}$ describe anisotropic "thermal diffusion" of the ions, since the parameter γ determines their mean energy in the nonequilibrium case. In an equilibrium medium, the coefficient of thermal diffusion can be either greater or less than zero.[2,3] It is natural to assume that the coefficients introduced above can also change sign depending on the collision integral, the value of E/N, and the other parameters of the problem.

In its structure, the expression (25) recalls the expression obtained under analogous conditions for the mean electron velocity,[11,12] although the expressions for determining the corresponding transport coefficients in the case of ions and electrons differ strongly. The main difference between the ion and electron fluxes is the appearance in (25) of terms proportional to $\partial\mathbf{e}/\partial t$ and $(\mathbf{e}\cdot\nabla)\mathbf{e}$. This last circumstance is due to the appreciable anisotropy of the ion velocity distribution function, which is absent in the case of electrons, for which $(\partial f_0/\partial v_\parallel)_v\approx 0$ and $D_{et}=D_{e\nabla}=0$.

Under certain conditions, the distribution function can depend not only on E/N but also on other parameters: the gas temperature, degree of ionization, fraction of excited particles, etc. The space or time variation of these parameters must give rise to additional ion fluxes, which can also be described using the approach developed above.

3. DETERMINATION OF TRANSPORT COEFFICIENTS FOR MODEL COLLISIONS

In the general case, the calculation of the ion transport coefficients in a gas under these conditions is a complicated numerical problem that is most often solved by the moment method or the Monte Carlo method.[1-3] These approaches are very formalized and directed toward obtaining a definite result. One obtains much greater physical transparency and simplicity in the approaches that are aimed at the solution of the Boltzmann equation for model collision integrals,[1-3,6,17] which can be applied to by no means all real systems and only in a restricted range of variation of the external conditions. The results of the application of these approaches to determine the ion transport coefficients introduced in the previous section are given in the Appendix.

One of these approaches, the so-called Lorentz model, has been used earlier[11] to describe electron transport. In particular, in the framework of this model analytic expressions were obtained for the coefficients when only elastic collisions are important and the scattering cross section varies as a power of the relative velocity of the colliding particles. These results can also be naturally applied to the description of the transport of light ions in a buffer gas consisting of heavy particles. However, this case is encountered relatively seldom in the case of ions. In the Appendix, we have obtained expressions for these coefficients for more typical ion transport models. They include the motion of ions in their own gas in a strong electric field, ion transport in the case of a constant collision frequency (Maxwell's model), and motion of heavy ions in a gas of light neutral particles (Rayleigh's model).

4. CONCLUSIONS

In this paper, we have considered ion transport having a thermal diffusion nature in weakly ionized gases and plasmas in a strong electric field when the ion energy distribution is nonequilibrium. Our results differ appreciably from the well-studied case with the equilibrium distribution function that is realized in the weak-field limit. First, as for ion diffusion, the fluxes are strongly anisotropic. Second, they can no longer be expressed in the form of a product of the corresponding coefficient and the gradient of an effective ion temperature that depends on T and E/N. In addition, in contrast to the previously studied case of electrons, for ions fluxes proportional to $\partial \mathbf{e}/\partial t$ and $(\mathbf{e}\cdot\nabla)\mathbf{e}$ also arise. A physical example of the realization of situations in which these additional fluxes can play a decisive role is that of a plasma in a rotating electric field and in a solenoidal field. In particular, circularly polarized rf or microwave radiation is of interest from the point of view of the stabilization of some discharge instabilities.[18–20] In this case, the field is constant in magnitude and varies in time only in its direction. The additional flux causes the instantaneous direction of the total ion flux to efficiently lag the field. When a homogeneous magnetic field varies sufficiently rapidly in accordance with a linear law in time, so that the electric solenoidal field is strong in the sense of this paper, the main role is played by the additional flux proportional to $(\mathbf{e}\cdot\nabla)\mathbf{e}$, which characterizes the "inertia" of the ion drift motion along the field.

It should be noted that physically the analyzed fluxes differ appreciably from the classical thermal diffusion fluxes that are initiated by an inhomogeneous background. In the above, we have studied a situation in which the background is homogeneous and the fluxes are due to variation of the electric field in space and time.

These ion transport processes can be important primarily in experiments with drift tubes made to measure the mobility and diffusion coefficients of ions in gases in a strong electric field.[2,3] The error of the measurement of, for example, the ion mobility is ~ 1, and an important contribution to it can come from inhomogeneity of E/N and other parameters. Indeed, in accordance with the estimate of Ref. 2, the ratio of the thermal diffusion flux of the ions to their drift flux under the conditions of the experiments can reach 1%. For heavy ions in a light gas, this ratio is even greater, since in this case the coefficients of longitudinal thermal diffusion are proportional to $m/M \gg 1$ (Rayleigh's model; M is the mass of the neutral particles).

Significant progress can be expected in this branch of physics if we can learn how to measure the ion transport coefficients introduced above. For example, the coefficient D_t can be obtained by analyzing experimental data for drift tubes when an alternating electric field is applied to the electrodes. Analysis of these coefficients together with the mobility of the ions and their diffusion coefficients would make it possible to obtain additional information about the cross sections and interaction potentials for ions interacting with neutral particles.

The effects studied above may also be important in a weakly ionized plasma. Although electron transport is frequently the most effective in an electron–ion plasma, the ambipolar fields that arise have the consequence that the total ambipolar diffusion and the thermal diffusion of the plasma are determined mainly by the slower ions.[21] In addition, more and more attention has been devoted in recent years to ion–ion plasmas,[22] the stationary characteristics and stability of which depend on ion transport. We note that such plasmas are not encountered only in strongly electronegative gases, in which almost all the electrons are attached to neutral particles. For example, in the positive column of a glow discharge there can be radial stratification of a plasma consisting of electrons and positive and negative ions.[23–25] Near the column axis, there is an ion–ion plasma, while near the column wall electrons and positive ions form the plasma.

APPENDIX

Motion of ions in their own gas in a strong electric field

Suppose that in the case of motion of ions in their own gas the main channel for scattering by neutral particles is resonant charge exchange leading to exchange of the velocities of the colliding particles. At the same time, the motions of the ions parallel and perpendicular to the field are "separated," and the ion drift velocity in a strong field appreciably exceeds the thermal velocity of the neutral particles ($mw_0^2 \gg T$). In the range of E/N values of interest, the charge exchange cross section σ can be assumed to be independent of the particle velocity. Then the collision integral, the distribution function of the ions, and their drift velocity in the uniform time-independent case are (Refs. 1–3, 17)

$$\bar{S}(f_0(v_\parallel)) = -v_\parallel \sigma f_0(v_\parallel), \tag{A1}$$

$$f_0(\mathbf{v}) = \sqrt{\frac{2mN\sigma}{\pi qE}} \exp\left(-\frac{mN\sigma}{2qE} v_\parallel^2\right) \theta(v_\parallel)\delta(\mathbf{v}_\perp), \tag{A2}$$

$$w_0 = \sqrt{\frac{2qE}{\pi mN\sigma}}, \tag{A3}$$

where $\theta(v_\parallel)$ is equal to unity for $v_\parallel > 0$ and zero for $v_\parallel < 0$, and δ is the delta function.

We first consider ion transport along the field. The solution of the corresponding equations with allowance for (A1)–(A3) can be written in the form

$$A(v_\parallel) = \frac{mw_0}{qE}\left(\frac{v_\parallel^3}{3\pi w_0^2} - \frac{v_\parallel}{2} + \frac{w_0}{6}\right),$$

$$B_\parallel(v_\parallel) = \frac{m}{2qE}\left[\left(\frac{v_\parallel^2}{\pi w_0^2} - 1\right)\frac{v_\parallel^2}{2} - \left(\frac{v_\parallel}{w_0} - \frac{\pi - 8}{8}\right) w_0^2\right],$$

$$F_{\perp}(v_{\parallel})=\frac{2mw_0(w_0-v_{\parallel})}{qE(v^2-v_{\parallel}^2)},\quad C=0,\quad F_{\parallel\perp}=0.$$

The corresponding coefficients are determined by integrating the functions given above. They are

$$D_{et}=0,\quad D_{e\nabla}=0,\quad D_{\gamma}=\frac{8-\pi}{8\pi}\frac{w_0}{N\sigma},$$

$$D_e=-\frac{\pi-2}{\pi}\frac{w_0}{N\sigma},\quad D_t=\frac{w_0}{3\pi N_\sigma}.$$

To determine the ion transport coefficient in the transverse direction, $D_{\gamma\perp}$, there is no need to solve Eq. (19). Instead, following the approach employed to find the transverse diffusion coefficient $D_{\perp}$ of ions,[1-3,17] we multiply both sides of this equation by $v_{\perp}$ and integrate it in the velocity space. We obtain

$$D_{\gamma\perp}=\frac{1}{N\sigma w_0}\frac{\partial\langle v_{\perp}^2\rangle}{\partial\ln\gamma}=0,$$

where the brackets denote averaging over the velocities. The coefficients $D_{\gamma\perp}$, $D_{e\nabla}$, and D_{et} vanish because the motion of the ions at right angles to the field does not depend on the parameter E/N.

The Maxwell and Rayleigh models of ion motion in a gas

We consider the transport processes discussed above using the so-called Maxwell and Rayleigh models of ion motion (Refs. 1–3, 17, and 26). In the first, one takes into account only elastic collisions of the ions with the neutral particles, and the interaction potential is assumed to be a polarization potential. Here the particle collision frequency $\nu=Ng\sigma$ does not depend on the relative velocity g. In the second model, one considers the transport of heavy ions in a gas of light neutral particles ($m/M\gg1$); in the case of a sufficiently strong electric field, this leads to a "needle-shaped" ion distribution function $f(\mathbf{v})$, for which the drift velocity w_0 is appreciably greater than the thermal velocity of both the neutral particles and the ions. In the Rayleigh model, one usually takes into account only elastic collisions with cross section that varies as a power on the velocity: $\sigma\propto g^{-\beta}$.

In the determination of the drift velocity w_0 and the diffusion coefficients $D_{\perp}$ and $D_{\parallel}$ under the assumptions made above, there is no need to find the distribution function $f(\mathbf{v})$ with allowance for the corrections for the inhomogeneity of the electron density. Instead, one can use the moment method, which gives the necessary collision integrals (Refs. 1–3, 17, 26, and 27). We apply a similar approach to obtain the required transport coefficients in the Maxwell and Rayleigh models.

Multiplying both sides of the equations for A, $B_{\perp}$, $B_{\parallel}$, C, and $F_{\perp}$ by $v_{\perp}$ or $v_{\parallel}$, integrating them over velocity, and simplifying the collision integrals by analogy with the case of ion diffusion (Refs. 1–3, 17, 26, and 27), we obtain

$$D_t=\frac{m+M}{(2-\beta)M}\frac{w_0}{\nu_1}\frac{\partial w_0}{\partial\ln\gamma},$$

$$D_{\gamma\perp}=\frac{m+M}{M\nu_1}\frac{\partial\langle v_{\perp}^2\rangle}{\partial\ln\gamma},$$

$$D_{\gamma\parallel}=\frac{m+M}{(2-\beta)M\nu_1}\frac{\partial}{\partial\ln\gamma}\left(\langle v_{\parallel}^2\rangle-\frac{w_0^2}{2}\right),$$

$$D_{et}=\frac{m+M}{M}\frac{w_0^2}{\nu_1},$$

$$D_e=\frac{m+M}{(2-\beta)M\nu_1}(\langle v_{\parallel}^2\rangle-w_0^2-\langle v_{\perp}^2\rangle),$$

$$D_{e\nabla}=\frac{m+M}{(2-\beta)M\nu_1}(\langle v_{\parallel}^2\rangle-\langle v_{\perp}^2\rangle),$$

where ν_1 is the momentum-transfer frequency of ion collisions with the neutral particles.

In the Maxwell model, $\beta=1$ and $\nu_1=1.105\nu$, where $\nu=2\pi qN(\alpha_p/\mu)^{1/2}$, α_p is the polarizability of the neutral particles, and μ is the reduced mass of such a particle and an ion. Here the drift velocity and the mean squares of the longitudinal and transverse velocities of the ions are[1-3,17,26]

$$w_0=\frac{m+M}{mM}\frac{eE}{\nu_1},$$

$$\langle v_{\parallel}^2\rangle=\frac{m+M}{m}\frac{\nu_1+(M/4m)\nu_2}{\nu_1+(3M/4m)\nu_2}w_0^2+\frac{T}{m},$$

$$\langle v_{\perp}^2\rangle=\frac{1}{2}\left(\frac{m+M}{m}w_0^2+\frac{3T}{m}-\langle v_{\parallel}^2\rangle\right).$$

where $\nu_2=0.772\nu$ is the viscosity collision frequency of the ions with the neutral particles.

In the Rayleigh model ($m\gg M$), the drift velocity is determined from the relation

$$\frac{qE}{MN}=w_0^2\sigma(w_0),$$

and $\langle v_{\parallel}^2\rangle$ and $\langle v_{\perp}^2\rangle$ are found from[17,26]

$$\langle v_{\parallel}^2\rangle=\frac{Mw_0^2}{(2-\beta)m}\left(1-\frac{\nu_2}{2\nu_1}\right)+w_0^2$$

and

$$\langle v_{\perp}^2\rangle=\frac{M\nu_2}{2m\nu_1}w_0^2.$$

In this model, $D_t\sim D_{\gamma\parallel}\sim D_{et}\sim mw_0^2/M\nu_1$ and $D_{\parallel}\sim D_{\perp}\sim w_0^2/\nu_1$ hold, i.e., certain coefficients that describe the ion fluxes of thermal diffusion nature are appreciably greater than the diffusion coefficients. This is because the ionic fluxes considered above are due to the nonlocality of the ion distribution function. Generally speaking, they are determined by the ion drift velocity. In contrast to them, the ion diffusion fluxes are determined by the random velocity,[17,26] which in the Rayleigh model is $\sim(m/M)^{1/2}$ times smaller than w_0.

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