On the dielectric function of an inhomogeneous medium

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The anisotropy of the dielectric function of an inhomogeneous medium is considered when it arises due to the presence of a preferred direction of the density gradient at each point in the medium. Expressions are obtained for the dielectric tensor of an inhomogeneous medium which take account of the anisotropy caused by the inhomogeneity. © *1995 American Institute of Physics.*

1. INTRODUCTION

At each point of an inhomogeneous medium, generally speaking, there exists a preferred direction associated with the existence of the density gradient. It is well known, for example, that a periodically inhomogeneous medium is birefringent for fields with wavelength much greater than the spatial period of the medium.[1] At the same time, in macroscopic electrodynamics the medium is sometimes considered inhomogeneous but isotropic. The applicability of such a quasi-isotropic approximation in the special case of geometric optics has been discussed by different authors (a review with references to the literature can be found in Ref. 2). It would seem that the most natural way to obtain the conditions of applicability of the quasi-isotropic approximation is by the appropriate derivation of the expression for the polarization of the material from a consideration of the local field acting on a molecule.[3–8] As is well known, the polarization of the material is directly related to the mean field acting on the molecule, and its relation to the mean macroscopic field is found from additional considerations taking account of the properties of the material. Only after finding the relations between the local field and the macroscopic field is it possible to obtain an expression for the dependence of the dielectric function on the density.

Such a study was carried out a few years ago by Sipe[9]; however, in the approximation that he used, anisotropy generally does not arise. Therefore it is of interest to clarify the accuracy to which the anisotropy of an equilibrium inhomogeneous medium can be neglected, treating the dielectric function of the medium as a scalar function of position, although at each such point there exists a preferred direction of the density gradient.

2. LOCAL FIELD IN AN INHOMOGENEOUS MEDIUM

The influence of the inhomogeneity of the medium on its dielectric function is simpler to trace out if we consider the relation between the local field acting on a molecule and the mean macroscopic field. A molecule of a medium located in an electric field acquires a dipole moment whose Fourier transform $\mathbf{d}(\mathbf{r},\omega)$ is related to the polarizability of the molecule $\alpha_{ij}(\omega)$ and the Fourier transform of the field acting on the molecule $\mathbf{E}(\mathbf{r},\omega)$:

$$d_i(\mathbf{r},\omega)=\alpha_{ij}(\omega)E_j(\mathbf{R},\omega). \tag{1}$$

The dipole moments induced in each molecule of the medium are sources of secondary fields. In order to calculate the local field, let us first consider the microscopic field $\mathbf{E}^{mic}$ acting on the molecule located at the point $\mathbf{R}$. This field is made up of the primary field $\mathbf{E}$ (no longer interacting with just one molecule of the medium) and the secondary fields of all the other particles. Therefore, for the Fourier transforms of the microscopic fields we can write

$$E_i^{mic}(\mathbf{R},\omega)=E_i^0(\mathbf{R},\omega)+4\pi\sum_{\mathbf{R}'} M_{ij}(\mathbf{R}-\mathbf{R}')\times E_j^{mic}(\mathbf{R},\omega), \tag{2}$$

where the sum is taken over all of the molecules of the medium except the one under consideration, and we have used the notation

$$M_{ij}(\mathbf{R})=\frac{1}{(2\pi)^3}\,\alpha_{is}(\omega)\times\int d^3q\,\frac{\exp(i\mathbf{q}\cdot\mathbf{R})(q^2\delta_{sj}-q_sq_j)}{q^2-(\omega/c)^2}. \tag{3}$$

The effective field acting on the molecule results from the addition of the fields of many molecules located in some region of space whose characteristic linear dimension l is large in comparison with intermolecular distances $n^{-1/3}$, but much smaller than the wavelength of the field:

$$n^{-1/3}\ll l\ll\lambda. \tag{4}$$

If these inequalities hold, then the field acting on the molecule results from the mutual cancellation of the fields of many molecules and its value is close to that of the acting field averaged over the positions of the other molecules. This latter field is commonly called the local field $\mathbf{E}^{loc}$. The equation for the local field can be found by averaging expression (2) over the positions of all molecules other than the one under consideration. Then the microscopic field in expression (2) can be replaced by the local field, and to find the average of the sum we multiply each of its terms by the probability $w(\mathbf{R}'';\mathbf{R})$ of finding a molecule at the relative position $\mathbf{R}''=\mathbf{R}-\mathbf{R}'$, and we then integrate over $\mathbf{R}''$. The dependence on $\mathbf{R}'$ in the terms of the sum then drops out, and the sum reduces to multiplying by the number of molecules $N-1=N$. It is convenient to introduce the notation

$$Nw(\mathbf{R}'';\mathbf{R})=n(\mathbf{R}')[1-f(\mathbf{R}'';\mathbf{R})],$$

where $n(\mathbf{R}')$ is the number density of the molecules in the inhomogeneous medium. Since the probability in any medium of a very small distance between two molecules tends to zero, the function $f(\mathbf{R}'';\mathbf{R})$ is equal to unity at $\mathbf{R}''=0$ in any medium, independent of $\mathbf{R}$. On the other hand, for $R''\gg n^{-1/3}$ the function $f(\mathbf{R}'';\mathbf{R})$ becomes negligible in any medium. This substantially limits the possible form of $f(\mathbf{R}'';\mathbf{R})$. It follows that the result of taking the average of expression (2) can be represented in the form

$$E_i^{loc}(\mathbf{R},\omega)=E_i^0(\mathbf{R},\omega)+4\pi\int d^3R''n(\mathbf{R}-\mathbf{R}'')[1 -f(\mathbf{R}'';\mathbf{R})]M_{ij}(\mathbf{R}'')E_j^{loc}(\mathbf{R}-\mathbf{R}'',\omega). \quad (5)$$

In order to relate the local field to the mean macroscopic field, we find the relation between the macroscopic field and the primary fields. By definition, the primary field satisfies Maxwell's equations in vacuum with the same current and charge densities as in the phenomenological Maxwell equations in the medium for the macroscopic field. Therefore, the difference between these equations does not contain currents or charges, and enables one to relate the Fourier transforms of the primary field, the macroscopic field, and the polarization of the medium:

$$E_i^0(\mathbf{k},\omega)=E_i(\mathbf{k},\omega)+4\pi P_i(\mathbf{k},\omega) -4\pi Q_{ij}(k,\omega)P_j(\mathbf{k},\omega), \quad (6)$$

where

$$Q_{ij}(\mathbf{k},\omega)=(k^2\delta_{ij}-k_ik_j)/[k^2-(\omega/c)^2]. \quad (7)$$

Transforming back to the fields themselves, without too much trouble we find

$$E_i^0(\mathbf{R},\omega)=E_i(\mathbf{R},\omega)+4\pi P_i(\mathbf{R},\omega)-4\pi\int d^3R''P_j(\mathbf{R} -\mathbf{R}'',\omega)\int\frac{d^3k}{(2\pi)^3}\,Q_{ij}(\mathbf{k},\omega)\exp(i\mathbf{k}\cdot\mathbf{R}''). \quad (8)$$

The latter equation can be used to eliminate the primary field from Eq. (5). However, it is more convenient first to introduce the local field in expression (8) in place of the polarization by way of the relation

$$P_i(\mathbf{R},\omega)=n(\mathbf{R})\alpha_{is}(\omega)E_s^{loc}(\mathbf{R},\omega). \quad (9)$$

Substituting relation (9) into Eq. (8) gives

$$E_i^0(\mathbf{R},\omega)=E_i(\mathbf{R},\omega)+4\pi n(\mathbf{R})\alpha_{is}E_s^{\mathrm{loc}}(\mathbf{R},\omega) -4\pi\int d^3R''n(\mathbf{R}-\mathbf{R}'')E_s^{\mathrm{loc}}(\mathbf{R}-\mathbf{R}'',\omega) \times M_{si}(\mathbf{R}''). \quad (10)$$

Eliminating the primary field from Eq. (5) with the help of relation (10), we easily obtain

$$E_i(\mathbf{R},\omega)=E_i^{\mathrm{loc}}(\mathbf{R},\omega)-4\pi n(\mathbf{R})\alpha_{ij}(\omega)E_j^{\mathrm{loc}}(\mathbf{R},\omega) +4\pi\int d^3R''n(\mathbf{R}-\mathbf{R}'') \times f(\mathbf{R}'';\mathbf{R})M_{ij}(\mathbf{R}'')E_j^{loc}(\mathbf{R}-\mathbf{R}'',\omega). \quad (11)$$

3. DIELECTRIC FUNCTION OF A WEAKLY INHOMOGENEOUS MEDIUM

We now consider an inhomogeneous medium whose density varies substantially over distances L large compared to the wavelength of the field. We can supplement (4) by writing

$$n^{-1/3}\ll l\ll\lambda\ll L. \quad (12)$$

In this case we can assume that the function

$$f(\mathbf{R}'';\mathbf{R})=\int d^3qf(\mathbf{q},\mathbf{R})\exp(i\mathbf{q}\cdot\mathbf{R}'') \quad (13)$$

is nonzero only over a range of R'' of order $n^{-1/3}$. The molecular number density $n(\mathbf{R}-\mathbf{R}'')$ and the local field $E^{loc}(\mathbf{R}-\mathbf{R}'',\omega)$ are essentially constant over intermolecular distances, which enables us to neglect $\mathbf{R}''$ in the arguments of these functions, and thus take them outside the integral. Hence, we can transform (11) for an inhomogeneous medium to the form

$$E_i(\mathbf{R},\omega)=E_j^{loc}(\mathbf{R},\omega)\{\delta_{ij}-4\pi n(\mathbf{R})[\alpha_{ij}(\omega) -\alpha_{is}(\omega)V_{sj}(\mathbf{R},\omega)]\}, \quad (14)$$

where

$$V_{sj}(\mathbf{R})=\int d^3qf(\mathbf{q},\mathbf{R})\left(\delta_{sj}-\frac{q_sq_j}{q^2}\right). \quad (15)$$

It should be emphasized that the foregoing equality $f(\mathbf{R}''=0;\mathbf{R})=1$ leads to the integral relation

$$\int d^3qf(\mathbf{q},\mathbf{R})=1,$$

which holds for any $\mathbf{R}$. In an inhomogeneous medium, the function $f(q,R)$ depends on the direction of the vector $\mathbf{q}$. We denote the function $f(\mathbf{q},\mathbf{R})$ averaged over all directions of the vector $\mathbf{q}$ by $\bar{f}(\mathbf{q},\mathbf{R})$, and set $f(\mathbf{q},\mathbf{R})=\bar{f}(\mathbf{q},\mathbf{R})+h(\mathbf{q},\mathbf{R})$. Substituting the latter relation into Eq. (15) noting that replacing $f(\mathbf{q},\mathbf{R})$ by $\bar{f}(\mathbf{q},\mathbf{R})$ in Eq. (15) gives

$$\int d^3q\bar{f}(\mathbf{q},\mathbf{R})\left(\delta_{ij}-\frac{q_iq_j}{q^2}\right)=\frac{2}{3}\,\delta_{ij}\int d^3qf(\mathbf{q},\mathbf{R}) =\frac{2}{3}\,\delta_{ij}, \quad (16)$$

leads to the expression

$$V_{ij}(\mathbf{R})=\frac{2}{3}\,\delta_{ij}-h_{ij}(\mathbf{R}), \quad (17)$$

where we have made use of the notation

$$h_{sj}(\mathbf{R})=\int d^3qh(\mathbf{q},\mathbf{R})\,\frac{q_sq_j}{q^2}. \quad (18)$$

In a homogeneous medium the quantity $h_{sj}(\mathbf{R})$ vanishes, but in a weakly inhomogeneous medium in which the inequalities (12) hold, h_{sj} $(\mathbf{R})$ is comparable to $l/L\ll\lambda/L$ in order of magnitude. Therefore if inequalities (12) hold, it is generally possible to neglect $h_{sj}(\mathbf{R})$. As a result, it follows from relation (17) that

$$E_i(\mathbf{R},\omega)=E_j^{\rm loc}(\mathbf{R},\omega)\left[\delta_{ij}-\frac{4\pi}{3}\,n(\mathbf{R})\alpha_{ij}(\omega)\right].\tag{19}$$

For a material with spherically symmetric molecules this relation simplifies:

$$\mathbf{E}(\mathbf{R},\omega)=\mathbf{E}^{\rm loc}(\mathbf{R},\omega)\left[1-\frac{4\pi}{3}\,n(\mathbf{R})\alpha(\omega)\right].\tag{20}$$

Hence it follows that in a weakly inhomogeneous medium with spherically symmetric molecules the dielectric function has the form

$$\varepsilon(\mathbf{R},\omega)=\frac{1+(8\pi/3)n(\mathbf{R})\alpha(\omega)}{1-(4\pi/3)n(\mathbf{R})\alpha(\omega)},\tag{21}$$

which reproduces Sipe's result.[9]

In a weakly inhomogeneous anisotropic medium, the form of the dielectric function becomes more complicated. It is convenient to use the crystal optics approximation, which treats the crystal as a homogeneous anisotropic medium, and which takes account of ordering of only the orientations of the molecules, not their spatial distribution. In particular, it does not allow for the difference between a solid crystal and a nematic liquid crystal.

We now consider an anisotropic single-axis medium in equilibrium in an external field directed along the principal optical axis. The electron density in such a medium depends on the coordinate along the principal optical axis. Such a medium will be single-axis, anisotropic, and inhomogeneous. In the crystal optics approximation it is sufficient in such a medium to take account of the ordered orientation of the molecules in the direction of the principal optical axis. Denoting the direction vector of this axis by the unit vector **e**, we can represent the polarizability of a molecule of the medium in the form

$$\alpha_{ij}(\omega)=a(\omega)(\delta_{ij}-e_ie_j)+b(\omega)e_ie_j.\tag{22}$$

The inhomogeneity of the material in the crystal optics approximation is described by the variation in the number density of the molecules, while their spatial order can be ignored. Then, for an inhomogeneous, single-axis material, the relation between the macroscopic field and the local field (14) can be inverted to yield

$$E_j^{loc}(\mathbf{R},\omega)=E_i(\mathbf{R},\omega)\left\{e_ie_j\,\frac{1}{1-(4\pi/3)n(\mathbf{R})b(\omega)}+(\delta_{ij}-e_ie_j)\,\frac{1}{1-(4\pi/3)n(\mathbf{R})a(\omega)}\right\}.\tag{23}$$

Hence, we obtain the dielectric function of an inhomogeneous, single-axis material in the following form:

$$\varepsilon_{ij}(\mathbf{R},\omega)=\varepsilon_e(\mathbf{R},\omega)(\delta_{ij}-e_ie_j)+\varepsilon_0(\mathbf{R},\omega)e_ie_j,$$

$$\varepsilon_e(\mathbf{R},\omega)=\frac{1+(8\pi/3)n(\mathbf{R})a(\omega)}{1-(4\pi/3)n(\mathbf{R})a(\omega)},$$

$$\varepsilon_0(\mathbf{R},\omega)=\frac{1+(8\pi/3)n(\mathbf{R})b(\omega)}{1-(4\pi/3)n(\mathbf{R})b(\omega)}.\tag{24}$$

4. DIELECTRIC FUNCTION OF AN INHOMOGENEOUS MEDIUM IN THE GENERAL CASE

We now consider an inhomogeneous medium in which the characteristic dimension L of the inhomogeneities is bounded only by the inequality

$$l\ll L\tag{25}$$

and inequality (4) holds. In this case, the influence of the inhomogeneities on the formation of the local field is, as before, not great, so that relation (14) between the local field and the macroscopic field still applies and the conditions for the derivation of the expression for V_{sj} (17) are still fulfilled. The quantity $h(\mathbf{q},\mathbf{R})$ is now comparable in order of magnitude to l/L, i.e., it is small in comparison with unity, but can be much larger than l/λ, and thus must be taken into account.

With the help of formula (17), relation (14) can be brought into the form

$$E_i(\mathbf{R},\omega)=\left[\delta_{ij}-\frac{4\pi}{3}\,n(\mathbf{R})\alpha_{ij}(\omega)\right]E_j^{loc}(R,\omega)+4\pi n(\mathbf{R})\alpha_{is}(\omega)h_{sj}(\mathbf{R})E_j^{loc}(\mathbf{R},\omega).\tag{26}$$

The quantity h_{sj} $(\mathbf{R})$ can be expressed in terms of the binary distribution function $F(\mathbf{R}',\mathbf{R})$:

$$h_{sj}(\mathbf{R})=\int\int\frac{d^3qd^3R''}{8\pi^3q^2}\,q_sq_j\,f(R'',\mathbf{R})\Big[\exp(-i\mathbf{q}\cdot\mathbf{R}'')-\overline{\exp(-i\mathbf{q}\cdot\mathbf{R}'')}\Big]=\frac{1}{(2\pi)^3}\int d^3R'[F(\mathbf{R}',\mathbf{R})-1]\int d^3q\,\frac{q_sq_j}{q^2}\left\{\frac{1}{q|\mathbf{R}-\mathbf{R}'|}\sin(q|\mathbf{R}-\mathbf{R}'|)-\exp[i\mathbf{q}(\mathbf{R}-\mathbf{R}')]\right\}.\tag{27}$$

In the derivation of expression (26) we have neglected terms quadratic in h_{sj} $(\mathbf{R})$. Therefore we can, to the same accuracy, write in place of Eq. (26)

$$W_{is}(\mathbf{R})(\delta_{sj}-4\pi n(\mathbf{R})\alpha_{sl}(\omega)h_{lk}(\mathbf{R})W_{kj}(\mathbf{R})]E_j(\mathbf{R},\omega)=E_i^{loc}(\mathbf{R},\omega),\tag{28}$$

where the tensor $W_{is}(\mathbf{R})$ is defined by the relation

$$W_{is}(\mathbf{R})\left\{\delta_{sj}-\frac{4\pi}{3}\,n(\mathbf{R})\alpha_{sj}(\omega)\right\}=\delta_{ij}.\tag{29}$$

Invoking the relation between the polarizability and the local field (9), it is straightforward to obtain from Eq. (28) an expression for the dielectric tensor of an inhomogeneous medium,

$$\varepsilon_{ij}(\mathbf{R},\omega)=\delta_{ij}+4\pi n(\mathbf{R})\alpha_{is}(\omega)W_{sj}(\mathbf{R})-16\pi^2n(\mathbf{R})\times\alpha_{is}(\omega)W_{sl}(R)\alpha_{lk}(\omega)h_{km}(\mathbf{R})W_{lj}(R).\tag{30}$$

In the case of material consisting of spherically symmetric molecules,

$$\alpha_{ij}(\omega)=\delta_{ij}\alpha(\omega),$$

$$W_{ij}(\mathbf{R})=\delta_{ij}[1-(4\pi/3)n(R)\alpha(\omega)]^{-1},$$

and formula (30) simplifies to

$$\varepsilon_{ij}(\mathbf{R},\omega)=\delta_{ij}\varepsilon(\mathbf{R},\omega)-h_{ij}(\mathbf{R})[\varepsilon(\mathbf{R},\omega)-1]^2$$
$$=\delta_{ij}\frac{1+(8\pi/3)n(\mathbf{R})\alpha(\omega)}{1-(4\pi/3)n(\mathbf{R})\alpha(\omega)}-h_{ij}(\mathbf{R})$$
$$\times\left|\frac{4\pi n(\mathbf{R})\alpha(\omega)}{1-(4\pi/3)n(\mathbf{R})\alpha(\omega)}\right|^2, \qquad (31)$$

where $\varepsilon(\mathbf{R},\omega)$ is the value of the dielectric function that is usually used to describe the inhomogeneous medium, i.e., disregarding anisotropy.

5. THE FIELD IN A ONE-DIMENSIONALLY INHOMOGENEOUS MEDIUM

Taking the anisotropy of an inhomogeneous medium into account substantially alters the Maxwell equations for the field in such a medium. Thus, for example, in a one-dimensionally homogeneous medium consisting of spherically symmetric molecules, and when the molecular number density varies only with z, the most general form of the tensor $h_{ij}(z)$ is

$$h_{ij}(z)=a(z)[\delta_{ij}-e_ie_j]+b(z)e_ie_j, \qquad (32)$$

and expression (31) for the dielectric tensor can be written as

$$\varepsilon_{ij}(z,\omega)=(\delta_{ij}-e_ie_j)\varepsilon_0(z,\omega)+e_ie_j\varepsilon_e(z,\omega), \qquad (33)$$

where we have made use of the notation

$$\varepsilon_0(z,\omega)=\varepsilon(z,\omega)-a(z)[\varepsilon(z,\omega)-1]^2,$$
$$\varepsilon_e(z,\omega)=\varepsilon(z,\omega)-b(z)[\varepsilon(z,\omega)-1]^2. \qquad (34)$$

Let the wave field have the form $\mathbf{H}(z)\exp(iqx-i\omega t)$, that is to say, let the direction of propagation of the wave lie in the xy plane. Maxwell's equations in such a medium have two linearly independent solutions, in one of which the components of the fields E_x, E_z, and H_y vanish, and in the other, the components H_x, H_z, and E_y vanish. In the limit of a homogeneous, anisotropic medium, the first solution goes over to the ordinary wave, and the second, to the extraordinary wave. In an inhomogeneous, anisotropic medium, the properties of the second solution depend substantially on the orientation of the direction of propagation relative to the principal optical axis. It is convenient to call the first solution, even in an inhomogeneous medium, the ordinary, or E-wave, and the second, the extraordinary, or H-wave. The ordinary wave obeys the same equation as in an inhomogeneous, isotropic medium, with the single exception that the dielectric function $\varepsilon(z,\omega)$ must now be replaced by the quantity $\varepsilon_0(z,\omega)$ defined by the first of Eqs. (34). The equation for the extraordinary wave differs from that for the H-wave in an inhomogeneous, isotropic medium, in particular, in that it contains two different characteristics of the medium: $\varepsilon_0(z,\omega)$ and $\varepsilon_e(z,\omega)$.

The equation for the magnetic field of such a wave $H=H_y$ has the form

$$\frac{d[(1/\varepsilon_0)(dH/dz)]}{dz}+\left[\left(\frac{\omega}{c}\right)^2-\frac{q^2}{\varepsilon_e}\right]H=0. \qquad (35)$$

Taking anisotropy into account can lead to qualitative differences in the behavior of the solutions. Thus, Ref. 10 presents the derivation by Försterling, who considered the reflection of an oblique incident wave from a layer of inhomogeneous, isotropic medium in which $\varepsilon(z)$ passes through a zero, and found that in contrast to the E-wave, in the reflection of an H-wave against the background of a decaying field near the zero of $\varepsilon(z)$, an anomalously abrupt amplification of the field takes place. However, the equation for the inhomogeneous, isotropic medium is obtained from the more general equation (35) by replacing $\varepsilon_0(z)$ and $\varepsilon_e(z)$ by $\varepsilon(z)$, which is valid only under certain conditions. Försterling's use of the approximation of an isotropic, inhomogeneous medium in equation (35) means that the difference $\varepsilon_0-\varepsilon_e$ is assumed to be small compared with any characteristic quantity of the problem, in particular, $|\varepsilon_0|$ and $|\varepsilon_e|$: $|\varepsilon_0-\varepsilon_e|\ll|\varepsilon_0|,|\varepsilon_e|$.

The functions $\varepsilon_0(\omega)$ and $\varepsilon_e(\omega)$ vanish, generally speaking, at different values of the frequency, and near the zeros of $\varepsilon_0(\omega)$ and $\varepsilon_e(\omega)$ the approximation of a homogeneous, isotropic medium is inapplicable. Hence it follows that near the zeros of the dielectric function it is necessary to take into account the anisotropy of the medium and to use Eq. (35) to determine the field of the H-wave.

Thus, Försterling's derivation, on the basis of which he concluded that the H-wave penetrates anomalously deeply behind the plane in which the dielectric function vanishes, was in error, since it was obtained outside the domain of applicability of the approximation of an isotropic, inhomogeneous medium, which he used.

6. RESULTS AND DISCUSSION

At every point in an inhomogeneous medium, there is a preferred direction of the density gradient. Therefore, the inhomogeneous medium, strictly speaking, is at the same time anisotropic. The important parameter determining the degree of anisotropy of the inhomogeneous medium is the ratio of the formation length of the local field l to the characteristic dimension L of the inhomogeneities. When the distance L over which the properties of the medium vary is small compared with l, e.g., on the surface of the material, the conditions for formation of the local field are altered, which induces a natural anisotropy of the surface layer, with a thickness of order l, even in an isotropic medium.[11]

If the characteristic dimension L of the inhomogeneities in the medium is larger than the formation length of the local field l and the required accuracy of the calculations is coarser than l/L, then the anisotropy of the inhomogeneous medium can be neglected, i.e., the medium can be taken to be isotropic.

If, on the other hand, one requires accuracy higher than l/L, then one must take account of the anisotropy and use the dielectric tensor (30) to describe such a medium, or, if the conditions of the problem permit, the special cases (31) or

(33). When considering effects directly due to the mere existence of the anisotropy, it is always necessary to allow for the tensor nature of the dielectric function.

Thus, for example, in a one-dimensional inhomogeneous medium whose density depends only on z, the conditions of propagation of an electromagnetic wave depend on the angle between the direction of propagation and the z axis, but in all other cases do not depend on this angle, in analogy with the ordinary and extraordinary waves in a homogeneous single-axis crystal. By neglecting the anisotropy in such a medium, we ignore the differences between these waves, which even in the presence of weak anisotropy will inevitably show up over a long enough path length. To account for such effects, it is necessary to allow for anisotropy.

Note that when the formation length of the local field l is comparable in magnitude to the characteristic size of the inhomogeneities, L, the relationship between the local field and the mean macroscopic field can be different at every point in the medium, so that it is necessary to calculate it at every individual point in the medium, based on the local nature of the inhomogeneities. For example, periodic variation of the electron density in a single crystal makes it possible to account for the influence of small inhomogeneities of electron density. This was done by Johnson by another method, namely by generalizing to optical frequencies an approach used to consider x-ray diffraction.[12]

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