Phase transitions in an exciton semiconductor and in a superfluid low-density Fermi liquid: Bose-liquid approach. Three and two-dimensional systems. General description at finite temperatures

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For a semiconductor close to the interband exciton instability, an effective boson action is constructed. It is shown that interactions with transition of particles from band to band lead to the appearance in the action of terms that break the global gauge invariance and, therefore, to the absence of superfluidity of the exciton liquid. An analogous Bose-liquid action with unbroken invariance with respect to a change in the phase of the Bose fields can be constructed for a superconductor with a low particle density. To describe the thermodynamics of a spatially homogeneous rarefied Bose liquid, a self-consistent approach taking into account the exact relationship between the self-energy functions is developed. The equations of the proposed theory do not contain infrared divergences, have a universal dimension-independent form, and make it possible to describe in a unified manner phase transitions in a Bose liquid with space dimension $D \geqslant 2$. The transition temperature in 3D and 2D exciton semiconductors is calculated. In addition, the temperature of Berezinskiĭ–Kosterlitz–Thouless transition for a 2D superfluid Bose system is found. In the case of a two-dimensional doped exciton semiconductor, it is shown that there exists a large region in the phase diagram in which the Fermi liquid of the excess particles is unstable with respect to phase demixing. © *1995 American Institute of Physics.*

1. INTRODUCTION

The possible instability of the ground state of a semiconductor leading to a phase transition of electron nature in the case when the exciton binding energy ε_{ex} exceeds the band gap ε_g was first demonstrated in Ref. 1 and was actively investigated for several years (see, for example, Refs. 2 and 3). It appears that at the present time this idea is receiving a new development in connection with the appreciable successes in the technology of growing semiconducting heterostructures with prespecified electron spectrum. Indeed, there are well-known systems such as superlattices of the second kind based on InAs/GaSb compounds in which variation of the thickness of the films makes it possible to vary over wide ranges the band gap and pass continuously from a semiconductor spectrum to a semimetal single-electron spectrum.[4] It is obvious that at a certain thickness we will certainly be able to satisfy the condition $\varepsilon_{ex} > \varepsilon_g$ of exciton instability. There have recently been reports of the observation of condensation of indirect excitons in heterostructures based on GaAs/AlAs (Ref. 5). The same phenomenon can evidently also be expected in the InAs/GaSb compounds. In addition, in the very near future we can expect the synthesis of new structures with controlled variation of the band gap or of the binding energy of electron–hole pairs.

These successes in the experimental domain require a reexamination of the existing status of the theory of the exciton instability in semiconductors. Indeed, despite the 30-year history of exciton semiconductors, there does not yet exist a physically sensible and well-behaved theory for nonzero temperatures permitting, in particular, the calculation of an important parameter such as the phase transition temperature. The generalization to the case $T \neq 0$ of the diagram approach of Refs. 2 and 3, which is rather complicated and cumbersome even at zero temperature, presents significant difficulties. Further, the calculation of the transition temperature T_0 in the standard mean field approximation leads to a manifestly unphysical result: $T_0 \sim \varepsilon_{ex} \sim \varepsilon_g$. This result is, in fact, entirely natural, since the usual mean field approach in this case is certainly not justified because there is no small interaction parameter in the semiconductor. In addition, all previous studies concerned isotropic three-dimensional systems, whereas the present experimental situation requires the construction of a theory for the case of an essentially anisotropic electron spectrum that is nearly two-dimensional or occupies an intermediate position between the 2D and 3D cases (size-quantized or layer systems). The latter requirement introduces additional complications on account of the increase in fluctuation effects with decreasing dimension.

Investigation of a 3D exciton liquid at $T=0$ in the limit $\varepsilon_{ex} - \varepsilon_g \ll \varepsilon_g$ (Ref. 2) shows that the system behaves like a weakly nonideal Bose liquid. Physically, it is obvious that this similarity must also be maintained at $T \neq 0$. In this paper, we propose an approach to the description of a semiconductor that is close to the exciton instability based on transformation of the original fermionic system to an effective Bose liquid of composite particles.

In Sec. 2, we analyze an effective low-energy Bose action of a semiconductor of arbitrary dimension D. The action has the standard form of the action of a rarefied Bose liquid with binary interaction, and the chemical potential λ of the bosons is fixed and equal to $\lambda = \varepsilon_{ex} - \varepsilon_g$. A similar Bose-liquid representation for a superconductor with low particle

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number was constructed earlier in the paper Ref. 6 of the authors. Thus, there is a close connection between the descriptions of an exciton semiconductor and a superconductor in the Schafroth limit, when the number of particles is so small that the mean separation between them becomes greater than the radius of the two-particle bound state. Indeed, these two systems are two realizations of an effective Bose liquid with given chemical potential and given particle number, respectively. However, there exists a difference, which consists of the absence of superfluidity of the exciton liquid due to the presence in semiconductors of interactions that violate the particle conservation law in each band and lift the degeneracy of the effective action with respect to the phase of the Bose fields. This question is considered in detail in Sec. 3.

The transition to Bose variables makes it possible to understand clearly and formulate the basic problems in the construction of the theory of exciton semiconductors at finite temperatures. To a large degree, these are associated with the complexity of the microscopic description of the thermodynamics of a Bose liquid over a wide temperature range, especially for systems of low dimensionality. Indeed, the diagram series of perturbation theory for a Bose liquid even in the $3D$ case converge poorly because of the singularities at small momenta in the diagrams of higher orders.

Nevertheless, already the first order (or the equivalent Bogolyubov approximation) for a three-dimensional rarefied Bose liquid gives a sensible result for T_0, which is equal to the condensation temperature of an ideal Bose gas (see, for example, Ref. 7). In a 2D Bose liquid, the first order is already divergent. In particular, there is a logarithmic divergence in the calculation of the density of the supercondensate particles. To compensate this divergence, Popov[8] proposed a procedure for taking into account the second order of the diagram perturbation theory. This theory made it possible to demonstrate qualitatively the possibility of formation of a superfluid density in the absence of a Bose condensate in the 2D system.

Nevertheless, such an approach cannot be recognized as satisfactory for our purposes on account of the impossibility of generalizing it to the three-dimensional (and, generally, $D>2$) case and the construction of a unified dimension-independent theory, which is needed for the description of systems with dispersion laws intermediate between 2D and 3D. The inapplicability of Popov's formalism for $D>2$ is basically due to the scheme of perturbation theory in Ref. 8, which does not permit a self-consistent determination of the temperature dependence of the self-energy functions.

To solve these difficulties, we develop in Sec. 4 a self-consistent approximation that is free of divergences for all dimensions of space and that makes it possible to describe in a unified manner the thermodynamics of a spatially homogeneous rarefied Bose gas for arbitrary $D\geqslant 2$. The two following sections of the paper are devoted to application of the developed formalism to the description of three- and two-dimensional low-density Bose liquids, respectively. We mention that there is good agreement between our results for the 2D system and the behavior of the XY model when allowance is made for irrotational fluctuations[9] (for more details, see Sec. 6) and also agreement of the transition temperature T_0 with the temperature obtained in Ref. 7 as a result of renormalization-group analysis of a 2D Bose liquid. We mention that Secs. 4, 5, and 6, which are devoted to the construction of a self-consistent approximation for a D-dimensional low-density Bose liquid, can be considered independently of the original problem of an exciton semiconductor or a superconductor with local pairs.

In Sec. 7, we consider a two-dimensional doped semiconductor that is close to the exciton instability. We show that both above and below the line of the phase transition to the exciton phase the phase diagram can contain regions in which $\partial\mu/\partial n<0$. This means that there is instability of the spatially homogeneous state of the Fermi liquid of the excess particles with respect to phase demixing. In this case, the nature of the phase demixing is associated with the suppression of the exciton instability and the decrease in the density of the excitons with the doping as a result of the Burstein shift of the exciton energy. Since the renormalization of the width of the forbidden band in an exciton semiconductor is directly related to the exciton density, the decrease in the latter with increasing number n of excess particles causes a lowering of the edge of the electron band and, possibly, a decrease of the potential μ with increasing n. It is this possibility that is realized in the case of the 2D spectrum. The considered mechanism belongs in the framework of the general philosophy of the band approach to the theory of phase demixing that has been developed by the authors in connection with high-temperature superconductivity during the last few years.[10,11] A detailed exposition of our point of view concerning this phenomenon, and also a review of some very varied experiments that obtain a natural interpretation in terms of phase demixing, can be found in Ref. 11. Here we merely mention that demixing of the electron fluid in a 2D doped semiconductor can have a direct bearing on copper oxide high-temperature superconductors, in which the role of the interband exciton instability is played by instability with respect to charge transfer from the copper to the oxygen. The existence of such instability for the CuO_2 plane was first demonstrated in Ref. 12.

2. EFFECTIVE LOW-ENERGY ACTION FOR A SEMICONDUCTOR WITH LONG-RANGE INTERACTION

We consider a semiconductor with band gap ε_g and masses in the conduction band and valence band equal to m_c and m_v, respectively. We write the partition function Z in the form of a functional integral over Grassmann fields $\psi_1(\mathbf{r},\tau)$ and $\psi_2(\mathbf{r},\tau)$, which describe the fermionic variables in the conduction band and in the valence band:

$$Z=\int D\bar{\Psi}D\Psi\ \exp(-S\{\bar{\Psi},\Psi\}),$$

$$\bar{\Psi}=(\psi_1^*,\psi_2^*), \tag{1}$$

where the action $S\{\bar{\Psi},\Psi\}$ has the form

$$S=S_0+S_{\text{int}}. \tag{2}$$

The term S_0 corresponds to the semiconductor without interaction,

$$S_0=\int_0^{1/T}d\tau\int d^D\mathbf{r}\left\{\psi_1^*\left[\partial_\tau-\frac{\nabla^2}{2m_c}+\frac{\varepsilon_g}{2}-\mu\right]\psi_1+\psi_2^*\times\left[\partial_\tau+\frac{\nabla^2}{2m_v}-\frac{\varepsilon_g}{2}-\mu\right]\psi_2\right\},\quad(3)$$

and the part of the action S_{int} includes all possible interelectron interactions. For simplicity, in this section we consider only interactions of density–density type, and we ignore the spin of the particles. In this case,

$$S_{\text{int}}=\int_0^{1/T}d\tau\int d^D\mathbf{r}d^D\mathbf{r}'\,\frac{1}{2}\sum_{i,j=1}^{2}V_{ij}\times(\mathbf{r}-\mathbf{r}')\psi_i^*(\mathbf{r})\psi_i(\mathbf{r})\psi_j^*(\mathbf{r}')\psi_j(\mathbf{r}'),\quad(4)$$

where $V_{11}(\mathbf{r})$ and $V_{22}(\mathbf{r})$ are the potentials of the intraband interaction, and $V_{12}(\mathbf{r})=V_{21}(\mathbf{r})$ is the potential of the interband density–density interaction. In the following section, we consider in detail the influence of the spin dependence, and of the interband interactions that are not taken into account in (4), and that violate conservation of particle numbers in each band and lift the degeneracy in S_{int} with respect to the relative phase of the fields $\psi_1(\mathbf{r},\tau)$ and $\psi_2(\mathbf{r},\tau)$.

The presence in the action (2) of the interband interaction leads to the possibility of bound exciton states. The energies ε_j and wave functions $\chi_j(\mathbf{r})$ of the excitons are determined by the following eigenvalue problem:

$$\left(-\frac{\nabla^2}{2m}+\varepsilon_j\right)\chi_j(\mathbf{r})=V_{12}(\mathbf{r})\chi_j(\mathbf{r}),\quad(5)$$

where $m=m_c\,m_v/M$ is the reduced mass and $M=m_c+m_v$ is the total mass of the two-particle state. If the binding energy ε_0 of the exciton ground state is so close to the band gap that

$$|\varepsilon_0-\varepsilon_g|/\varepsilon_g\ll 1\quad(6)$$

the semiconductor can be described in terms of an effective Bose liquid of composite particles. In this section, we shall not dwell on the derivation of the effective Bose action for the interelectron density–density interaction (4). For the special case $V_{11}(\mathbf{r})=V_{22}(\mathbf{r})=V_{12}(\mathbf{r})=e^2/\mathbf{r}$, detailed calculations can be found in Ref. 13. In addition, the corresponding derivation is largely analogous to the procedure for obtaining the Bose action for a superconductor in the Schafroth limit that was proposed by the authors of the present paper in Ref. 6. The derivation of the low-energy action of the exciton liquid in the presence in S_{int} of interband interactions with transitions of particles from band to band, and also with allowance for spin effects, contains some specific features and will be considered in the following section.

The formal procedure of the transition from the original Fermi representation in the action (2) to the Bose representation consists of the introduction by means of the Hubbard–Stratonovich transformation of intermediate Bose fields that decouple the four-fermion terms in S_{int} (4). Subsequent integration over the Fermi fields leads to an effective action that depends only on the Bose fields $\varphi_j(\mathbf{r},\tau)$ of the excitons. If the condition (6) holds, the exciton density is low, and this makes possible a restriction to only binary exciton collisions. This means that in the expansion of the action in powers of the Bose field, terms higher than φ_j^4 can be ignored. If, as usual, the excited states of the excitons are sufficiently well separated in energy from the ground state, $(\varepsilon_g-\varepsilon_j)/|\varepsilon_g-\varepsilon_0|\gg 1$, then the main contribution to the low-energy Bose action comes solely from the Bose field $\varphi_0(\mathbf{r},\tau)\equiv\varphi(\mathbf{r},\tau)$ corresponding to the excitons in the ground state (s states). As a result, the effective action takes the form

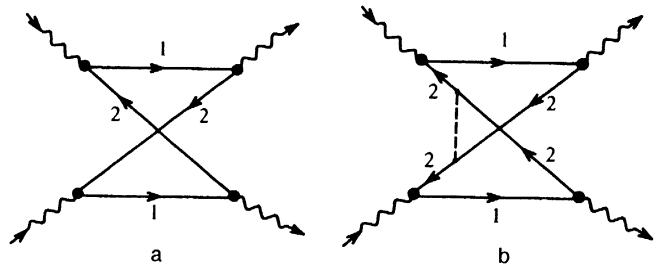

FIG. 1. Diagrams for bare vertex t_0 of the boson–boson interaction.

$$S_{\text{eff}}=\int d\tau d^D\mathbf{r}\left\{\varphi^*\left(\partial_\tau-\frac{\nabla^2}{2M}-\lambda\right)\varphi+\frac{1}{2}t_0\varphi^*\varphi\varphi^*\varphi\right\},\quad\lambda=\varepsilon_g-\varepsilon_0.\quad(7)$$

The spatial scale t_0 of the boson–boson interaction is determined by the smallest scale of the problem—the radius $\mathbf{r}_0=(2m\varepsilon_0)^{-1/2}$ of the exciton state. This makes it possible to write the interaction in local form and subsequently, in the investigation of the effective Bose system, the finite range of the interaction can be taken into account by a cutoff of the momentum integrals at the necessary places.

The physical nature of the exciton–exciton interaction is clear from the diagrams for t_0 in Fig. 1. In the process of collision, the excitons decay with subsequent exchange of their constituent Fermi particles. If only the interband interaction V_{12} is present in the system, only the process represented in Fig. 1a is possible. Such a process leads to boson–boson repulsion. If the intraband interaction V_{ii} is also present, there is a contribution from the diagram of Fig. 1b with interaction of the fermions in the intermediate state. The resulting expression for t_0 has the form

$$t_0=2\sum_{\mathbf{p},\mathbf{k}}\left\{\chi_{0,\mathbf{p}}^3V_{12}(\mathbf{p}-\mathbf{k})\chi_{0,\mathbf{k}}-\frac{1}{2}\chi_{0,\mathbf{p}}^2[V_{11}(\mathbf{p}-\mathbf{k})+V_{22}(\mathbf{p}-\mathbf{k})]\chi_{0,\mathbf{k}}^2\right\},\quad(8)$$

where $\chi_{0,\mathbf{p}}$ is the Fourier transform of the exciton wave function in the s state. For $V_{12}=V_{ii}$, the expression (8) is obtained, for example, in Ref. 13. Since it is precisely this case that is most naturally realized in semiconductors, we analyze the expression for the boson–boson interaction for identical potentials of the interband and intraband interactions. Then the expression (8) can be written in the symmetric form

$$t_0=\sum_{\mathbf{p},\mathbf{k}}V(\mathbf{p}-\mathbf{k})\chi_{0,\mathbf{p}}\chi_{0,\mathbf{k}}(\chi_{0,\mathbf{p}}-\chi_{0,\mathbf{k}})^2>0.\quad(9)$$

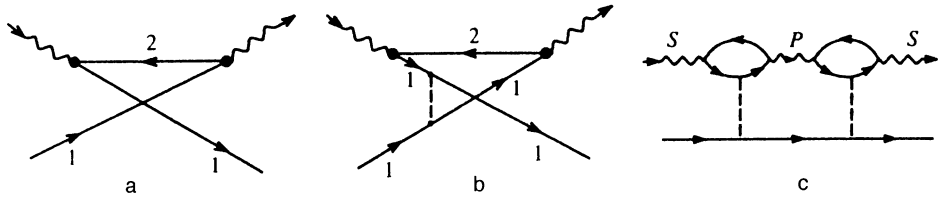

FIG. 2. Diagrams for the bare vertex of the fermion–boson interaction.

Thus, for arbitrary potential $V(\mathbf{p})$, the exciton–exciton interaction always corresponds to repulsion, thereby ensuring stability of the Bose liquid of the composite particles.

We show that for the potential $V(\mathbf{r})$ of an electron–electron interaction of arbitrary range,

$$t_0 \propto r_0^{D-2}/m, \tag{10}$$

where the coefficient of proportionality is of order unity and is determined by the particular form of $V(\mathbf{r})$. For the Coulomb potential, the estimate (10) is obvious on dimensional grounds due to the presence of a unique characteristic scale—the Bohr radius $\mathbf{r}_0$. In particular, the calculation of (9) for $D=2$, 3 and $V(\mathbf{r})-e^2/\mathbf{r}$ leads to the result

$$t_0^{3D}=\frac{13\pi}{3}\frac{\mathbf{r}_0}{m}, \quad t_0^{2D}=\frac{4\pi}{m}\left(1-\pi^2\frac{315}{4096}\right)\approx 0.96\frac{\pi}{m}. \tag{11}$$

The value of t_0^{3D} in (11) was first found in Ref. 2. The corresponding values for a point potential of the interelectron interaction have the form

$$t_0^{3D}=2\pi\frac{r_0}{m}, \quad t_0^{2D}=\frac{2\pi}{m}. \tag{12}$$

Note that for $D\geqslant 2$ in the case of a potential $V(\mathbf{r})$ of short range R the contribution of the intraband interaction to t_0 [the diagram in Fig. 1b and the second term in (8) corresponding to it] is negligibly small with respect to the parameter R/r_0. Comparison of the expressions for the boson–boson interaction in the case of the potentials of infinite (11) and zero (12) range leads us to believe that the estimate (10) is valid at least in the cases of the 2D and 3D systems, which are the ones in which we are interested.

Thus, the effective action of the semiconductor close to the exciton instability has the standard form of the action of a Bose liquid, and the chemical potential λ of the bosons is fixed and equal to

$$\lambda=\varepsilon_0-\varepsilon_g. \tag{13}$$

Earlier,[6] the authors of the present paper showed that a Fermi liquid with attraction between the particles in the low-density limit can also be described in terms of a Bose-liquid action of the type (7) with chemical potential λ of the bosons equal to twice the electron chemical potential μ. The role of the bosons in the latter case is played by bound states of two electrons with opposite spins. Thus, the exciton semiconductor and the superconductor with low particle number are two different realizations of an effective Bose liquid with, respectively, given chemical potential and given particle number.

Note that the global gauge invariance of the action (7), which in the case of a superconductor reflects the basic symmetry requirements, is for a semiconductor exclusively a consequence of the neglect in S_{int} (4) of interaction terms that involve a transition of particles from band to band. We consider the role of these interactions, and also the difference between a semiconductor and a superconductor associated with the freedom in the spin structure of an electron–hole pair, in Sec. 2. However, there exists one further difference that leads to new physical effects (see Sec. 7), namely the possible inequality of the number of electrons and holes in a doped semiconductor.

In the presence of a certain number n of excess particles (for definiteness, we consider n-type doping), we obtain a liquid of unpaired fermions that interact with the Bose liquid described by the action (7).

We analyze the possible mechanisms of fermion–boson interaction. As will be shown below, the dominant effect is always the fermion–boson repulsion due to the effect that leads to the Burstein shift: excess particles, occupying states of the conduction band, reduce the phase space that participates in the exciton formation. As a result, the exciton energy is reduced by an amount proportional to the electron density: $\delta\varepsilon_0\approx-\gamma_0 n$. Therefore, in second-quantized form, we must obtain a correction to the action of the form $\gamma_0\psi^*\psi\varphi^*\varphi$ (ψ is the fermion field), and this corresponds to repulsive scattering of the fermion by the boson.

The diagram corresponding to the interaction mechanism described above is shown in Fig. 2a. The scattering process consists of decay of the exciton in the intermediate state and replacement of the electron that is in the exciton by the incident electron. The presence of the intraband interaction leads to the appearance of the process shown in Fig. 2b. This scattering process corresponds to an effective attraction. The resulting vertex of the fermion–boson interaction corresponding to the diagrams in Fig. 2a and 2b can be obtained, for example, in the framework of a formalism analogous to that of Ref. 13:

$$\gamma_0=\sum_{\mathbf{k}}\{V(\mathbf{k})\chi_0(0)\chi_0(\mathbf{k})-V(\mathbf{k})\chi_0^2(\mathbf{k})\}>0. \tag{14}$$

The positivity of γ_0 corresponding to the fermion–boson repulsion is obvious from the structure of the integrand in (14).

One further possible interaction mechanism is associated with the interaction that arises between the electron charge and the London multipole moments in the case of virtual polarization of the exciton. This process corresponds to an effective attraction and proceeds through an intermediate excited state of the exciton (the diagram in Fig. 2c). The main contribution is made by the p state that is closest in energy.

In the dipole approximation, this interaction was considered in Ref. 14. In the general case, the vertex of the multipole–charge interaction has the form

$$\gamma_{m-e}=\int \frac{d^D\mathbf{k}}{(2\pi)^D}\sum_j \frac{|\Gamma_j(\mathbf{k})|^2V^2(\mathbf{k})}{k^2/2m_r+\varepsilon_0-\varepsilon_1},$$

$$m_r=\frac{m_c M}{m_c+M}, \tag{15}$$

where ε_1 is the energy of the excited p state, and m_r is the reduced fermion–boson mass. The index j labels the degenerate p states and takes the values $j=x,y,z$ for $D=3$ and $j=x,y$ for $D=2$. The quantity $\Gamma_j(\mathbf{k})$ corresponds to the triangle vertex of the electron–exciton interaction with transition of the exciton from the s to the p state (see Fig. 2c):

$$\Gamma_j(\mathbf{k})=\int d^D\mathbf{r}\chi_0(\mathbf{r})\chi_{1,j}(\mathbf{r})e^{i\mathbf{kr}}. \tag{16}$$

In (16), $\chi_{1,j}(\mathbf{r})$ is the wave function of the p state. It is obvious that γ_{m-e} will have the maximum value in the case of the long-range Coulomb interelectron interaction $V(\mathbf{r})$. We show that even in this case the multipole–charge attraction for $D=2$ and 3 is much less that the repulsive interaction γ_0. The value of γ_0 calculated in accordance with (14) for $V(\mathbf{r})=e^2/\mathbf{r}$ in the 2D and 3D systems is

$$\gamma_0^{2D}=\left(1-\frac{3\pi}{16}\right)\frac{4\pi}{m},\quad \gamma_0^{3D}=\frac{12\pi r_0}{m}. \tag{17}$$

We find the attractive potential from (15) and (16). For the 2D system, we obtain the expression

$$\gamma_{m-e}^{2D}=\frac{\pi}{12m}\left(\frac{9}{16}\right)^3 I\left(\frac{m_r}{2m}\right),$$

$$I(\alpha)=4\alpha\int_0^\infty \frac{dx}{(x+\alpha)(x+1)^5}. \tag{18}$$

Using the obvious upper bound of the integral in (18), $I(\alpha)<1$, and also Eqs. (17), we find the ratio of the repulsive potential γ_0^{2D} to the attractive potential γ_{m-e}^{2D}:

$$\frac{\gamma_0^{2D}}{\gamma_{m-e}^{2D}}>48\left(1-\frac{3\pi}{16}\right)\left(\frac{16}{9}\right)^3\approx 111\gg 1. \tag{19}$$

Making similar calculations for the 3D case, we obtain

$$\frac{\gamma_0^{3D}}{\gamma_{m-e}^{3D}}=\frac{3^{11}\sqrt{3}}{2^{15}}\sqrt{\frac{m}{m_r}}\approx 10\frac{\sqrt{m_v(m_v+2m_c)}}{m_v+m_c}. \tag{20}$$

It follows from (20) that for $D=3$ the attraction becomes comparable with the repulsion provided m_c exceeds m_v by a factor of more than 100. However, in two-dimensional systems the multipole–charge attraction is negligibly small for arbitrary ratio m_c/m_v [see (19)]. Thus, for the majority of physically sensible situations the electron–exciton interaction corresponds to repulsion. At the same time, the total fermion–boson action takes the form

$$S_{\text{eff}}=S_B+S_{FB}, \tag{21}$$

where S_B is determined by the expression (7) and corresponds to the bosonic part of the action, while S_{FB} describes the fermions in the conduction band and the fermion–boson interaction:

$$S_{FB}=\int d\tau d^D\mathbf{r}\left\{\psi^*\left(\partial_\tau-\frac{\nabla^2}{2m_c}-\mu\right)\psi+\gamma_0\psi^*\psi\varphi^*\varphi\right\}. \tag{22}$$

In (22), ψ is the fermionic field corresponding to the excess particles in the conduction band, and μ is the electron chemical potential measured from the bottom of the band. For γ_0, as for t_0 in (7), the following estimate holds for arbitrary $V(\mathbf{r})$:

$$\gamma_0\propto r_0^{D-2}/m. \tag{23}$$

In particular, for $V(\mathbf{r})$ of infinite range in the cases $D=2$ and 3 we have the expression (17), and in the case of a point potentials of the interelectron interaction $V(\mathbf{r})$ the calculation of γ_0 in accordance with (14) gives

$$\gamma_0^{2D}=4\pi/m, \tag{24}$$

$$\gamma_0^{3D}=8\pi r_0/m. \tag{25}$$

Thus, the general form of the effective low-energy action of a semiconductor close to the exciton instability does not depend on the specific form of the potential of the interelectron interaction, which determines only the numerical coefficient in the expressions for the boson–boson (10) and fermion–boson (25) interactions.

3. BOSE ACTION FOR SEMICONDUCTOR WITH INTERBAND INTERACTION OF GENERAL FORM. ALLOWANCE FOR LIFTING OF THE PHASE DEGENERACY

It follows from the results of the previous section that the low-energy action for a semiconductor with interband interaction of the density–density type (7) is identical to the corresponding action for a superconductor with low particle density (see our study of Ref. 6). The invariance of the action with respect to phase rotations leads to the occurrence of a Goldstone branch of vibrations and to superfluidity. Such behavior is entirely natural for superconductors, but it is well known that the exciton liquid in semiconductors does not possess the property of superfluidity. This is due to the fact that in real systems there are always interactions with transitions of particles from band to band, and these lift the degeneracy with respect to the difference phase of the fields $\psi_1(\mathbf{r},\tau)$ and $\psi_2(\mathbf{r},\tau)$ in S_{int}. This last leads to the appearance of a gap in the spectrum of collective excitations and violation of the criterion for superfluidity.

Thus, allowance for the additional terms of the interband interaction must lead to the violation of global gauge invariance of the action (7). Bearing in mind the final remark in the previous section, we shall assume a point interaction.

For a two-band system, the most general expression for S_{int} is

$$S_{\text{int}}=-\int d\tau d^D\mathbf{r}\sum_{\mu,\nu}\frac{1}{2}g_\mu^\nu(\bar\psi\tau_\nu\times\sigma^\mu\psi)^2,$$

$$\bar{\psi}=(\psi_{1\uparrow}^{*}\psi_{1\downarrow}^{*}\psi_{2\uparrow}^{*}\psi_{2\downarrow}^{*}). \tag{26}$$

In (26), $\sigma_0=1$, σ_j $(j=1,2,3)$ are the Pauli matrices, and $\tau_\nu=\sigma_\nu/2$ are isotopic matrices that act on the space of band indices. The symbol $\times$ denotes the direct product of matrices.

It follows from the results of Sec. 1 that to establish the form of the boson action in the given case we can, without loss of generality of the results, retain in S_{int} only the interband interaction. Then g_μ^ν are nonvanishing only for $\nu=1,2$. The invariance with respect to the spin rotations imposes the condition $g_1^\nu=g_2^\nu=g_3^\nu$.

Thus, there remain four independent coupling constants. To eliminate the four-fermion terms in (26), it is necessary to introduce four real Bose fields—two scalar, Σ_0^1 and Σ_0^2, corresponding to the singlet exciton states, and two vector fields with components Σ_j^1 and Σ_j^2 $(j=1,2,3)$ corresponding to the triplet excitons. We represent these fields in the complex form

$$\Sigma_s=\Sigma_0^1+i\Sigma_0^2, \quad \boldsymbol{\Sigma}_t=\boldsymbol{\Sigma}_1+i\boldsymbol{\Sigma}_2. \tag{27}$$

After the Hubbard–Stratonovich transformation, we obtain

$$S=\int d\tau d^D\mathbf{r}\left\{\bar{\psi}M\psi+\frac{(\Sigma_s+\Sigma_s^*)^2}{2g_0^1}-\frac{(\Sigma_s-\Sigma_s^*)^2}{2g_0^2}+\frac{(\boldsymbol{\Sigma}_t+\boldsymbol{\Sigma}_t^*)^2}{2g_1^1}-\frac{(\boldsymbol{\Sigma}_t-\boldsymbol{\Sigma}_t^*)^2}{2g_1^2}\right\}, \tag{28}$$

$$M=\begin{bmatrix}\partial_\tau-\dfrac{\nabla^2}{2m_c}+\dfrac{\varepsilon_g}{2} & \Sigma_s+\boldsymbol{\Sigma}_t\sigma \\ \Sigma_s^*+\boldsymbol{\Sigma}_t^*\sigma & \partial_\tau+\dfrac{\nabla^2}{2m_\nu}-\dfrac{\varepsilon_g}{2}\end{bmatrix}\equiv\hat{G}^{-1}+\hat{\Sigma}, \tag{29}$$

where $\hat{G}^{-1}$ is the diagonal part of the matrix M corresponding to the inverse Green's function of the semiconductor without interaction, and $\hat{\Sigma}$ is the off-diagonal part of M describing the exciton Bose fields. Note that in the mean field approximation the Bose variables (27) correspond to the four types of exciton condensate that are possible for a two-band system and are well known in the theory of an exciton dielectric (see, for example, Ref. 15). In our case, these are fluctuating variables with respect to which functional integration is performed.

Since Σ_s and Σ_t occur in the same manner in the action (29), we can without loss of generality consider just one channel. For definiteness, we consider only the singlet Bose field (omitting then the index s of Σ_s). We go over from the coupling constants to the corresponding binding energies ε_μ^ν:

$$\frac{1}{g_\mu^\nu}=\int\frac{d^D\mathbf{p}}{(2\pi)^D}\frac{1}{p^2/2m+\varepsilon_\mu^\nu}.$$

In the case $|\varepsilon_g-\varepsilon_\mu^\nu|/\varepsilon_g\ll 1$, we obtain for the part of the action (28) that does not depend on the Fermi variables (28)

$$2\Delta_0^{-2}\left(\varepsilon_g-\frac{\varepsilon_0^1+\varepsilon_0^2}{2}\right)\Sigma^*\Sigma+2\Pi(0)\Sigma^*\Sigma+\frac{1}{2}\Delta_0^{-2}(\varepsilon_0^1-\varepsilon_0^2)\times(\Sigma^{*2}+\Sigma^2),$$

where

$$\Pi(0)=\int\frac{d^D\mathbf{p}}{(2\pi)^D}\frac{1}{p^2/2m+\varepsilon_g}.$$

The Gaussian integral with respect to the Fermi variables can be calculated in the standard manner (see, for example, the corresponding calculations for a superconductor in Ref. 6). The part of this integral quadratic in the Bose variables is

$$\frac{1}{2}\operatorname{Tr}(\hat{G}\hat{\Sigma}^*\hat{G}\hat{\Sigma})=T\sum_{\varepsilon,q}2\Delta_0^{-2}\left\{-\Pi(0)-i\varepsilon+\frac{q^2}{2M}\right\}\Sigma^*\Sigma.$$

We introduce the renormalized Bose field $\varphi_q=\sqrt{2}\Delta_0^{-1}\Sigma(q)$. Then with allowance for the two previous expressions, we finally obtain for the effective action

$$S_{\text{eff}}=\int d\tau d^D\mathbf{r}\left\{\varphi^*\left(\partial_\tau-\frac{\nabla^2}{2M}-\lambda\right)\varphi+\frac{1}{4}\delta\varepsilon(\varphi^{*2}+\varphi^2)+\frac{1}{2}t_0(\varphi^*\varphi)^2\right\},$$

$$\lambda=\varepsilon_g-\frac{\varepsilon_0^1+\varepsilon_0^2}{2}; \quad \delta\varepsilon=\varepsilon_0^1-\varepsilon_0^2. \tag{30}$$

The final term in the action (30), which describes the boson–boson interaction, arises from the term of fourth order in the expansion in powers of the Bose fields $\hat{\Sigma}$ of the total action (28) after integration over the Fermi variables. The potential of the boson–boson interaction t_0 in (30) is determined by the expression (12).

Thus, because of the difference of the coupling constants in the channels corresponding to the real and imaginary components of the exciton field, the action contains a term that breaks the invariance with respect to a change in the phase of the field φ. In real semiconductors, interactions with the transition of particles from band to band, to the extent of which the phase fixing energy $\delta\varepsilon$ is not equal to zero, are usually weak by virtue of the orthogonality of Bloch functions in different bands. In contrast, the interaction of density–density type is not weak. Therefore, in the majority of cases the phase fixing energy must be small. It is such a situation that we consider below. More precisely, we require that

$$\delta\varepsilon/2\lambda\ll 1. \tag{31}$$

Note that despite the smallness of the parameter (31) the breaking of phase symmetry leads to important physical consequences. First, superfluidity disappears even at $T=0$. Second, as was demonstrated in Ref. 3 for $T=0$ and $D=3$, when the considered interactions are taken into account a phase transition with formation of a condensate is a first-order transition. However, we show below that allowance for fluctuations leads to a first-order transition even in a standard Bose

liquid with phase-degenerate action. Therefore, for $D=3$ this circumstance is unimportant, and, as a consequence, the weak phase fixing does not affect the thermodynamic properties of $3D$ systems. The most important physical changes occur for $D=2$. Indeed, it is well known that in two-dimensional degenerate systems at all $T\neq 0$ there is no long-range order (in the sense of the presence of a true Bose condensate). In this case, the phase transition entails the emergence of "rigidity" of the system ρ_s and a change in the nature of the decrease of the binary correlation function from exponential to power-law behavior below the critical temperature. The lifting of the degeneracy immediately leads to the possibility of formation of a condensate and long-range order. Nevertheless, a formal description can be constructed in such a way that the "condensate" density will not occur at all in the final equations and, as in the 3D case, the dynamics of both degenerate 2D systems and systems with weak phase fixing can be described in the framework of a unified scheme.

4. SELF-CONSISTENT APPROXIMATION FOR *D*-DIMENSIONAL BOSE LIQUID ($D\geqslant 2$)

We have established that both an exciton semiconductor and a low-density Fermi liquid with attraction reduce for arbitrary dimension D to an effective Bose liquid in the limit (7) of low particle density. In the given case, the presence of the low-density parameter, which is determined by the ratio of the range of the interaction potential to the mean separation between the particles, is guaranteed by the smallness of $r_0\sqrt{2M\lambda}\sim(\lambda/\varepsilon_0)^{1/2}\ll 1$.

Thus, it is necessary to have a physically sensible formalism suitable for describing a low-density Bose liquid in a wide range of temperatures, including the phase-transition temperature. In this section, we construct a self-consistent approximation that makes it possible to describe in a unified manner phase transitions in a Bose liquid of arbitrary dimension $D\geqslant 2$.

We proceed from the standard action (7) for the Bose liquid. In the first stage of the calculations, we renormalize the vertex t_0 of the boson–boson interaction. Following Popov,[8] we introduce an intermediate momentum $\tilde{p}$ such that $\tilde{\varepsilon}=\tilde{p}^2/2M$ satisfies the inequality $\lambda<\tilde{\varepsilon}\ll\varepsilon_0$. We integrate over the range of variables $k>\tilde{p}$. As a result, we obtain an action of the form (7) in which the bare interaction t_0 is replaced by the renormalized t and all momentum integrals are truncated at the upper limit by $\tilde{p}$. The renormalization consists of dressing t_0 by a series of ladder diagrams corresponding to the scattering of two bosons by each other, i.e., t is determined by the expression

$$t=\frac{t_0}{1+t_0\Pi},\quad \Pi=\int_{\tilde{p}<|\mathbf{k}|<r_0^{-1}}\frac{d^D\mathbf{k}}{(2\pi)^D}\frac{M}{k^2}. \tag{32}$$

The expression for the polarization operator Π in (32) is written down in the simplest case of a step dependence on k of the potential t_0. Note that in this case, as usual, the ladder diagram series is distinguished by virtue of the low particle density. Taking into account the definition (9) of t_0, we can obtain the following expression for the renormalized interaction potential for $D>2$:

$$t^{D>2}=\frac{r_0^{D-2}}{m}\frac{\alpha}{1+\beta M/m}, \tag{33a}$$

where the numerical coefficients α and β are of order unity and are determined by the actual form of the bare interaction potential. For $D=2$, the polarization operator in (32) is logarithmically large at low momenta. Therefore, the parameters of the bare interaction disappear altogether from the expression for the renormalized potential t:

$$t^{2D}=\frac{4\pi}{M}\ln^{-1}\frac{\varepsilon_0}{\tilde{\varepsilon}},\quad \varepsilon_0=1/(2mr_0^2). \tag{33b}$$

Thus, it follows from (33) that in the low-density limit (r_0 is small) for any dimension $D\geqslant 2$ we have an effective theory of a Bose liquid with weak interaction. The Hamiltonian of the system has the form

$$H=\int d^D\mathbf{r}\left\{\varphi^*(\xi_{\mathbf{k}}-\lambda)\varphi+\frac{1}{2}t_0\varphi^*\varphi\varphi^*\varphi\right\},\quad \xi_{\mathbf{k}}=\frac{\hat{\mathbf{k}}^2}{2M}. \tag{34}$$

The weakness of the interaction makes it possible in the subsequent stages to use perturbation theory with respect to t or an approximation of mean field type. However, as was already noted in the Introduction, the existing diagram methods for describing the thermodynamics of a Bose liquid are not suitable for our purposes because of the important differences in the formalism for $D=2$ and $D>2$ and the impossibility of self-consistent determination of the self-energy functions in two-dimensional systems.

We proceed as follows. For $T<T_c$, we separate in the field φ the "condensate" (c-number) part φ_0:

$$\varphi=\varphi_0+\psi,\quad \langle\psi\rangle=0,\quad \varphi_0=\varphi_0^*=\sqrt{\rho_0}. \tag{35}$$

Note that the separation of φ_0 by no means indicates the presence of a true Bose condensate as macroscopic occupancy of a single lowest level. The "condensate" part φ_0 must be looked upon in the general case as the square root of a large number of particles in states with momenta $k<q_0\ll\sqrt{2M\lambda}$. A bare "condensate" defined in such a manner and generating in a 2D system a superfluid density was first introduced by Popov in his diagram theory of a 2D Bose liquid. An appreciable difference between a bare "condensate" and a true one arises only in two- and one-dimensional systems, in which the density of the true Bose condensate is identically equal to zero for $T\neq 0$. The subsequent theory must be constructed in such a way that neither the lowest cutoff momentum q_0 nor the density of the bare "condensate" ρ_0 occurs in the final equations. With allowance for (35), the Hamiltonian density h can be rewritten as follows:

$$h=-\lambda\rho_0+\frac{1}{2}t\rho_0^2+\psi^*(\xi_{\mathbf{k}}-\lambda)\psi+2t\rho_0\psi^*\psi$$

$$+\frac{1}{2}t\rho_0(\psi^*\psi^*+\psi\psi)+t\sqrt{\rho_0}(\psi^*\psi^*\psi+\psi^*\psi\psi)$$

$$+h_{\text{int}},\quad h_{\text{int}}=\frac{1}{2}t\psi^*\psi^*\psi\psi. \tag{36}$$

In the self-consistent field approximation, we introduce a bilinear decoupling in the interaction terms of (36). With allowance for (35), the penultimate term does not contribute. We write the final four-fermion term h_{int} in the form

$$h_{\text{int}}^{SF}=2ts\psi^*\psi+\frac{1}{2}td(\psi^*\psi^*+\psi\psi)+\Phi,$$

where $s=\langle\psi^*\psi\rangle$, $d=\langle\psi^*\psi^*\rangle$. The potential part of the Hamiltonian of the self-consistent field Φ is determined from the condition $\langle h_{\text{int}}^{SF}\rangle=\langle h_{\text{int}}\rangle$. As a result, we obtain for Φ

$$\Phi=-ts^2-\frac{1}{2}td^2.$$

Introducing the normal, Σ, and anomalous, Δ, self-energy parts in accordance with

$$\Sigma=2t(\rho_0+s),\quad \Delta=t(\rho_0+d),$$

we obtain for the Hamiltonian of the self-consistent approximation the expression

$$H^{SF}=H_0^{SF}+\int d^D\mathbf{r}\left\{\psi^*(\xi_{\mathbf{k}}-\lambda+\Sigma)\psi\right.$$

$$\left.+\frac{1}{2}\Delta(\psi^*\psi^*+\psi\psi)\right\}, \tag{37}$$

$$H_0^{SF}=-\lambda\rho_0+\frac{1}{2}t\rho_0^2-\frac{1}{2t}(\Delta-t\rho_0)^2-\frac{1}{4t}(\Sigma-2t\rho_0)^2.$$

We diagonalize H^{SF} (37) by a standard canonical transformation for Bose operators. Averaging the obtained expression over the grand canonical ensemble, we find the thermodynamic potential of the system Ω:

$$\Omega=\langle H^{SF}\rangle=H_0^{SF}+\int\frac{d^D\mathbf{k}}{(2\pi)^D}\left\{\frac{1}{2}(E_{\mathbf{k}}-\xi_{\mathbf{k}}+\lambda-\Sigma)\right.$$

$$\left.+T\ln(1-\varepsilon^{-E_{\mathbf{k}}/T})\right\}, \tag{38}$$

where E_k is the spectrum of quasiparticles:

$$E_k=\sqrt{(\xi_{\mathbf{k}}-\lambda+\Sigma+\Delta)(\xi_{\mathbf{k}}-\lambda+\Sigma-\Delta)}.$$

In the expression (38) for Ω we require fulfillment of the exact relation well known in the theory of a Bose liquid that connects the self-energy functions Σ and Δ at zero momentum and ensures the acoustic nature of the excitation spectrum (see, for example, Ref. 16):

$$\Sigma-\Delta=\lambda. \tag{39}$$

This step, which appears simple at first glance, has a deep meaning and is the fundamental point in the construction of our approximation. We eliminate Σ in (38) by means of the identity (39) and determine ρ_0 from the usual condition of extremality of the thermodynamic potential Ω ($d\Omega/d\rho_0=0$). As a result, we obtain

$$t\rho_0=\Delta.$$

Finally, we have the following expression for the thermodynamic potential, which depends only on the anomalous self-energy function Δ:

$$\Omega_<=-\frac{\lambda^2-\Delta^2}{4t}-\frac{\lambda\Delta}{2t}+\int\frac{d^D\mathbf{k}}{(2\pi)^D}\left\{\frac{1}{2}(E_{\mathbf{k}}-\xi_{\mathbf{k}}-\Delta)\right.$$

$$\left.+T\ln(1-e^{-E_{\mathbf{k}}/T})\right\}. \tag{40}$$

In the expression (40), $E_{\mathbf{k}}=\sqrt{\xi_{\mathbf{k}}(\xi_{\mathbf{k}}+2\Delta)}$ is the Bogolyubov quasiparticle spectrum. Minimizing (40) with respect to Δ, we obtain a self-consistency equation determining the dependence of Δ:

$$\Delta=\lambda-2t\int\frac{d^D\mathbf{k}}{(2\pi)^D}\frac{1}{2}\left\{\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}(1+2n_{\mathbf{k}})-1\right\}. \tag{41}$$

In (41), $n_{\mathbf{k}}=n(E_{\mathbf{k}}/T)$ is the Bose distribution function. We find the equation for the total density ρ of the Bose particles from the well-known thermodynamic relation

$$\rho=-\frac{\partial\Omega}{\partial\lambda}=\frac{\lambda+\Delta}{2t}. \tag{42}$$

Alternatively, with allowance for the self-consistency equation (41)

$$\rho=\frac{\lambda}{t}-\int\frac{d^D\mathbf{k}}{(2\pi)^D}\frac{1}{2}\left\{\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}(1+2n_{\mathbf{k}})-1\right\}. \tag{42a}$$

Thus, Eqs. (40) and (41) completely determine the thermodynamics of the spatially homogeneous Bose liquid below the transition point. The expressions obtained have universal form for any space dimension and for arbitrary D are free of divergence at small momenta.

The proposed procedure for deriving the equations—the imposition of the condition (39) on the expression for the thermodynamic potential (38) before its minimization with respect to the self-consistent field Δ—differs fundamentally from the standard mean field approximation. As a rather cumbersome analysis shows, it corresponds in diagram language to effecive allowance for the vertex corrections to the Green's functions that are obtained by replacing in all possible ways two "condensate" lines by particle lines in the superfield diagrams for Ω. We note that in the second order in t the equations for ρ (42a) and Ω (40) are identical to the equations obtained by Popov with allowance for the second order of diagram perturbation theory.[8] However, the complete system of equations differs appreciably due to the presence of Eq. (41), which determines a self-consistent change in the anomalous self-energy function Δ as a function of T. This change is determined by the higher (in t) orders of perturbation theory that are contained in our form of the theory and leads to fundamental physical consequences. The following sections of the paper are devoted to the analysis of these consequences.

Using the expression for the renormalized vertex t, we can show that in the considered low-density limit it is meaningful to retain only the temperature-dependent part of the integrals in (40)–(42). Then the expressions we obtain for the thermodynamic potential below the transition point, $\Omega_<$, the total density ρ, and the self-consistency equation for Δ simplify somewhat:

$$\Omega_< = -\frac{\lambda^2-\Delta^2}{4t} - \frac{\lambda\Delta}{2t} + T\int \frac{d^D\mathbf{k}}{(2\pi)^D} \ln(1-e^{-E_\mathbf{k}/T}), \tag{43a}$$

$$\rho = \frac{\lambda}{t} - \int \frac{d^D\mathbf{k}}{(2\pi)^D} \frac{\xi_\mathbf{k}}{E_\mathbf{k}} n_\mathbf{k} \equiv \frac{\lambda+\Delta}{2t}, \tag{43b}$$

$$\Delta = \lambda - 2t\int \frac{d^D\mathbf{k}}{(2\pi)^D} \frac{\xi_\mathbf{k}}{E_\mathbf{k}} n_\mathbf{k}. \tag{43c}$$

The phase transition temperature T_0 is determined by the condition $\Delta=0$. For $T>T_0$, the thermodynamic potential $\Omega_>$ is given by

$$\Omega_> = -\frac{\Sigma^2}{4t} + T\int \frac{d^D\mathbf{k}}{(2\pi)^D} \ln(1-e^{-(\xi_\mathbf{k}-\lambda+\Sigma)/T}). \tag{44}$$

The self-energy function Σ is determined by the minimization of (44).

To conclude this section, we formulate equations for the case of a Bose liquid with fixed number of particles (ρ =const). We express the chemical potential λ in terms of ρ by means of the second equation of (43b). After a Legendre transformation

$$F_< = \Omega_< + \lambda\rho$$

we find the free energy $F_<$ for $T<T_0$:

$$F_< = t\rho^2 + \frac{\Delta^2}{2t} - \Delta\rho + T\int \frac{d^D\mathbf{k}}{(2\pi)^D} \ln(1-e^{-E_\mathbf{k}/T}). \tag{45a}$$

From the condition $\partial F/\partial\Delta=0$, we obtain an equation for Δ:

$$\Delta = t\rho - t\int \frac{d^D\mathbf{k}}{(2\pi)^D} \frac{\xi_\mathbf{k}}{E_\mathbf{k}} n_\mathbf{k}. \tag{45b}$$

Comparison of (45b) and (43c) shows that the solution for ρ=const can be obtained from the solution for λ=const by the successive substitutions

$$2t\to t, \quad \text{and then} \quad \lambda\to t\rho. \tag{46}$$

5. BOSE CONDENSATION IN THREE-DIMENSIONAL SYSTEMS: FIRST-ORDER PHASE TRANSITION

It is well known that in the mean field approximation the phase transition in a 3D Bose liquid is a second-order phase transition, as it apparently should be in the case of spontaneous symmetry breaking in a degenerate system. However, there exist cases in which allowance for fluctuations in systems with phase degeneracy leads to a change in the transition order. For example, it was shown in Ref. 17 that in an ordinary superconductor the transition is always of first order because of the fluctuations in the electromagnetic field. A similar result was obtained for a model of a semimetal with congruent Fermi surfaces of the electrons and holes in Ref. 18, in which allowance for the correlation corrections to the mean field approximation led to replacement of a second-order transition by a first-order one.

In this section, we show that in a three-dimensional Bose liquid the fluctuation effects contained in the equations formulated above also lead to a phase transition to a condensed state that is weakly of the first order.

We write the self-consistency equation (43c) for $D=3$ in the form

$$\Delta = \lambda - \frac{(2MT)^{3/2}}{2\pi^2} tJ\left(\frac{2\Delta}{T}\right),$$

$$J(x) = \int_0^\infty \frac{\xi^{3/2}d\xi}{\sqrt{\xi^2+\xi x}(e^{\sqrt{\xi^2+\xi x}}-1)}. \tag{47}$$

In the special case $\lambda=\varepsilon_0-\varepsilon_g$, this equation determines the phase diagram of a $3D$ exciton semiconductor. To establish the nature of the phase transition, it is sufficient to investigate the solution (47) at small Δ. Calculating the asymptotic behavior of the integral in (47) when $2\Delta/T\ll 1$, we obtain an equation valid near the transition point:

$$\lambda - \Delta = t\frac{(2MT)^{3/2}}{2\pi^2}\left(\frac{1}{2}\sqrt{\pi}\zeta - \pi\sqrt{\frac{2\Delta}{T}}\right), \tag{48}$$

where $\zeta\equiv\zeta(3/2)$ is the Riemann zeta function. The low-temperature solution with $\Delta\neq 0$ disappears abruptly when the $\lambda-\Delta$ straight line touches the function determined by the right-hand side of (48). From the condition of equality of the derivatives with respect to Δ of the left- and right-hand sides of the self-consistency equation, we find that this occurs at

$$\Delta = \Delta_c \equiv \frac{t^2M^3}{\pi} T_0^2. \tag{49}$$

To determine the temperature T_0 of the first-order phase transition, we substitute Δ_c (49) in Eq. (48). We then find

$$T_0 = T_B\left[1 + \frac{4Mt}{3\zeta^{4/3}}\left(\frac{4\lambda}{t}\right)^{1/3}\right], \tag{50}$$

$$T_B = \frac{\pi}{2M}\left(\frac{4\lambda}{t\zeta}\right)^{2/3}. \tag{50a}$$

The dependence of the anomalous self-energy function Δ on T is shown in Fig. 3. In the range of temperatures $T_B<T<T_0$, there exist two branches of solutions of Eq. (47). To the solution with smaller Δ (lower branch), there corresponds a maximum of the thermodynamic potential. The high-temperature (normal) phase can exist (as a metastable one for $T<T_0$) all the way to $T=T_B$. It is at this point that we have vanishing of $\Sigma-\lambda$, which plays the role of an effective chemical potential of the normal Bose liquid [see (44)].

The temperature of the phase transition in the exciton semiconductor is obtained by substituting $\lambda=\varepsilon_0-\varepsilon_g$ in (50).

For a system with a given number of particles, we find by means of the substitutions (46)

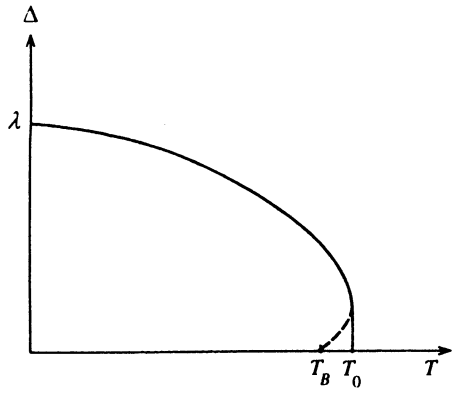

FIG. 3. The dependence $\Delta(T)$ for a three-dimensional system. The dashed curve shows the unstable branch of solutions.

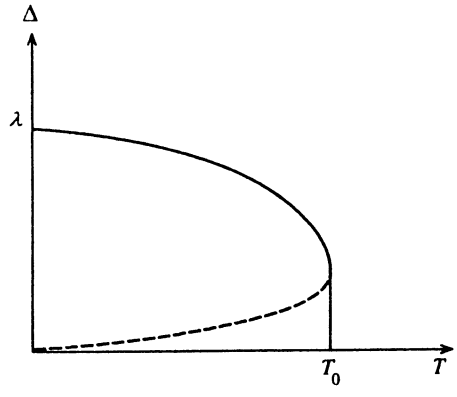

FIG. 4. The dependence $\Delta(T)$ for a two-dimensional Bose liquid.

$$T_0=T_B\left[1+\frac{4Mt}{3\zeta^{4/3}}\rho^{1/3}\right],\quad T_B=\frac{2\pi}{M}\left(\frac{\rho}{\zeta}\right)^{2/3}. \tag{51}$$

Note that the lower limit of existence T_B (51) of the normal phase is equal to the well-known condensation temperature of an ideal Bose gas.

The smallness of $(T-T_0)/T_0$ in the region in which the solution (47) is two-valued enables us to assert a first-order transition in a $3D$ Bose liquid in the standard language of a Landau functional. The expression for the functional Ω near the transition point can be readily recovered from Eq. (48) or obtained by direct expansion of the general expression (43a) at small Δ and $D=3$. Introducing the order parameter φ_0 as the square root of the condensate density in accordance with the expression $\Delta=t\varphi_0^2$, we obtain

$$\Omega\{\varphi_0\}=\frac{3}{2}\lambda\frac{T-T_B}{T_B}\varphi_0^2-\frac{4}{3\pi}(Mt)^{3/2}T_B\varphi_0^3+\frac{1}{2}t\varphi_0^4.$$

The appearance in this last equation of the cubic invariant obviously indicates a first-order phase transition.

6. PHASE TRANSITIONS IN A TWO-DIMENSIONAL BOSE LIQUID

We find the solution of the self-consistency equation (43c) in a two-dimensional system. In the limit $2\Delta/T\ll 1$, this equation has the form

$$\Delta=\lambda-\frac{Mt}{\pi}T\ln\frac{T}{2\Delta}. \tag{52}$$

Note that in the $D=2$ renormalized vertex (33b) the cutoff energy $\tilde{\varepsilon}$ can be replaced to logarithmic accuracy by λ. With allowance for this, we define the dimensionless interaction potential

$$\tilde{t}=\frac{Mt}{\pi}\approx\frac{4}{\ln(\varepsilon_0/\lambda)}. \tag{53}$$

It is the smallness of this quantity ($\tilde{t}\ll 1$) that will serve as the criterion of applicability of the theory we are to develop. We introduce the dimensionless variables

$$x=\frac{2\Delta}{T},\quad \tilde{\lambda}=\frac{2\lambda}{T}$$

and in these equations write down the self-consistency equation

$$\tilde{\lambda}-x=2\tilde{t}\ln\frac{1}{x}. \tag{54}$$

At a certain point $x=x_0$, the straight line $\tilde{\lambda}-x$ touches the function $2\tilde{t}\ln(1/x)$. This point corresponds to a first-order phase transition from the low-temperature ($\Delta\neq 0$) to the high-temperature ($\Delta=0$) phase. Equating the derivatives on the right- and left-hand sides of (54), we find the value of x_0:

$$x_0\equiv\frac{2\Delta_c}{T_0}=2\tilde{t}, \tag{55}$$

and then the critical temperature:

$$T_0=\lambda\left\{\tilde{t}\ln\frac{e}{2\tilde{t}}\right\}^{-1}=\frac{1}{4}\lambda\ln\frac{\varepsilon_0}{\lambda}\left\{\ln\left(\frac{e}{8}\ln\frac{\varepsilon_0}{\lambda}\right)\right\}^{-1}. \tag{56}$$

In the second equation of (56), we have used (53). The discontinuity of the anomalous self-energy function at the transition is determined by

$$\Delta_c=\lambda\left\{\ln\frac{e}{2\tilde{t}}\right\}^{-1}. \tag{57}$$

In contrast to three-dimensional systems, the solution of the self-consistency equation for $D=2$ has two branches at all $T<T_0$, and in accordance with this the high-temperature phase with $\Delta=0$ exists (as a metastable phase when $T<T_0$) right down to $T=0$. The dependence $\Delta(T)$ is shown in Fig. 4. The asymptotic behaviors of the upper branch of $\Delta(T)$ (corresponding to the global minimum of Ω) near zero and near T_0 are

$$\Delta(T)\simeq\lambda\left[1-\tilde{t}\left(\frac{T}{\lambda}\right)^3\right]\quad\text{as } T\to 0,$$

$$\Delta(T)\simeq\Delta_c\left[1+\left(2\ln\frac{e}{2\tilde{t}}\right)^{1/2}\sqrt{\frac{T_0-T}{T_0}}\right]\quad\text{as } T\to T_0.$$

Note that results analogous to ours (abrupt appearance of anomalous mean values and the existence of the normal phase in a metastable state down to $T=0$ for $D=2$) were obtained in Ref. 19 for an exciton semiconductor with maxi-

mally strong phase fixing, when the Bose field can be assumed to be real. However, the approach used by the authors, which in a certain sense is phenomenological, does not make it possible to take into account correctly the renormalization of the vertices, which plays a cardinal role in two-dimensional systems.

The transition temperature in a liquid with a fixed number of particles can be obtained from (56) by the substitutions (46). Noting, in addition, that for ρ=const the dimensionless vertex is to logarithmic accuracy

$$\tilde{t} \approx 2 \ln^{-1}\left(\frac{1}{r_0\rho^{1/3}}\right),$$

we finally obtain

$$T_0=\frac{2\pi\rho}{M} \ln^{-1}\left[\frac{e}{2} \ln\left(\frac{1}{r_0\rho^{1/3}}\right)\right]. \tag{58}$$

In systems with broken phase invariance (exciton semiconductor), the phase transition at $T=T_0$ that we have described corresponds to the formation of a true condensate and long-range order in the absence of superfluidity. This transition acquires a different physical content in superfluid systems (Bose liquid, superconductor with small number of particles).

In the construction of our approximation, we have taken into account only irrotational, or "anharmonic" in the terminology of Ref. 9, fluctuations. In the framework of such a treatment, a superfluid density ρ_s appears at the temperature T_0, and the nature of the decrease of the correlation function changes from an exponential one at $T>T_0$ to a power-law decrease at $T<T_0$. The superfluid density can be determined in the usual manner:

$$\rho_s=\rho-\rho_n,$$

where the density of the normal component is readily found by calculating the mean momentum of the Bose liquid in a moving coordinate system:

$$\rho_n=\int \frac{d^2\mathbf{k}}{(2\pi)^2} \xi_{\mathbf{k}}\left(-\frac{\partial n_{\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}}\right).$$

Using the expression for the total density (43b) and the self-consistency equation (43c) for $D=2$, we find

$$\rho_s=\frac{\Delta}{t}-\int \frac{d^2\mathbf{k}}{(2\pi)^2}\left\{\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}} n_{\mathbf{k}}-\frac{\xi_{\mathbf{k}}}{T}\frac{e^{E_{\mathbf{k}}/T}}{(e^{E_{\mathbf{k}}/T}-1)^2}\right\}. \tag{59}$$

The integral in (59) is a regular function of Δ/T and tends to zero at small Δ/T. Therefore, near the phase transition temperature T_0 we can write

$$\rho_s \simeq \Delta(T)/t. \tag{60}$$

Thus, as the temperature is lowered to the point T_0 a finite superfluid density $\rho_s(T)$ equal to Δ_c/t arises abruptly. The dependence $\rho_s(T)$ is similar to the function $\Delta(T)$ in Fig. 4.

Note that the abrupt occurrence of ρ_s in the given case agrees very well with the dependence $\rho_s(T)$ due to irrotational fluctuations in the XY model[9] and differs qualitatively from the corresponding dependence in Popov's diagram theory,[8] in which the absence of a self-consistency procedure leads to a smooth vanishing of ρ_s at the transition point.

Allowance for solenoidal excitations changes the picture of the phase transition. It is known that in 2D-degenerate systems a transition with a change in the rate of decrease of the binary correlation function and the occurrence of true superfluidity occurs at the temperature T_s through the Berezinskiĭ–Kosterlitz–Thouless (BKT) mechanism by the pairing of vortices that also exist above the transition temperature.[20] The superfluid density at the transition point $\rho_s(T_s)$ and the critical temperature are related by the universal equation[21]

$$\frac{\rho_s(T_s)}{2MT_s}=\frac{1}{\pi}. \tag{61}$$

Using Eq. (60) and the expressions (56) for T_0 and (57) for Δ_c, we calculate this ratio at $T=T_0$:

$$\frac{\rho_s(T_0)}{2MT_0}=\frac{1}{2\pi}.$$

Comparison of the last two equations shows that the temperature T_s of the BKT superfluid transition is less than the temperature T_0 of the disappearance of ρ_s due to the irrotational fluctuations. In the range of temperatures $T_s<T<T_0$, ρ_s (59) denotes the coefficient of "rigidity" of the system introduced by Berezinskiĭ. The presence of the "rigidity" of ρ_s ensures the existence above T_s of isolated quantum vortices and an acoustic branch of excitations that transmit interaction between the vortices. We can estimate T_s if we know the relation (61) and use the relationship (60) between ρ_s and Δ and the self-consistency equation (54). As a result, we obtain

$$\frac{T_0-T_s}{T_0}=\ln^{-1}\frac{e^2}{2\tilde{t}}=\ln^{-1}\left[\frac{e^2}{2}\ln\left(\frac{1}{r_0\rho^{1/3}}\right)\right].$$

Thus, the width of the temperature region between T_s and T_0 decreases as the density tends to zero. Therefore, in a large temperature range $0<T<T_s$ the $\rho_s(T)$ (59) that we have found corresponds to the true density of the superfluid component of the rarefied Bose liquid.

To conclude this section, we find the temperature dependence of the total density $\rho(T)$ in systems with given chemical potential λ (for example, for a semiconductor, where $\lambda=\varepsilon_0-\varepsilon_g$).

At temperatures $T<T_0$, the dependence of the density on the temperature is determined by the expression (43b). Therefore, as the transition point is approached from below ρ takes the value

$$\rho_- \equiv \rho(T_0-0)=\frac{\lambda+\Delta_c}{2t}.$$

At $T>T_0$, the thermodynamic potential $\Omega_>$ of the system has the form (44), and in accordance with this total density of the particles in the normal state is

$$\rho=\frac{\Sigma}{2t}\equiv\frac{\lambda+\theta}{2t}. \tag{62}$$

In (62), we have introduced the parameter $\theta=\Sigma-\lambda$. The self-energy function Σ is determined by miminizing $\Omega_>$:

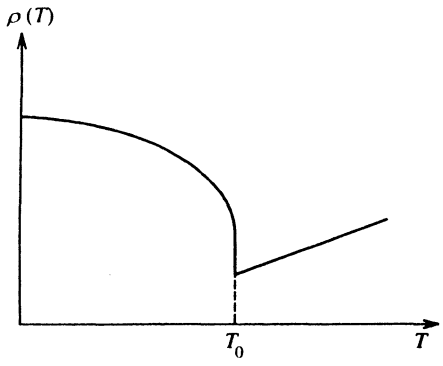

FIG. 5. Temperature dependence of the total density (means square of the field φ) in systems with given chemical potential.

$$\Sigma = 2t \int \frac{d^2\mathbf{k}}{(2\pi)^2} n\left(\frac{\xi_{\mathbf{k}} - \lambda + \Sigma}{T}\right).$$

Near the transition, $\theta/T \ll 1$. Calculating the asymptotic behavior of the integral in the last equation, we obtain the following self-consistency equation for $\theta(T)$:

$$\theta = -\lambda + \tilde{t} T \ln \frac{T}{\theta}. \tag{63}$$

The limiting value of the density when T_0 is approached from above is determined by the value of $\theta_c = \theta(T_0)$ at the transition point:

$$\rho_+ \equiv \rho(T_0 + 0) = \frac{\lambda + \theta_c}{2t}.$$

The equation for θ_c is readily obtained by using the relation $\Delta_c / T_0 = \tilde{t}$ and the expression (57) for Δ_c:

$$\eta = \ln \frac{2}{e\eta}, \quad \eta = \frac{\theta_c}{\Delta_c}.$$

The solution of this equation gives $\eta = \theta_c / \Delta_c \approx 0.463$. Thus, at the transition point the total density has a discontinuity equal to

$$\delta\rho = \rho_- - \rho_+ = 0.537 \frac{\Delta_c}{2\tilde{t}} \equiv 0.537 \frac{M}{2\pi} T_0. \tag{64}$$

The solution of Eq. (63) for $T > T_0$ shows that above T_0 the density ρ is a linear function of T:

$$\rho(T) - \rho_+ = \frac{1}{2} \frac{m\eta}{1+\eta} \ln\left(\frac{e}{2\tilde{t}}\right)(T - T_0).$$

The functional form of $\rho(T)$ is shown in Fig. 5. It is interesting to note a possible analogy with magnetic systems, in which the mean density $\langle \varphi^* \varphi \rangle$ represents the mean square of the magnetic moment $\langle \mathbf{m}^2 \rangle$. The dependence of this quantity on T must be similar to $\rho(T)$ in Fig. 5.

7. DEMIXING OF THE ELECTRON LIQUID IN A TWO-DIMENSIONAL DOPED SEMICONDUCTOR

In this section, we investigate the behavior of a semiconductor close to the exciton instability when it contains a certain nonvanishing concentration n of excess particles. It was shown above (see Sec. 2) that the description of such a system reduces to the solution of the problem of fermions interacting with Bose exciton degrees of freedom in the conduction band. One of the manifestations of this interaction is a possible instability of the spatially homogeneous state of the Fermi liquid with respect to the formation of macroscopic regions with enhanced concentration of particles, i.e., with respect to phase demixing. The effective action of the doped semiconductor was constructed in Sec. 2 and is determined by Eqs. (21), (7), and (22). The origin of the phase demixing is associated with the fermion–boson repulsion [the final term in S_{FB} (22)]. Indeed, the presence of a nonvanishing fermion density leads to an effective decrease in the boson chemical potential λ and, as a consequence, to a decrease in the density of the Bose particles. This effect is a manifestation of the suppression of the exciton instability by doping that results from the decrease in the exciton binding energy as a result of the Burstein shift. Since the same fermion–boson repulsion term determines the renormalization of the edge of the conduction band, which is proportional to the density of the bosons, the decrease in this density as a result of the doping leads to a lowering of the edge of the electron band. Thus, the increase in the number of electrons in the band gives rise to a decrease in the exciton density and a resultant lowering of the band edge. If this lowering occurs more rapidly than the growth of the electron chemical potential μ due to the occupation of the band, a situation is realized in which the chemical potential μ decreases with increasing particle number n, i.e., we have $\partial\mu/\partial n < 0$, which means there is absolute instability of the spatially homogeneous state of the system.

We investigate the problem in the framework of the self-consistent approximation developed above (see Sec. 4). Integration over the rapidly varying degrees of freedom in the first stage leads to the replacement of the bare potentials of the boson–boson, t_0, and fermion–boson, γ_0, interactions by the renormalized quantities t and γ in the action (21). As in Sec. 4, the renormalization of t_0 consists of the summation of the ladder diagrams corresponding to the scattering of two bosons by each other. Therefore, for the considered case $D = 2$

$$t = \frac{4\pi}{M} \ln^{-1} \frac{\varepsilon_0}{\lambda} = \frac{\pi}{M} \tilde{t}.$$

Similarly, γ_0 is renormalized by the dressing of the bare potential by the ladder series describing scattering of a fermion by a boson:

$$\gamma = \frac{\gamma_0}{1 + \gamma_0 \Pi_{FB}} \cong \frac{2\pi}{m_r} \ln^{-1} \frac{\varepsilon_0}{\lambda} = \frac{\pi}{2m_r} \tilde{t}. \tag{65}$$

In the expression (65), Π_{FB} is the polarization operator in the fermion–boson scattering channel, and m_r is the reduced fermion–boson mass:

$$m_r = \frac{m_c M}{m_c + M}.$$

Subsequent calculations are similar to those made in Sec. 4. The decoupling of the fermion–boson interaction term leads,

first, to renormalization of the boson chemical potential. Namely, it leads to replacement of λ by a Λ equal to

$$\Lambda = \lambda - \gamma n. \tag{66}$$

Second, it leads to the appearance of the electron self-energy part Σ_F, which determines a shift of the edge of the electron band that is proportional to the density ρ of the Bose particles:

$$\Sigma_F = \gamma\rho.$$

The electron chemical potential is determined from the equation for the fermion number:

$$n = 2\int \frac{d^2\mathbf{k}}{(2\pi)^2}\left[\exp\left\{\frac{k^2/2m_c - \mu + \gamma\rho}{T}\right\} + 1\right]^{-1}.$$

The two-dimensionality of the spectrum makes it possible to calculate the integral in the last equation exactly and find μ explicitly:

$$\mu = \gamma\rho + T\ln(e^{\pi n/m_c T} - 1). \tag{67}$$

The second term in the expression (67) for μ increases monotonically with decreasing n and describes the increase in the chemical potential with doping as a result of occupation of the band. The first term in (67) corresponds to the change in μ due to the renormalization of the edge of the conduction band resulting from the fermion–boson interaction. The total boson density ρ is described by the expressions of the previous section with replacement of the bare chemical potential λ by the renormalized Λ (66). Therefore, the first term always decreases with increasing n and makes a negative contribution to $\partial\mu/\partial n$, creating the prerequisites for a sign change of it.

We calculate the derivative of the chemical potential μ with respect to the particle number n and show that it can be negative both in the high-temperature normal phase ($\Delta = 0$) and in the ordered phase with $\Delta \neq 0$. The line $T_0(n)$ in the phase diagram that separates these two phases can be obtained from the expression (56) for T_0:

$$T_0(n) = \frac{1}{4}\Lambda\ln\frac{\varepsilon_0}{\Lambda}\left\{\ln\left(\frac{e}{8}\ln\frac{\varepsilon_0}{\Lambda}\right)\right\}^{-1}.$$

Below this line $\Delta \neq 0$, and ρ is determined by the expression (43b). Therefore

$$\frac{\partial\mu}{\partial n} = \frac{\gamma}{2t}\left(-\gamma + \frac{\partial\Delta}{\partial n}\right) + \frac{\pi}{m_c}\frac{1}{1 - e^{-\pi n/m_c T}}.$$

We express $\partial\Delta/\partial n$ in terms of $\Delta(n,T)$, differentiating with respect to n the self-consistency equation (52) (naturally, after the replacement in it of λ by Λ). Finally, we obtain for $\partial\mu/\partial n$

$$\frac{\partial\mu}{\partial n} = -\frac{\pi}{m_c}\left\{\frac{M}{8m_c}\left(1 + \frac{m_c}{M}\right)^2\tilde{t}\,\frac{2T_0\Delta - T\Delta_c}{T_0\Delta - T\Delta_c} - \frac{1}{1 - e^{-\pi n/m_c T}}\right\}. \tag{68}$$

Since the dimensionless vertex $\tilde{t}$ (55) is a small parameter of the theory, at low temperatures $T \ll T_0(n)$ a sign reversal of $\partial\mu/\partial n$ is possible only in the presence of strongly differing masses M of the boson and m_c of the fermion, or, equivalently , when the electron and hole have very different masses. However, on the approach to the transition point the denominator in the first term tends to zero, and, therefore, for any M/m_c there always exists below the line $T = T_0(n)$ a region in which $\partial\mu/\partial n$ and the homogeneous state is absolutely unstable with respect to phase demixing.

Above the phase transition line, the total density ρ changes in accordance with the expression (62). Calculating the derivative $\partial\theta/\partial n$ by means of the expression (63), we find for $\partial\mu/\partial n$ in the disordered phase

$$\frac{\partial\mu}{\partial n} = -\frac{\pi}{m_c}\left\{\frac{M}{8m_c}\left(1 + \frac{m_c}{M}\right)^2\tilde{t}\,\frac{T\Delta_c}{T_0\theta + T\Delta_c} - \frac{1}{1 - e^{-\pi n/m_c T}}\right\}.$$

The last expression shows that in the normal phase too there can exist a region of instability of the spatially homogeneous state but only if the boson mass is much greater or much less than the fermion mass.

8. CONCLUSIONS

In this paper, we have proposed a self-consistent description of phase transitions in systems that admit representation in terms of an effective rarefied Bose liquid. For a semiconductor close to the exciton instability we have constructed an effective low-energy action. This action has the standard form of the action for a Bose liquid with repulsive short-range interaction. We have shown that for semiconductors the symmetry with respect to rotation of the phase of the Bose field is always broken by the presence in real systems of interactions with transitions of particles from the conduction band to the valence band.

The equations of the self-consistent theory developed in this paper for the rarefied Bose liquid do not contain divergences at small momenta and have a universal dimension-independent form. This last fact makes it possible to describe in a unified manner the thermodynamics of a spatially homogeneous Bose liquid in dimensions $D \geqslant 2$. For exciton systems and superconductors, our approach makes it possible to describe accurately the limit with respect to the particle density that is the opposite of BCS theory (or, more precisely, the limit with respect to the ratio of the two-particle bound state to the mean separation between the particles). The existence of the two correct asymptotic behaviors with respect to the density is necessary for understanding the physics in the intermediate region, to which many real systems (including high-temperature superconductors) correspond. In addition, our results may also have a wider domain of applicability, since bosonic degrees of freedom arise naturally in many problems in the physics of strongly correlated systems.

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