Mechanism of formation of pulses of self-induced transparency in the presence of Kerr nonlinearity

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The formation of soliton-like pulses in resonantly absorbing media in the presence of nonresonant Kerr nonlinearity is investigated analytically. Under the assumption of coherent interaction of the field with two-level atoms, solutions are found in the form of stationary phase-modulated pulses. The dependences of the pulse parameters on the form of broadening of the resonance line and on the detuning between the carrier frequency and the transition frequency are investigated. In contrast to "pure" self-induced transparency, the existence of pulses of a stationary shape is possible in a restricted range of powers and durations. A detailed description is given of a new mechanism of soliton formation based on mutual compensation of the phase self-modulation due to the effect of the Kerr nonlinearity, and the nonlinear resonant dispersion of the group velocity due to the interaction of the pulse with resonant atoms. Simultaneous interaction of a pulse with resonant and nonresonant nonlinearities can be realized in a single-mode optical fiber with resonant ion impurities. To avoid effects due to linear dispersion of the group velocity, the pulse carrier frequency must be in the region of zero dispersion of the fiber. The results of the theory are applied to a calculation of an experimental scheme of a fiber laser for the generation of self-induced transparency solitons proposed by Nakazawa *et al.*, for which the conditions listed above are satisfied. It is found that obtaining a stable pulse train of solitons is problematic unless there is appreciable modification of the experimental arrangement. © *1996 American Institute of Physics.* [S1063-7761(96)00501-4]

1. INTRODUCTION

The generation of ultrashort light pulses and the study of their properties is a rapidly developing field of laser physics. We consider the two main mechanisms of formation of optical solitons. The attention of scientists is mainly concentrated on investigating solitons of the nonlinear Schrödinger equation (NSE solitons) as the main carriers of information in fiber-optic communication lines. The soliton formation mechanism is based on the mutual compensation of the linear dispersion of the group velocity and the nonlinearity of the medium due to the nonlinear phase self-modulation of a pulse.[1]

However, a detailed description of the nonlinear interaction of optical pulses with matter is severely restricted by the requirement that the pulse carrier frequency be far from all resonant frequencies of the medium. In the case of strong resonant interactions, the response of the medium to an applied field is described by Bloch's system of equations.[2] In the framework of the two-level approximation and for sufficiently short pulses (pulse duration much shorter than all relaxation times), the self-consistent system of Maxwell–Bloch equations also has solutions in the form of stable solitary waves—self-induced transparency solitons.[3,4] The mechanism of formation of these solitons differs from the NSE mechanism and is associated with coherent absorption of photons at the pulse leading edge and their stimulated reemission at the trailing edge. A self-induced transparency soliton can be obtained in at least two different ways:

1) an external source (laser) is used to form an initial radiation pulse that, passing through the resonant medium, acquires the shape of a self-induced transparency soliton;[4]

2) a cell with coherent absorber is placed within a laser cavity,[5] which for a suitable choice of the conditions can itself be a source of self-induced transparency solitons. The possibility of using the self-induced transparency effect as a mode-locking mechanism in a laser with a coherent absorber was established theoretically in Refs. 5–9. A program of experimental realization of the generation of self-induced transparency solitons is described in Ref. 10.

Undoubted interest attaches to the question of the simultaneous action of both soliton formation mechanisms in a field in the form of a short pulse. So far as we know, the problem of the propagation of an intense optical pulse in a medium with resonant and nonresonant linearities and dispersion of the group velocity was first posed in Ref. 11 and then considered in more detail in Ref. 12. In the framework of this model, it was found that the existence of soliton solutions is possible only in the case of exact fulfillment of a relationship between the dispersion of the group velocity, the coefficient of nonlinearity of the refractive index, and the dipole moment of the resonant transition.[1)] This condition is too stringent and difficult to implement for the media usually employed.[15,16]

A physical situation in which there is simultaneous realization of the three effects—linear dispersion of the group velocity, nonlinearity of the refractive index, and resonant nonlinearity—can be modeled by introducing impurities in the form of resonant atoms within an optic fiber. In such a model, one is justified in considering the possible coexistence of self-induced transparency solitons and NSE solitons. Moreover, the solution of the problem is of interest from two

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points of view.[12] First, the use of optic fibers as guiding system for the self-induced transparency solitons makes it possible to avoid the destructive action of diffraction[17,18] and observe the development of "pure" self-induced transparency over appreciable distances: $L>160L_{ab}$, where L is the length of the medium, and L_{ab} is the absorption length (see Ref. 16). Second, some of the pulse energy is irreversibly lost during the time of the transition process, going over into excitation energy of the medium and then being dissipated in the surrounding space by relaxation processes. This makes it possible to obtain purer pulses than in the case when the mechanism of Schrödinger soliton formation takes place.

However, we have noted above that the practical realization of the desired NSE solitons and the self-induced transparency solitons encounters a serious hindrance in the form of a stringent condition imposed on the parameters of the medium. In order to obtain at least a partial solution to the problem and observe the effect of self-induced transparency in "pure" form, it was proposed[10,16] to suppress Schrödinger soliton formation by shifting the pulse carrier frequency into the region of zero dispersion of the waveguide. However, the experiments of Refs. 10 and 16 reveal only partial agreement with the theory of the effect. In this paper, it will be shown that the discrepancies that arise can be attributed to the effect of nonresonant nonlinearity on the formation of self-induced transparency solitons. That this effect can be important follows if only because in the presence of only nonresonant nonlinearity stable propagation of pulses of stationary shape is impossible. Moreover, we shall show that the competition between the resonant and Kerr nonlinearities leads to the appearance of critical values of the power and duration of the pulse that establish a boundary of the region of existence of solutions in the form of solitary waves.

Thus, we encounter the problem of the simultaneous interaction of a short pulse of the electromagnetic field with two forms of optical nonlinearity of completely different nature. Such an investigation was undertaken for the first time in Refs. 19 and 20 in connection with a study of the properties of chirped pulses in the self-induced transparency effect. However, the solution of the problem used the assumption that the nonresonant nonlinearity was weak, and this strongly restricted the applicability of the theory. A theoretical investigation free of the assumption that the nonresonant nonlinearity makes a small contribution is presented below. The investigation includes the derivation of analytic solutions for the envelope of the pulse, its phase, and the components of the Bloch vector, which are different from the solutions found in Ref. 20. New exact relationships are established between the parameters of the soliton-like pulse. The conclusion of Ref. 20 that pulses of stationary shape can exist even in the presence of nonresonant nonlinearity is also modified. We show that this is true only for pulses that are not too powerful ($\mathscr{W}<\mathscr{W}_{cr}$) and not too short ($\tau>\tau_{cr}$). A conclusion of Ref. 20 that remains unchanged is that phase modulation of the field is inescapable in the presence of nonlinearity of Kerr type.

In this paper, we not only present new analytic results and make a detailed comparison of them with the results of Matulic and Eberly[20] but also reveal for the first time the physical mechanisms of formation of soliton-like pulses in the case of simultaneous action of two forms of optical nonlinearity on the pulse. The stationary pulse shape is maintained by a balance between the phase self-modulation due to the action of the Kerr nonlinearity and the nonlinear dispersion of the group velocity induced by the interaction of the pulse with the resonant medium. The main difference from the NSE mechanism is the nonlinear nature of the dispersion of the group velocity and its variability over the region occupied by the pulse spectrum. The conclusions of the theory are used to explain the results of the experiments of Refs. 10 and 16. It should be noted in this connection that the parameters of the medium and field in Refs. 10 and 16 were such that the perturbation theory of Ref. 20 cannot be applied. We show that the experimental scheme for a laser for generating self-induced transparency solitons is unsuitable for this purpose on account of the destructive effect of the Kerr nonlinearity.

2. STATIONARY PULSES IN A MEDIUM WITH HOMOGENEOUS BROADENING

2.1. Basic equations

We write down the system of Maxwell–Bloch equations, which give a self-consistent description of the interaction of a field with resonant atoms. To the right-hand side of the wave equation, we add terms that take into account the effect of the nonresonant atoms, namely, the dispersion of the group velocity and the nonlinearity of the refractive index of the medium:

$$\frac{\partial}{\partial v}E-\sigma\frac{\partial}{\partial u}E=\frac{i}{\vartheta}\langle P\rangle-i\mu\frac{\partial^2}{\partial u^2}E-i\nu|E|^2E, \tag{1}$$

$$\frac{\partial}{\partial u}P=-i\Delta\Omega\tau P-i\vartheta EN,$$

$$\frac{\partial}{\partial u}N=\frac{i}{2}\vartheta(EP^*-E^*P), \tag{2}$$

where $u=(t-z/V)/\tau$, $v=z/L_{ab}$, $\sigma=L_{ab}(1/V-1/c)/\tau$, $\mu=L_{ab}/L_D$, $\nu=L_{ab}/L_{NL}$, $\vartheta=A_0\tau$, and A_0 is the pulse amplitude. The field and resonant part of the polarization are written as

$$\mathscr{E}(z,t)=A_0\frac{\hbar}{d}E(u,v)\exp[-i(\omega_0t-kz)], \tag{3}$$

$$\mathscr{P}(z,t)=ndP(u,v)\exp[-i(\omega_0t-kz)]. \tag{4}$$

Here ω_0 is the frequency of the resonant transition; $E(u,v)$ and $P(u,v)$ are the complex envelopes of the field and the resonant part of the polarization. The functions $E(u,v)$ and $P(u,v)$ contain a phase, and it will be convenient for us to avoid writing explicitly the offset between the field frequency and the frequency of the resonance transition in the equation for the polarization, and instead adopt for the phase

the initial conditions $\varphi \xrightarrow[t\to-\infty]{} \text{const}$, where the constant is the offset. In the Bloch equations (2), we have omitted the terms responsible for the relaxation of the inversion and the polarization, i.e., we consider pulses that are appreciably shorter than both relaxation times. The symbol $\langle ... \rangle$ denotes averaging over the ensemble of atoms:

$$\langle ... \rangle = \frac{1}{\pi g(0)} \int_{-\infty}^{\infty} g(\Delta\Omega) ... d\Delta\Omega\tau.$$

We approximate the line shape by a Gaussian,

$$g(\Delta\Omega) = \frac{T^*}{\sqrt{\pi}} \exp[-(\Delta\Omega T^*)^2],$$

where $(T^*)^{-1}$ is the spectral half-width of the inhomogeneously broadened transition.

In this section, we shall be interested in solutions in the form of pulses with spectrum much broader than the nonresonantly broadened spectral profile of the absorption (the so-called sharp-line limit): $\tau^{-1} \gg T^*$, i.e., each individual atomic transition is accurately tuned to the resonant frequency ω_0. Therefore, the characteristic absorption length L_{ab} differs from the usual definition by the replacement of T^* by τ:

$$L_{ab} = \left[\frac{2\pi\omega_0 d^2 n}{c\hbar\eta_0} \tau \right]^{-1} \tag{5}$$

where n is the concentration of the resonant atoms, d is the dipole moment, and c is the velocity of the pulse in the medium with refractive index η_0. In writing down (1), we have taken into account the dependence of the refractive index of the nonresonant medium on the field intensity: $\eta_{nr} = \eta_0 + \eta_2 I$. The characteristic length over which the Kerr nonlinearity develops is determined by the expression

$$L_{NL} = \frac{\lambda A_{\text{eff}}}{2\pi\eta_2} \mathscr{W}, \tag{6}$$

where η_2 is the coefficient of nonlinearity of the refractive index, A_{eff} is the effective area of the mode, and $\mathscr{W}$ is the power of the pulse. The dispersion length L_D is written as follows:

$$L_D = \frac{1}{2} \frac{\tau^2}{k''}, \tag{7}$$

where k'' is the dispersion of the group velocity.

The existence of conservation laws for the system of equations (1) and (2) greatly simplifies the solution of the current problem. In the absence of relaxation of the elements of the density matrix, the length of the Bloch vector is conserved:

$$N^2 + |P|^2 = 1. \tag{8}$$

The exchange of energy between the field and medium takes place in accordance with another conservation law:

$$\frac{\partial}{\partial u} \left\{ \frac{\vartheta^2}{2} \sigma |E|^2 - N + i \frac{\vartheta^2}{2} \mu \left(E \frac{\partial}{\partial u} E^* - E^* \frac{\partial}{\partial u} E \right) \right\} = \frac{\vartheta^2}{2} \frac{\partial}{\partial v} |E|^2 \tag{9}$$

Finally, the field and polarization are connected by a third conservation law:

$$\frac{\partial}{\partial u} \left\{ i(PE^* + EP^*) - \vartheta \left[i \frac{\nu}{2} |E|^4 + i\mu \left| \frac{\partial}{\partial u} E \right|^2 + \frac{1}{2} \left(E^* \frac{\partial}{\partial v} E - E \frac{\partial}{\partial v} E^* \right) \right] \right\} = \frac{\vartheta}{2} \frac{\partial}{\partial v} \left(E \frac{\partial}{\partial u} E^* - E^* \frac{\partial}{\partial u} E \right) \tag{10}$$

2.2. Solution in the form of a solitary wave

In what follows, we shall solve the problem under the assumption that the dispersion length is large compared with L_{ab} and L_{NL} and it can be assumed that $\mu \approx 0$. Such an approximation will be correct if the pulse carrier frequency is right at the region of zero dispersion of the optic fiber. When dispersion cannot be ignored ($\mu \neq 0$), the problem becomes more complicated, and its solution can be found in Refs. 11–15. We decompose the field and polarization into real amplitude and phase parts:

$$E(u,z) = e(u) \exp\{-i[\varphi(u) - \delta k z]\}, \tag{11}$$

$$P(u,z) = [p(u) + iq(u)] \exp\{-i[\varphi(u) - \delta k z]\}. \tag{12}$$

The form of the expressions (11) and (12) means that we restrict ourselves to a search for only self-wave solutions of Eqs. (1) and (2). Moreover, among these we choose only those that vanish at infinity:

$$E \xrightarrow[u\to\pm\infty]{} 0.$$

Although solutions in the form of pulses of stationary shape are not the only ones among the solitary waves that propagate without absorption, they can nevertheless be called the most fundamental solutions, since in the absolute majority of cases the remaining forms of solitary waves evolve to a state with conserved shape. (As an exception to this rule, we may mention some forms of 0π pulses; see Ref. 21.)

A correction δk to the propagation constant k arises if the pulse carrier frequency is different from the frequency of the atomic transition. The value of δk must be determined and expressed in terms of the dispersion relation $\delta k(\delta\omega)$, where $\delta\omega$ is the offset. Solving Eqs. (1) and (2) and using the conservation laws (9) and (10), we obtain expressions for the inversion and components of the polarization as functions of the field $e(u)$:

$$N(u) = -1 + \frac{\vartheta^2}{2} \sigma e^2(u), \tag{13}$$

$$p(u) = \frac{\vartheta}{2} \left[\delta k L_{ab} e(u) + \frac{\nu}{2} e^3(u) \right] \tag{14}$$

$$q(u)=\vartheta\sigma\frac{\partial}{\partial u}e(u). \tag{15}$$

In writing down the expressions (13)–(15), we have taken into account the fact that the medium is in the unexcited state before the arrival of the pulse:

$$N\xrightarrow[z\to+\infty]{}-1,\quad P\xrightarrow[z\to+\infty]{}0.$$

We use the conservation law (8) and write down a first-order differential equation for finding the profile of the envelope $e(u)$:

$$\left(\frac{\partial}{\partial u}e\right)^2=ae^2-be^4-ce^6, \tag{16}$$

$$a=\frac{\sigma-(\delta kL_{ab})^2/4}{\sigma^2},\quad b=\left(\frac{\vartheta}{2}\right)^2+\frac{\delta kL_{ab}}{4\sigma^2}\nu,$$

$$c=\frac{\nu^2}{16\sigma^2}. \tag{17}$$

For very weak fields ($e\ll 1$), we obtain exponential growth of $e(u)$ with the rate a. In accordance with the meaning of the self-wave variable u, it is precisely u that determines the velocity and duration of the pulse. Therefore, the constant a must be set equal to unity. The solution of Eq. (16) can be obtained in the form of an explicit function:

$$e(u)=\left[\cosh^2 u+\frac{b-1}{2}+\frac{b^2+4c-1}{4}e^{2u}\right]^{-1/2} \tag{18}$$

From the field normalization condition $\max[e(u)]=1$, we find a relationship between the parameters b and c:

$$b+c=1. \tag{19}$$

Finally, we represent the field envelope in the simplest form by shifting the origin $u=0$ to the maximum of the pulse:

$$u\to\left[u-\frac{1}{2}\ln(1+c)\right].$$

This procedure is valid, since the original system of equations possesses translational symmetry with respect to the wave coordinate. Thus, we obtain

$$e(u)=\left[\frac{1}{(1+c)\cosh^2 u-c}\right]^{1/2} \tag{20}$$

In the absence of nonresonant nonlinearity, the parameter c is equal to zero. In this case, we are dealing with the self-induced transparency effect in a "pure" form, and, as was to be expected, the envelope is a hyperbolic secant; see (20). If it is assumed that the nonresonant nonlinearity is weak and we take into account the parameter ν to a power not higher than the first,[2)] as was done in Ref. 20 in the framework of perturbation theory, then we also obtain an envelope in the form $e(u)=\operatorname{sech} u$. The complete identity with the shape of the self-induced transparency solitons is explained by the fact that the parameter c differs from zero by an amount $\propto\nu^2$, which is ignored in the perturbation theory.

Because of the nonlinearity of the refractive index, the in-phase part of the polarization $p(u)$, namely, the second term in (14), becomes the source of nontrivial phase modulation (i.e., a modulation that is different from a simple frequency shift). Substituting the expression (14) and the expansion (11) in Eq. (1) for the field, we find

$$\varphi=\frac{\partial}{\partial u}\varphi=-\frac{\delta kL_{ab}}{2\sigma}-\frac{3\nu}{4\sigma}e^2(u). \tag{21}$$

The first term in (21) does not vanish as $u\to\infty$ and is the frequency offset normalized by the pulse duration. Thus, there is a relationship between the correction δk to the wave number and the offset $\delta\omega$, i.e., the dispersion relation

$$\Delta=\delta\omega\tau=-\frac{\delta kL_{ab}}{2\sigma}. \tag{22}$$

Using (22) and the condition $a=1$, we write down for the pulse velocity the expression

$$V^{-1}=c^{-1}+\frac{L_{ab}\tau}{1+\Delta^2}. \tag{23}$$

It is interesting to note that the pulse velocity is completely determined by the mechanism of self-induced transparency and is not changed by the presence of nonresonant nonlinearity. The same result was obtained in the framework of perturbation theory in Ref. 20.

Returning to the expression (21), we see that the conclusion drawn in Ref. 20 of the unavoidability of phase modulation under the influence of Kerr nonlinearity is correct. Phase self-modulation is manifested most radically in the relationship between the amplitude and duration of the pulse [see (19) and (17)]:

$$\left(\frac{\vartheta}{2}\right)^2-\frac{\nu}{2}\Delta(1+\Delta^2)+\frac{\nu^2}{16}(1+\Delta^2)^2=1. \tag{24}$$

2.3. The case of exact resonance

It is readily noted that when the field is tuned to exact resonance with the transition ($\Delta=0$), the relationships between the duration of the pulse and its amplitude [see (24)] in the theory of "pure" self-induced transparency and in perturbation theory[20] are identical, whereas the present theory predicts a difference. Figure 1 shows the shape of the pulse envelope $e(u)$ for all three theories for the case $\Delta=0$ of exact resonance. Generally speaking, an expansion in a series with respect to the parameter ν becomes invalid near resonance. It can be seen from the expression (24) that for $\nu>8|\Delta|/(1+\Delta^2)$ the third term exceeds the second, although the requirement of internal consistency of perturbation theory presupposes the opposite relationship. This contradiction was not noted in Ref. 20, but it can be eliminated if the limits of applicability of perturbation theory are restricted by the inequality

$$\nu\ll\frac{8|\Delta|}{1+\Delta^2}.$$

We now make a more detailed investigation of the properties of the solution (20) in the case $\Delta=0$ and rewrite Eq. (24) in the form

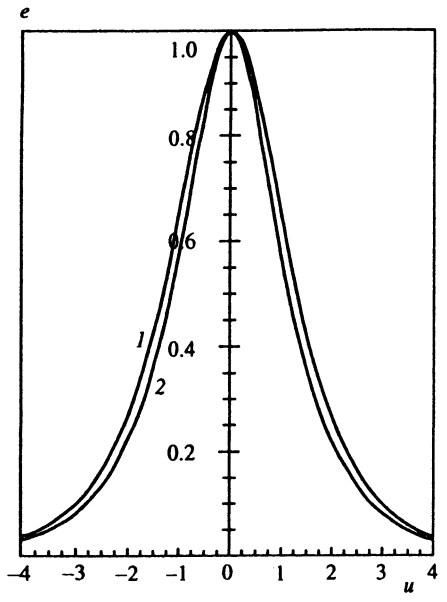

FIG. 1. Shape of pulse envelope $e(u)$. Curve 1 represents the "pure" effect of self-induced transparency; the same shape is obtained in perturbation theory (for small values of ν). Curve 2 gives the exact theory based on the combined effect of self-induced transparency and phase self-modulation, $\nu=2\sqrt{2}$.

$$\mathscr{W}_0^{\cdot 2}+\mathscr{T}_0^4\mathscr{W}_0^{\cdot}-2\mathscr{T}_0^2=0. \tag{25}$$

Here $\mathscr{W}_0^{\cdot}$ and $\mathscr{T}_0$ are the normalized power and duration of the pulse:

$$\mathscr{W}_0^{\cdot}=\frac{\mathscr{W}^{\cdot}}{\mathscr{W}_{cr}^{\cdot}},\quad \mathscr{T}_0=\frac{\tau}{\tau_{cr}},$$

$$\mathscr{W}_{cr}^{\cdot}=\frac{1}{8}c\eta_0\varepsilon_0 A_{eff}\frac{2}{\tau_{cr}^2}\frac{\hbar^2}{d^2},$$

$$\tau_{cr}=\left[\frac{c\eta_0\varepsilon_0 A_{eff}\hbar^2/8d^2}{\sqrt{2}(\lambda A_{eff}/2\pi\eta_2)(2\pi\omega_0\eta_0 d^2 n/c\hbar)}\right]^{1/3} \tag{26}$$

The physical meaning of $\mathscr{W}_{cr}^{\cdot}$ and τ_{cr} is explained in Fig. 2. In contrast to the self-induced transparency effect in "pure" form, a stationary solution in a medium with two types of nonlinearity does not exist for all values of the power and duration. The critical values $\mathscr{W}_{cr}^{\cdot}$ and τ_{cr} are the maximum values of the power and the minimum duration for which a stationary propagation regime can still be achieved. Analysis of the transient processes of establishment of stationary form of the field when a pulse enters a medium with arbitrary parameters is beyond the scope of the problem considered here, although there is undoubted interest in an estimate of the extent to which nonresonant nonlinearity affects the threshold conditions for the occurrence of a solitary wave and the processes of breakup of pulses with area $>3\pi$. The dashed curve in Fig. 2 is the branch corresponding to the unstable solution. This solution does not satisfy the obvious asymptotic behavior—in the absence of nonresonant nonlinearity, $\nu\to 0$, it does not go over into a classical 2π pulse. It can also be seen from Fig. 2 that for the same value of the power the Kerr nonlinearity leads to pulses of shorter duration than for the case of "pure" self-induced transparency. In

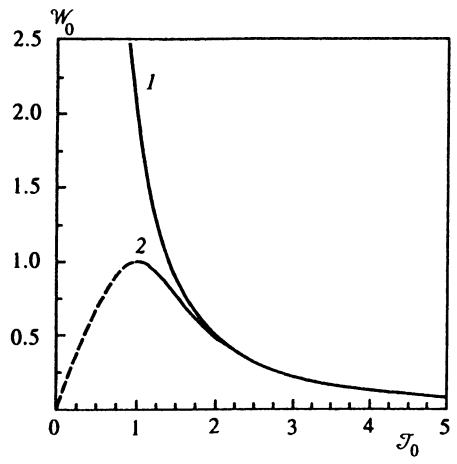

FIG. 2. Graph of dependence of the power of a pulse on its duration. Curve 1 corresponds to the effect of "pure" self-induced transparency: $\nu=0$. Curve 2 takes into account the existence of nonresonant nonlinearity: $\nu\neq 0$. The dashed curve is the branch corresponding to the unstable solution. The power and duration are normalized in accordance with (26).

Fig. 1, we show for comparison the shape of the classical self-induced transparency solitons and the envelope of the soliton-like pulse in the form (20) with the shortest possible duration $\tau=\tau_{cr}$ at a given level of the power.

2.4. The case of arbitrary offset

We now analyze the parameters of the stationary solution, taking into account the offset between the pulse carrier frequency and the center of the absorption line. We rewrite Eq. (24), using the notation introduced in (26):

$$\mathscr{W}_0^{\cdot 2}+\frac{\mathscr{T}_0^4-2\sqrt{2}\Delta(1+\Delta^2)\mathscr{T}_0}{(1+\Delta^2)^2}\mathscr{W}_0^{\cdot}-\frac{2\mathscr{T}_0^2}{(1+\Delta^2)^2}=0. \tag{27}$$

The dependence of the power of the pulse on its duration is qualitatively the same as for the case of exact resonance. As the offset is changed, only the position of the maximum of the curve is shifted. The coordinates of the maximum as a function of the offset, $\mathscr{W}_{max}^{\cdot}(\Delta)$ and $\mathscr{T}_{min}(\Delta)$, can be found from the graphs in Fig. 3. If the carrier frequency of the field is shifted to the red relative to the resonance, $\Delta<0$, the maximum power of the stationary solution rapidly decreases:

$$\mathscr{W}_{max}^{\cdot}\underset{\Delta\to-\infty}{\propto}1/\Delta^2.$$

and simultaneously there is an increase in the value of the minimum duration:

$$\mathscr{T}_{min}\underset{\Delta\to-\infty}{\propto}|\Delta|.$$

If the shift of the carrier frequency is toward the violet, $\Delta>0$, then with increasing offset the maximum power increases, reaching asymptotically the limiting value $\mathscr{W}_0^{\cdot}\xrightarrow[\Delta\to+\infty]{}3/\sqrt[3]{4}\approx 1.890$; at the same time, the minimum duration increases as $\mathscr{T}_{min}\propto\Delta$. The asymmetry of the curves with respect to the sign of the offset is explained by the

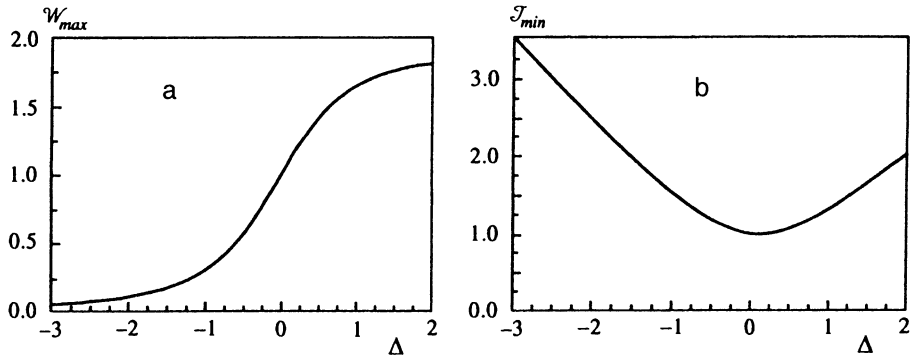

FIG. 3. Maximum power (a) and minimum duration (b) of the stationary solution (in the presence of Kerr nonlinearity) as a function of the offset. For positive values of the offset, the $\mathscr{W}_{max}(\Delta)$ tends asymptotically to the value 1.89.

difference in the dispersion properties of the nonlinear resonant medium to the right and left of the transition frequency. This will be a subject of discussion in Sec. 4.

The dependences shown in Fig. 3 confirm the obvious conclusion that a stationary solution cannot exist in the presence of only nonlinearity of Kerr type. With increasing distance from resonance, the effect of the self-induced transparency disappears, and the region of existence of solutions in the form of solitary waves becomes narrower and narrower. However, it should be noted here that as we displace the pulse carrier frequency we move away from the point of zero dispersion of the fiber, and allowance for the dispersion properties of the medium becomes important. Sufficiently far from resonance, the soliton formation mechanism acquires a different nature and is described by the nonlinear Schrödinger equation.

To compare the results of the exact theory and perturbation theory when there is offset between the carrier frequency and the transition frequency, we examine Fig. 4. It shows the dependence of the power of the pulse on the duration for $\Delta=-1$ and $\Delta=+1$ for the three theories considered. For negative values of the offset, perturbation theory gives qualitatively and quantitatively correct results. Large discrepancies are observed at positive offsets: perturbation theory predicts infinite growth of the power at finite values of the duration. This contradiction is eliminated in the exact theory. Quite generally, near the singular point

$$\mathscr{T}_0=[2\sqrt{2}|\Delta|(1+\Delta^2)]^{1/3}$$

perturbation theory is invalid, as we noted above.

Besides the concepts of critical power and critical duration (26), it is helpful to introduce the concept of the maximum dimensionless parameter ν_{max}, which, if exceeded, would signify breakdown of the stationary propagation regime. Introducing the quantity

$$\nu_{cr}=L_{ab}(\tau_{cr})/L_{NL}(\mathscr{W}_{cr})$$

for the case of exact resonance, we can write down an expression for ν_{max} as a function of the offset:

$$\nu_{max}=\frac{L_{ab}(\mathscr{T}_{min})}{L_{NL}(\mathscr{W}_{max})} \tag{28}$$

In the case of exact resonance, $\nu_{max}=\nu_{cr}=2\sqrt{2}$.

Figure 5 shows the dependence $\nu_{max}(\Delta)$. It explains the proximity of the curves for the exact theory and perturbation theory when $\Delta=-1$ and their strong difference when $\Delta=+1$ (see Fig. 4). We see that for the same absolute magnitude of the offset, the greatest possible value of the parameter ν differs strongly depending on the sign of the offset: for positive values of the offset, ν can take values so large than they are outside the limit of applicability of perturbation theory; in the case of negative offset, the maximum possible values of ν are much less, and perturbation theory gives satisfactory results.

It should be noted that the condition $\nu<\nu_{max}$ is necessary but not sufficient for the existence of self-induced transparency solitons. Sufficiency is ensured by the simultaneous fulfillment of two inequalities: $\tau>\mathscr{T}_{min}$, $\mathscr{W}<\mathscr{W}_{max}$.

All the comparisons and discussions presented above were basically concerned with pulses having a power close to the maximum value, when the Kerr nonlinearity has a strong effect on the pulse dynamics. If the pulse is weak, the

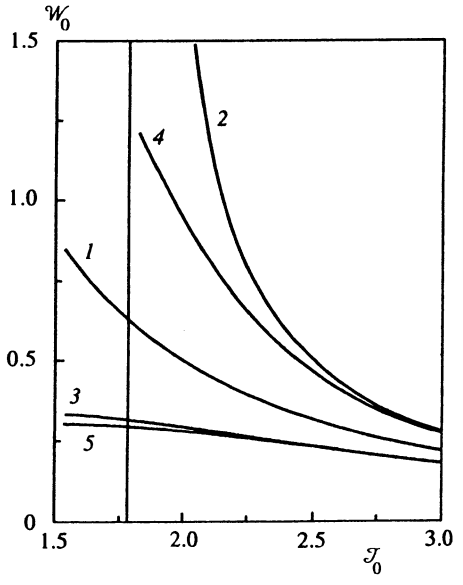

FIG. 4. Dependence of pulse power on duration for two values of the offset: $\Delta=\pm1$. Curve 1 gives the effect of the "pure" self-induced transparency, $\nu=0$. Curves 2 and 3 correspond to perturbation theory for $\Delta=+1$ and $\Delta=-1$, respectively. Curves 4 and 5 give the result of the exact theory for $\Delta=+1$ and $\Delta=-1$, respectively. The vertical line passes through the singular point $\mathscr{T}_0\approx1.78$ and is an asymptote for curve 2. The power and duration are normalized in accordance with (26).

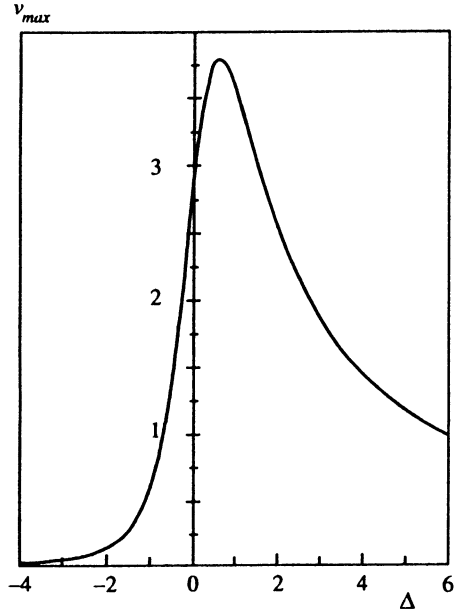

FIG. 5. Dependence of the dimensionless parameter ν_{max} on the dimensionless offset Δ.

effect of the Kerr nonlinearity is slight, and the results of all three theories—the exact theory, perturbation theory, and the theory of self-induced transparency—agree. This corresponds to the asymptotic merging of the curves in Figs. 2 and 4 in the limit $\mathscr{T}_0 \to \infty$.

3. PULSES IN A MEDIUM WITH INHOMOGENEOUS BROADENING

We consider the different limiting case in which the inhomogeneously broadened line of the resonance transition is appreciably broader than the pulse spectrum: $(T^*)^{-1} \gg \tau^{-1}$. At the same time, the definition of the absorption length is changed:

$$L_{ab} = \left[\frac{2\pi^2 \omega_0 d^2 n}{c\hbar\eta_0} T^* \right]^{-1}. \tag{29}$$

It is no longer possible to find a solution of the system (1)–(2) for $\nu \neq 0$ in analytic form. In the limit $\nu \to 0$, we are dealing with the "pure" effect of self-induced transparency, and the solution of this problem is a pulse without phase modulation with envelope in the form sech u (Ref. 4), and the solution can be found by representing the polarization in factorized form: $q(\Delta\Omega,u) = F(\Delta\Omega)\tilde{q}(u)$. However, the presence of nonresonant nonlinearity leads to the occurrence of phase modulation, and polarization expressed in factorized form is no longer a solution of the problem. In such a situation, it is sensible to seek an approximate solution by representing the population difference as a power series in the field amplitude. This method was developed in Ref. 22 in an application to the problem of the propagation of ultrashort pulses in an absorber without the use of the approximation of slowly varying phases and amplitudes, and it was generalized to the case of arbitrary offsets in Ref. 23. To simplify the exposition, we shall assume that the field is tuned to exact resonance with the medium, i.e.,

$$\dot{\varphi} \xrightarrow[u \to \pm\infty]{} 0, \quad \delta k L_{ab} = 0.$$

Solving Eqs. (1) and (2), we can readily obtain an expression for the phase of the field:

$$\varphi = -\frac{3}{4}\frac{\nu}{\sigma} e^2 + \frac{1}{\sigma\vartheta^2}\frac{1}{e^2}\langle \Delta\Omega \tau N \rangle. \tag{30}$$

It can be seen from Eq. (30) that if we know the functional dependence $N = N(\Delta\Omega, e)$, we can determine $\dot{\varphi}(e)$ and reduce the problem to the solution of a second-order nonlinear differential equation for the field $e(u)$. We represent $N(\Delta\Omega, e)$ as an expansion in powers of the field intensity:

$$N(\Delta\Omega, e) = -1 + \sigma\vartheta^2 \sum_{n=0}^{\infty} \frac{f_{2n}(\Delta\Omega)}{2n+2} e^{2n+2}. \tag{31}$$

From Eqs. (2) there follows an analogous expansion for the absorption part of the polarization $q(\Delta\Omega, e)$:

$$q(\Delta\Omega, e) = \sigma\vartheta\dot{e} \sum_{n=0}^{\infty} f_{2n}(\Delta\Omega) e^{2n}. \tag{32}$$

For the purposes of this paper, it is sufficient to consider the first two terms of the series $n = 0,1,\ldots$. Substituting (31) into (30), we obtain the required expression for the phase $\dot{\varphi}(e)$ of the field as a function of the amplitude:

$$\dot{\varphi}(e) = \left[-\frac{3}{4}\frac{\nu}{\sigma} + \frac{1}{4}\langle \Delta\Omega \tau f_2(\Delta\Omega) \rangle \right] e^2. \tag{33}$$

The expansion (32) and Eq. (1) for the field are compatible only under the condition that all the spectral response functions beginning with $n = 1$ are odd:

$$\langle f_{2n}(\Delta\Omega) \rangle = 0, \quad n = 1,2,\ldots \tag{34}$$

Substituting (33) and (11) into (1) and using the condition (34), we find

$$\langle p(\Delta\Omega, u) \rangle = \sigma\vartheta \left[\langle \Delta\Omega \tau f_0(\Delta\Omega) \rangle e + \frac{1}{4}\left(\frac{\nu}{\sigma} + \langle \Delta\Omega \tau f_2(\Delta\Omega) \rangle \right) e^3 \right]. \tag{35}$$

Similarly, we obtain an expression for $\langle \Delta\Omega \tau p(\Delta\Omega, u) \rangle$:

$$\langle \Delta\Omega \tau p(\Delta\Omega, u) \rangle = \sigma\vartheta \left[\langle (\Delta\Omega\tau)^2 f_0(\Delta\Omega) \rangle e + \left(\frac{3}{4}\frac{\nu}{\sigma} - \frac{1}{4}\langle \Delta\Omega \tau f_2(\Delta\Omega) \rangle \right) \times \left(\frac{1}{3}\langle \Delta\Omega \tau f_0(\Delta\Omega) \rangle e^3 + \frac{1}{5}\langle \Delta\Omega \tau f_2(\Delta\Omega) \rangle e^5 \right) \right]. \tag{36}$$

Using (31)–(36), we write down the final equation for the field amplitude:

$$\dot{e}^2=\hat{a}e^2-\hat{b}e^4-\hat{c}e^6, \tag{37}$$

$$\hat{a}=\frac{\langle 1\rangle/\sigma-\langle(\Delta\Omega\tau)^2 f_0(\Delta\Omega)\rangle}{\langle f_0(\Delta\Omega)\rangle},$$

$$\hat{b}=\frac{1}{2}\left(\frac{\nu}{\sigma}-\frac{1}{3}\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle\right)\frac{\langle\Delta\Omega\tau f_0(\Delta\Omega)\rangle}{\langle f_0(\Delta\Omega)\rangle}+\left(\frac{\vartheta}{2}\right)^2,$$

$$\hat{c}=\frac{1}{16}\left(\frac{\nu}{\sigma}-\frac{1}{3}\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle\right)\times\frac{9\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle/5+\nu/\sigma}{\langle f_0(\Delta\Omega)\rangle}. \tag{38}$$

It is remarkable that in the derivation of Eq. (37) we have not made any additional approximations besides (31), and in the limit $\tau \ll T^*$ Eq. (37) goes over exactly into Eq. (16) with the same coefficients:

$$a=\hat{a}, \quad b=\hat{b}, \quad c=\hat{c} \quad \text{for } \tau \ll T^*.$$

It should also be mentioned that retention of the following terms in the expansion (31) leads merely to an increase in the degree of the polynomial on the right-hand side of (37), and the solution can again be found, at least by quadratures.

To obtain a solution in closed form, it is necessary to know the values of the spectral response functions $f_0(\Delta\Omega)$ and $f_2(\Delta\Omega)$ averaged with a weight over the inhomogeneously broadened profile. The detailed procedure for finding these quantities is described in the Appendix. We use the results obtained in the Appendix, having calculated all the necessary integrals in the limit of a broad absorption line, $\tau \gg T^*$:

$$\langle f_0(\Delta\Omega)\rangle=1, \quad \langle\Delta\Omega\tau f_0(\Delta\Omega)\rangle=0,$$

$$\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle=-\nu. \tag{39}$$

Then the expressions for $\hat{a},\hat{b},\hat{c}$ simplify to

$$\hat{a}=1, \quad \hat{b}=\left(\frac{\vartheta}{2}\right)^2, \quad \hat{c}=-\frac{1}{15}\nu^2. \tag{40}$$

Using the normalization condition (19) and making the substitutions $b\to\hat{b}$ and $c\to\hat{c}$, we write down an equation that relates the power and duration of the pulse:

$$\nu^2-2\sqrt{15}\mathcal{T}^2\nu+15=0. \tag{41}$$

We have here introduced the dimensionless parameter $\mathcal{T}$ in accordance with

$$\mathcal{T}=\frac{\tau}{\hat{\tau}_{cr}}, \quad \hat{\tau}_{cr}=\sqrt{\frac{1}{\sqrt{15}}\frac{c\varepsilon_0\eta_0 A_{\text{eff}}\hbar^2/d^2}{\lambda A_{\text{eff}}/2\pi\eta_2}}L_{ab}. \tag{42}$$

The time $\hat{\tau}_{cr}$ is the minimum pulse duration, below which the stationary propagation regime is impossible. An upper limit of the pulse power can be determined similarly:

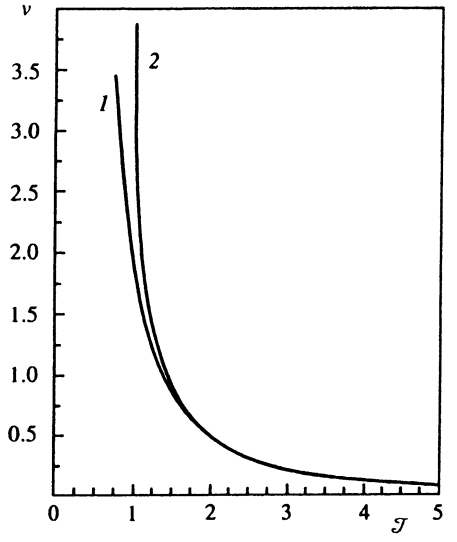

FIG. 6. Dependence of the power of a pulse on its duration for a medium with inhomogeneously broadened absorption line. Curve 1 corresponds to the effect of "pure" self-induced transparency. Curve 2 takes into account the presence of nonresonant nonlinearity: $\nu\neq 0$. Instead of the power, the ordinate gives the dimensionless parameter ν, which is proportional to the power.

$$\hat{\mathscr{W}}_{cr}=\sqrt{15}\frac{\lambda A_{\text{eff}}/2\pi\eta_2}{L_{ab}}. \tag{43}$$

The nature of the dependence of the power of the pulse on its duration is illustrated by Fig. 6, where for convenience of subsequent analysis we have plotted along the ordinate not the pulse power $\mathscr{W}$, but the dimensionless parameter ν proportional to it; in fact $\nu=\sqrt{15}\mathscr{W}/\hat{\mathscr{W}}_{cr}$ and, therefore, max $\nu=\sqrt{15}$. The qualitative nature of the dependence in Fig. 6 is the same as in the case of a homogeneously broadened absorption line—the curve has a discontinuity. However, whereas for a homogeneously broadened line allowance for phase self-modulation leads to pulses of a shorter duration than for the "pure" effect of self-induced transparency at the same pulse power, for the inhomogeneously broadened line the situation is reversed—the phase self-modulation leads to an increase in the duration.

4. MECHANISMS OF FORMATION OF SOLITON-LIKE PULSES

In the above, we have considered in detail the problem that takes into account the joint effect of phase self-modulation and self-induced transparency on the process of pulse formation. The very possibility of stationary pulses presupposes the existence of a mechanism that compensates the effect of the phase self-modulation. We show that for a medium with a homogeneously broadened line, compensation is achieved by the nonlinear resonant dispersion of the group velocity that arises under the influence of self-induced transparency. For an inhomogeneously broadened line, compensation is achieved by the dispersion of the group velocity induced by the pulse in the resonant medium by means of the Kerr nonlinearity. In this section, on the basis of an analysis of the competition between phase self-modulation, self-

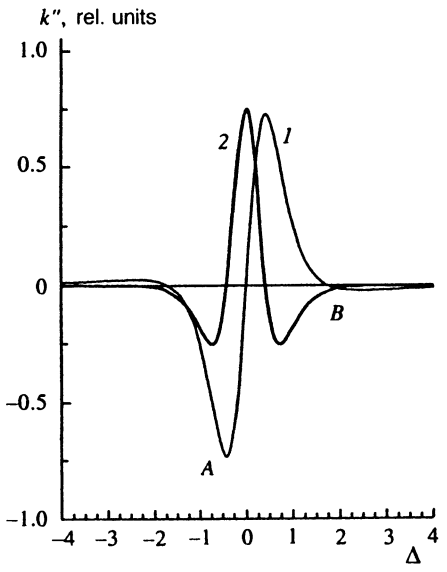

FIG. 7. Nonlinear resonant dispersion of the group velocity for a homogeneously broadened absorption line (curve *1*) and for an inhomogeneously broadened absorption line (curve *2*) as a function of the normalized offset Δ. The letters *A* and *B* indicate the positions of local minima of the dispersion. The scale of the ordinate is arbitrary.

induced transparency, and nonlinear dispersion of the group velocity, we explain the main features detected: the nature of the phase modulation of the pulse, the reasons for the existence of critical values of the power and pulse duration, the asymmetric dependence of the pulse parameters on the sign of the offset, etc.

We consider a medium with a homogeneously broadened line, making the assumption that the pulse carrier frequency is near resonance. The dependence of the refractive index on the intensity for $\eta_2>0$ (the inequality is valid for the majority of media employed) leads to a displacement of the "red" frequencies in the spectrum of the field to the leading edge of the pulse and to a corresponding displacement of the "blue" frequencies to its trailing edge. The resulting asymmetry can be compensated by the anomalous dispersion of the group velocity, $\partial^2 k/\partial\omega^2<0$, which forces the "blue" components to move faster than the "red" ones, and, thus makes the pulse narrower with time. The graph of the resonant dispersion of the group velocity obtained for pure self-induced transparency is shown in Fig. 7 (curve *1*). It completely mimics the dependence of the linear dispersion of the group velocity on the frequency for a homogeneously broadened absorption line, except that the homogeneous lifetime T_2 is replaced here by the pulse duration τ. It can be seen from Fig. 7 that the best conditions for pulse propagation, i.e., $\min(\partial^2 k/\partial\omega^2)$, are created to the left of the resonance in the immediate vicinity of the center of the line, $\Delta\approx-0.4$ (point *A* in Fig. 7). At the same time, it may appear that the dependence in Fig. 5 contradicts the assertion we have made, since the "transmission maximum" (i.e., max ν, see Fig. 5) is at a "blue" shift, $\Delta\approx+0.7$, i.e., it lies in the region of large values of the normal dispersion of the group velocity. In fact, there is no contradiction: the true pulse carrier frequency is determined by the expression

$$\omega_{\text{av}}=\omega_0+\int_{-\infty}^{\infty}\frac{\partial\varphi}{\partial t}e^2(t)dt\Big/\int_{-\infty}^{\infty}e^2(t)dt, \tag{44}$$

and, using (21), we find

$$\omega_{\text{av}}=\left(\omega_0+\frac{\Delta}{\tau}\right)-\nu\frac{1+\Delta^2}{2\tau}=\omega-\nu\frac{1+\Delta^2}{2\tau}. \tag{45}$$

In the expression (45), we have identified two components of the offset—the components that depend and do not depend on the magnitude of the Kerr nonlinearity; ω is the pulse carrier frequency in the absence of the Kerr nonlinearity. Substituting the values of $\nu_{\max}$ and $\Delta(\nu_{\max})$ into (45), we find that in reality a "transmission maximum" is found for pulses with mean carrier frequency ω_{av} that is shifted to the "red" and is in immediate proximity to the maximum of the anomalous dispersion (point *A* in Fig. 7). The fact that ω_{av} and the coordinate of the point *A* do not exactly coincide is explained by the asymmetry of the dispersion curve near the point *A* and incomplete allowance for the dispersion properties of the medium, since phase self-modulation makes an additional contribution to the in-phase nonlinear (in the field) part of the polarization $p(u)$, which we have ignored in our simplified interpretation.

It can be asserted that by virtue of the effective compression of the pulse due to the dispersion of the group velocity, it has a shorter duration than a 2π pulse in "pure" self-induced transparency (see Fig. 1). Perfect compensation of the phase self-modulation by the resonant dispersion of the group velocity for a pulse of stationary shape is confirmed by the absence of a linear chirp in the expression for the phase $\varphi(u)$ [see (21)] when it is expanded in powers of u near the maximum of the field:

$$\varphi(u)=\varphi\Big|_{u=0}+\frac{d\varphi}{du}\Big|_{u=0}u+\frac{1}{2}\frac{d^2\varphi}{du^2}\Big|_{u=0}u^2+\ldots, \tag{46}$$

and from (21) we find $d^2\varphi/du^2|_{u=0}=0$. The lack of a linear chirp in the phase-modulated pulse has the consequence that it cannot be compressed in time when transmitted through an ordinary dispersion delay line—a fiber with quadratic dispersion or a diffraction grating. On the other hand, the nonconstancy of the resonant dispersion of the group velocity is the reason for the occurrence of time aberrations, i.e., the existence of superlinear chirps in the expansion (46).

Partial compensation of the aberrations, for example, quenching of the quadratic chirp, is possible if a combined grating–prism compressor is used[24] (see also Refs. 25 and 26). Simultaneously, an appreciable shortening of the pulse duration can be achieved.

Far from resonance ($|\Delta|\gg1$), the behavior of the dispersion curve differs qualitatively depending on the sign of the offset. Thus, a pulse with carrier frequency shifted to the "red" ($\Delta\ll-1$) enters the region of normal dispersion, and compensation of the phase self-modulation becomes very problematic. Figure 3 illustrates the rapid narrowing of the region of stability of stationary pulses: the critical power decreases as the square of the offset, and the critical duration increases linearly with increasing offset. Substituting $\nu=\nu_{\max}(\Delta)$ for $\Delta\ll-1$ in the expression (45), we obtain

$$\omega_{av}=\left(\omega_0+\frac{\Delta}{\tau}\right)-\frac{2}{3\Delta}=\omega-\frac{2}{3\Delta}. \tag{47}$$

It can be seen from (47) that there is a tendency for the frequency to be pushed into the region of minimum values of the normal dispersion, i.e., to the "red." Simultaneously, the phase modulation is "quenched," and this leads to a narrowing of the pulse spectrum, and thus to a decrease in the efficiency of the dispersion spreading in the region of normal dispersion.

If the pulse carrier frequency is shifted far to the "blue" ($\Delta\gg 1$), the form of the solution is radically changed. With increasing offset, the critical power increases, reaching asymptotically the maximum value (see Fig. 3); the critical duration increases as in the case of a "red" shift, but at a rate that is lower by a factor $\sqrt[3]{2}$. The fact that the domain of existence of solutions is appreciably broader than in the left-hand wing of the absorption line is explained by the reversal of the sign of the dispersion (see Fig. 7). Substituting $\nu=\nu_{max}(\Delta)$ for $\Delta\gg 1$ in the expression (45), we obtain

$$\omega_{av}=\left(\omega_0+\frac{\Delta}{\tau}\right)-\frac{3\Delta}{4}=\omega-\frac{3\Delta}{4}, \tag{48}$$

i.e., there is again a tendency for the pulse carrier frequency to be pushed into the region of $\min(\partial^2 k/\partial\omega^2)$, in the given case to the point B in Fig. 7. In addition, in the case of motion in the direction of higher frequencies the phase modulation increases, this being explained by the need for a broadening of the pulse spectrum in order to increase the efficiency of dispersive compression.

The mechanism of pulse formation described above can be compared with the NSE mechanism. In both, the region of normal dispersion is unsuitable for the existence of solutions in the form of pulses, whereas in the region of anomalous dispersion the process of soliton formation is maintained. However, a qualitative difference of the resonance mechanism is associated with the nonlinear origin of the dispersion of the group velocity and with the strongly nonmonotonic variation of its magnitude through the region occupied by the pulse spectrum, whereas in the NSE model the dispersion of the group velocity is assumed to be constant. This difference is manifested in the phase modulation of the pulse and in the tendency for the frequency to be pulled in the direction of $\min(\partial^2 k/\partial\omega^2)$.

We note one further difference of the resonance mechanism of soliton formation from the NSE mechanism—the critical values of the power and pulse duration. In the first place, this difference is due to the slow growth of the nonlinear resonant dispersion of the group velocity with decreasing duration ($\propto\tau^{-1}$). At the same time, the phase self-modulation increases in proportion to the instantaneous intensity, and this, by virtue of the relation $(A_0\tau)^2=4$, which holds for a classical 2π pulse, corresponds to a growth $\propto\tau^{-2}$ of the phase self-modulation. Thus, the existence of the critical values of the power and duration of the pulse is associated with the dominant role of the regime of self-induced transparency, which requires conservation of the area under the envelope near a value equal to 2π.

If the absorption line is inhomogeneously broadened, the mechanism of soliton formation is different from the one described above—the nonlinear dispersion of the group velocity is due in a resonant medium to the nonlinearity of the refractive index and not to self-induced transparency. In the limit of a broad line ($\tau\gg T^*$) in the regime of self-induced transparency, each atom separately is the source of the in-phase part of the polarization $p(\Delta\Omega)$, but since the vibrations of the atoms on opposite sides of the central frequency are added out of phase, the total contribution from all atoms of the medium, $\langle p(\Delta\Omega)\rangle$, is zero. Therefore, the refractive index under the absorption profile does not change, and therefore the magnitude of the nonlinear resonant dispersion of the group velocity due to self-induced transparency is constant and numerically equal to zero. From this, we conclude that self-induced transparency and the dispersion associated with it cannot lead to compensation of phase self-modulation in the case of an inhomogeneously broadened line.

Inclusion of nonlinearity of the refractive index leads to the appearance of phase self-modulation. Phase self-modulation can be compensated by anomalous dispersion of the group velocity, which in the given case can also be interpreted as nonlinearity of the refractive index.

To show this, we use (33) for the phase and Eq. (39) to obtain the dispersion relation

$$\Delta k(t,z)=-\frac{\partial\varphi}{\partial z}=-\frac{\nu}{V\tau}e^2(u),$$

which takes the usual form of the dependence $\Delta k(\omega)$ after a Fourier transformation:

$$\Delta k(\omega)=-\frac{\nu}{V\tau^2}\int_{-\infty}^{\infty}e^2(t)\exp(i\omega t)dt.$$

The behavior of $\Delta k(\omega)$ is shown in Fig. 7. Maxima of the anomalous dispersion are situated symmetrically on both sides of the pulse carrier frequency ω_0. It is readily seen that with increasing pulse power, the depth of the maximum of the anomalous dispersion and its position relative to the central frequency increase approximately linearly. The mean carrier frequency ω_{av} is determined in accordance with the expression [see (44)]

$$\omega_{av}=\omega_0-\frac{\nu}{\tau}\int_{-\infty}^{\infty}e^4(t)dt\Big/\int_{-\infty}^{\infty}e^2(t)dt$$

$$\text{for}\quad \omega=\omega_0,$$

from which it can be seen that there is a quadratic growth of the offset with increasing pulse power. Thus, the velocity of the pulse carrier frequency appreciably exceeds the velocity of the maxima of the anomalous dispersion of the group velocity, and this leads to the impossibility of mutual compensation of the phase self-modulation and the dispersion of the group velocity at high power. This mechanism is responsible for the existence of critical values of the power and duration of a stationary pulse propagating in a medium with an inhomogeneously broadened line.

We have restricted ourselves to the approximation of a maximally broad line, and therefore the pulse parameters are insensitive to the offset of the carrier frequency of the field

from the line center. However, as soon as the pulse spectrum reaches the edge of the inhomogeneously broadened absorption profile, the nonlinear dispersion induced by the self-induced transparency becomes nonvanishing and leads to a change in the critical values of the pulse power and duration. At even larger offsets, the nature of the broadening becomes unimportant, and then the results of the theory for a homogeneously broadened line are valid.

We emphasize once more that the nature of the dispersion of the group velocity is different for the two considered forms of broadening. In the case of a homogeneously broadened line, nonlinear dispersion is induced by a pulse in the resonant medium and does not depend on the existence of nonresonant impurities; its maximum value is determined by the concentration of the absorbing atoms. For a maximally broad inhomogeneously broadened line, dispersion also arises through the nonlinear interaction of a pulse with the resonant atoms, but now its value is directly proportional to the magnitude of the Kerr nonlinearity and is equal to zero in a purely resonant medium.

5. COMPARISON OF THEORY AND EXPERIMENT

We now consider in more detail an analysis of experimental data on the propagation of self-induced transparency solitons in optical fibers[16] and experiments on the generation of self-induced transparency solitons.[10] Using realistic values of the physical parameters, we show that the effect of Kerr nonlinearity on the formation of a self-induced transparency soliton in fibers doped with ions of rare-earth elements may lead to an appreciable modification in the shape of the pulse, and even to its destruction.

5.1. Experiments on the propagation of pulses of self-induced transparency

In experiments on the propagation of self-induced transparency solitons,[16] the pulse source was an erbium glass laser giving pulses of duration $\tau \approx 200$–300 ps. A Pockels cell was used to isolate a single pulse. The wavelength 1.534 μm coincided with the point of zero dispersion of the erbium fiber, into which the initial pulse from the laser was directed. A special cryogenic system was used to cool the fiber to 4.2 K in order to increase the homogeneous lifetime T_2 from 1 ps to 10 ns. Thus, coherence of the interaction of the input pulse with the erbium fiber was ensured. For the precise details of the experiment, a description of the detection system, etc., we refer to the study itself.[16] We merely mention that the authors investigated the propagation of pulses in lines satisfying $\alpha_{ab}L \approx 41.2$, 82.4, 165 ($\alpha_{ab}$ is the linear absorption coefficient,[3)] and $L = 1.5$, 3, 6 m are the lengths of the fiber sections) and reliably observed the breakup of pulses of area $3\pi < \theta < 5\pi$ into a pair of 2π pulses. A fourfold narrowing of the pulses was observed, and in a number of cases this was accompanied by a simultaneous increase in the intensity. The authors of Ref. 16 analyzed the delay of the pulses at the output and showed that there was agreement with the predictions of the theory of self-induced transparency.

Following the aims of the present paper, we shift the emphasis of the experiments of Ref. 16 to the detection of nonresonance effects. The experimental conditions, $T^* = 0.073$ ps $\ll \tau \approx 300$ ps $\ll T_2 = 10$ ns, do not correspond to the theoretical model of propagation of pulses in an inhomogeneously broadened absorber. Using the expressions (42) and (43), we calculate the critical values of the power and duration:

$$\hat{\mathscr{W}}_{cr} = 63.6 \text{ kW}, \quad \hat{\tau}_{cr} = 16.4 \text{ ps}. \tag{49}$$

We have here used data of the experiment of Ref. 16: $d = 1.6 \cdot 10^{-32}$ C·m, $\lambda = 1.53 \cdot 10^{-6}$ m, $\eta_2 = 3.2 \cdot 10^{-20}$ m^2/W, $\alpha_{ab} = 27.5$ m^{-1}, and $D = 10^{-5}$ m is the diameter of the core. In Ref. 16, the power of a 2π pulse of duration $\tau_{sit} = 280$ ps was calculated: $\mathscr{W}_{sit} = 107$ W. Both of these values, τ_{sit} and $\mathscr{W}_{sit}$, are far from the critical values, and the stationary regime of propagation can be treated from the point of view of pure self-induced transparency. The nonlinear length in this case is $L_{NL} = 5.6$ m, and the parameter ν is small: $\nu = 6.5 \cdot 10^{-3}$.

In the experiment, a narrowing of the pulses by almost four times was observed during the nonstationary transformations. Under these conditions, one must expect an appreciable increase in intensity (by a factor of about 16, but because of the loss in the fiber this number is somewhat less). Nevertheless, these values remain an order of magnitude less than the critical values, and nonresonance effects do not play a significant role. However, it was specially noted in Ref. 16 that for input power 300 W, a pulse of indefinite shape was detected at the output of a 6-m segment of waveguide (see Fig. 3f in Ref. 16). Estimating the nonlinear length for this value of the power as $L_{NL} = 1.99$ m, we see that over a length of 6 m appreciable nonresonance effects can accumulate. More detailed conclusions cannot be presented, since the theory we have developed describes only the stationary regime of propagation.

5.2 Experiments on the generation of self-induced transparency pulses

The results of the theory can be applied with a high degree of certainty and accuracy to the description of the experiments on the generation of self-induced transparency solitons in Ref. 10. The point is that obtaining stable self-induced transparency pulses by means of mode locking presupposes the reproduction of a pulse of stationary shape in each passage through the cavity, and we can argue in terms of a stationary propagation regime. The conclusions obtained in the present paper for the propagation regime are valid for generalization to the regime of pulse generation within a cavity if a unidirectional ring laser with small transmission losses is used, and if the pulse length is much shorter than the cavity length. This last condition is satisfied when a large number of modes participate in the generation. These conditions were satisfied in the experiments of Ref. 10.

We briefly describe the design of the experiment. The unidirectional ring laser consists of two main parts: a 100-m segment of erbium fiber is pumped at room temperature by a titanium–sapphire laser (0.98 μm) and serves as an amplifying medium; a 3-m segment of erbium fiber, cooled to 4.2 K, is used as a coherent intracavity absorber. In order to obtain lasing at the resonant wavelength 1.53 μm, an optical filter of 1 nm bandwidth is placed within the cavity. The homoge-

neous lifetimes of the amplifier ($T_{2g}\approx 1$ ps) and absorber ($T_{2p}\approx 10$ ns) make it possible to ensure coherent interaction of the pulse with the absorber if the pump power is sufficiently high for the pulse to have a duration shorter than T_{2p}. Under such conditions, one can expect the formation of a 2π pulse of self-induced transparency within the cavity. However, as theoretical calculations show,[5,7–9] this does not always occur. We consider the main points.

To obtain stable mode locking, it is necessary to create conditions that suppress amplification of weak fluctuations of the field in the part of the cavity in which there is no pulse at the given time, i.e., the laser must actually be below the threshold for the occurrence of cw lasing. At the same time, generation of pulses with duration less than T_{2p} can be realized under these conditions, since they are subject to weak absorption and for them the total gain in the cavity is positive. We conclude that the concentration of absorbing atoms must be above a certain minimum value $N_{\min}$ for the weak-field gain of the cavity to be less than zero. On the other hand, no matter how small the losses of the pulse due to the finite value of T_{2p}, they impose an upper limit on the concentration of the absorber, above which stable mode locking is disrupted, because losses predominate over the gain for a field in the form of a 2π pulse. Thus, stable generation of pulses of self-induced transparency is possible at absorber concentrations in the range $N_{\min}< N<N_{\rm cr}$. Omitting the details (which can be found in Ref. 9), we note that the absorber concentration in the experiments of Ref. 10 lies in this range, and therefore the condition of stability is satisfied.

Preliminary estimates show the possibility of achieving stable generation of self-induced transparency pulses. To obtain pulses of duration 280 ps, the peak power is 107 W, and for a laser of length 135 m (pulse repetition rate 1.53 MHz) the mean power level is accordingly 80 mW; see Ref. 10. As the authors of Ref. 10 varied the pump power from 70 mW to 170 mW, they observed a transition from a Q-switched regime to a mode-locking regime. The achieved minimum pulse width of 12–17 ns was still greater than T_{2p}, corresponding to a predominance of saturated absorption and not coherent effects during pulse formation. No experiments were carried out at higher pump powers, and the regime of generation of 2π pulses was not reached. Using the results of the present theory, we now assess the possibility of obtaining self-induced transparency solitons in the experimental scheme proposed in Ref. 10.

Above all, we note that the amplifier line is much broader than the pulse spectrum, and the role of the active medium reduces to compensation of the linear losses in the cavity and the incoherent losses in the absorber.[7] The main role in the formation of the field profile is played by the coherent absorber. The validity of such a model of the role of the amplifying and absorbing media is proved in Refs. 7 and 8. Thus, assuming that the density of the intracavity field is entirely sufficient to maintain the regime of stationary generation of 2π pulses, we can reduce the problem of generation of self-induced transparency solitons in a fiber laser to the problem of pulse propagation in a lossless, coherent resonant nonlinear waveguide. The role of the phase self-modulation that arises because of the dependence of the refractive index of the optical fiber on the intensity in the formation of a self-induced transparency soliton can be elucidated by comparing the nonlinear length L_{NL} and the absorption length L_{ab}. The procedure for calculating them is different from the one given above in the discussion of the experiments of Ref. 16. During a round trip through the cavity, the pulse passes through two segments of the waveguide doped with erbium ions of total length 103 m. At the same time, for the given range of durations, the Kerr nonlinearity is instantaneous, and during the time of a round trip of 660 ns is restored to its original value. For $\tau_{\rm sit}=280$ ps and $\mathscr{W}_{\rm sit}=107$ W, we calculate the nonlinear length divided by unit length of the cavity:

$$L_{NL}=5.6\ \text{m}\cdot\frac{135\ \text{m}}{103\ \text{m}}=7.34\ \text{m}$$

The segment of the 3-m erbium waveguide cooled to 4.2 K and used as absorber is characterized by a non-instantaneous response. The spontaneous relaxation time of the absorber ($T_1=10$ ms) is long compared with the round trip time through the cavity ($T_R=660$ ns). Under these conditions, the difference between the absorber occupancies that arises from the energy deposited by the passing pulse does not have sufficient time to relax to its equilibrium value before the arrival of the next pulse. Thus, the absorption coefficient of a weak field within the cavity will decrease until it reaches a new value $(\alpha_{ab})_{\rm sat}$. In the stationary regime, the pulse encounters the same steady value of the absorption coefficient $(\alpha_{ab})_{\rm sat}$ after each round trip through the cavity. The value of $(\alpha_{ab})_{\rm sat}$ must be determined, and depends on the incoherent losses of the pulse during one trip and the number of pulses that pass through the absorber during time T_1. In the absence of the field, the equilibrium value of the absorption coefficient is established in accordance with

$$\alpha_{ab}(t)=\alpha_{ab}-[\alpha_{\rm ab}-\alpha_{ab}(t=0)]\exp(-t/T_1), \tag{50}$$

where α_{ab} is the unsaturated absorption coefficient, and $\alpha_{ab}(t=0)$ is the absorption coefficient immediately after a pulse has passed. Therefore, having made a round trip through the cavity, the pulse encounters an absorption coefficient $(\alpha_{ab})_{\rm sat}$ determined by

$$(\alpha_{ab})_{\rm sat}=\alpha_{ab}-[\alpha_{ab}-\alpha_{ab}(t=0)]\exp[-(T_R-\tau)/T_1]. \tag{51}$$

The equation for $\alpha_{\rm ab}(t=T_R)$ is

$$\alpha_{ab}(t=T_R)=\alpha_{ab}-[\alpha_{ab}-\alpha_{ab}(t=0)]\exp(-T_R/T_1)-(\alpha_{ab})_{\rm sat}(\Delta N), \tag{52}$$

where ΔN is the change in the population difference (which is assumed normalized to unity) produced by the energy deposited by the pulse in the medium. Assuming steady-state generation $\alpha_{ab}(t=0)=\alpha_{ab}(t=T_R)$, we find the required value of the saturated absorption coefficient:

$$(\alpha_{\rm ab})_{\rm sat}=\alpha_{\rm ab}\left[1+\frac{\Delta N}{\exp(T_R/T_1)-1}\right]^{-1}\frac{3\ \text{m}}{135\ \text{m}}. \tag{53}$$

The value of the saturated absorption coefficient is calculated for unit length of the cavity; for this reason, the correction factor on the right-hand side is introduced.

Despite the fact that the pulse duration is much shorter than the phase memory time T_{2p} of the medium, the ratio τ/T_{2p} remains finite. Therefore, the propagation of the pulse in the absorber is accompanied by irreversible transfer of field energy to the medium, and the absorber remains in the excited state after the pulse has passed. Knowing the absorption coefficient of the pulse energy, we can calculate the population difference ΔN.[4)]

In Ref. 4, a law of damping of the energy of a pulse of duration $\tau \ll T_{2p}$ is derived; under the assumption $T_{2p} \ll T_1$, it has the form

$$\frac{1}{\Gamma}\frac{d\Gamma}{dv} = -\frac{\tau}{\pi g(0)}\frac{\tau}{T_{2p}}\int_{-\infty}^{+\infty}\left\{\frac{1}{1+(\Delta\Omega\tau)^2} - \frac{2}{3}\frac{1}{[1+(\Delta\Omega\tau)^2]^2}\right\} g(\Delta\Omega)d(\Delta\Omega). \tag{54}$$

For an absorber with a broad inhomogeneously broadened line ($\tau \gg T^*$), (54) simplifies to

$$\frac{1}{\Gamma}\frac{d\Gamma}{dv} = -\frac{2}{3}\frac{\tau}{T_{2p}}. \tag{55}$$

Thus, we have found the relative change in the difference of the absorber populations that results from the energy deposited by the pulse after it has passed:

$$\Delta N = \frac{2}{3}\frac{\tau}{T_{2p}}. \tag{56}$$

No matter how low the excitation level of the medium, it plays an important role in the dynamics of generation in lasers with solid-state absorber, since the absorbing medium is saturated after T_1/T_R passes through the cavity, and the total energy of saturation may be appreciable.

We obtain the final expression for the absorption length in the cavity by substituting the derived value of ΔN into (53):

$$(L_{ab})_{\text{sat}} = L_{ab}\left[1 + \frac{2}{3}\frac{\tau/T_{2p}}{\exp(T_R/T_1)-1}\right]\frac{135\text{ m}}{3\text{ m}}. \tag{57}$$

In addition to the derivation given above, we note that in the regime of self-induced transparency, the pulse may be appreciably delayed in the absorber. In the calculations, we have not taken into account the increase in the time of passage through the cavity that arises from this, since for the experimental scheme considered, this correction is small (less than 1% of T_R).

By means of (57), we calculate the intracavity absorption length, $(L_{ab})_{\text{sat}}=472$ m. For a laser with coherent absorber, the parameter ν has the value $\nu=64.3$, which is appreciably greater than the critical value $\nu=4$ for an absorber with an inhomogeneously broadened line. We arrive at an important conclusion: it is necessary to change the experimental design used in Ref. 10 in order to generate a train of self-induced transparency solitons. We do not give specific prescriptions for the modification, believing that all the necessary information regarding this can be extracted from the computational scheme presented above.

6. CONCLUSIONS

In this paper, we have obtained analytically a steady solution in the form of a stationary phase-modulated pulse propagating in a medium with resonant and nonresonant nonlinearities. We have found the conditions for the existence of such a solution and have investigated in detail the dependences of the pulse parameters on the properties of the propagation medium: the form of broadening of the resonant transition, the offset between the pulse carrier frequency and the transition frequency, and the relative importance of the resonant and nonresonant nonlinearities. As one of the main conclusions of the paper, we can identify the calculation of the critical values of the pulse power and duration that bound the region of existence of soliton-like pulses. This is a feature that has no analog in pure self-induced transparency. In a real physics experiment, the present model corresponds to propagation of an ultrashort optical pulse in a single-mode fiber doped with resonant impurities under the condition that the pulse carrier frequency is in the neighborhood of the point of zero dispersion of the fiber. The results of the theory are applied to the discussion of experiments on the propagation and generation of self-induced transparency solitons,[10,16] and we have shown that in a laser the effect of absorber saturation due to incoherent losses of the field and the noninstantaneous nature of the spontaneous relaxation processes may lead to an appreciable growth in importance of the nonresonant nonlinearity and, as a consequence, to termination of the soliton generation regime. In particular, the theory predicts the impossibility of generating self-induced transparency solitons if the geometry of the experiments of Ref. 10 is used.

Immediately before this paper was sent to the editors, the authors found the abstract of the paper of Chi and Barone in Bull. Phys. Soc. Amer. **16**, 71 (1971). Unfortunately, we were not able to find published results of the investigation. Judging from the text of the abstract, the report contained a solution to a problem similar to the one that we have considered in Sec. 2 of the present paper. Being unable to compare the results of the two theories, we have presented in Sec. 2 the complete solution of the problem and we apologize in advance to Chi and Barone for a possible duplication of some details.

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APPENDIX. SPECTRAL RESPONSE FUNCTION $f_2(\Delta\Omega)$

Using (31), (32), and (33), we find the expression for the dispersive part of the polarization:

$$p(\Delta\Omega,u)=\sigma\vartheta\Big\{\Delta\Omega\tau f_0(\Delta\Omega)+\Big[\frac{1}{3}f_0(\Delta\Omega)\Big(\frac{3}{4}\frac{\nu}{\sigma}-\frac{1}{4}\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle\Big)+\frac{1}{3}\Delta\Omega\tau f_2(\Delta\Omega)+\frac{1}{5}\Big(\frac{3}{4}\frac{\nu}{\sigma}-\frac{1}{4}\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle\Big)\times f_2(\Delta\Omega)e^2\Big]e^2\Big\}. \tag{A1}$$

We substitute (A1), (31), (32), and (33) into the equation for the polarization (2) and obtain a nonlinear second-order differential equation for the field $e(u)$:

$$f_0(\Delta\Omega)\ddot{e}+f_2(\Delta\Omega)(\ddot{e}e^2+2\dot{e}^2e)=\Big(\frac{1}{\sigma}-(\Delta\Omega\tau)^2\times f_0(\Delta\Omega)\Big)e-\Big\{\frac{4}{3}\Delta\Omega\tau f_0(\Delta\Omega)\Big(\frac{3}{4}\frac{\nu}{\sigma}-\frac{1}{4}\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle\Big)+\frac{1}{3}(\Delta\Omega\tau)^2f_2(\Delta\Omega)+\frac{\vartheta^2}{2}f_0(\Delta\Omega)\Big\}e^3+\ldots \tag{A2}$$

In the expansions (31) and (32), we have restricted ourselves to the first two terms of the series, and therefore in the adopted approximation it will be correct to seek an expression for the spectral response function under the assumption that the field has the shape of a 2π pulse:

$$e=\operatorname{sech} u, \quad \vartheta=2. \tag{A3}$$

Substituting (A3) in (A2) and equating the coefficients of e, we obtain for the spectral response function $f_0(\Delta\Omega)$ the expression

$$f_0(\Delta\Omega)=\frac{1}{1+(\Delta\Omega\tau)^2}, \quad \sigma=1. \tag{A4}$$

In deriving (A4), we have used the normalization condition

$$\max[\,f_0(\Delta\Omega)]=1.$$

Similarly, we equate the coefficients of e^3:

$$f_2(\Delta\Omega)=-\frac{\Delta\Omega\tau f_0(\Delta\Omega)}{9+(\Delta\Omega\tau)^2}(3\nu-\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle). \tag{A5}$$

Finally, we find an expression for the spectral response function $f_2(\Delta\Omega)$:

$$f_2(\Delta\Omega)=-\frac{3\nu}{1-I}\frac{\Delta\Omega\tau}{[1+(\Delta\Omega\tau)^2][9+(\Delta\Omega\tau)^2]}, \tag{A6}$$

$$I\equiv\Big\langle\frac{(\Delta\Omega\tau)^2}{[1+(\Delta\Omega\tau)^2][9+(\Delta\Omega\tau)^2]}\Big\rangle. \tag{A7}$$

Then, accordingly,

$$\langle\Delta\Omega\tau f_2(\Delta\Omega)\rangle=-\frac{3\nu I}{1-I}. \tag{A8}$$

Note that the form of the expression (A6) agrees with the requirement that the function $f_2(\Delta\Omega)$ must be odd with respect to $\Delta\Omega$.

[1)]The more detailed investigations of Refs. 13 and 14 showed that there is an entire set of conditions for the three listed parameters of the medium for which the existence of stable solutions in the form of 2-hump, 3-hump, etc., pulses is possible.

[2)]In this section, when speaking of perturbation theory, we mean precisely this condition.

[3)]The relationship between the absorption coefficient and the absorption length is given by $\alpha_{ab}=1/L_{ab}$; by L_{ab} we mean the absorption length introduced in accordance with (29).

[4)]To take the damping of the field into account, a term $(-P\tau/T_{2p})$ has been introduced phenomenologically on the right-hand side of Eq. (2) for the polarization to describe the damping of the macroscopic polarization in time T_{2p} (see Ref. 2).

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