Observation of Moffat eddies in permalloy thin-film elements

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Electron-microscopic observation of the domain structure of permalloy thin-film elements reveals the existence of a geometric progression of magnetic-flux vortices within acute-angled protuberances of the elements, which are identified with Moffat eddies in a viscous fluid. © *1995 American Institute of Physics.*

1. INTRODUCTION

The domain structure of thin-film magnetic microcircuit elements has been investigated by Lorentz electron microscopy. These investigations were performed for the purpose of studying the magnetization reversal of elements in advanced circuits for memory devices under the action of an applied field, and results of general physical significance were obtained. It was shown on the basis of the experimental material that vortex magnetic-flux structures, whose behavior is similar to the flow of a viscous fluid, form in discrete magnetic elements. Analogies between hydrodynamic and magnetic fluxes were first noted in Ref. 1 in connection with the vortex theory of cross-tie domain walls. The problem of such walls was considered in an infinite film, and the solution obtained corresponded to familiar solutions from hydrodynamics.

In the present work the flux in a restricted element is considered with the boundary condition $B_n=0$, which is analogous to the condition for streamline flow and, as will be shown below, is valid in the case of weak anisotropy. It will then be shown that the vortex structures observed in magnetic elements whose shell contains sharp corners correspond to Moffat eddies in the problem of the streamline flow of a viscous fluid past a corner. It is interesting that Moffat eddies, which were predicted theoretically in Ref. 2 as an interpretation for one of the solutions of the Stokes equation for the problem cited, have not been observed with certainty in hydrodynamic flows with sharp corners and have been observed only in some similar problems, such as streamline flow past a cavity.[3]

2. EXPERIMENTAL RESULTS

Uniform permalloy films containing 81% Ni and 19% Fe and having a thickness of 40 nm were deposited by an electron-beam method on specially prepared substrates with a temperature of the heating panel equal to 200 °C. The films had a coercive force $H_c=1.2$ Oe, an anisotropy field $H_a=2.7$ Oe, and practically zero magnetostriction. Individual microcircuit elements were fabricated from the uniform films by photolithography and separated from the substrate. Then they were placed in the magnetization-reversing device of a UÉMV-100K electron microscope. The magnetization-reversing device made it possible to perform the investigations in constant and variable fields with strengths equal to 0–875 Oe. The images of the samples investigated in the defocused operating mode (below the plane of the film) were recorded on photographic plates. The elements measured ~ 15 μm in the transverse direction.

It is known[4] that slight defocusing of the electron beam makes it possible to observe the domain structure of thin films with anisotropy in their plane. Figures 1a–1c show the domain structure of a ring-shaped permalloy element. White and black lines are seen. They represent Néel walls of both polarities, circular Bloch lines in the form of points on these walls, and a magnetic fine structure ("magnetization ripples"). The direction of the magnetic flux was perpendicular to the ripple lines. The distribution of the magnetic flux reconstructed from the ripple lines reveals the presence of vortex structures (Fig. 1b).

It was previously established that the domain structure of the same element can be different in the absence of an applied field.[5] In the present case the vortex structures in the sharp corners of the element are present in all modifications of the basic structure (Figs. 1a–1c). The investigations performed showed that the application of an external field stronger than 20 Oe results in saturation of the element and in disappearance of the vortex regions in its central zone (Fig. 2). However, it is seen that the vortices in the sharp corners of the element remain (Fig. 2). The application of an external variable field likewise does not completely suppress the vortices in the sharp corners of the elements.

Such stability of the vortex formations in applied fields indicates that the presence of the vortices is related to the properties of the magnetic flux itself; therefore, it was postulated that the observed vortex structures are Moffat eddies, which appear when a magnetic flux flows past a sharp corner.

3. THEORETICAL INTERPRETATION

To confirm the hypothesis advanced above, it must be shown that the starting equations and boundary conditions for magnetic and hydrodynamic flows are identical. In the case of a hydrodynamic flow, the vortex solution appears in the following manner.

In 1964 Moffat[2] theoretically investigated the problem of the streamline flow of a viscous fluid past a sharp corner. It is known that this problem has a classical solution, which is obtained using conformal mappings:[6]

$$u=r^n \cos n\varphi, \quad v=r^n \sin n\varphi. \tag{1}$$

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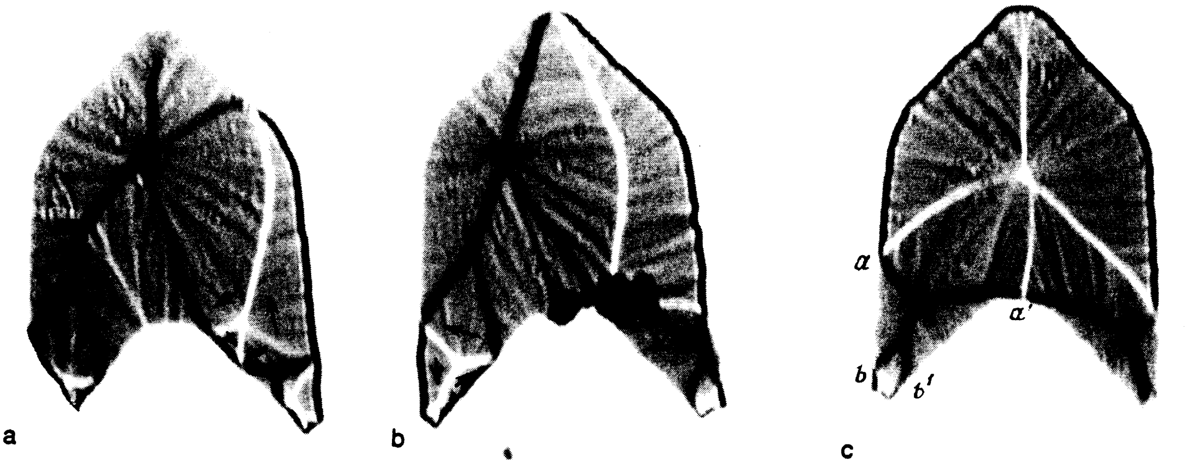

a b c

FIG. 1. Domain structure obtained by Lorentz electron microscopy from different modifications of "ring-shaped" permalloy elements measuring ~ 15 μm. The direction of the magnetic flux is perpendicular to the lines of the magnetic fine structure (it is marked by arrows). Vortex regions are seen within the acute-angled protuberances of the elements and are identified with Moffat eddies in the present work. The dimensions of the vortices were measured along the aa' and bb' lines (c) by convention.

This solution is real, and for $n=2$, it describes an ordinary family of hyperbolas for the lines of flow of the fluid. There is also a known solution for supersonic flows having a separatrix, which can be a weak discontinuity, a shock wave, etc. in different problems.[7]

Solving the Stokes equation in the form $\nabla^4\psi=0$, which describes flow without a separatrix, for the present problem, Moffat[2] investigated the solution of the form

$$f_\lambda(\vartheta)=A\cos\lambda\vartheta+C\cos(\lambda-2)\vartheta, \tag{2}$$

where λ is any number, real or complex. The condition for flow around the object, $v_n=0$, for $\vartheta=\pm\alpha$ leads to the equations for λ:

$$\sin 2\mu\alpha=\mu\sin 2\alpha, \quad \mu=\lambda-1. \tag{3}$$

These equations do not have real solutions for $2\alpha<146°$. The complex solution $\mu=p+iq$ leads to two real equations:

$$\begin{cases}\sin\xi\cosh\eta=-k\xi,\\ \cos\xi\sinh\eta=-k\eta,\end{cases} \tag{4}$$

where we have introduced the notation $\xi=2\alpha p$, $\eta=2\alpha q$, $k=\sin 2\alpha/2\alpha>0$. The solution of (4) exists in the range $(2n-1)\pi<\xi_n<(2n-1/2)\pi$ and can be written as

$$\lambda_n=1+(2\alpha)^{-1}(\xi_n+i\eta_n). \tag{5}$$

Moffat interpreted (5) as an infinite sequence of eddies converging at the edge of the corner, whose dimensions and intensities decrease in a geometric progression with the factors $\exp(\pi/\eta)$ and $\exp(\pi\xi/\eta)$, respectively.

Thus, Moffat eddies appear as a realization of one of the solutions of the Stokes equation in the form $\nabla^4\psi=0$ in the case of the flow of a viscous fluid past a sharp corner under the condition $2\alpha<146°$. On the other hand, it was shown in Ref. 1 that the properties of a magnetic flux in films of the permalloy type are similar in some respects to the properties of a viscous fluid. We use the equations for the magnetic induction $\mathbf{B}(x,y)$ in the plane of a film[1]

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}=0, \quad \frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=2\omega(x,y), \tag{6}$$

where ω is the vorticity. If we introduce the stream function ψ for $\mathbf{B}$ according to the relation

$$d\psi=-B_y dx+B_x\,dy, \tag{7}$$

in the case of $\omega=\text{const}$ (according to Ref. 1, this case is realized for a circular Bloch line; as is seen from Figs. 1a–1c, there are circular Bloch lines at the center of the vortex formations), the stream function $\psi(x,y)$ will satisfy the Stokes equation $\nabla^4\psi=0$. In addition, the condition $B_n=0$ holds on the edges of a restricted permalloy element. This condition was obtained from an analysis of the experimental data for thin magnetic structures on the edges of elements of various shape.[5] It is not difficult to see that this condition is

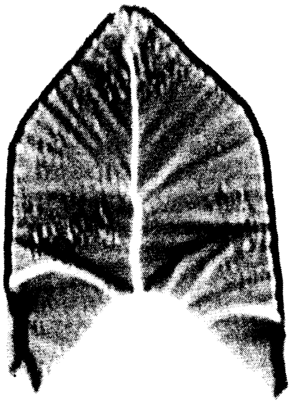

FIG. 2. Domain structure of a "ring-shaped" element after saturation in an applied field stronger than 20 Oe. The vortex regions vanished in the central part of the element, but were maintained in the acute-angled protuberances.

similar to the condition for streamline flow. It should be noted that this condition is valid for magnetic thin-film elements only in the limit $\beta \rightarrow 0$, where $\beta = H_a/M$ and H_a is the field of the anisotropy of the film.

It was also confirmed experimentally that the dimensions of the vortices in the protuberances of the ring-shaped elements decrease in a geometric progression. To this end seven vortex trains, in which the number of vortices ranged from 2 to 4, were chosen from the available material for study. The dimensions of each vortex (which has the form of an irregular quadrangle) were measured along lines aa' and bb' (Fig. 1c) in the vortex cell by convention. The measured values of the ratio r_n/r_{n+1} ranged from 2.0 to 3.1 in the series containing three or four vortices, i.e., it varied only slightly. The mean value for all the vortices in five series was 2.66 ± 0.30, which is quite close to the value of $e = 2.71....$ In the two cases in which there were only two vortices, this ratio was equal to 3.0 and 4.5, i.e., it was somewhat higher. In the latter case the ratio of the dimensions of the vortices was possibly influenced by the applied magnetic field.

4. DISCUSSION OF RESULTS

In Ref. 3, which contains a review of the available investigations of the flow of viscous fluids with the appearance of vortices similar to Moffat eddies, flows with low Reynolds numbers were discussed for the most part. According to Ref. 1, the Reynolds numbers for a magnetic flux in permalloy films are high ($\sim 10^6$); however, no investigation of Moffat eddies has apparently been performed for flows with such values of R. Therefore, the question of the complete validity of the comparison of flows with Reynolds numbers of different orders of magnitude remains open. Nevertheless, a cautious statement can be made on the basis of the work in Ref. 8, where flows were investigated in a rectangular cavity containing Moffat eddies in the range of Reynolds numbers from 20 to 4000. According to the data presented in Ref. 8, the dependence of the relative dimensions of the eddies on R is such that when R increases to 10^3, the picture becomes identical to flows with low Reynolds numbers.

Thus, it can be stated that the appearance of the geometric progression of eddies predicted by Moffat when a stream of a viscous fluid flows past a sharp corner is realized in the case of a magnetic flux.

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