Improved energy transfer in free-electron lasers by trapping electron oscillators

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The process of amplifying the electromagnetic wave in a free-electron laser with an oppositely directed focusing field and a wiggler (undulator) field of right-hand polarization is considered numerically and analytically. A special law for intensifying the wiggler field at the entrance to the interaction region, which makes it possible to distribute all the beam electrons in a series of phase bunches separated by 2π with respect to the wave of the longitudinal force, is proposed. The trapping of electron bunches by a high-frequency electromagnetic field is utilized to increase the efficiency of the device. The possibility of trapping electron bunches as a result of profiling the parameters of the laser is demonstrated using critera for stable motion. The Lyapunov theory is used to analyze the stability of the electron motion. Parameters close to the experimental values are numerically simulated by the large-particle method, and a laser efficiency equal to 55% (as opposed to 22% for a uniform wiggler field) is obtained. © *1995 American Institute of Physics.*

1. INTRODUCTION

The problem of optimizing the energy transfer and increasing the efficiency is common to most electronic devices with a prolonged interaction (for example, traveling-wave tubes) that employ an electron beam as the working medium. The approach traditionally used is to provide for conditions under which an electron bunch formed during the interaction with the electromagnetic field would remain in the decelerating phase of the electromagnetic wave for as long as possible. Variation of the parameters of the electrodynamic system in which the electromagnetic wave propagates through the interaction region is usually proposed for this purpose. Since the electron beam slows during energy transfer, as it imparts energy to the high-frequency wave, the phase velocity of the wave (or a spatial harmonic), which is synchronous with the beam, should decrease with distance in the interaction region.

Oversized waveguides or quasioptical resonators are employed as the wave-conducting systems in free-electron lasers; therefore, such a possibility is practically unrealizable in them. However, a similar result can be achieved by varying the field strength of a wiggler (undulator) and/or a focusing field along the length of the interaction region. The fact is that the electrons move along stationary trajectories characterized by a definite ratio between the transverse and longitudinal velocities in the interaction region of a laser. This ratio is determined by the amplitudes of the magnetic fields used at a given value of the electron energy. Thus, the decrease in the longitudinal velocity of an electron beam appearing during energy transfer can be compensated by varying the ratio between the amplitudes of the magnetic fields as a function of distance in the interaction region.

It is convenient to use profiling of the wiggler field for this purpose, since the value of the transverse velocity is most sensitive to it, and the required result can be achieved by relatively weak variation of this field. Among the experimental implementations of this method for increasing the efficiency of free-electron lasers, the work in Ref. 1, where an efficiency of 34% was achieved, and in Ref. 2 is noteworthy.

However, in those investigations there were no results of fundamental importance which could serve as a basis to devise a law for optimizing the magnetic field profiles in order to maximize the efficiency.

The present work is devoted to optimizing the energy transfer in free-electron lasers on the basis of the phenomenon of the trapping of electron oscillators by the field of a high-frequency wave.

The starting equations which describe the one-dimensional model of a free-electron laser are formulated in Sec. 2. In Sec. 3 the conditions under which the bunches formed are trapped by the field of an electromagnetic wave are analyzed on the basis of Lyapunov's theory of the stability of motion. Optimum magnetic field profiles in free-electron lasers can be obtained from the stability criteria. A method for calculating the bunching of the electron beam is developed on the basis of numerical simulation in Sec. 4. As a result, all the electrons of an initially unmodulated electron beam can be distributed in a series of phase bunches separated from one another by 2π owing to the special form of the wiggler field at the entrance to the interaction region. The nonlinear regime of a free-electron laser with profiling of the wiggler magnetic field is numerically simulated in Sec. 5. The efficiency of such a laser which employs an electron beam with an energy of 1.88 MeV reaches 55% when the profiling is optimal.

2. STARTING ASSUMPTIONS

The basic idea underlying the approach is the proposition that a relatively "loose" electron bunch forms and then the stable equilibrium position of that bunch is shifted toward the decelerating phase of the electromagnetic (more precisely, combination) wave by profiling the magnetic fields. No electron in the bunch is in precise synchronism with the electromagnetic wave, but each oscillates within the phase width of the bunch, which can be quite large. Such an electron cannot leave the bunch due to the presence of a sufficiently high potential barrier in the field of the electromagnetic wave, and, thus, all the electrons in the bunch are trapped by the electromagnetic wave.[3] Energy is thus transferred from the electron bunch as a whole to the electromagnetic wave. If the oscillatory motion of the electrons is stable, the energy-transfer process is not limited by such typical phenomena as, for example, an increase in the spread of electron velocities in the bunch, and, in principle, the efficiency of the interaction between the beam and the high-frequency electromagnetic wave can reach the highest values allowed by the physical operating principles of free-electron lasers.

In solving this problem we restrict ourselves to the paraxial approximation, i.e., we shall not take into account the transverse dependence of the static fields of the laser.

We consider a stream of electrons propagating along the z axis in the fields of a free-electron laser

$$\mathbf{B}_{st}=B_w(z)[\mathbf{e}_x \cos(k_w z)+\mathbf{e}_y \sin(k_w z)]+\mathbf{e}_z B_0$$

in the presence of an electromagnetic wave of right-hand polarization

$$\mathbf{A}_{hf}=A[\mathbf{e}_x \cos(\omega t-kz)+\mathbf{e}_y \sin(\omega t-kz)],$$

where $B_w(z)$ and B_0 are the amplitudes of the wiggler field and the focusing magnetic fields, respectively, A is the amplitude of the vector potential of the electromagnetic wave $k_w=2\pi/\lambda_w$, and λ_w is the spatial period of the wiggler field.

We restrict ourselves to consideration of the case in which the high-frequency space-charge forces in the electron beam can be neglected in comparison with the forces of the static magnetic fields. In this case the equation of motion of the electrons has the form

$$\frac{d\mathbf{v}}{dt}=-\frac{e}{m\gamma}\left(\mathbf{E}+\frac{\nabla\times\mathbf{B}}{c}-\frac{\mathbf{v}\cdot(\mathbf{v}\cdot\mathbf{E})}{c^2}\right),$$

$$\frac{d\gamma}{dt}=-\frac{e}{mc^2}\,\mathbf{v}\cdot\mathbf{E},$$

where $\gamma=1/\sqrt{1-(v_x^2+v_y^2+v_z^2)/c^2}$ is the relativistic factor, $\mathbf{E}=-(1/c)(\partial\mathbf{A}/\partial t)$, $\mathbf{B}=\mathbf{v}\times\mathbf{A}$, $\mathbf{A}=\mathbf{A}_{st}+\mathbf{A}_{hf}$, and $\mathbf{A}_{st}$ is the component of the vector potential corresponding to the static fields of the free-electron laser.

Going over to a coordinate system rotating together with the wiggler field

$$\mathbf{e}_1=\mathbf{e}_x \cos(k_w z)+\mathbf{e}_y \sin(k_w z), \tag{1}$$

$$\mathbf{e}_2=\mathbf{e}_x \sin(k_w z)-\mathbf{e}_y \cos(k_w z), \quad \mathbf{e}_3=\mathbf{e}_z,$$

and supplementing the original system with the equation of a slowly varying amplitude of an electromagnetic wave,[4] we can obtain:

$$\begin{aligned}
\frac{dp_1}{dZ}&=\left(\frac{f_0}{p_3}-1\right)p_2-\alpha_s\left(\frac{\gamma}{p_3}-\frac{ck}{\omega}\right)\sin\theta,\\
\frac{dp_2}{dZ}&=-\left(\frac{f_0}{p_3}-1\right)p_1+f_w-\alpha_s\left(\frac{\gamma}{p_3}-\frac{ck}{\omega}\right)\cos\theta,\\
\frac{d\gamma}{dZ}&=-\frac{1}{p_3}\,\alpha_s(p_1\sin\theta+p_2\cos\theta),\\
\frac{d\theta}{dZ}&=g\left(\frac{\gamma}{p_3}-\frac{ck}{\omega}\right)-1,\\
\frac{d\alpha_s}{dZ}&=\frac{j_0}{mc^3k_w^2/e}\,\frac{\omega}{ck}\int_0^{2\pi}\frac{p_1\sin\theta+p_2\cos\theta}{p_3}\,d\theta_0,
\end{aligned} \tag{2}$$

where $Z=k_w z$, $f_{w,0}=\Omega_{w,0}/k_w c$, $\Omega_{w,0}=eB_{w,0}/mc$, $\theta=\omega t-(k+k_w)z$, and $\theta_0\in[0...2\pi]$ denotes the initial phases of the electrons with respect to the electromagnetic wave,

$$\alpha_s=\frac{eA}{k_w mc^2}\,\frac{\omega}{c},\quad p_{1,2,3}=\frac{v_{1,2,3}}{c}\,\gamma,$$

the p_i are the normalized electron momenta satisfying the relation $p_3=\sqrt{\gamma^2-1-p_1^2-p_2^2}$; $j_0=en_0e_0$ is the current density of the beam, and $g=\omega/ck_w$ is a frequency conversion factor.

We note that in the absence of an electromagnetic field ($A=0$) this system at once provides a particular solution corresponding to the stationary trajectories

$$v_1=v_\perp,\quad v_2=0,\quad v_3=v_\parallel. \tag{3}$$

3. CONDITIONS FOR THE MOTION STABILITY OF ELECTRON BUNCHES

We analyze the conditions for the stability of electron bunches moving in the fields of a free-electron laser under the assumption that the high-frequency modulation of the transverse electron velocities can be neglected, i.e., that we can set

$$v_x=v_\perp(z)\cos(k_w z),\quad v_y=v_\perp(z)\sin(k_w z),$$

where, in the general case, $v_\perp(z)$ is a slow function of the longitudinal coordinate z as a consequence of the profiling of the parameters of the free-electron laser.

We introduce the new variable $X=\omega t-\int_0^z h_0 dz$, where $h_0=k+k_w$. Differentiating it with respect to t and using the last of Eqs. (1), we obtain the following system

$$\frac{dX}{dt}=h_0(v_{\rm ph}-v_z),\quad \frac{d\gamma}{dt}=-\alpha_s k_w v_\perp \sin(X), \tag{4}$$

where $v_{\rm ph}=\omega/h_0$ is the phase velocity of the wave of the longitudinal force.

To investigate the stability of the solutions of the system (4) (i.e., to reveal the conditions for trapping electron oscillators) we use Lyapunov's theory.[5] Introducing the new variables

$$X_1=X+\varphi_0,\quad X_2=\gamma_1-\gamma,$$

where

$$\sin\varphi_0=\frac{v_{ph}}{\alpha_s k_w v_\perp}\frac{d\gamma_1}{dz},\quad \gamma_1=\frac{1}{\sqrt{\mu-v_{ph}^2/c^2}},$$

$$\mu=1-\frac{v_\perp^2}{c^2},$$

and assuming that φ_0 is an adiabatic function of z (i.e., φ_0 is constant throughout the oscillation period of an oscillator), we transpose the system (4) to the coordinate origin:

$$\frac{dX_1}{dt}=h_0[v_{ph}-v_z(X_2)],\tag{5}$$

$$\frac{dX_2}{dt}=\alpha_s k_w v_\perp\left\{\sin(X_1-\varphi_0)+\sin\varphi_0-\frac{\sin\varphi_0}{v_{ph}}\times[v_{ph}-v_z(X_2)]\right\},$$

where

$$v_z(X_2)=c\sqrt{\mu-\frac{1}{(\gamma_1-X_2)^{-2})}}.$$

We analyze the stability of the solution $X_1=X_2=0$ of the system (5) under the assumption that h_0, v_{ph}, $v_\perp$, γ_1, and α_s are adiabatic functions of z.

We introduce the function

$$W_1=\int_0^{X_2}[v_{ph}-v_z(x_2)]dx_2-\alpha_s k_w v_\perp/h_0[\cos(\varphi_0)-\cos(X_1-\varphi_0)+X_1\sin(\varphi_0)].\tag{6}$$

The derivative of this function with respect to time taken with consideration of the system (5) equals

$$\frac{dW_1}{dt}=W_2=-\alpha_s k_w v_\perp\frac{\sin\varphi_0}{v_{ph}}[v_{ph}-v_z(X_2)]^2\tag{7}$$

It is seen that the functions $W_1,W_2=0$ when $X_1=X_2=0$. The function W_1 will be a Lyapunov function, if it has a definite sign in a certain vicinity around the coordinate origin (in this case it is required that $W_1>0$). The equilibrium position will be stable, if $W_2\leqslant 0$. These conditions hold if

$$\int_0^{X_2}[v_{ph}-v_z(x_2)]dx_2>0,\tag{8}$$

$$v_\perp\cos\varphi_0<0,\tag{9}$$

$$|\sin\varphi_0|<\frac{1-\cos X_1}{\sqrt{(1-\cos X_1)^2+(X_1-\sin X_1)^2}},\tag{10}$$

$$v_\perp\sin\varphi_0\geqslant 0.\tag{11}$$

An analysis reveals that the condition (8) is satisfied automatically for any parameters. The conditions (9) and (11) characterize the position of the equilibrium point with respect to the traveling wave and indicate that the trapping of a bunch is possible only in cases in which the equilibrium position is located close to the transition from the accelerating phase of the field to its decelerating phase. The most stringent condition for the stability of the motion of electron bunches is (10). This condition must hold for all the electrons in a bunch. We introduce the notation

$$\xi(X_1)=\frac{1-\cos X_1}{\sqrt{(1-\cos X_1)^2+(X_1-\sin X_1)^2}}.$$

The potential well has the form of a cosine [$\xi\sim\cos X_1$]; therefore, its phase width equals 2π. In the range $0\leqslant|X_1|\leqslant\pi$ the function $\xi(X_1)$ is a monotonically decreasing function of $|X_1|$; therefore, the inequality (10) is valid for all electrons in a bunch, if it holds for the electron oscillator with the largest amplitude. Let $|X_1|_{\max}$ be the largest possible deviation of such an oscillator from its equilibrium position over the entire working length of the laser after the buncher. We set $\xi=\xi(|X_1|_{\max})$. Then the inequality (10) will hold for all oscillators in a bunch, if

$$|\sin\varphi_0|=\xi.\tag{12}$$

It is known *a priori* that $\xi(X_1)$ varies in the range $0<\xi(X_1)<1$. Therefore, the value of ξ in Eq. (12) can be "guessed" by selecting it in the range indicated [we note that there is always a certain maximum value of ξ, above which the stability of the oscillators in a bunch begins to break down; for any value of ξ below that limiting value the profiling laws obtained from (12) will produce stable motion of all the electrons in the bunch].

As can be seen from the conditions (8)–(11), electrons in a free-electron laser can be trapped, in principle, both by profiling the wiggler field or the solenoid field and by varying the spatial period of the wiggler field or the electromagnetic wave. We restrict ourselves to the derivation of only the profiling law for the wiggler field, since the value of the transverse velocities of the electrons in a free-electron laser is most sensitive to it.

To obtain the profiling law for the wiggler field we rewrite the previously introduced quantity $\sin\varphi_0$ in terms of the dimensionless variables:

$$\sin\varphi_0=\frac{p_{ph}}{\alpha_s p_\perp}\frac{d\gamma_1}{dZ}=\frac{p_{ph}}{\alpha_s\gamma_1}\frac{dp_\perp}{dZ},\tag{13}$$

where $\gamma_1=\sqrt{1+p_\perp^2+p_{ph}^2}$, and $p_{ph}=v_{ph}\gamma/c$. Using (12), replacing the derivative of $p_\perp$ in (13) by the derivative of f_w, and averaging over the initial entry phases of the electrons, we obtain

$$\frac{df_w}{dZ}=\frac{\xi\alpha_s}{2\pi}\int_0^{2\pi}\frac{\gamma_1}{p_{ph}}\frac{f_0-p_3+p_1(p_1+f_w)/p_3}{p_3}d\theta_0.\tag{14}$$

The profiling law (14) for the wiggler field was derived assuming that electron bunches already exist. Thus, it would be useful to examine the question of their formation.

4. PRELIMINARY BUNCHING OF ELECTRONS

For efficient laser operation, the electrons at the entrance to the interaction region must follow stationary trajectories. Then, in the oscillation region, which usually extends over five to six spatial periods of the wiggler field, the amplitude of the wiggler field varies from 0 to the nominal value corresponding to the synchronous value of the longitudinal velocity. When the wiggler field increases in such a manner, it is not possible to distribute all the electrons in a temporally periodic series of phase bunches, which is one of the necessary conditions for their subsequent trapping. An obvious reason for this is that some of the electrons entering the accelerating phase of the electromagnetic wave inevitably get out of synchronization with the wave. Therefore, we attempted to juxtapose the electron buncher with the segment where the electrons move along stationary trajectories.

It was shown in Ref. 6 that the configuration of a laser with an oppositely directed focusing magnetic field is more stable against the spread of the transverse velocities of the beam electrons at the entrance to the interaction region appearing as a result of nonadiabatic strengthening intensification of the wiggler field at the entrance to the interaction region. Moreover, such a configuration is efficient even when there is abrupt (steplike) rise in the wiggler field at the entrance to the interaction region. Therefore, it can be asserted that for any (up to 5%) variations of the wiggler field in the buncher segment, practically all the electrons enter the interaction region. For this reason all the calculations are performed for a laser with an oppositely directed focusing magnetic field.

We placed the buncher segment in five spatial periods λ_w of the wiggler field. A simple idea was proposed to achieve 100% bunching of the electron beam. The electrons entering the accelerating phase of the electromagnetic wave must be slowed slightly. Then the electrons in the decelerating phase will not necessarily get out of synchronization with the field. To implement this idea the wiggler field was varied in the following manner at the entrance to the interaction region. Over a segment measuring $2\lambda_w$ the amplitude of the field increased from 0 to the nominal value (corresponding to exact synchronism) according to a $\sim\sin^2(\pi z/4\lambda_w)$ law. This was followed by a linear increase in the field to a value which is several percent greater than the nominal value in the $2\lambda_w \leqslant z \leqslant 3.5\lambda_w$ segment and then by linear variation of the amplitude to a value slightly smaller than the nominal value in the $3.5\lambda_w \leqslant z \leqslant 5\lambda_w$ segment.

The numerical calculations were performed using Eqs. (2). Figure 1 illustrates the dependence of the wiggler field on the dimensionless longitudinal coordinate Z. It is seen that a series of phase bunches with a width of order π separated by 2π can form when the amplitude of the wiggler field varies in this manner. It should be emphasized that all the electrons of the original unmodulated stream can be utilized in this case and that, in contrast to traditional bunchers, the electrons which have left one period of the rf field participate in the creation of the bunch in the next period of the field, thereby making a positive contribution to the overall efficiency.

The calculations showed that when the amplitude of the

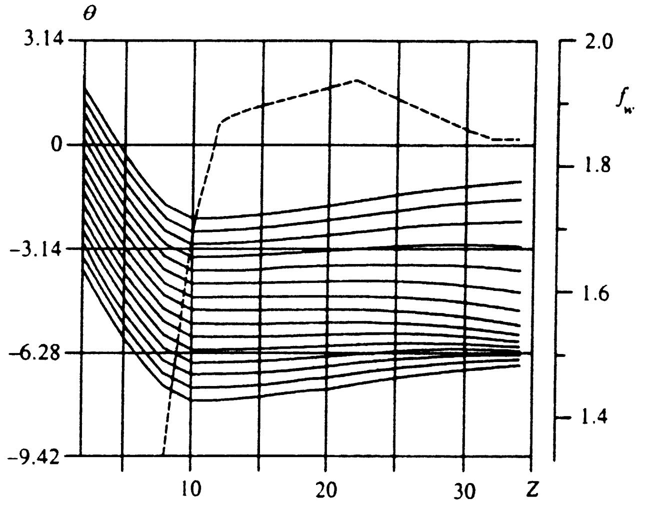

FIG. 1. Dependence of the normalized wiggler field f_w in the buncher segment (dashed line; the nominal value $f_w = 1.88$) and of the phases of the electrons θ (solid lines) on the dimensionless length Z. The initial amplitude of the electromagnetic wave $\alpha_s(0) = 0.02$.

electromagnetic wave is increased by a factor up to 1.5, values of the amplitude of the wiggler field above or below the nominal value can vary within several percent without significant impairment of the quality of the bunches obtained. When the initial amplitude of the electromagnetic field is increased by a factor of 2 or more, rapid separation of the electrons entering the accelerating and decelerating phases occurs; the characteristics of the buncher change, but its design concept remains unchanged.

It should be noted that Eq. (14) does not depend on the bunching method and that only the optimal parameter ξ depends on the phase width of the bunch produced in the buncher.

5. RESULTS OF NUMERICAL SIMULATION

We use the theory developed above to investigate the nonlinear regime of a free-electron laser with the following parameters: an energy of the electron beam $\varepsilon = 1.88$ MeV, which corresponds to $\gamma = 4.75$, a focusing magnetic field $B_0 = -1.4$ kG, a nominal value of the wiggler field $B_w = 2.8$ kG, a wiggler field period $\lambda_w = 7.2$ cm, and a working wavelength of 6.2 mm. As we have already stated above, we restrict ourselves to the case in which the trapping of the electrons is accomplished by profiling the wiggler field. The numerical simulation of the system (2) was performed using particles. It should be noted that the choice of the point at which the profiling begins is very important, in principle. In the present case, since, according to the idea underlying the method, it is not necessary to create a compact electron bunch, the profiling begins immediately after the buncher. Numerical calculations showed that with linear profiling of the wiggler field the efficiency can increase up to 40%, as opposed to 22% for the case of a uniform wiggler field (Fig. 2, dashed line).

To achieve the optimal profiling regime, the system (2) was supplemented by Eq. (14). Since the value of ξ is not

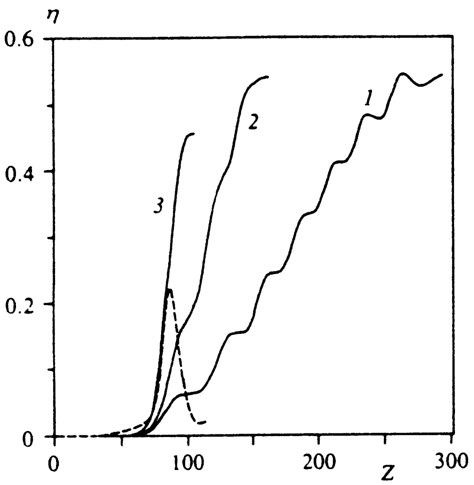

FIG. 2. Dependence of the efficiency η of a free-electron laser on the dimensionless length Z: *1)* $\xi=0.01$, *2)* $\xi=0.04$, *3)* $\xi=0.07$. The initial amplitude of the electromagnetic wave is $\alpha_s(0)=0.02$.

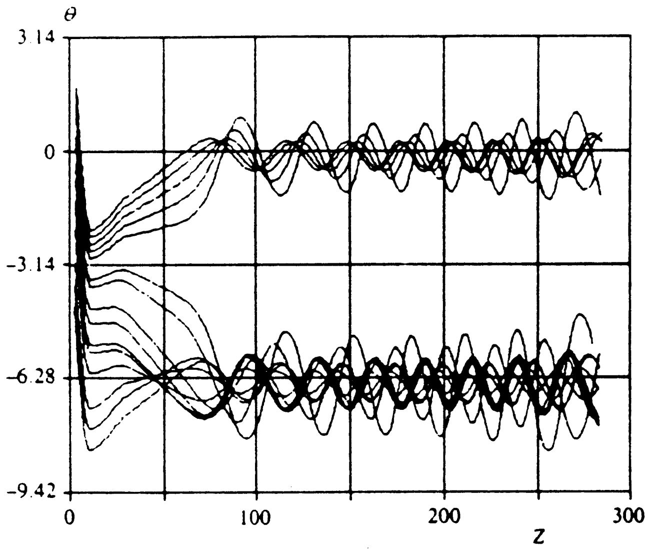

FIG. 4. Dependence of the electron phase θ on the dimensionless length Z.

known at the onset, it is sought for the optimal profiling law by scanning the range indicated above. Figures 2 and 3 present plots of the dependence of the wiggler field and the efficiency of the free-electron laser, respectively, on Z for various values of ξ. It is seen that under the optimal profiling law corresponding to $\xi=0.01$, the efficiency of the laser reaches 55%. The length of the interaction region increases slightly, but remains experimentally feasible. Note that as ξ varies from 0.01 to 0.07, the efficiency varies only slightly (from 55% to 45%), but the length of the interaction decreases sharply (by a factor of 3), i.e., a compromise between raising the efficiency and lowering the gain of the laser per unit length is always possible. It is seen from Fig. 4 that all the electrons forming the phase bunches participate in the energy-transfer process with the electromagnetic wave and that the phase width of the bunches remains of order π over the entire interaction region.

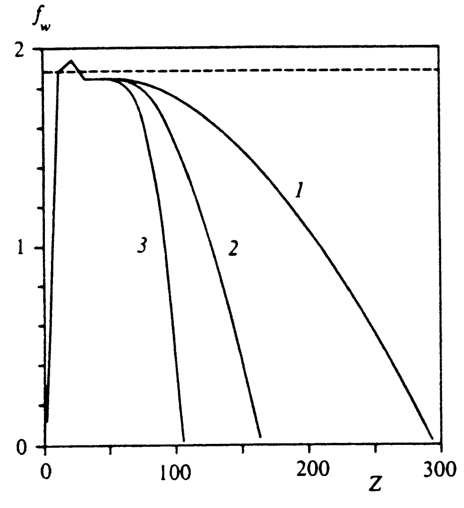

FIG. 3. Dependence of the normalized wiggler field f_w on the dimensionless length Z: *1)* $\xi=0.01$, *2)* $\xi=0.04$, *3)* $\xi=0.07$. The dashed line corresponds to the nominal value of f_w.

6. CONCLUSIONS

Thus, an investigation of the possibility of optimizing energy transfer in a free-electron laser with an oppositely directed focusing field on the basis of the trapping of electron bunches by the field of the electromagnetic wave has been described in this communication. Intensification of the wiggler field at the entrance to the interaction region according to a special law, which can be determined by numerical simulation, has been proposed to achieve 100% bunching of the electron beam. Lyapunov's theory of stability has been used to derive criteria for trapping bunches, which served as a basis for obtaining the optimal profiling law for the wiggler field. Particle simulations have confirmed that the high-frequency modulation of the transverse velocities of the electrons can be neglected in deriving the criteria for trapping electron bunches. A value for the laser efficiency in the millimeter range equal to 55%, which is close to the limiting possible value allowed by the physical operating principles of the device, was achieved. In analogy to the nonrelativistic devices,[3] such lasers can be called autophase free-electron lasers.

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